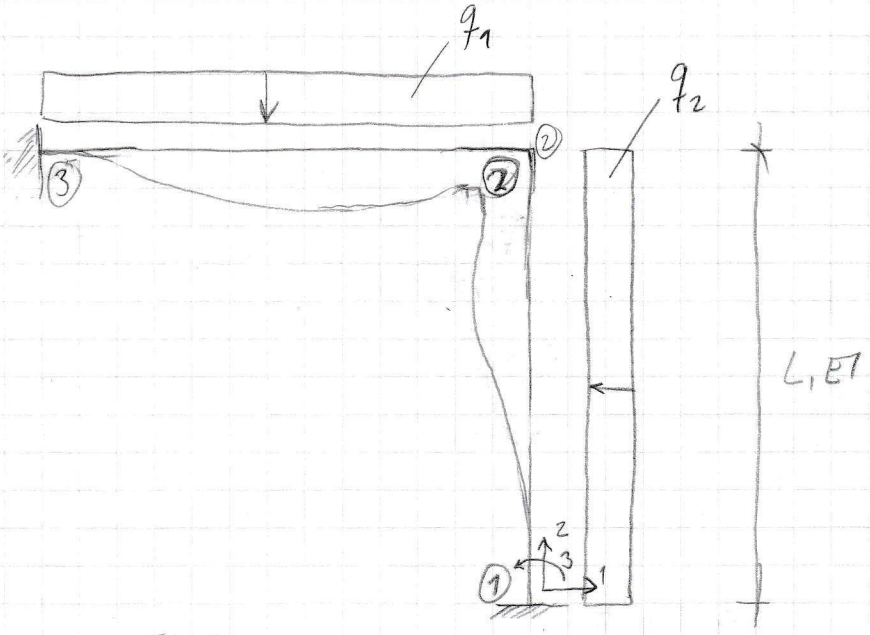
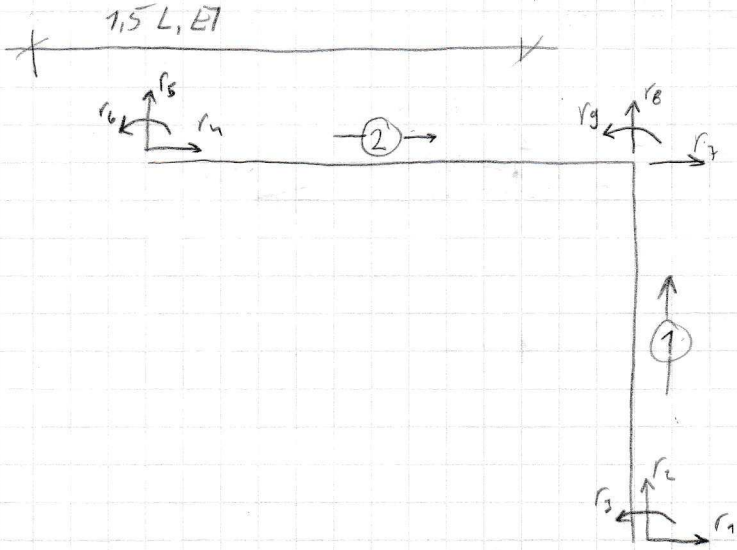
25.2.2016
Harj. 3

Lujuusoppi 2



$L = 4\,m$
$q_1 = 45\ kN/m$
$q_2 = 9\ kN/m$

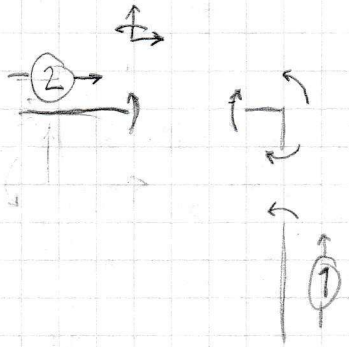
$E = 210\ GPa$
$I = 3 \cdot 10^{-5}\ m^4$



$r_1 = 0$
$r_2 = 0$
$r_3 = 0$
$r_4 = 0$
$r_5 = 0$
$r_6 = 0$
$r_7 = r_7$
$r_8 = r_8$
$r_9 = r_9$

Topologia

①	1	2	3	7	8	9
②	4	5	6	7	8	9



① ②

Nod 1
1	P_1	+ 0	$= R_1$
2	P_2	0	$= R_2$
3	P_3	0	$= R_3$

Nod 2
1	P_4	P_4	$= R_7$
2	P_5	P_5	$= R_8$
3	P_6	P_6	$= R_9$

Nod 3
1	0	P_1	$= R_4$
2	0	P_2	$= R_5$
3	0	P_3	$= R_6$

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$$A = \begin{bmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} C & -S & 0 & 0 & 0 & 0 \\ S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & -S & 0 \\ 0 & 0 & 0 & S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Jäykkyysmatriisi globaalissa koordinaatistossa

$K_r = R - R^0 \qquad K^e = A^T \bar{K}^e A$

Elementti 1

$\varphi = 90° \Rightarrow \cos 90° = 0, \quad \sin 90° = 1$

$$\Rightarrow A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^T \bar{K}^e = \begin{bmatrix} 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

$$K^e = A^T \bar{K}^e A = \begin{bmatrix} \frac{12EI}{L^3} & 0 & -\frac{6EI}{L} & -\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} \\ 0 & \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 \\ -\frac{6EI}{L^2} & 0 & \frac{4EI}{L} & \frac{6EI}{L^2} & 0 & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} \\ 0 & -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 \\ -\frac{6EI}{L^2} & 0 & \frac{2EI}{L} & \frac{6EI}{L^2} & 0 & \frac{4EI}{L} \end{bmatrix}$$

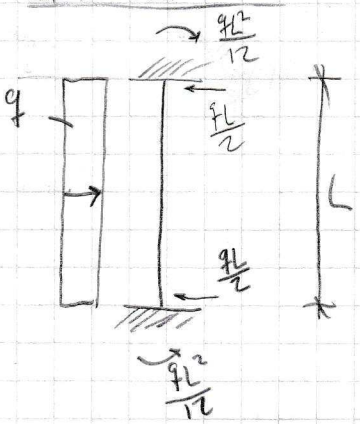
Elementti 2

$\varphi = 0° \Rightarrow \cos 0° = 1,\ \sin 0° = 0$

$$\Rightarrow A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = I \qquad A^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = I$$

$$K^e = I\bar{K}^e I = \bar{K}^e = \begin{bmatrix} \frac{EA}{1{,}5L} & 0 & 0 & -\frac{EA}{1{,}5L} & 0 & 0 \\ 0 & \frac{12EI}{(1{,}5L)^3} & \frac{6EI}{(1{,}5L)^2} & 0 & -\frac{12EI}{(1{,}5L)^3} & \frac{6EI}{(1{,}5L)^2} \\ 0 & \frac{6EI}{(1{,}5L)^2} & \frac{4EI}{1{,}5L} & 0 & -\frac{6EI}{(1{,}5L)^2} & \frac{2EI}{1{,}5L} \\ -\frac{EA}{1{,}5L} & 0 & 0 & \frac{EA}{1{,}5L} & 0 & 0 \\ 0 & -\frac{12EI}{(1{,}5L)^3} & -\frac{6EI}{(1{,}5L)^2} & 0 & \frac{12EI}{(1{,}5L)^3} & -\frac{6EI}{(1{,}5L)^2} \\ 0 & \frac{6EI}{(1{,}5L)^2} & \frac{2EI}{1{,}5L} & 0 & -\frac{6EI}{(1{,}5L)^2} & \frac{4EI}{1{,}5L} \end{bmatrix}$$

Lukitusvoimat



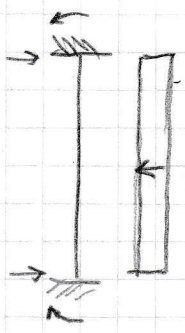
$P^\circ = A^T \bar{P}^\circ$

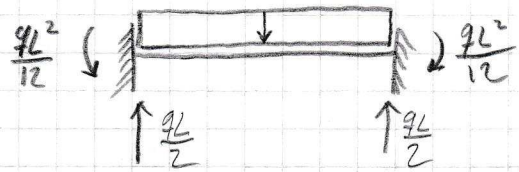
$\bar{P}^\circ$

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \begin{bmatrix} 0 \\ \frac{qL}{2} \\ \frac{qL^2}{12} \\ 0 \\ \frac{qL}{2} \\ -\frac{qL^2}{12} \end{bmatrix}$$

$A^T\bar{P}^\circ = P^\circ$

$$\begin{bmatrix} -qL/2 \\ 0 \\ qL^2/12 \\ -qL/2 \\ 0 \\ -qL^2/12 \end{bmatrix}$$





$$\begin{bmatrix} 0 \\ qL/2 \\ qL^2/12 \\ 0 \\ qL/2 \\ qL^2/12 \end{bmatrix}$$

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$$
\begin{bmatrix}
\frac{12EI}{L^3} & 0 & -\frac{6EI}{L} & 0 & 0 & 0 & -\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} \\
0 & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{L} & 0 \\
-\frac{6EI}{L^2} & 0 & \frac{4EI}{L} & 0 & 0 & 0 & \frac{6EI}{L^2} & 0 & \frac{2EI}{L} \\
0 & 0 & 0 & \frac{EA}{1{,}5L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{12EI}{(1{,}5L)^3} & \frac{6EI}{(1{,}5L)^2} & 0 & -\frac{12EI}{(1{,}5L)^3} & \frac{6EI}{(1{,}5L)^2} \\
0 & 0 & 0 & 0 & \frac{6EI}{(1{,}5L)^2} & \frac{4EI}{1{,}5L} & 0 & -\frac{6EI}{(1{,}5L)^2} & \frac{2EI}{1{,}5L} \\
-\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & -\frac{EA}{1{,}5L} & 0 & 0 & \frac{EA}{1{,}5L}+\frac{12EI}{L^3} & 0+0 & 0+\frac{6EI}{L^2} \\
0 & -\frac{EA}{L} & 0 & 0 & -\frac{12EI}{(1{,}5L)^3} & -\frac{6EI}{(1{,}5L)^2} & 0+0 & \frac{12EI}{(1{,}5L)^3}+\frac{EA}{L} & -\frac{6EI}{(1{,}5L)^2}+0 \\
-\frac{6EI}{L^2} & 0 & \frac{2EI}{L} & 0 & \frac{6EI}{(1{,}5L)^2} & \frac{2EI}{1{,}5L} & 0+\frac{6EI}{L^2} & \frac{6EI}{(1{,}5L)^2}+0 & \frac{4EI}{1{,}5L}+\frac{4EI}{L}
\end{bmatrix}
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ r_7 \\ r_8 \\ r_9 \end{bmatrix}
=
\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ 0 \\ 0 \\ 0 \end{bmatrix}
-
\begin{bmatrix} -\frac{qL}{2} \\ 0 \\ qL^2/12 \\ 0 \\ \frac{q1{,}5L}{2} \\ \frac{q(1{,}5L)^2}{12} \\ -\frac{qL}{2}+0 \\ 0+\frac{q1{,}5L}{2} \\ -\frac{qL^2}{12}+\frac{q(1{,}5L)^2}{12} \end{bmatrix}
$$

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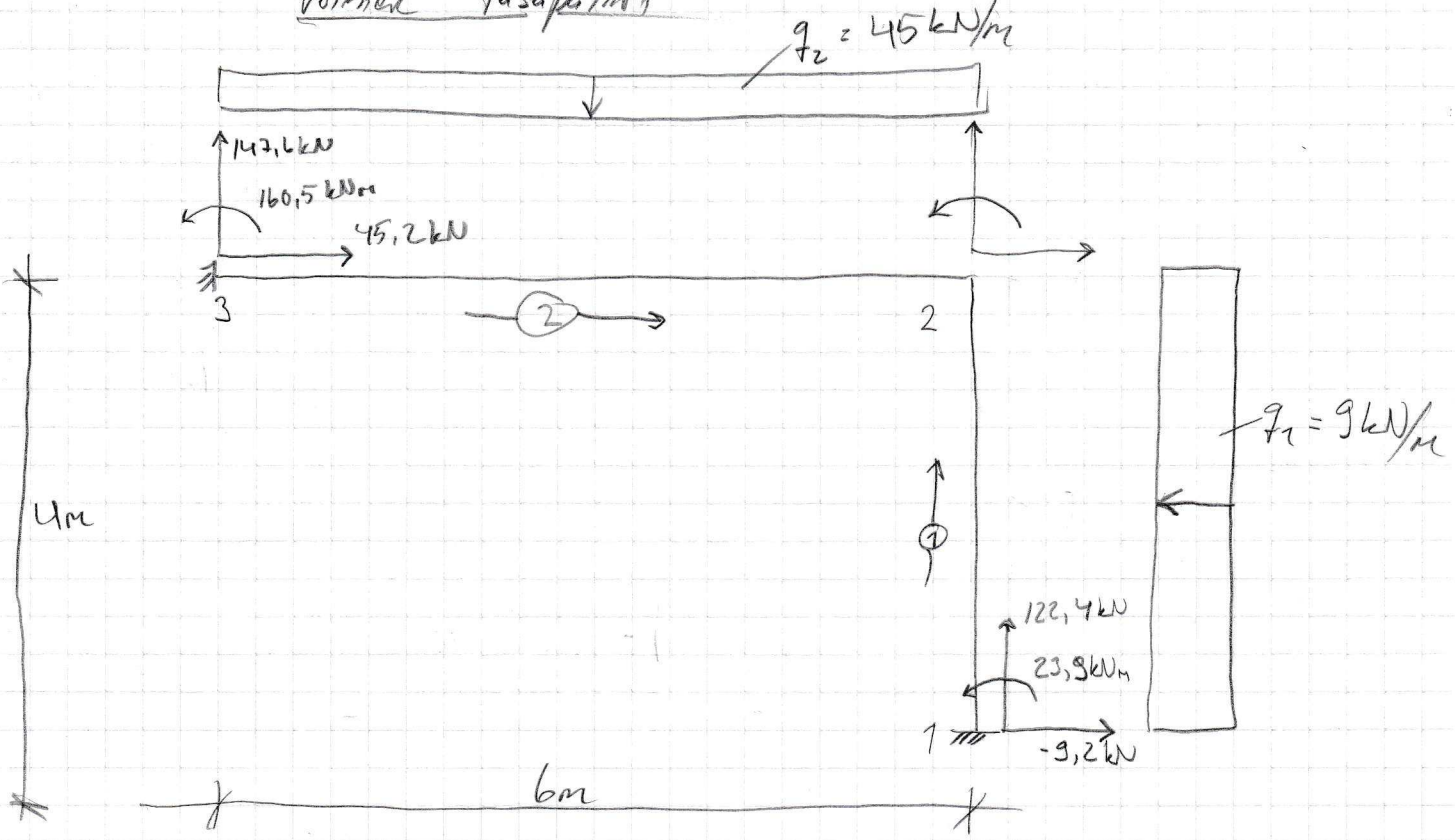
Voimien tasapaino



$\curvearrowleft$: $-23{,}9\,kNm - 160{,}5\,kNm + 147{,}6\,kN \cdot 6m + 45{,}2\,kN \cdot 4m - 45\,kN/m \cdot \frac{6^2 m^2}{2} - 9\,kN/m \cdot \frac{4^2 m^2}{2}$

$= 0 \Rightarrow$ OK!

$\rightarrow$: $-9{,}2\,kN - 9\,kN/m \cdot 4m + 45{,}2\,kN = 0 \Rightarrow$ OK!

$\uparrow$: $122{,}4\,kN + 147{,}6\,kN - 45\,kN/m \cdot 6m = 0 \Rightarrow$ OK!