Adaptive Minority Game

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1 Introduction

In the economic field at the end of the 20th century, people gradually discovered that some assumptions about agents have limitations, such as complete rationality and perfect information. As a complex system, many small disturbances in the financial market may be amplified in the results. Minority Game was proposed in this context. Physicists start with a basic scenario: in the market, if supply exceeds demand, the buyer's market is an advantage; conversely, if demand exceeds supply, the seller's market is an advantage. Therefore, the minority tends to have higher returns. In the study of Minority Game, in order to make the model closer to reality, we make the following assumptions about the agent:
1. The agent has a limited grasp of global information and can only get feedback through interaction with the environment. In our Minority Game model, that means each agent can only get the historical information about which party (0 or 1) wins. Details are in the next section.
2. The agent has limited ability and can only remember limited information, so the old information will be overwritten by the new information. In our Minority Game model, that means each agent will only remember a limited number of historical information, and this memory will be updated in each round.

2 Theory

2.1 Basic Minority Game

In Basic Minority Game, we build the following model:
1. There are N agents in total, N is an odd number, so there will always be a minority. Each agent can remember the winning faction of the past M games (represented as a binary string of length M).
2. A strategy includes responses to all possible historical situations (all together 2^M kind of historical situations). In total, there are 2^{2^M} strategies in the strategy space. Each agent has K (K>1) strategies.
3. Give the agents the binary string of memory, the agents will choose the strategy with the highest ranking (if there are multiple highest strategies, randomly choose one from them), and act based on this strategy.
4. After the action, the number of 0 and 1 is counted. The party that is the minority wins.
5. All the agents add 1 point to their strategies that predict the stage correctly.
6. Repeat the above steps for a certain number of times.

2.2 Adapative Minority Game

Adaptive Minority Game adds some steps to Basic Minority Game: Count the number of wins of each agent. Every τ steps, the fraction r $(0 < r < 1)$ agents with the least number of wins adjust their strategies. The adjustment method is: select two strategies from their own, cross them to get two new strategies, and use these two new strategies to replace two strategies in the original strategy set. According to the different implementations of the adjustment method, we can further divide the Adapative Minority Game into four types:
1. The hybrid "parents" are the two best performing strategies, and the generated "children" replace the "parents".
2. The hybrid "parents" are two random strategies, and the generated "children" replace the "parents".
3. The hybrid "parents" are the two best performing strategies, and the generated "children" replace the two worst performing strategies.
4. The hybrid "parents" are two random strategies, and the generated "children" replace the two worst performing strategies.
By performing this kind of adaptation, we are expecting that on average, each agent will perform better. In other words, the average winning rate of agents will increase, which means that more people will win in a game, which means that the number of minority agents will move closer to $0.5N$. In this sense, we will use experimental data in the next section to prove that the third adaptation method, which uses the children from the best parents to substitute the worst two strategies, can best achieve our goal. In addition, we will also explore the impact of different parameters on the entire game situation.

2.3 Measuring indicators

We mainly measure the following indicators as the basis for judging the situation information:

Attendance at time t represents for the number of agents choosing 1 at time t.

Performance of an agent at time t is calculated by the cumulative number of wins of the agent at time t - the average cumulative number of wins of all the agents at time t.

Best Agent is the agent who wins the most times.

Win Rate of an agent at time t is the cumulative number of wins of the agent divided by the total simulation times t.

3 Results

3.1 Computational results of Basic Minority Game

When the memory length (M) is small, we find that the volatility of attendance in Basic Minority Game is large, which means that it is easier to have the situation that only a few people benefit (see Fig. 1).

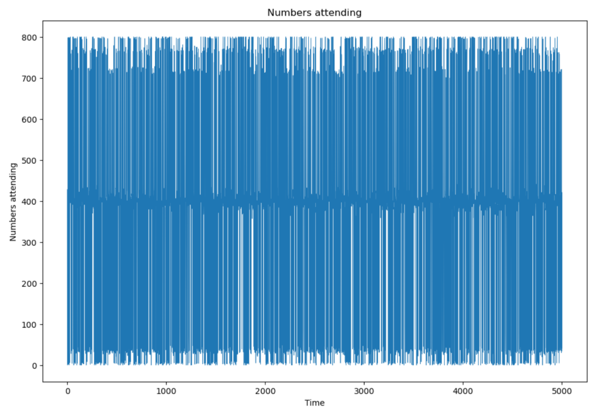

Fig. 1. Attendance graph with $N = 801$, simulation time $= 5000$, memory length $M = 2$ and the number of strategies for each player $K = 10$

At least in these 5000 iterations, we did not see any signs of convergence in attendance. The problem lies in the strategy selection mechanism. We print out the best agent's strategy score information after 5000 iterations (For details on the correspondence between strategy numbers and specific contents, see Appendix A.):

Table 1. Strategies used by the best player and their scores

Strategy	Score
7	2501
1	2501
10	2499
14	2499
2	2500
4	2500
12	2499
15	2500
6	2500
13	2500

We found that the scores of the various strategies are not very different. On the one hand, this shows that there is a lot of randomness in the process of the agent choosing the best strategy to act; on the other hand, it also shows that the strategies themselves are not good or bad. Let's take a look at the scores of the worst agent's strategies after 5000 iterations:

Table 2. Strategies used by the worst player and their scores

Strategy	Score
0	2500
3	2501
10	2499
11	2500
5	2501
13	2500
9	2500
7	2501
8	2499
12	2499

We found that not only are the strategy scores within the same agent not very different; even between the best agent and the worst agent, the strategy scores are also not very different. Note that a strategy score is the number of times it correctly predicts the situation. This shows that the ability of different agents to correctly predict the situation is not much different. However, if we plot the performance graph, we can see that there is a considerable difference between the performance of the best agent and the worst agent:

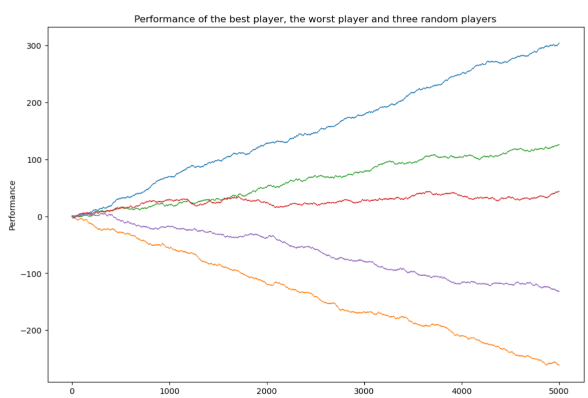

Fig. 2. Performance graph with $N = 801$, simulation time $= 5000$, memory length $M = 2$ and the number of strategies for each player $K = 10$

In the figure, the top curve represents the best agent, the bottom curve represents the worst agent, and the three in the middle are random agents. We found that in this figure, the curves of the best agent and the worst agent have a clear trend. In 5,000 simulations, the difference in their number of wins is almost 600, accounting for 12% of the total number of games. And this gap will expand as the number of simulations increases, almost proportional to the number of simulations.

We try to increase the memory length to enhance the difference between different strategies and try to see if this will reduce the overall fluctuation. From the following figure, the situation has indeed improved:

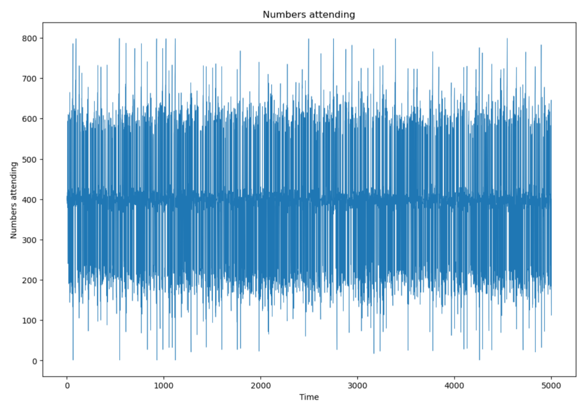

Fig. 3. Attendance graph with $N = 801$, simulation time $= 5000$, memory length $M = 4$ and the number of strategies for each player $K = 10$

We found that when $M = 4$, the volatility of attendance is indeed reduced. This is also reflected in the performance graph:

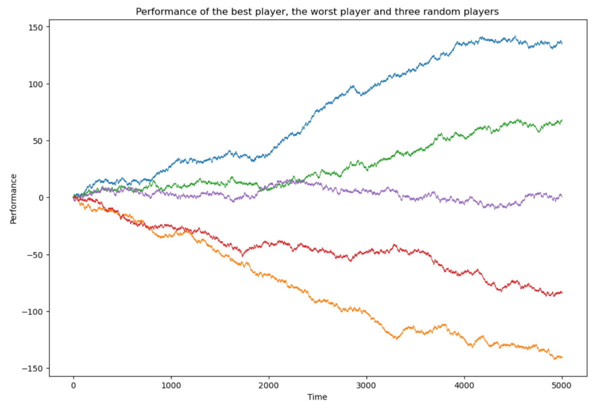

Fig. 4. Performance graph with $N = 801$, simulation time $= 5000$, memory length $M = 4$ and the number of strategies for each player $K = 10$

We can see that after 5000 iterations, the gap between the best agent and the worst agent is reduced to about 300, which is almost half of that when $M = 2$. However, when we look at the strategy scores, we find that the situation has not changed significantly:

Table 3. Strategies used by the worst player and their scores

Strategy	Score
36013	2499
28802	2501
63974	2501
10020	2501
45462	2500
24750	2499
54410	2500
29442	2502
33235	2501
26873	2499

Table 4. Strategies used by the worst player and their scores

Strategy	Score
19191	2499
52614	2501
27564	2500
30870	2499
55598	2500
58758	2501
43613	2498
699	2499
63441	2501
15181	2500

The reason for this phenomenon is that regardless of whether a strategy is the strategy selected by the agent in the current game, as long as it successfully predicts the result, its score will $+1$. On average, the probability of each strategy being awarded in each game is 0.5. Therefore, with enough simulations, the score of each strategy will approach $\frac{thenumberofsimulations}{2}$. From this we find that in the Basic Minority Game, adjusting M will only scale the results, and this scaling can never reach our ideal low volatility situation. Therefore, we introduce the concept of adapt and observe the resulting changes.

3.2 Computational results of Adapative Minority Game

We tried to reproduce the four attendance graphs corresponding to the four different adaptation methods in the textbook. Indeed, in all four methods, the attendance graph showed some convergence to the mean, but it could never converge to the level of fluctuation close to zero as in the textbook.

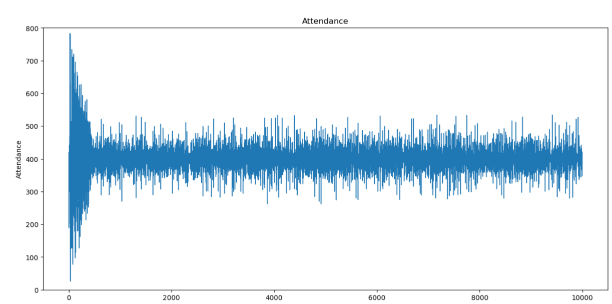

Fig. 5. Attendance graph with $N = 801$, simulation time $= 10000$, memory length $M = 4$ and the number of strategies for each player $K = 16$. Adapting rate $=$ per 60 steps. The worst 40% agents do this modification.

In this graph, the children are produced from the best parents and substitute the worst two strategies. Since the book does not give the scoring mode after adjusting the strategy, we tried many methods. The method we used to generate this graph was to reset all the strategy scores of each adjusted agent to zero. We also tried two methods:
1. Keep the scores of the unadjusted strategies unchanged and assign the new strategy the highest score of the old strategies.
2. Keep the scores of the unadjusted strategies unchanged and let the new strategy inherit the score of the strategy it replaced.
However, the volatility of these two methods is greater than the volatility of this graph. Therefore, this graph was once our volatility convergence limit.
However, later we changed the way to reset the strategy scores when doing the modification and get the graph below:

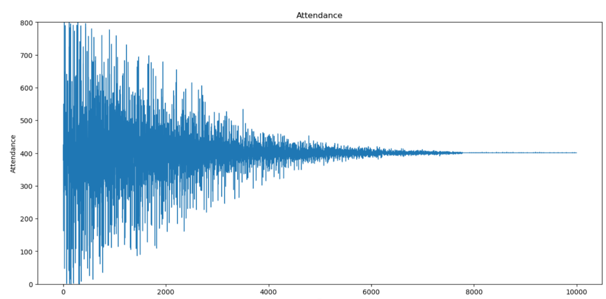

Fig. 6. Attendance graph with $N = 801$, simulation time $= 10000$, memory length $M = 4$ and the number of strategies for each player $K = 16$. Adapting rate $=$ per 60 steps. The worst 40% agents do this modification.

Our new approach is: after the worst batch of agents have updated their strategies, we reset the scores of all strategies of everyone to 0. This approach sounds incredible, because those agents in a dominant position do not seem to need to reset their strategy scores. But this approach actually has a potential benefit for them.

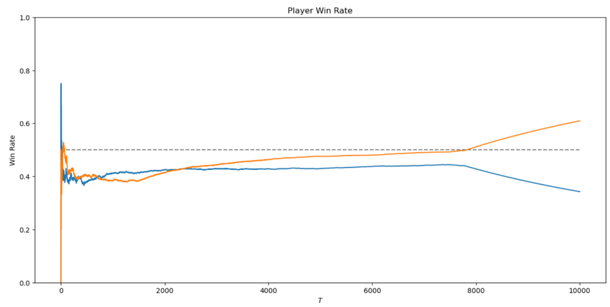

Fig. 7. The orange line gives the winning rate of the best agent, and the blue line represents for the worst. Reset scheme: reset all.

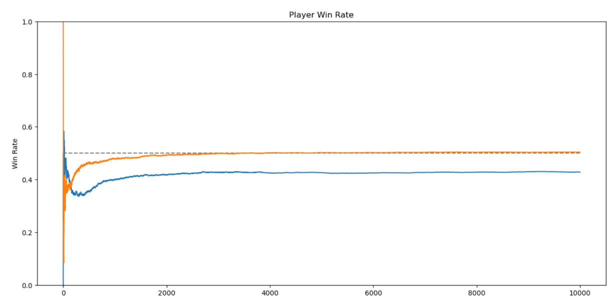

Fig. 8. The orange line gives the winning rate of the best agent, and the blue line represents for the worst. Reset scheme: reset the worst only.

Fig. 7 shows the win rate graph of the best agent and the worst agent under the situation that everyone will reset their strategy scores to zero after the worst batch of agents modified their strategies. Fig. 8 is the graph that only those who modified their strategies will reset their strategy scores to 0. We can see that in Fig. 8, the best winning rate is growing slowly and can only achieve around 0.5. However, in Fig. 7, the best winning rate start to grow rapidly after exceeding a certain point and have a value much higher than 0.5. Given different payoff functions, the agents who are in advantages may also choose to reset their strategy scores to seek the possibility of obtaining greater benefits.
Relate Fig. 7 and Fig. **??**, we discover that the point where the fluctuation of attendance becomes very small and the point where the winning rate starts to 'bifurcate' is almost the same point. This reveals that the smaller attendance fluctuation is not because everyone's strategy has become better, so everyone's benefits have increased, but because of the inadaptive behavior of some agents.

Their forced "concessions" increase the benefits of the entire system.
We noticed that in the Basic Minority Game, different parameters have an overall scaling effect on the fluctuation of the attendance. In the Adaptive Minority Game with the 'global reset' scheme, although the fluctuation of the attendance always tends to be close to zero, different parameters still have an impact on the winning rate of the agent. Next, we will explore the impact of different memory lengths M, different number of strategies K and different strategy selection mechanisms on the winning rate.

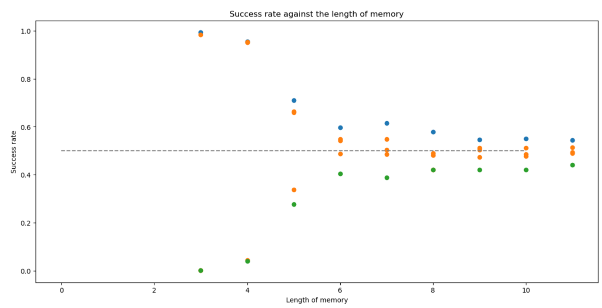

Fig. 9. The blue scatters are the best agents, green scatters are the worst agents and the orange scatters are random agents. Here the success rate is simply the winning rate.

We found that as M increases, the difference in winning rate between different agents decreases, and finally stabilizes within 0.2. Note that our previous case was run when $M = 4$, which is when the difference is relatively large. This figure tells us that when M is large, the global reset scheme will not be enabled because the advantaged player finds that doing so does not have higher potential benefits. Therefore, we then focus on the impact of M on the winning rate under the local reset scheme (especially when M is large).

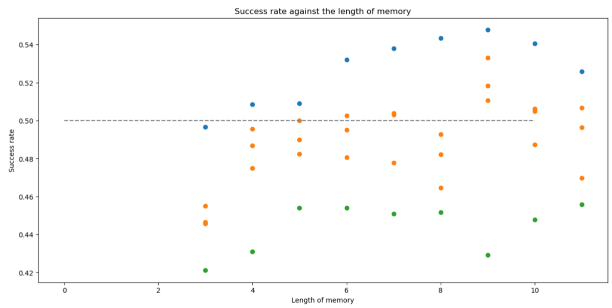

Fig. 10. The blue scatters are the best agents, green scatters are the worst agents and the orange scatters are random agents. Here the success rate is simply the winning rate.

Note that the difference in winning rate represented by the spacing of the y-axis is small. Therefore, combined with Fig. 9, under the local reset scheme, M has little effect on the winning rate. Under large M, the dominant agent will choose not to reset its strategy score for stability.
Next, let's look at the impact of different K on the winning rate:

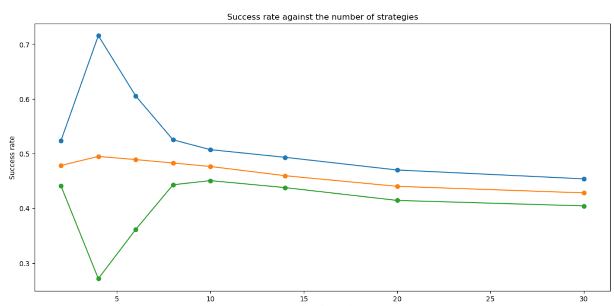

Fig. 11. The blue line is the best agent, green line is the worst agent and the orange line is random agent. Here the success rate is simply the winning rate. $N = 1001$, $M = 5$, $T = 5000$. Global reset.

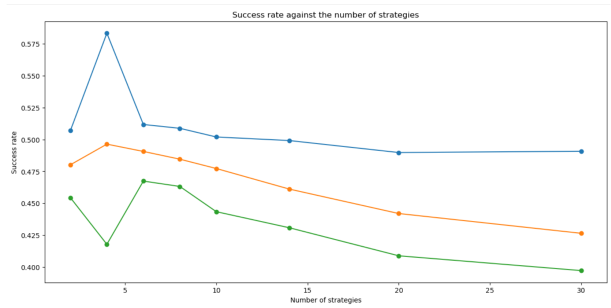

Fig. 12. The blue line is the best agent, green line is the worst agent and the orange line is random agent. Here the success rate is simply the winning rate. $N = 1001$, $M = 5$, $T = 5000$. Local reset.

Contrary to our expectation that the larger K is, the better the result is, we found that K has an optimal value. After exceeding this optimal value, too many choices make agents very confused. We also ran a graph of winning rate against K in the Basic Minority Model, which is exactly the same in shape as Fig. 11 and Fig. 12. For the Basic Minority Model, we drew a scatter plot of Switching rate against Winning rate:

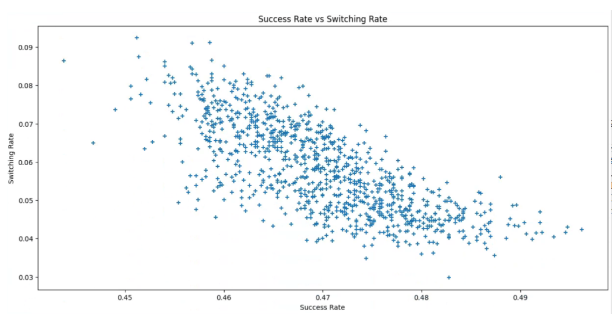

Fig. 13. Switching rate against winning rate in Basic Minority Game.

where switching rate is the times that the agent switch to a different strategy divided by the total time. From this graph, we can see that the switching rate and winning rate are negatively correlated. In other words, those agents with a high winning rate have a relatively simple strategy. On the other hand, those agents who repeatedly struggle between different strategies are misled by their strategy scores and are far away from the correct strategy. Therefore, increasing the threshold of the switch strategy may be a way to increase the winning rate of disadvantaged agents. However, when we draw the switching rate scatter plot of the Adaptive Minority Game, we find that the situation is different:

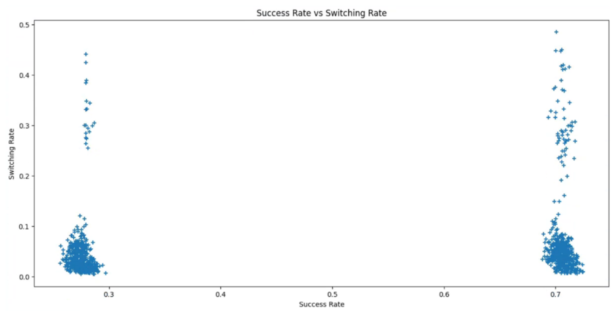

Fig. 14. Switching rate against winning rate in Adaptive Minority Game. $N = 1001, M = 5, K = 6$, global reset.

First, we were surprised to find that all agents were divided into two camps. From the large blank space in the winning rate, we can conclude that the agents in each camp have a high degree of consistency in their actions. Secondly, we found that there is no correlation between switching rate and winning rate. Considering that agents will replace the worst strategy with a variation of the dominant strategy, we speculate that the strategies held by each agent are quite similar. Therefore, no matter how it switches between strategies, it does not change its action, nor does it change its winning rate.

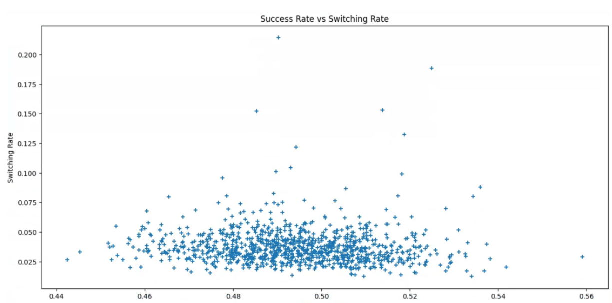

Fig. 15. Switching rate against winning rate in Adaptive Minority Game before the attendance convergent.

This is a scatter plot of switching rates before attendance convergence under other parameters. We find that all switching rates are distributed in a lower range and are not correlated with winning rate. We speculate that each agent has a relatively fixed set of dominant strategy at this time, and constantly uses this small set of dominant strategy variants to replace poorly performing strategies.

4 Conclusion

In Basic Minority Game, the performance of agents depends largely on the innate random strategy selection. However, in Adaptive Minority Game, agents can change their strategies. Through the analysis of the above process, we found that the strategy adjustment mode of agents in AMG may be that a few excellent strategies drive out the majority of inferior strategies, resulting in the convergence of agents in the end, and the group with fewer people becomes the winner.

In addition, the study also found that parameters such as memory length and number of strategies have an important impact on the agent's performance, especially on the relationship between the diversity of strategy choices and agent performance.

Adaptive minority games provide important insights into understanding agent behavior in complex markets, and future research can further explore the impact of different parameters on game outcomes to optimize strategy adjustment mechanisms.

References

1. Sinha, S. (2011). Econophysics: an introduction. Wiley-VCH.
2. Challet, D., Marsili, M., & Zhang, Y.-C. (2005). Minority games. Oxford University Press.

A Mapping from strategy to its number

Take $M = 2$ as an example. There are altogether $2^M = 4$ kinds of historical situation: 00, 01, 10, 11. Number each historical situation in the following way:

$$\{\mathrm{i}\}_{10} = \{Binary\ string\ of\ Historical\ Situation\ i\}_2$$

For example, 00 is historical situation 0, 01 is historical situation 1, 10 is historical situation 2, 11 is historical situation 3.

Then, strategy 0001 means:

If the input is historical situation 0, the output is 1.

If the input is historical situation 1, the output is 0.

If the input is historical situation 2, the output is 0.

If the input is historical situation 3, the output is 0.

And strategy 1001 means:
If the input is historical situation 0, the output is 1.
If the input is historical situation 1, the output is 0.
If the input is historical situation 2, the output is 0.
If the input is historical situation 3, the output is 1.

Then we map the strategy to its number in the same way, so that:
Strategy No.4 = Strategy 0100
Strategy No.5 = Strategy 0101.