Problem Set 5. Solutions.

May 4, 2022

1

Consider rolling two fair dice, one green and one red. Let R be the random variable which determines the face value of the red die, and G be the random variable which determines the face value of the green die.
(a) What is the population (possible values) for R, G, and R + G?
(b) What is the probability distribution function for R, G, and R + G?

Solution. a. Populations for R, G, R+G are $\{1, 2, 3, 4, 5, 6\}$, $\{1, 2, 3, 4, 5, 6\}$, $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ respectively.

b. R and G have uniform distribution. R+G has distribution $\mathbb{P}(R + G = x) = (6 - |x - 7|)/36$.

2

Consider the random variables R and G as in Problem 1. What is E(R), E(G), and E(R + G)?

Solution. E(R) = E(G) = 3.5, E(R+G) = E(R)+E(G)=7.

3

A card is drawn at random from a standard deck of 52 cards.
(a) What is the probability that it is a black ace or a red queen?
(b) What is the probability that it is a face card or a black card?
(c) What is the probability that it is neither a heart or a queen?

Solution. a. 4/52 = 1/13
b. ((number of red face cards) + (number of black cards))/52 = (6+26)/52 = 32/52 = 8/13
c. 1 - (number of hearts + number of non-heart queens)/52 = 1 - 16/52 = 9/13.

4

In a lottery every week, 2,000,000 tickets are sold for $1 apiece. Say 4000 of these tickets pay off $30 each, 500 pay off $800 each, and one ticket pays off $1,200,000, and no ticket pays off more than one prize.
(a) What is the expected value of the winning amount for a player with a single ticket?
(b) What is the expected value of the winning amount for a player with five tickets?

Solution. a. (4,000*30+500*800+1,200,000)/2,000,000 = 0.86
b. 5*0.86 = 4.3

5

On Wednesday, check the available winnings for the MegaMillions lottery on the website https://www.walottery.com/JackpotGames/MegaMillions.aspx. Compute the expected value of buying one ticket.

Solution. Depending on the exact amount of the jackpot, the expected value is around 0.38-2 = -1.62.

6

One complaint about online roulette games is that they are not always fair. One user said "within the first 5 minutes of downloading this app I placed a $500 bet on black 8 times in a row and 6 times it was red". Let's say the person played 100 times in 5 minutes. Construct a function in python called AppTester that makes a list of 100 spins, and returns TRUE if there is a consecutive subsequence of length 8 which contains 6 red numbers, otherwise returns FALSE. Test your function enough to know it is working. Record the TRUE/FALSE results of 10 experiments in a list named P4. Show your python code and P4.

Solution. see https://github.com/Vilin97/math381/blob/master/hw5/appTester.py

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Use your function AppTester to find a good approximation to the probability that in 100 spins we get a consecutive subsequence of length 8 containing 6 red numbers. Say what your approximation to the probability is and show the python code you used to compute it.

Solution. The probability is approximately 0.965

8

In a certain country, the probability is 49/50 that a randomly selected fighter plane returns from a mission without mishap. Mia argues that this means there is one mission with a mishap in every 50 consecutive flights. She concludes that if fighter pilot Woody returns safely from 49 consecutive missions, he should quit and return home before the fiftieth mission. Is Mia right? Explain why or why not.

Solution. Mia's reasoning is incorrect: returning from the 50th mission is independent of the success of the first 49 missions. However, no one should be sent to do a task, where the probability of death is 1/50, so I'd say Woody should quit nevertheless.