LUND UNIVERSITY

FMAN-45: Machine Learning

Lecture 12:

Reinforcement Learning II

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~~Lecture 1 - Introduction and basic concepts~~

~~Lecture 2 - Probability theory and linear regression~~

~~Lecture 3 - Optimization theory and methods~~

~~Lecture 4 - Methods for linear classification~~

~~Lecture 5 - Support vector machines~~

~~Lecture 6 - Clustering and dimensionality reduction~~

~~Lecture 7 - Neurons and Feed-Forward Networks~~

~~Lecture 8 - Convolutional Neural Networks~~

~~Lecture 9 - Recurrent Neural Networks~~

~~Lecture 10 - Reinforcement learning ; an introduction~~

Lecture 11 - Reinforcement learning ; going deeper

Lecture 12 - Generative models

Lecture 13 - Deep learning in practice with Python APIs

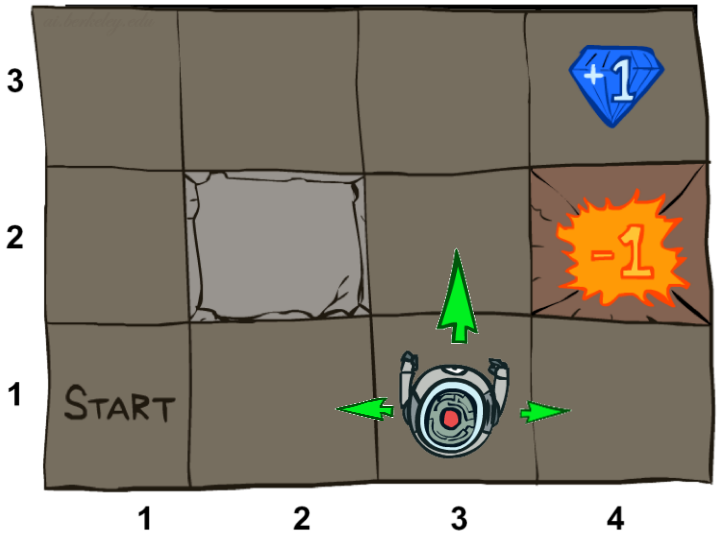
Basics

Classical

Modern

Last Time: Markov Decision Processes (MDPs)

- An MDP is defined by:
  - A set of states $s \in S$
  - A set of actions $a \in A$
  - A transition function $T(s, a, s')$
    - Probability that a from s leads to s', i.e., $P(s' | s, a)$
    - Also called the *model* or the *dynamics*
    - Similar to successor function, except may be stochastic
  - A reward function $R(s, a, s')$
    - Sometimes just R(s) or R(s')
    - May be non-deterministic → extend dynamics model $P(s' | s, a)$ into $P(r, s' | s, a)$ [although not very common]
  - A start state
  - Maybe a terminal state



- MDPs are **non-deterministic search problems**

Last Time: Optimal Quantities

- The optimal value of a state s:

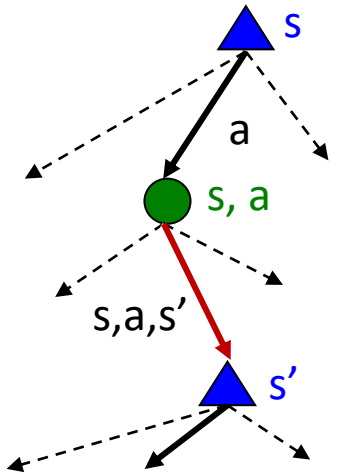
  $V^*(s)$ = expected utility starting in s and acting optimally

- The optimal value of a Q-state (s,a):

  $Q^*(s,a)$ = expected utility starting out having taken action a from state s and (thereafter) acting optimally

- The optimal policy:

  $\pi^*(s)$ = optimal action from state s



s is a *state (max node)*

(s, a) is a *q-state (chance node)*

(s,a,s') is a *transition*

Last Time: The Bellman Optimality Equations

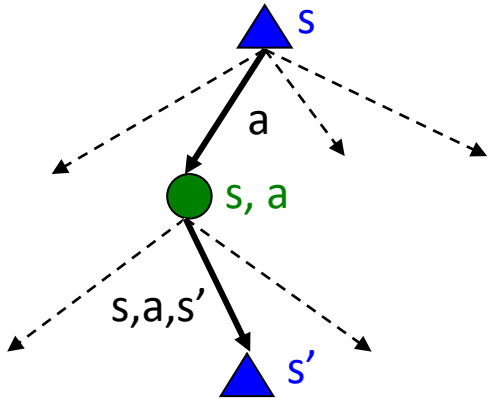
- Definition of "optimal utility" is a simple one-step lookahead relationship amongst optimal utility values

$$V^*(s) = \max_a Q^*(s,a)$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s') \right]$$

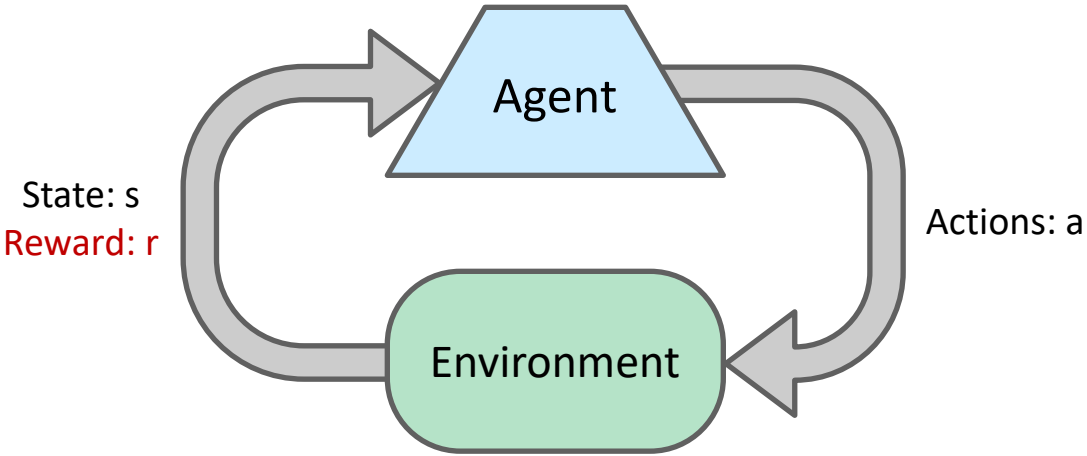
$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s') \right]$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q^*(s',a') \right]$$



- These are the Bellman optimality equations, and they **characterize optimal values**

Last Time: Reinforcement Learning (RL)



- Basic idea:
  - Receive feedback in the form of rewards
  - Agent's utility is defined by the reward function
  - Must ***learn*** to act so as to maximize expected rewards
  - All learning is based on observed samples of outcomes!

Last Time: Reinforcement Learning (RL)

- Passive vs. active
  - **Both:** Dynamics of MDP unknown --- *T(s,a,s')* and *R(s,a,s')* unknown
  - **Passive:** Given *fixed* policy π, goal is to learn state values *V(s)*. No choice about actions, just execute policy and learn from experience.
  - **Active (what we're mostly interested in):** Agent chooses actions, with the goal of **learning optimal policy** π and maybe state values *V(s)*. ***Exploration vs. exploitation***

- Model-based (MB) vs. model-free (MF):
  - **MB:** First learn MDP quantities *T(s,a,s')* and *R(s,a,s').* Then use MDP algorithms (e.g. policy iteration) to find policy. **Pros:** Often more sample efficient. **Cons:** The "goodness" of the policy is bounded by the "goodness" of the model.
  - **MF:** Do not explicitly try to learn model dynamics *T* and *R*. Instead directly learn the policy / state-values, e.g. using Q-learning. Pros/Cons are "opposite" to those of MB

Last Time: Q-Learning

- Q-Learning: sample-based Q-value iteration

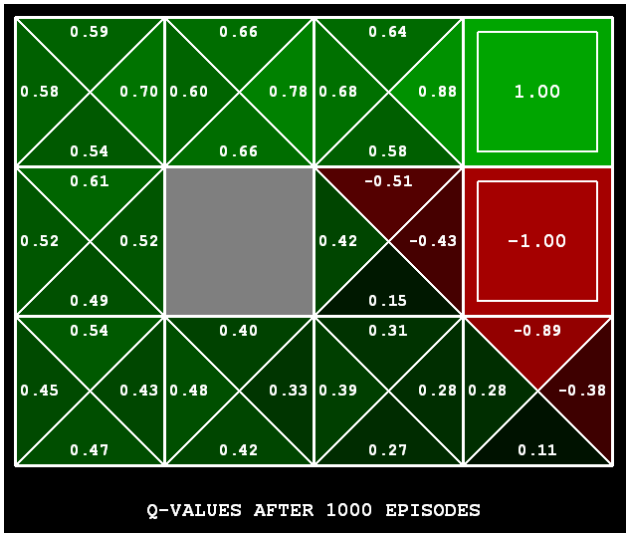
$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

- Learn Q(s,a) values as you go
  - Receive a sample *(s,a,s',r)*
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate:

$$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

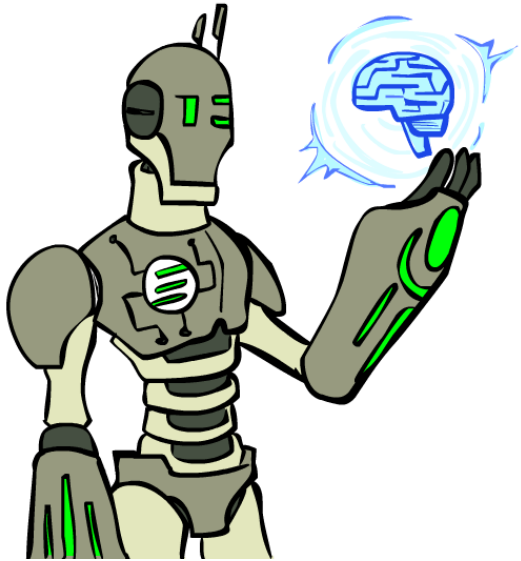
  - Incorporate the new estimate into a running average:

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\,[sample]$$



Today

- Intermediate / averaged lookahead
- Deep function approximation in RL → **deep reinforcement learning (DRL)**
- **Learning optimal policy π^* directly**, without intermediate step of learning *Q** or *V**, using **policy gradients**



Partial slide credits:

- **Dan Klein and Pieter Abbeel, *CS188 Intro to AI* at UC Berkeley (http://ai.berkeley.edu);**
- **David Silver, *COMPM050/COMPGI13 Reinforcement Learning*, UCL**
- **Richard Sutton and Andrew Barto, *Reinforcement Learning: An Introduction***
- **Andrey Karpathy, blog post on Reinforcement Learning: http://karpathy.github.io/2016/05/31/rl/**

Evolution of *(s, a, r, s')*-transitions in MDPs

- How a trajectory evolves in MDP given policy $\pi(a \mid s)$ and dynamics $p(r, s' \mid s, a)$
- Sample first action: $a_0 \sim \pi(* \mid S_0 = s_0)$
- The reward r_1 and next state s_1 are obtained as: $r_1, s_1 \sim p(*,* \mid S_0 = s_0, A_0 = a_0)$
- A trajectory of transitions thus evolves as:

$$s_0 \xrightarrow{\pi(*|s_0)} a_0 \xrightarrow{p(*,*|s_0,a_0)} r_1, s_1 \xrightarrow{\pi(*|s_1)} a_1 \xrightarrow{p(*,*|s_1,a_1)} r_2, s_2 \xrightarrow{\pi(*|s_2)} \dots$$

$$\tau = (s_0, a_0, r_1, s_1, a_1, r_2, s_2, \dots)$$

- Slightly informal notation: $\tau \sim \pi$ "trajectory sampled from policy", although in reality of course also depends on model dynamics p

Bellman Equations written as Expectations

- Last lecture we looked at the state-value Bellman equation (similar for *Q(s,a)*):

$$V^{\pi}(s) = \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V^{\pi}(s')]$$

- This can be written more generally (and with slightly different notation, see Sutton and Barto), allowing stochastic policies and rewards:

$$\begin{aligned} v_{\pi}(s) &= \sum_{a} \pi(a|s) \sum_{r,s'} p(r, s'|s, a) \left[r + \gamma v_{\pi}(s')\right] \\ &= \mathbb{E}_{\pi(*|s)} \left\{ \sum_{r,s'} p(r, s'|s, A) \left[r + \gamma v_{\pi}(s')\right] \right\} \\ &= \mathbb{E}_{\pi(*|s)} \left\{ \mathbb{E}_{p(*,*|s,A)} \left\{ R + \gamma v_{\pi}(S') \right\} \right\} \end{aligned}$$

- **Remark:** We cannot change the transition probability *p*, only the policy π → often written simply as $v_{\pi}(s) = \mathbb{E}_{\pi(*|s)} \left\{ R + \gamma v_{\pi}(S') \right\}$ or even $v_{\pi}(s) = \mathbb{E}_{\pi} \left\{ R + \gamma v_{\pi}(S') \right\}$

More Notation and Definitions (Sutton and Barto)

$$G_t \doteq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots \quad = \quad \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

$$v_\pi(s) \doteq \mathbb{E}_\pi[G_t \mid S_t = s] = \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \;\middle|\; S_t = s\right]$$

$$q_\pi(s,a) \doteq \mathbb{E}_\pi[G_t \mid S_t = s, A_t = a] = \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \;\middle|\; S_t = s, A_t = a\right]$$

Clearer notation would be: $v_\pi^t(s)$, $q_\pi^t(s,a)$

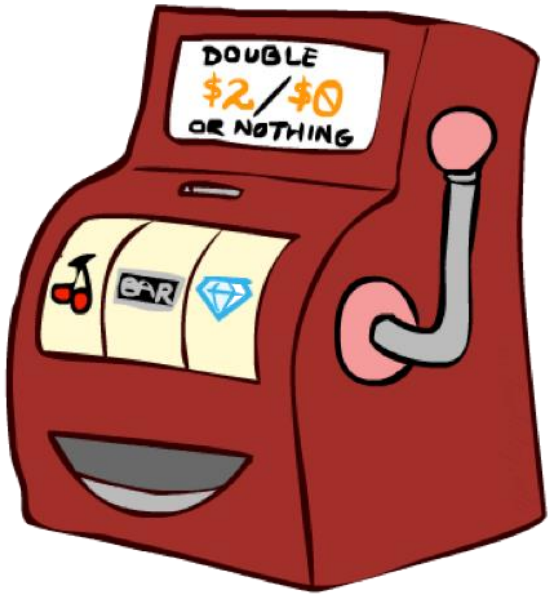
$$s_0 \xrightarrow{\pi(*|s_0)} a_0 \xrightarrow{p(*,*|s_0,a_0)} r_1, s_1 \xrightarrow{\pi(*|s_1)} a_1 \xrightarrow{p(*,*|s_1,a_1)} r_2, s_2 \xrightarrow{\pi(*|s_2)} \dots$$

$$\tau = (s_0, a_0, r_1, s_1, a_1, r_2, s_2, \dots)$$

Monte-Carlo i.e. Direct Evaluation

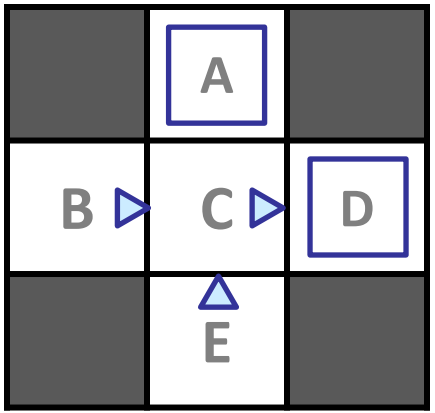
- Goal: Compute values for each state under π

- Idea: Average together observed sample values
  - Act according to π
  - Every time you visit a state, write down what the sum of discounted rewards turned out to be
  - Average those samples G_t



Example: Monte-Carlo (MC) Evaluation

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

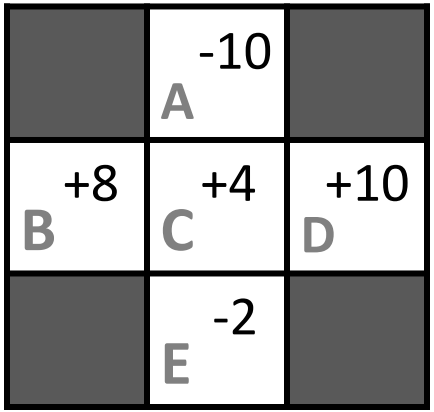
Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Output Values



Incremental Mean Update

Sample x, mean μ

$$
\begin{aligned}
\mu_k &= \frac{1}{k}\sum_{j=1}^{k} x_j \\
&= \frac{1}{k}\left(x_k + \sum_{j=1}^{k-1} x_j\right) \\
&= \frac{1}{k}\left(x_k + (k-1)\mu_{k-1}\right) \\
&= \mu_{k-1} + \frac{1}{k}\left(x_k - \mu_{k-1}\right)
\end{aligned}
$$

Incremental Monte-Carlo Updates

- Update $V(s)$ incrementally after episode $S_1, A_1, R_2, \dots, S_T$
- For each state S_t with return G_t

$$N(S_t) \leftarrow N(S_t) + 1$$
$$V(S_t) \leftarrow V(S_t) + \frac{1}{N(S_t)}(G_t - V(S_t))$$

- In non-stationary problems, it can be useful to track a running mean, *i.e.* forget old episodes – use a learning rate:

$$V(S_t) \leftarrow V(S_t) + \alpha(G_t - V(S_t))$$

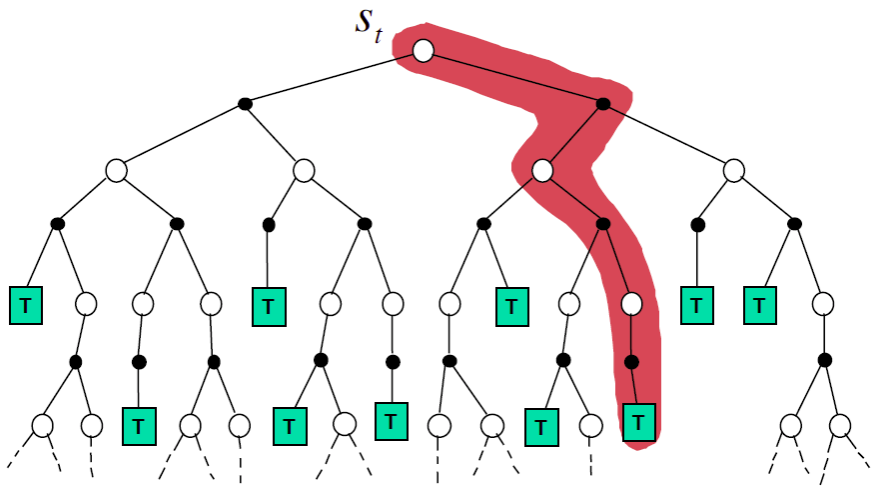
Monte-Carlo (MC) Policy Evaluation Update

To evaluate state s

- **First** / **every** time-step t when state s is visited in an episode:
  - Increment counter: $N(s) \leftarrow N(s) + 1$
  - Increment total return: $S(s) \leftarrow S(s) + G_t$
  - Estimate value by mean return: $V(s) \leftarrow \frac{S(s)}{N(s)}$
- By law of large numbers

  $V(s) \rightarrow v_\pi(s)$ as $N(s) \rightarrow \infty$
- Unbiased estimate, but high variance!

$$V(S_t) \leftarrow V(S_t) + \alpha\,(G_t - V(S_t))$$



Monte-Carlo (MC) Policy Evaluation

- The goal is to **learn v_π from episodes of experience under policy π**

$$S_1, A_1, R_2, ..., S_k \sim \pi$$

- The return is the total discounted reward

$$G_t = R_{t+1} + \gamma R_{t+2} + ... + \gamma^{T-1} R_T$$

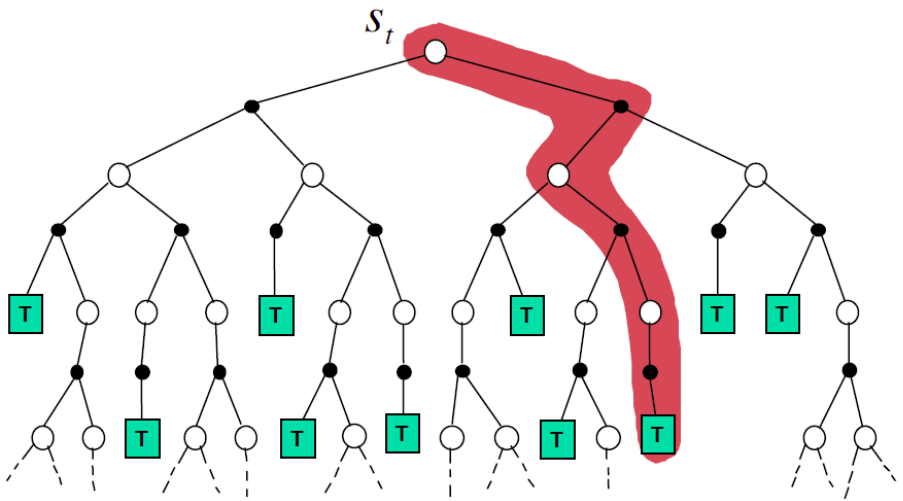
- The value function is the expected return

$$v_\pi(s) = \mathbb{E}_\pi \left[G_t \mid S_t = s \right]$$

- **Monte-Carlo** policy evaluation uses **empirical mean return** instead of expected return

$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t - V(S_t) \right)$$

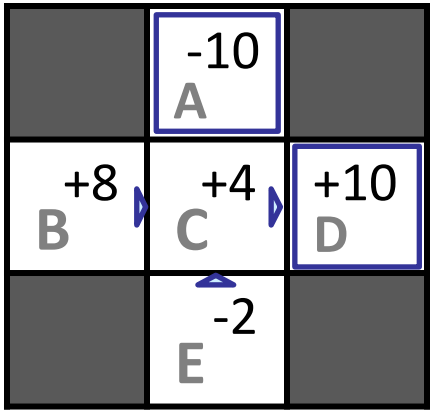
$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t - V(S_t) \right)$$



Pros and Cons of Monte-Carlo (MC) Evaluation

- **What's good about MC evaluation?**
  - Easy to understand
  - Eventually computes the correct average values, using just sample transitions
- **What bad about it?**
  - It wastes information about state connections
  - Each state must be learned separately (i.e. we only use samples that pass a state)
  - So, it takes a long time to learn

Output Values



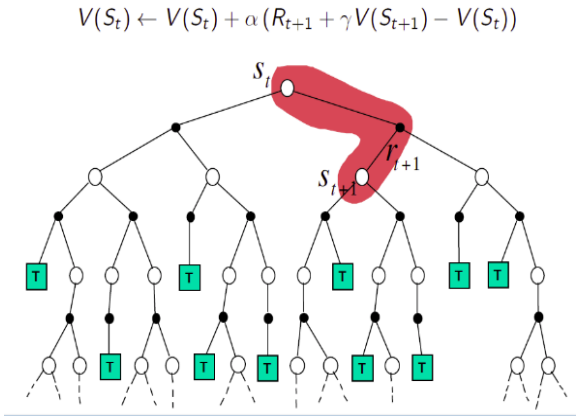
If B and E both go to C under this policy, how can their values be different?

Temporal Difference (TD) Learning

Same goal as before: **Learn v_π online from experience under policy π**

- Simplest **temporal-difference learning algorithm, TD(0):**
  - Update $V(S_t)$ toward estimated return $R_{t+1} + \gamma V(S_{t+1})$

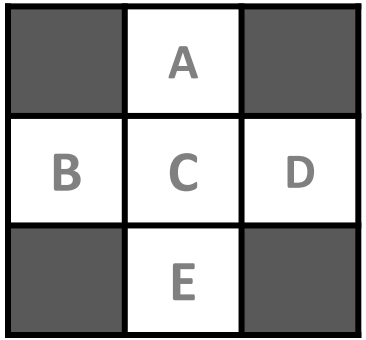
$$V(S_t) \leftarrow V(S_t) + \alpha \left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\right)$$



  - $R_{t+1} + \gamma V(S_{t+1})$ is called the **TD target**
  - $\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$ is called the **TD error**

Example: Temporal Difference Learning

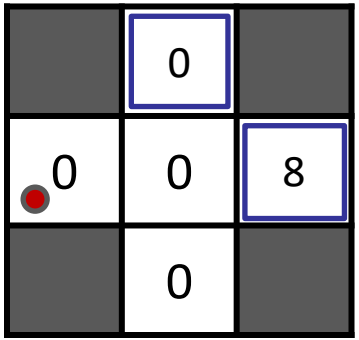
States



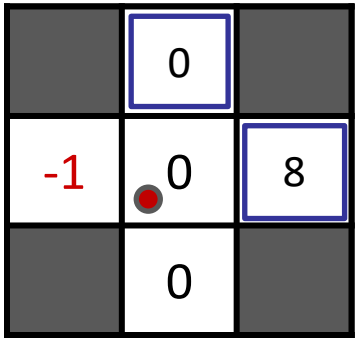
Assume: $\gamma = 1$, $\alpha = 0.5$

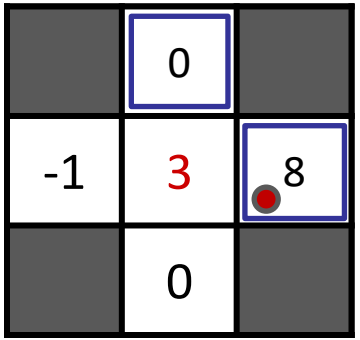
Observed Transitions

B, east, C, -2



C, east, D, -2

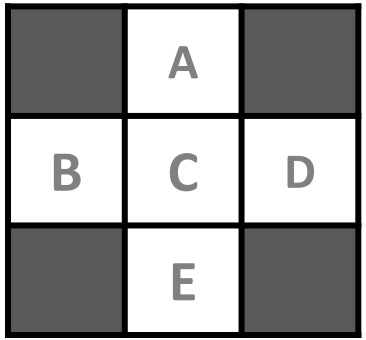




$$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + \alpha\left[R(s,\pi(s),s') + \gamma V^{\pi}(s')\right]$$

Example: Temporal Difference Learning

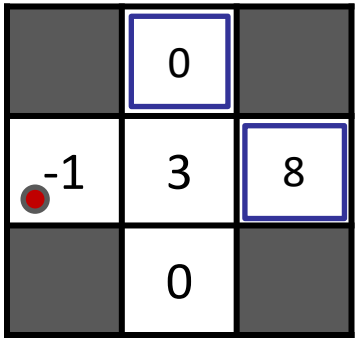
States



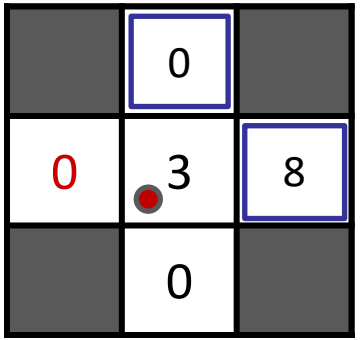
Assume: $\gamma = 1$, $\alpha = 1/2$

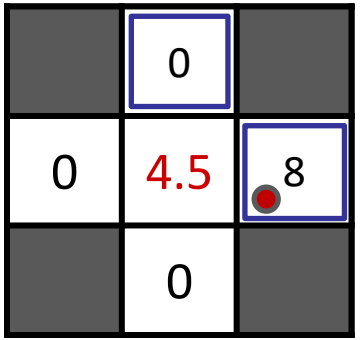
Observed Transitions

B, east, C, -2



C, east, D, -2

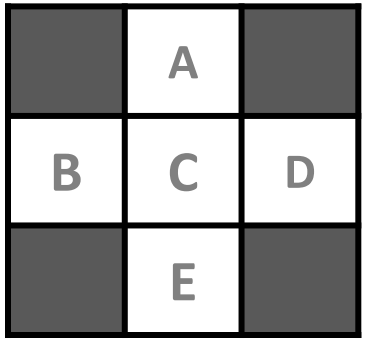




$$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + \alpha\left[R(s,\pi(s),s') + \gamma V^{\pi}(s')\right]$$

Example: Temporal Difference Learning

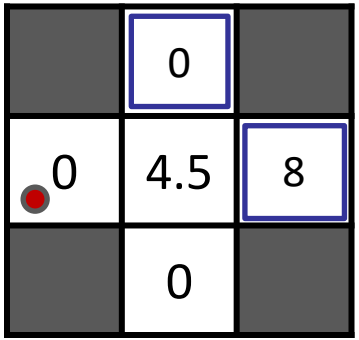
States



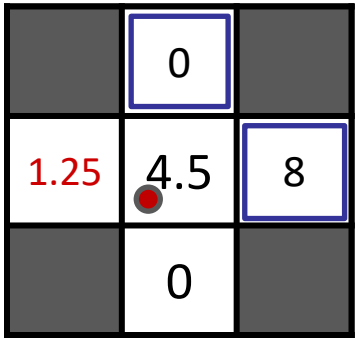
Assume: $\gamma = 1$, $\alpha = 1/2$

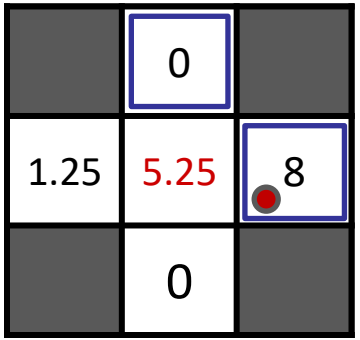
Observed Transitions

B, east, C, -2



C, east, D, -2

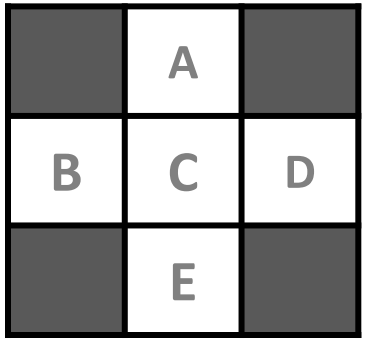




$$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + \alpha\left[R(s,\pi(s),s') + \gamma V^{\pi}(s')\right]$$

Example: Temporal Difference Learning

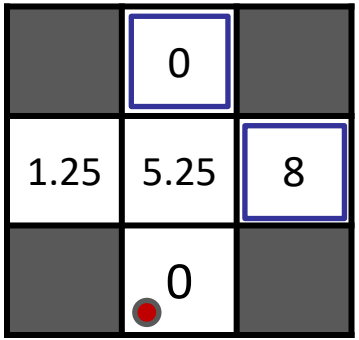
States



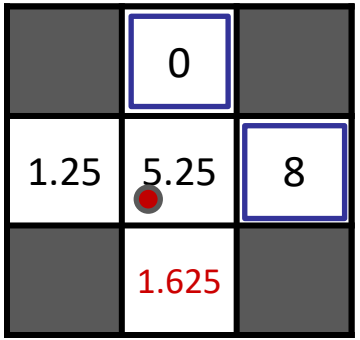
Assume: $\gamma = 1$, $\alpha = 1/2$

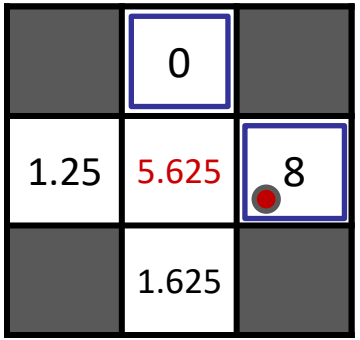
Observed Transitions

E, North, C, -2



C, east, D, -2





$$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + \alpha\left[R(s,\pi(s),s') + \gamma V^{\pi}(s')\right]$$

Temporal Difference vrs Monte Carlo Update

- **Temporal-difference learning algorithm, TD(0):**
  - Update $V(S_t)$ toward estimated return $R_{t+1} + \gamma V(S_{t+1})$

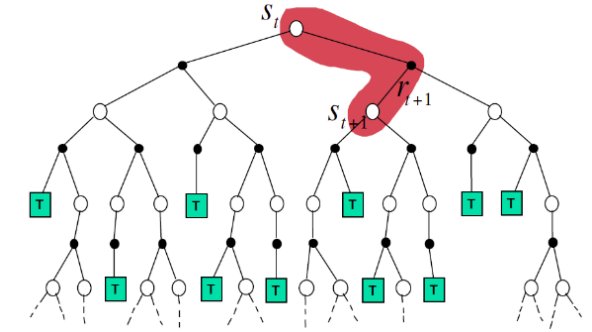
$$V(S_t) \leftarrow V(S_t) + \alpha\,(R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$

- **Monte-Carlo, MC:**
  - Update value $V(S_t)$ toward actual return G_t

$$V(S_t) \leftarrow V(S_t) + \alpha\,(G_t - V(S_t))$$

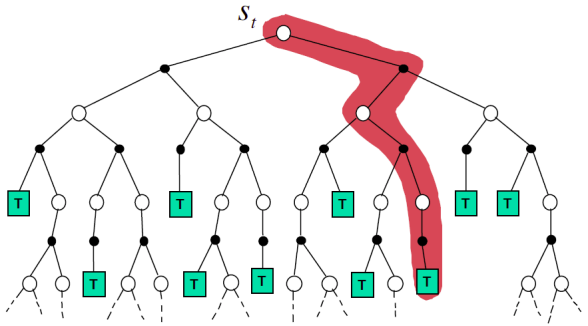
- **Note: MC and TD(0) equally applicable for *Q(s,a)* as for *V(s)* – state-value function *V* used here for concreteness**

$V(S_t) \leftarrow V(S_t) + \alpha\,(R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$



TD(0) ↑ vs. MC ↓

$V(S_t) \leftarrow V(S_t) + \alpha\,(G_t - V(S_t))$



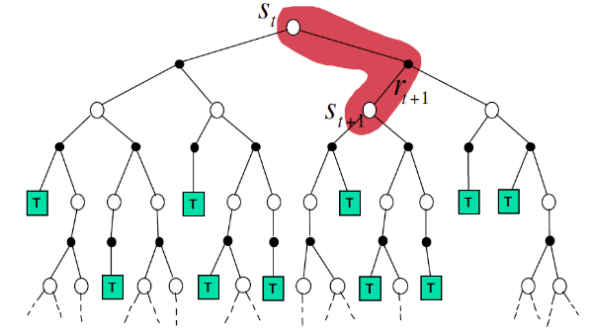
Bias Variance Trade-offs

- Return $G_t = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-1} R_T$ is **unbiased** estimate of $v_\pi(S_t)$
  - We observe sample of **actual reward**
- True TD target $R_{t+1} + \gamma v_\pi(S_{t+1})$ is **unbiased** estimate of $v_\pi(S_t)$
  - Simply the **Bellman equation** for the state-value function
- TD target $R_{t+1} + \gamma V(S_{t+1})$ is **biased** estimate of $v_\pi(S_t)$
  - $V(S_{t+1})$ is **not actually observed** ("estimate based on an estimate")
- TD target is much **lower variance** than the return
  - Return depends on **many** random actions, transitions, rewards
  - TD target depends on **one** random action, transition, reward

TD vs. MC

- **TD has low variance, some bias**
  - Usually more efficient than MC
  - TD(0) converges to $v_\pi(s)$, but not always with function approximation
  - Can be sensitive to initial value
  - Works in non-episodic / continuing environments
  - Fully exploits underlying MDP assumption

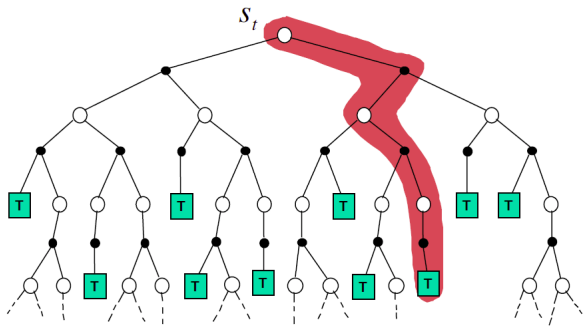
$$V(S_t) \leftarrow V(S_t) + \alpha\left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\right)$$



TD(0) ↑ vs. MC ↓

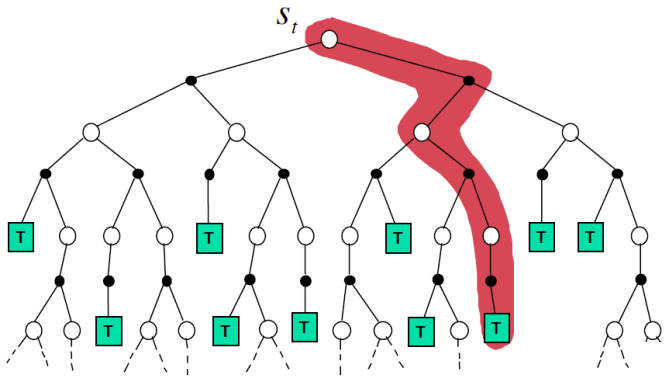
- **MC has high variance, zero bias**
  - Good convergence properties even with function approximation
  - Not very sensitive to initial value
  - Very simple to understand and use
  - Only works in episodic / terminating environments
  - Does not exploit underlying MDP assumption

$$V(S_t) \leftarrow V(S_t) + \alpha\left(G_t - V(S_t)\right)$$



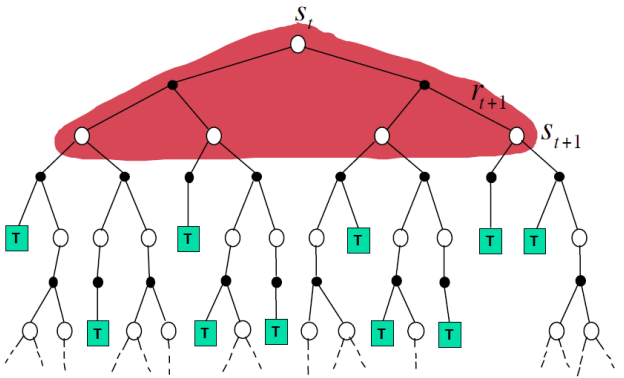
MC vs. TD vs. DP (Dynamic Programming / Bellman)

$$V(S_t) \leftarrow V(S_t) + \alpha\left(G_t - V(S_t)\right)$$



MC

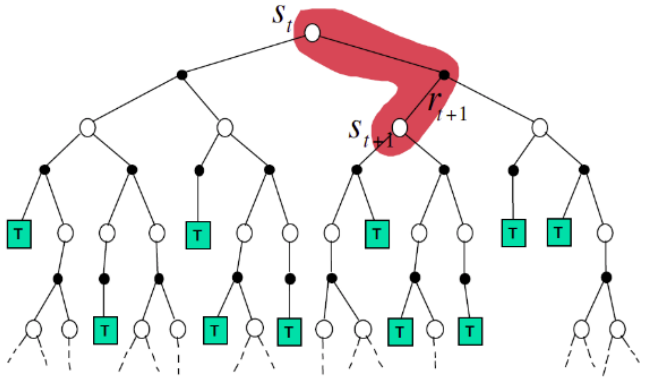
$$V(S_t) \leftarrow \mathbb{E}_\pi\left[R_{t+1} + \gamma V(S_{t+1})\right]$$



DP/ Policy evaluation

TD(0)

$$V(S_t) \leftarrow V(S_t) + \alpha\left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\right)$$

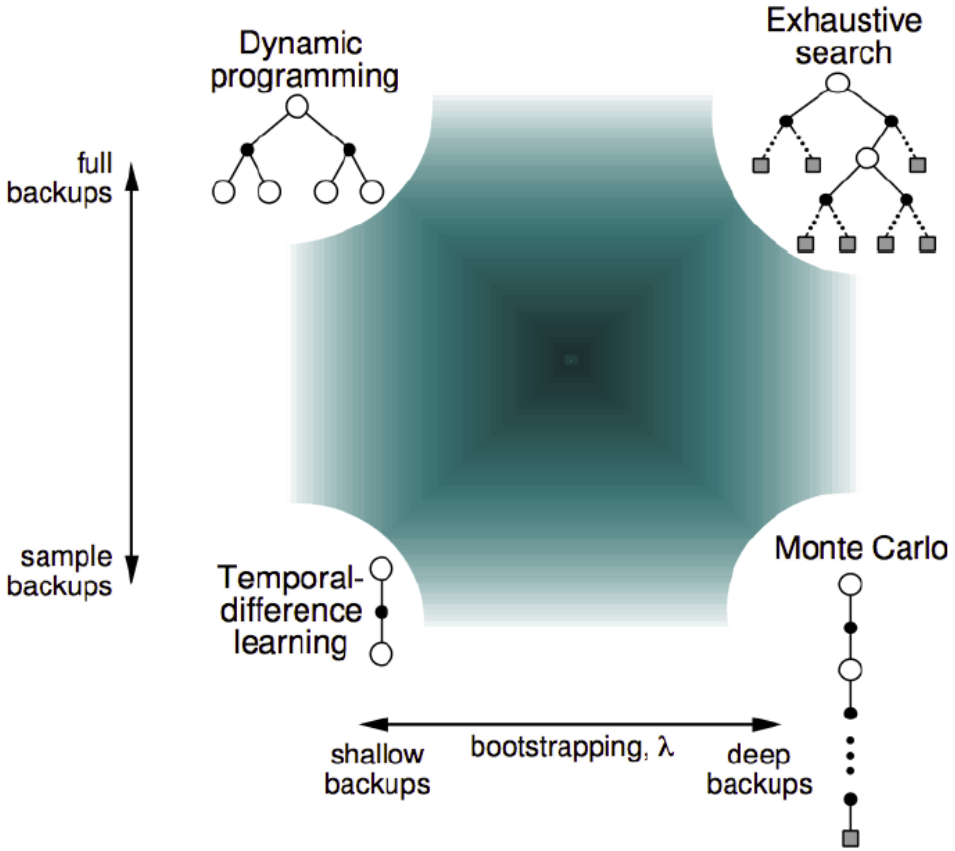


Sampling vs. Bootstrapping

- **Sampling: update samples an expectation**
  - MC samples
  - TD samples
  - DP does not sample

- **Bootstrapping: update involves an estimate**
  - MC does not bootstrap
  - TD bootstraps
  - DP bootstraps

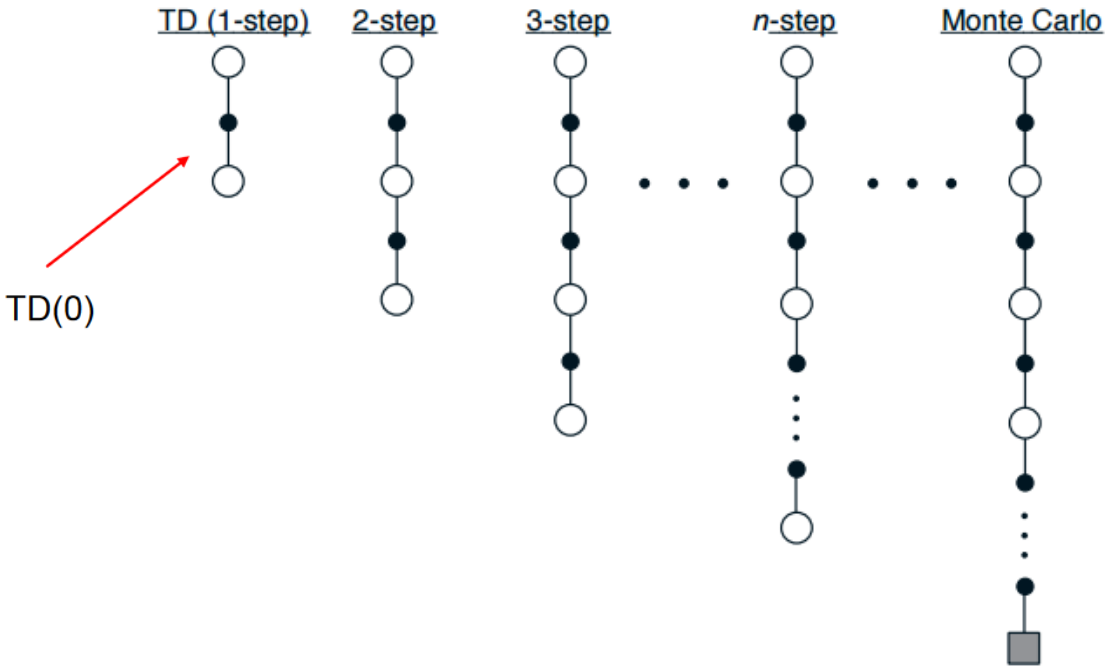
Unified View of Reinforcement Learning



How much lookahead when learning?

- So far (today and last lecture) we have considered only two ”extremes”:
- 1) Monte-Carlo (MC) estimation (**full lookahead**):
  - We update estimates of *V(s)* or *Q(s,a)* only at the very **end of episodes**
  - Only works for **episodic / terminating environments**
  - Does not exploit underlying MDP assumption → **may be better when the system isn't truly an MDP**
  - **High variance, but unbiased**
- 2) Temporal difference learning, e.g. Q-learning (**one-step lookahead – TD(0)**):
  - Update estimate of *V(s)* or *Q(s,a)* after **every single step**
  - Update based on **immediate reward + bootstrapping** (i.e. incorporate current estimate of the values one step into the future)
  - Works in **continuing / non-terminating environments**
  - Fully exploits underlying MDP assumptions → **often better-performing when MDP assumption is true**
  - **Low variance, but can have large bias**
- **3) Today: Can we do something in between to balance the respective benefits?**

Multi-Step TD



Spectrum: **one-step backups of TD(0) → termination-only backups of MC**. In between are the ***n*-step backups**, based on ***n*** **steps of real rewards and the estimated value of the *n*th next state**, appropriately discounted

n-Step Return

- Consider the following n-step returns for $n = 1,2,\ldots,\infty$:

$$
\begin{aligned}
n = 1 \quad (TD) \quad & G_t^{(1)} = R_{t+1} + \gamma V(S_{t+1}) \\
n = 2 \quad & G_t^{(2)} = R_{t+1} + \gamma R_{t+2} + \gamma^2 V(S_{t+2}) \\
\vdots \quad & \vdots \\
n = \infty \quad (MC) \quad & G_t^{(\infty)} = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-1} R_T
\end{aligned}
$$

- Define the n-step return

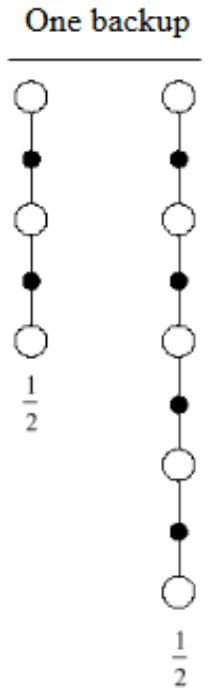
$$G_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{n-1} R_{t+n} + \gamma^n V(S_{t+n})$$

- n-step temporal-difference learning

$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t^{(n)} - V(S_t) \right)$$

Averaging n-Step Returns

- Can average *n*-step returns for different *n*
- E.g. average 2-step and 4-step returns $\frac{1}{2}G^{(2)} + \frac{1}{2}G^{(4)}$



- A valid update not just toward any n-step return, but toward **any of their averages**

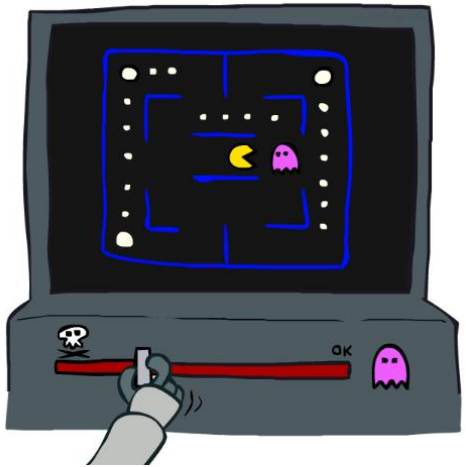
- Any set of returns can be averaged, even an infinite set, as long as the **weights on the component returns are positive and sum to 1**.

Function Approximation for Reinforcement Learning

- Last time we looked at **tabular** Q-learning for finding action-value function *Q(s,a)*
- Also looked at **approximate** Q-learning with ***linear*** **function approximation**

$$Q_{\boldsymbol{w}}(s,a) = w_1 f_1(s,a) + \cdots + w_n f_n(s,a)$$

- Possible also for the state-value function *V(s)*, $V_{\boldsymbol{w}}(s)$
- **Why function approximation?**
  - Too many states and / or actions to fit in memory
  - Too slow to learn each state- / state-action value separately
  - Generalize from seen to unseen states
- We'll focus on **differentiable function approximators**
  - Linear
  - (Deep) neural networks



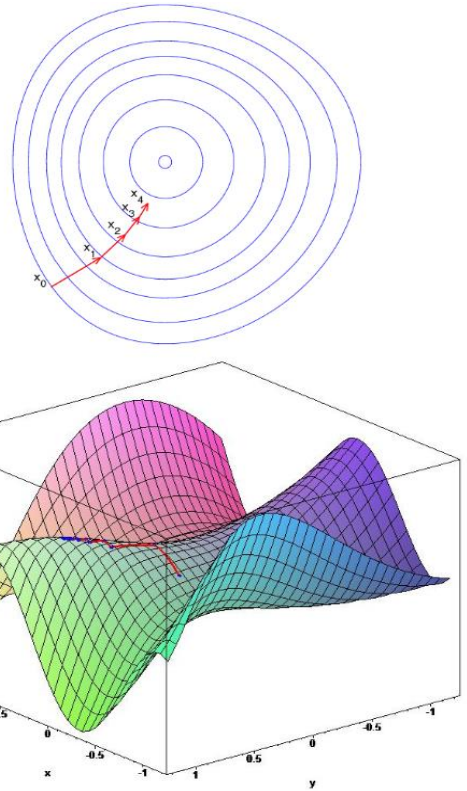
Gradient Descent (again)

- Let *J(**w**)* be a differentiable function of parameter vector ***w***
- The gradient of *J(**w**)* is:

$$\nabla_{\mathbf{w}} J(\mathbf{w}) = \begin{pmatrix} \frac{\partial J(\mathbf{w})}{\partial \mathbf{w}_1} \\ \vdots \\ \frac{\partial J(\mathbf{w})}{\partial \mathbf{w}_n} \end{pmatrix}$$

- We (locally) minimize *J(**w**)* by adjusting ***w*** in the direction of the gradient with a step-size parameter α:

$$\Delta \mathbf{w} = -\frac{1}{2}\alpha \nabla_{\mathbf{w}} J(\mathbf{w})$$



Value Function Approximation via Stochastic Gradient Descent (SGD)

- **Goal:** Find parameters ***w*** minimizing MSE between approx. and true value functions

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[(v_\pi(S) - \hat{v}(S, \mathbf{w}))^2 \right]$$

- Gradient descent finds a local minimum

$$\begin{aligned} \Delta \mathbf{w} &= -\frac{1}{2} \alpha \nabla_{\mathbf{w}} J(\mathbf{w}) \\ &= \alpha \mathbb{E}_\pi \left[(v_\pi(S) - \hat{v}(S, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) \right] \end{aligned}$$

- ***In practice Stochastic gradient descent samples the gradient***

$$\Delta \mathbf{w} = \alpha (v_\pi(S) - \hat{v}(S, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w})$$

- Expected update is equal to full gradient update
- However, often high variance

What About the True Value Function $v_\pi(s)$?

- On previous slide, assumed true value function was given (”supervised learning”)

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[(v_\pi(S) - \hat{v}(S, \mathbf{w}))^2 \right]$$

- **RL is not supervised learning**! The closest thing to supervision is the **rewards**

- In practice, we use a *target* estimate of $v_\pi(s)$

  - For MC, the target is the return G_t

$$\Delta \mathbf{w} = \alpha(G_t - \hat{v}(S_t, \mathbf{w}))\nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w})$$

  - For TD(0), the target is the TD target $R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w})$

$$\Delta \mathbf{w} = \alpha(R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w}) - \hat{v}(S_t, \mathbf{w}))\nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w})$$

Linear Value Function Approximation

- Represent value function by a **linear combination of features:**

$$\hat{v}(S, \mathbf{w}) = \mathbf{x}(S)^\top \mathbf{w} = \sum_{j=1}^{n} \mathbf{x}_j(S) \mathbf{w}_j$$

- The **objective function is then quadratic** in the parameters ***w***

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[(v_\pi(S) - \mathbf{x}(S)^\top \mathbf{w})^2 \right]$$

- **SGD converges to global optimum**
- **Update rule is really simple** (”step-size ***x*** pred. error ***x*** feature value”):

$$\nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) = \mathbf{x}(S)$$

$$\Delta \mathbf{w} = \alpha (v_\pi(S) - \hat{v}(S, \mathbf{w})) \mathbf{x}(S)$$

Linear Function Approximation for Q-Learning

Represent value function by a **linear combination of features:**

$$\hat{Q}(s,a;\boldsymbol{w}) = w_1 f_1(s,a) + \cdots + w_n f_n(s,a) = \boldsymbol{w}^\top \boldsymbol{f}(s,a)$$

The **objective function is then quadratic** in the parameters ***w***

$$J(\boldsymbol{w}) = \mathbb{E}_\pi \left\{ \left(Q(s,a) - \hat{Q}(s,a;\boldsymbol{w}) \right)^2 \right\} = \mathbb{E}_\pi \left\{ \left(Q(s,a) - \boldsymbol{w}^\top \boldsymbol{f}(s,a) \right)^2 \right\}$$

Gradient with respect to **w is:**

$$\nabla_{\boldsymbol{w}} J(\boldsymbol{w}) = -2\mathbb{E}_\pi \left\{ \left(Q(s,a) - \boldsymbol{w}^\top \boldsymbol{f}(s,a) \right) \boldsymbol{f}(s,a) \right\}$$

Update rule:

$$\boldsymbol{w} \leftarrow \boldsymbol{w} + \alpha \left(Q(s,a) - \boldsymbol{w}^\top \boldsymbol{f}(s,a) \right) \boldsymbol{f}(s,a)$$

Estimating the unknown True Q-values

What about $Q(s,a)$?

- We can **bootstrap** (just like for tabular Q-learning!)

$$Q(s,a) \approx r + \gamma \max_{a'} \hat{Q}(s', a'; \boldsymbol{w})$$

- Replacing the true value Q(s,a) by the bootstrapped value in the weight update:

$$\boldsymbol{w} \leftarrow \boldsymbol{w} + \alpha \left(r + \gamma \max_{a'} \hat{Q}(s', a'; \boldsymbol{w}) - \hat{Q}(s, a; \boldsymbol{w}) \right) \boldsymbol{f}(s,a)$$

Linear Function Approximation for Q-Learning

$$Q(s,a) \approx w_1 f_1(s,a) + \cdots + w_n f_n(s,a) = \boldsymbol{w}^\top \boldsymbol{f}(s,a)$$

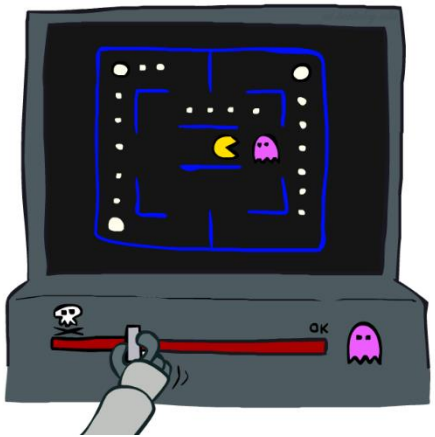
- **Q-learning with linear Q-functions:**

$$\text{transition} = (s, a, r, s')$$

$$\text{difference} = \left[r + \gamma \max_{a'} Q(s', a')\right] - Q(s,a)$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha\,[\text{difference}] \quad \text{Exact Q's}$$

$$w_i \leftarrow w_i + \alpha\,[\text{difference}]\, f_i(s,a) \quad \text{Approximate Q's}$$



- **Intuitive interpretation:**
  - **Adjust weights of *active* features**
  - E.g., if something unexpectedly bad happens, **blame the features that had significant magnitude**: disprefer all states with that state's features

Experience Replay

- Stochastic gradient descent as described so far can be very **sample inefficient**
- **Key problem:** we are only using the very latest experience (transition) when updating our weights. Recall e.g. the linear Q-learning update rule using $\text{transition} = (s, a, r, s')$

$$\boldsymbol{w} \leftarrow \boldsymbol{w} + \alpha \left(r + \gamma \max_{a'} \hat{Q}(s', a'; \boldsymbol{w}) - \hat{Q}(s, a; \boldsymbol{w}) \right) \boldsymbol{f}(s, a)$$

- Why may it be problematic only using the very latest experience in weight update?
  - **Sample inefficient:** An experience is only used once. It can be costly to gain experiences, and would like to use them several times (similar to training for several epochs on training data in supervised learning)
  - **Non-I.I.D:** Convergence analyses (and empirical performance!) often require training samples to be i.i.d. Clearly, consecutive experiences are highly dependent!

Stochastic Gradient Descent with Experience Replay

- Given value function approximation $\hat{v}(s, \mathbf{w}) \approx v_\pi(s)$
- Together with **experience buffer** $\mathcal{D}$ consisting of **<*state, value*>** pairs

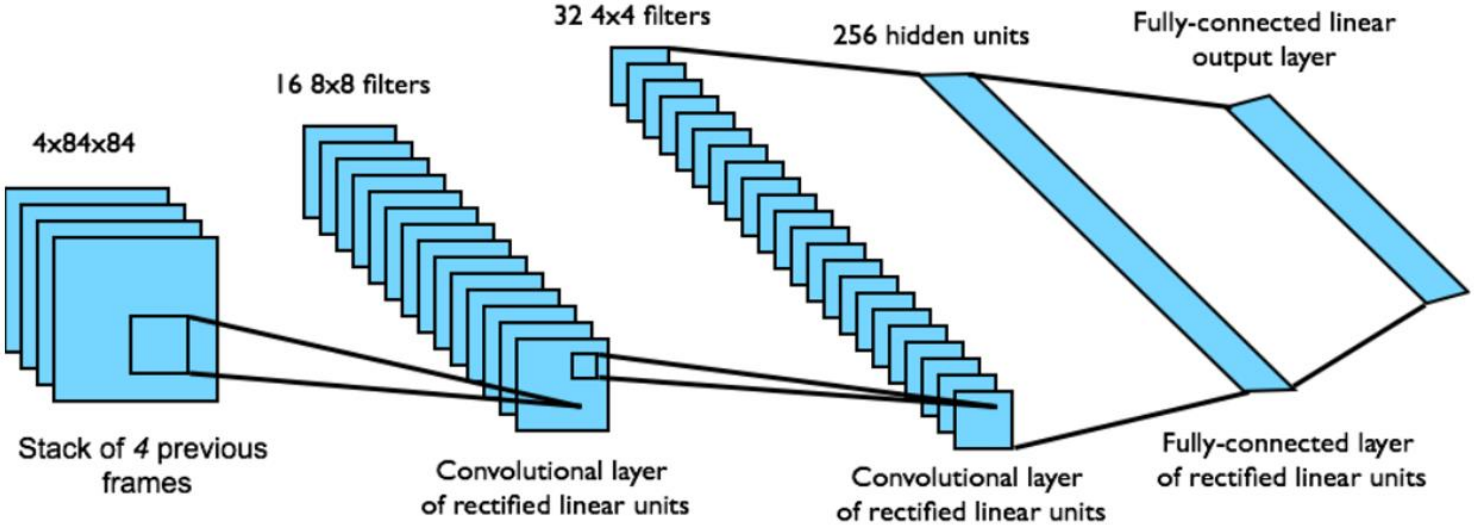
$$\mathcal{D} = \{\langle s_1, v_1^\pi \rangle, \langle s_2, v_2^\pi \rangle, ..., \langle s_T, v_T^\pi \rangle\}$$

- Recall MSE-objective: $J(\mathbf{w}) = \mathbb{E}_\pi \left[(v_\pi(S) - \hat{v}(S, \mathbf{w}))^2 \right]$
- **Repeat:**
  1. Sample **<*state, value*>** pair from experience (”replay an experience”): $\langle s, v^\pi \rangle \sim \mathcal{D}$
  2. Apply stochastic gradient descent update: $\Delta \mathbf{w} = \alpha(v_\pi(S) - \hat{v}(S, \mathbf{w}))\nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w})$

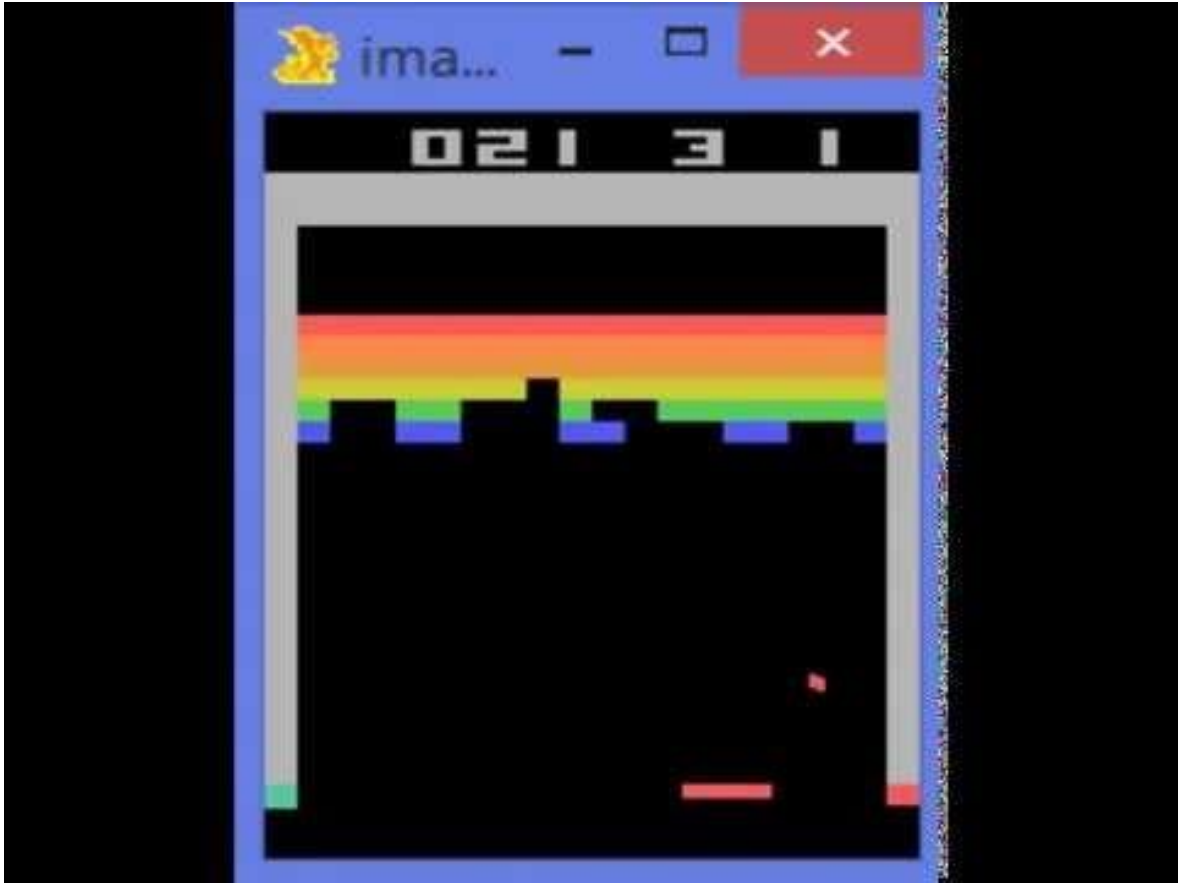
- Similar to supervised learning, e.g. image classification. There we have dataset of images – corresponding here to $\mathcal{D}$ -- and we sample images, using SGD to update neural network weights. ***In RL, the learner also collects the dataset!***

Deep Q-Networks (DQNs)

- Use **deep neural network** instead of linear weights for approximating *Q(s,a)*
- The fully-connected linear output layer has one channel for each action
  → **approximation of** *Q(s,a)* **for each action** *a* **simultaneously**
- Below DQN architecture was used by Google DeepMind to achieve super-human performance on several Atari games:



DQN for Playing Atari



DQN – Key Concepts

- DQN uses **experience replay** and **fixed Q-targets (separate target network)**
- Take action a_t according to ε-greedy policy (see previous lecture)
- Store transition $(s_t, a_t, r_{t+1}, s_{t+1})$ in the replay memory (experience buffer $\mathcal{D}$)
- Sample random mini-batch of transitions *(s,a,r,s')* from $\mathcal{D}$
- Compute Q-learning targets w.r.t. old, fixed parameters $\boldsymbol{w}^-$
- Optimise MSE between Q-network and Q-learning target:

$$J(\boldsymbol{w}) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}} \left\{ \left(r + \gamma \max_{a'} \hat{Q}(s', a'; \boldsymbol{w}^-) - \hat{Q}(s, a; \boldsymbol{w}) \right)^2 \right\}$$

- Every *C*:th step, set $\boldsymbol{w}^- = \boldsymbol{w}$

DQN – Full Algorithm

Algorithm 1 Deep Q-learning with experience replay

Initialize replay memory D to capacity N
Initialize action-value function Q with random weights θ
Initialize target action-value function $\hat{Q}$ with weights $\theta^- = \theta$
for episode 1, M **do** Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do**
    With probability ε select a random action a_t
    otherwise select $a_t = \arg\max_a Q(\phi(s_t), a; \theta)$
    Execute action a_t in the emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store experience $(\phi_t, a_t, r_t, \phi_{t+1})$ in D
    Sample random minibatch of experiences $(\phi_j, a_j, r_j, \phi_{j+1})$ from D
    Set $y_j = \begin{cases} r_j & \text{if episode terminates at step } j+1 \\ r_j + \gamma \max_{a'} \hat{Q}(\phi_{j+1}, a'; \theta^-) & \text{otherwise} \end{cases}$
    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ with respect to the weights θ
    Every C steps reset $\hat{Q} = Q$
  end for
end for

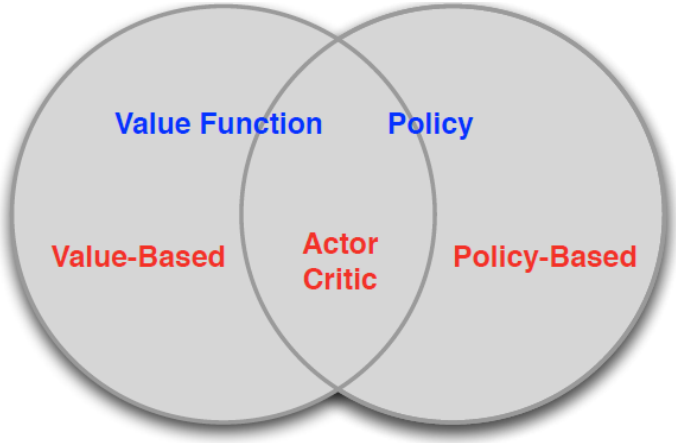
https://www.cs.toronto.edu/~vmnih/docs/dqn.pdf

Policy-Based Reinforcement Learning (PB-RL)

- Have looked at (exact / approxmiate) approaches for finding *V(s)*, *Q(s,a)*
- For example, from *Q(s,a)* a policy is obtained as $\pi(s) = \arg\max_a Q(s,a)$
- Above paradigm for finding a policy falls into ***value-based RL***
- Now we'll **directly parametrize the policy** $\pi_{\boldsymbol{\theta}}(a \mid s) = P(a \mid s; \boldsymbol{\theta})$
  - Use **stochastic policy to enable exploration** (instead of ε-greedy approach, as for Q-learning). Also, most PB-RL algorithms assume stochastic policy to work.
- This is ***policy-based RL***
- Focus will still be on **model-free reinforcement learning**

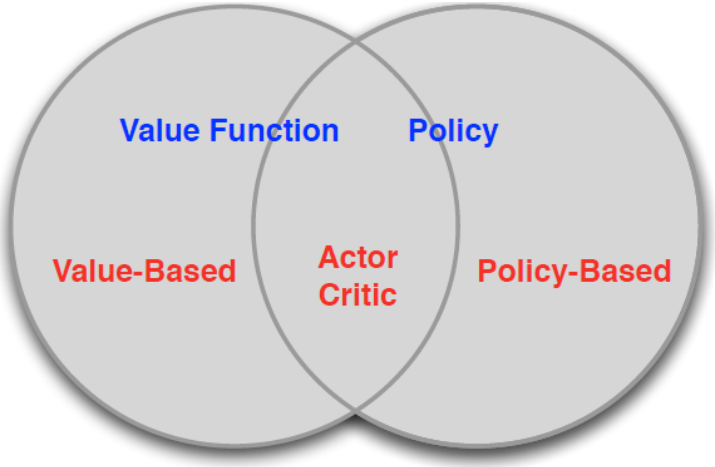
Value-Based vs. Policy-Based RL

- **Value-based**
  - Learn *V(s)* or *Q(s,a)*
  - Policy extracted from *V(s)* or *Q(s,a)*
- **Policy-based**
  - No intermediate learning of value functions
  - Directly learn the policy $\pi_{\boldsymbol{\theta}}(a \mid s)$
- **Actor-Critic = Combination**
  - Learn *V(s)* or *Q(s,a)* (**CRITIC**)
  - Learn policy $\pi_{\boldsymbol{\theta}}(a \mid s)$ (**ACTOR**), often more efficiently than in pure policy-based approach



Properties of Policy-Based RL

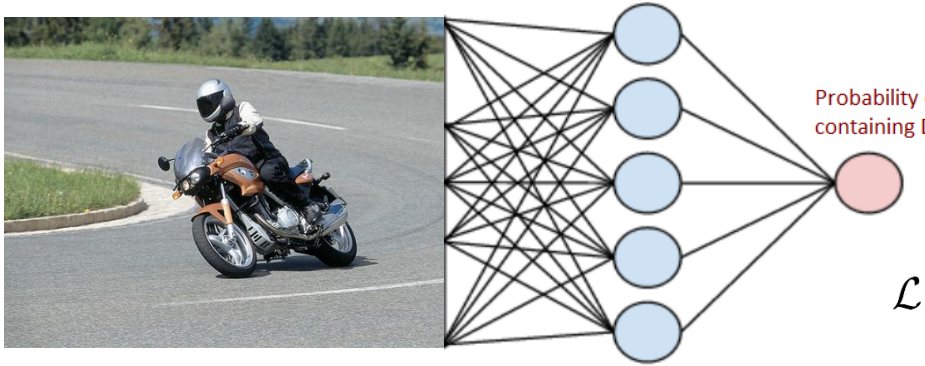
- **Advantages**
  - Better convergence properties
  - Effective in high-dimensional or continous action-spaces
  - Can learn stochastic policies
  - Actually more intuitive approach if you understand supervised learning



- **Disadvantages**
  - Typically converges to a local instead of global optimum (however also true for value-based RL with function approximation)
  - Evaluating policy (expected reward) is typically inefficient and high variance

Recall: Supervised Learning (in Deep Networks)

- **”Does the below image contain a dog?” Answer:** *y = 0 -- no dog!*
- The ”decision” if it contains a dog is **soft**, i.e. a probability from **differentiable** neural network layers (we can make small changes in the parameters to obtain small changes in the output prediction – no hard decisions needed in training!)
- Also, no actor (agent) affecting ”next states” and ”returns” (losses)



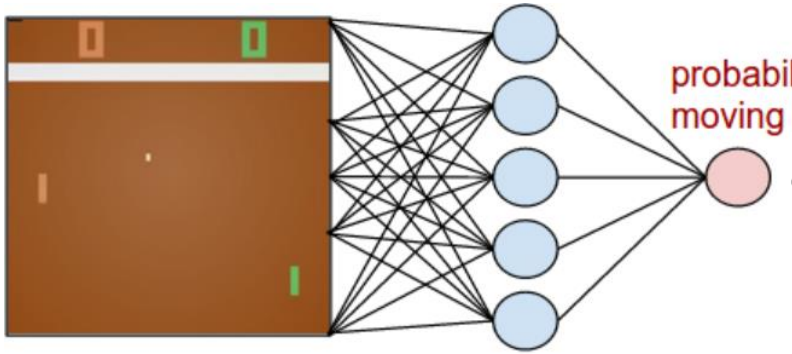
$$\sigma(\mathrm{NN}(\boldsymbol{x};\boldsymbol{\theta})) = \hat{y}_{\boldsymbol{\theta}}$$

$$\mathcal{L}(\boldsymbol{\theta}) = -\left[y \log \hat{y}_{\boldsymbol{\theta}} + (1-y)\log(1-\hat{y}_{\boldsymbol{\theta}})\right]$$

$$J(\boldsymbol{\theta}) = -\mathcal{L}(\boldsymbol{\theta}) = \left[y \log \hat{y}_{\boldsymbol{\theta}} + (1-y)\log(1-\hat{y}_{\boldsymbol{\theta}})\right] \cdot (+1)$$

Intuition of Policy-Based RL

- **"Should I move up, to increase chances of winning?" Ans:** How should I know?!
- Above question may be difficult to answer in supervised learning framework
  - What would the loss function be? What about training data?
  - Can use (possibly very delayed) reward, such as +1 for winning -1 for losing the game
- Note that the *probability* of moving up can be produced via differentiable neural network layers, but a ***hard decision*** must be made both in training and testing, typically by sampling (~) from the action probability distribution



$$\sigma(\mathrm{NN}(\boldsymbol{x};\boldsymbol{\theta})) = \hat{y}_{\boldsymbol{\theta}} \xrightarrow{\text{sample action (up / down)}} \tilde{y}$$

$$J(\boldsymbol{\theta}) = -\mathcal{L}(\boldsymbol{\theta}) = ??$$

Objective function for policy-based RL

- **Goal:** Find best parameters $\boldsymbol{\theta}$ for stochastic policy $\pi_{\boldsymbol{\theta}}(a \mid s)$

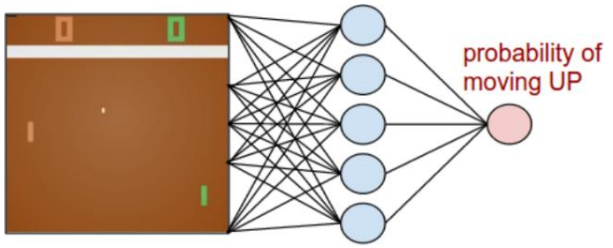
$$s_0 \xrightarrow{\pi_{\boldsymbol{\theta}}(*|s_0)} a_0 \xrightarrow{p(*,*|s_0,a_0)} r_1, s_1 \xrightarrow{\pi_{\boldsymbol{\theta}}(*|s_1)} a_1 \xrightarrow{p(*,*|s_1,a_1)} r_2, s_2 \xrightarrow{\pi_{\boldsymbol{\theta}}(*|s_2)} \ldots a_{T-1} \xrightarrow{p(*,*|s_{T-1},a_{T-1})} r_T, s_T$$

$$\tau = (s_0, a_0, r_1, s_1, a_1, r_2, s_2, \ldots, a_{T-1}, r_T, s_T)$$

- Assume: $\pi_{\boldsymbol{\theta}}(a \mid s)$ stochastic and differentiable when nonzero

- **Key question:** How to measure quality of $\pi_{\boldsymbol{\theta}}$?

- **Proposal:** $J(\boldsymbol{\theta}) = \mathbb{E}_{\pi_{\boldsymbol{\theta}}} \left\{ \sum_{t=0}^{T-1} \gamma^t R_{t+1} \middle| S_0 = s_0 \right\}$



Maximizing Proposed Objective Function

- **Objective function:** Expected (discounted) cumulative return from initial state by following policy $\pi_{\boldsymbol{\theta}}(a \mid s)$

$$J(\boldsymbol{\theta}) = \mathbb{E}_{\pi_{\boldsymbol{\theta}}} \left\{ \sum_{t=0}^{T-1} \gamma^t R_{t+1} \middle| S_0 = s_0 \right\}$$

- As usual, use the gradient as search direction: **gradient ascent**!

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \nabla_{\boldsymbol{\theta}} \mathbb{E}_{\pi_{\boldsymbol{\theta}}} \left\{ \sum_{t=0}^{T-1} \gamma^t R_{t+1} \middle| S_0 = s_0 \right\}$$

- **Problem:** That gradient looks complicated! What to do?
- **Solution:** We'll use some "tricks" introduced on the next slides

The Problem: The Gradient of the Expectation

- For now let's just look at integrating over the first triplet (s,a,s')

$$\nabla_\Theta \mathbb{E}_{\pi_\Theta}\left[\sum_{t=0}^{T-1} R_{t+1}\right] = \nabla_\Theta \int_{s,a,s'} \mu(s)p(s'|s,a)\pi_\Theta(a|s)\sum_{t=0}^{T-1} R_{t+1}ds,da,ds'$$

- We can move the gradient into the integral because only the policy depends on θ.

$$\int_{s,a,s'} \mu(s)p(s'|s,a)\nabla_\Theta\pi_\Theta(a|s)\sum_{t=0}^{T-1} R_{t+1}ds,da,ds'$$

Issue: We cannot use Monte-Carlo to evaluate this because the policy is not a distribution anymore.

The Log Derivative Trick

- Multiply and divide by the policy

$$\int_{s,a,s'} \mu(s)p(s'|s,a)\frac{\pi_\Theta(a|s)}{\pi_\Theta(a|s)}\nabla_\Theta\pi_\Theta(a|s)\sum_{t=0}^{T-1} R_{t+1}ds,da,ds'$$

$$= \int_{s,a,s'} \mu(s)p(s'|s,a)\pi_\Theta(a|s)\frac{\nabla_\Theta\pi_\Theta(a|s)}{\pi_\Theta(a|s)}\sum_{t=0}^{T-1} R_{t+1}ds,da,ds'$$

$$= \int_{s,a,s'} \mu(s)p(s'|s,a)\pi_\Theta(a|s)\nabla_\Theta\log(\pi_\Theta(a|s))\sum_{t=0}^{T-1} R_{t+1}ds,da,ds'$$

Trick 2: Getting rid of those MDP dynamics

- Back to looking at full trajectory

$$s_0 \xrightarrow{\pi_{\boldsymbol{\theta}}(*|s_0)} a_0 \xrightarrow{p(*,*|s_0,a_0)} r_1, s_1 \xrightarrow{\pi_{\boldsymbol{\theta}}(*|s_1)} a_1 \xrightarrow{p(*,*|s_1,a_1)} r_2, s_2 \xrightarrow{\pi_{\boldsymbol{\theta}}(*|s_2)} \ldots a_{T-1} \xrightarrow{p(*,*|s_{T-1},a_{T-1})} r_T, s_T$$

$$\nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\tau) = \nabla_{\boldsymbol{\theta}} \log \left(\mu(s_0) \prod_{t=0}^{T-1} \pi_{\boldsymbol{\theta}}(a_t|s_t) p(r_{t+1}, s_{t+1}|s_t, a_t) \right)$$

$$= \nabla_{\boldsymbol{\theta}} \left(\log \mu(s_0) + \sum_{t=0}^{T-1} [\log \pi_{\boldsymbol{\theta}}(a_t|s_t) + \log p(r_{t+1}, s_{t+1}|s_t, a_t)] \right)$$

$$= \nabla_{\boldsymbol{\theta}} \sum_{t=0}^{T-1} \log \pi_{\boldsymbol{\theta}}(a_t|s_t) = \sum_{t=0}^{T-1} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t|s_t)$$

- **Great!** We don't need to know / estimate the MDP dynamics!

Putting it all together → Policy Gradient

- **Finally a practical form for the policy gradient:**

$$\nabla_{\boldsymbol{\theta}}\mathbb{E}_{\pi_{\boldsymbol{\theta}}}\left\{\sum_{t=0}^{T-1}\gamma^t R_{t+1}\middle| S_0=s_0\right\}=\mathbb{E}_{\pi_{\boldsymbol{\theta}}}\left\{\underbrace{\left(\sum_{t=0}^{T-1}\nabla_{\boldsymbol{\theta}}\log\pi_{\boldsymbol{\theta}}(a_t|s_t)\right)}_{\nabla_{\boldsymbol{\theta}}\log\pi_{\boldsymbol{\theta}}(\tau)}\underbrace{\left(\sum_{t=0}^{T-1}\gamma^t R_{t+1}\right)}_{\text{"}g(\tau)\text{"}}\middle| S_0=s_0\right\}$$

- **We used these two tricks:**

$$\nabla_{\boldsymbol{\theta}}\mathbb{E}_{f_{\boldsymbol{\theta}}}\left\{g(x)\right\}=\mathbb{E}_{f_{\boldsymbol{\theta}}}\left\{\nabla_{\boldsymbol{\theta}}\log f_{\boldsymbol{\theta}}(x)g(x)\right\}$$

$$\nabla_{\boldsymbol{\theta}}\log\pi_{\boldsymbol{\theta}}(\tau)=\sum_{t=0}^{T-1}\nabla_{\boldsymbol{\theta}}\log\pi_{\boldsymbol{\theta}}(a_t|s_t)$$

- **Now we can perform gradient steps**

Past Rewards Unaffected by Future Actions

$$\nabla_{\boldsymbol{\theta}} \mathbb{E}_{\pi_{\boldsymbol{\theta}}} \left\{ \sum_{t=0}^{T-1} \gamma^t R_{t+1} \middle| S_0 = s_0 \right\} = \mathbb{E}_{\pi_{\boldsymbol{\theta}}} \left\{ \left(\sum_{t=0}^{T-1} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t|s_t) \right) \left(\sum_{t=0}^{T-1} \gamma^t R_{t+1} \right) \middle| S_0 = s_0 \right\}$$

$$= \mathbb{E}_{\pi_{\boldsymbol{\theta}}} \left\{ \sum_{t=0}^{T-1} \left(\nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t|s_t) \left(\sum_{t'=0}^{T-1} \gamma^{t'} R_{t'+1} \right) \right) \middle| S_0 = s_0 \right\}$$

$$= \mathbb{E}_{\pi_{\boldsymbol{\theta}}} \left\{ \sum_{t=0}^{T-1} \left(\nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t|s_t) \left(\sum_{t'=t}^{T-1} \gamma^{t'} R_{t'+1} \right) \right) \middle| S_0 = s_0 \right\}$$

- Intuition of last equality: **Past rewards are unaffected by future actions**! When assessing action $a_t \sim \pi_{\boldsymbol{\theta}}(* \mid s_t)$ it makes sense to only account for what happens **onwards in time!**

Sample-Based Policy Gradient

$$\nabla_{\boldsymbol{\theta}} \mathbb{E}_{\pi_{\boldsymbol{\theta}}} \left\{ \sum_{t=0}^{T-1} \gamma^t R_{t+1} \middle| S_0 = s_0 \right\} = \mathbb{E}_{\pi_{\boldsymbol{\theta}}} \left\{ \sum_{t=0}^{T-1} \left(\nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t|s_t) \left(\sum_{t'=t}^{T-1} \gamma^{t'} R_{t'+1} \right) \right) \middle| S_0 = s_0 \right\}$$

$$\approx \frac{1}{M} \sum_{i=1}^{M} \sum_{t=0}^{T-1} \left(\nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t^i|s_t^i) \left(\sum_{t'=t}^{T-1} \gamma^{t'} r_{t'+1}^i \right) \right)$$

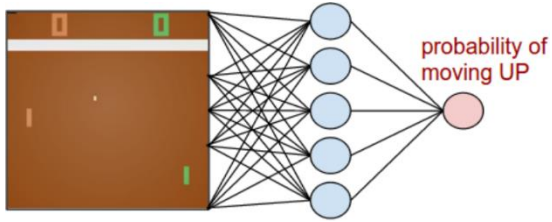
- Above: **Monte-Carlo estimate** of policy gradient (unbiased, noisy)
- Analogous to TD-learning, could use other way to estimate future discounted reward. In fact, the **policy gradient theorem** states:

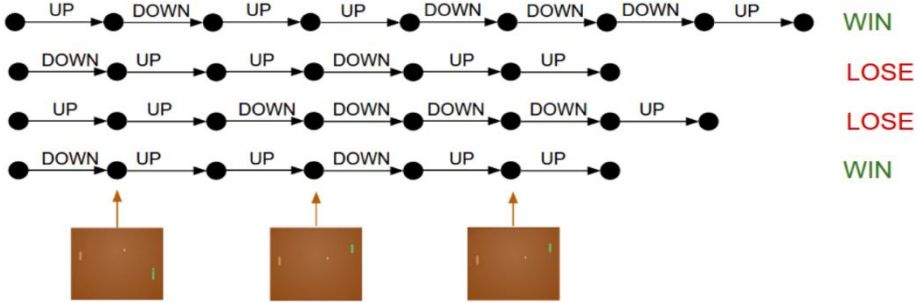
$$\nabla_{\boldsymbol{\theta}} \mathbb{E}_{\pi_{\boldsymbol{\theta}}} \left\{ \sum_{t=0}^{T-1} \gamma^t R_{t+1} \middle| S_0 = s_0 \right\} = \mathbb{E}_{\pi_{\boldsymbol{\theta}}} \left\{ \sum_{t=0}^{T-1} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t|s_t) Q^{\pi_{\boldsymbol{\theta}}}(s_t, a_t) \middle| S_0 = s_0 \right\}$$

Interpreting the policy gradient

$$\nabla_{\boldsymbol{\theta}} \mathbb{E}_{\pi_{\boldsymbol{\theta}}} \left\{ \sum_{t=0}^{T-1} \gamma^t R_{t+1} \middle| S_0 = s_0 \right\} \approx \frac{1}{M} \sum_{i=1}^{M} \sum_{t=0}^{T-1} \left(\nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t^i | s_t^i) \left(\sum_{t'=t}^{T-1} \gamma^{t'} r_{t'+1}^i \right) \right)$$

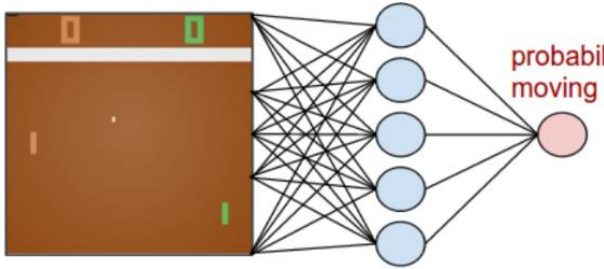
- Run M trajectories given current policy $\pi_{\boldsymbol{\theta}}(a \mid s)$. When taking action $a_t^i \sim \pi_{\boldsymbol{\theta}}(* \mid s_t^i)$ in i:th trajectory, see how it goes (onwards in that trajectory). Update $\boldsymbol{\theta}$ to increase (log-)probability of actions yielding positive future cumulative return, and decrease otherwise





Relating derived policy gradient to initial example

- **”Should I move up, to increase chances of winning?”**



$$\sigma(\mathrm{NN}(\boldsymbol{x};\boldsymbol{\theta})) = \hat{y}_{\boldsymbol{\theta}} \xrightarrow{\text{sample action (up / down)}} \tilde{y}$$

$$J(\boldsymbol{\theta}) = [\tilde{y}\log\hat{y}_{\boldsymbol{\theta}} + (1-\tilde{y})\log(1-\hat{y}_{\boldsymbol{\theta}})] \cdot \left(\sum_j r_j\right)$$

- It holds that

$$\begin{aligned}\log\hat{y}_{\boldsymbol{\theta}} = \log\sigma\left(\mathrm{NN}(\boldsymbol{x};\boldsymbol{\theta})\right) &= \text{log-prob. of action "move up" given current frame } \boldsymbol{x} \\ &= \log\pi_{\boldsymbol{\theta}}(a_t = 1|s_t) \ (\boldsymbol{x} \text{ is the state } s_t \text{ here})\end{aligned}$$

Relating derived policy gradient to initial example

$$J(\boldsymbol{\theta}) = [\tilde{y} \log \hat{y}_{\boldsymbol{\theta}} + (1 - \tilde{y}) \log(1 - \hat{y}_{\boldsymbol{\theta}})] \cdot \left(\sum_j r_j \right)$$

- Assume that

$$\tilde{y} = 1 \text{ (sampled aciton: "move up", where } \tilde{y} \text{ is the action } a_t \text{ here)}$$

- Then

$$J(\boldsymbol{\theta}) = [1 \cdot \log \hat{y}_{\boldsymbol{\theta}} + (1 - 1) \cdot (1 - \log \hat{y}_{\boldsymbol{\theta}})] \cdot \left(\sum_j r_j \right) = \log \pi_{\boldsymbol{\theta}}(a_t = 1 | s_t) \cdot \left(\sum_{t'=t}^{T-1} \gamma^{t'} R_{t'+1} \right)$$

- With gradient

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t = 1 | s_t) \cdot \left(\sum_{t'=t}^{T-1} \gamma^{t'} R_{t'+1} \right)$$

- Compare to prev. slides

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) \approx \frac{1}{M} \sum_{i=1}^{M} \sum_{t=0}^{T-1} \left(\nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a_t^i | s_t^i) \left(\sum_{t'=t}^{T-1} \gamma^{t'} r_{t'+1}^i \right) \right)$$

Reading

***Reinforcement Learning: An Introduction (2nd edition)*, Sutton & Barto 2016**

- **Chapter 7.1**
  - n-step TD Prediction
- **Chapter 9.1 – 9.4, 9.6**
  - Value Function Approximation
  - Stochastic Gradient Descent
  - Artificial Neural Networks
- **Chapter 13.1 – 13.3**
  - Policy Gradient Methods