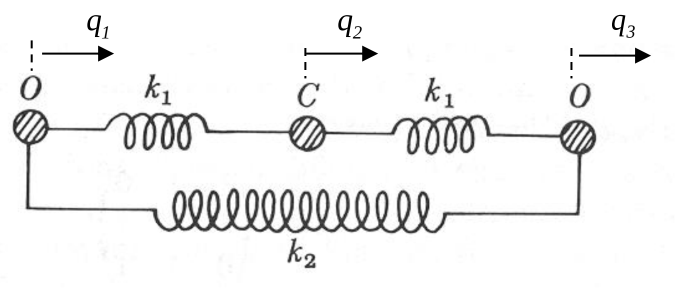
MECHANICAL VIBRATIONS - Examination task 2

Name: …………………………………..

Problem 2:2 (optional)

Find the mode shapes and eigenfrequencies for the linear vibrations of the CO_2 molecule (that is, vibrations in line with the molecule, see figure below). The mass of the oxygen atom is denoted by m_O and the mass of the carbon atom by m_C. The inter-atomic forces are represented by elastic springs with spring constants k_1 and k_2, respectively, according to the figure below.



Solution:

Name:..

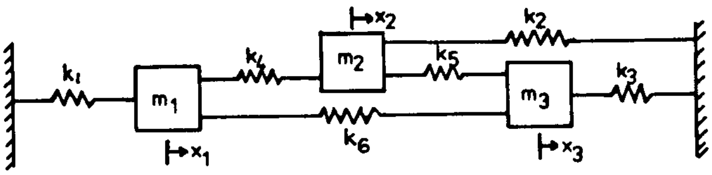
Problem 2:3

Consider the following model of an un-damped mechanical system consisting of three masses connected with springs according to the figure below.

$$m_1 = 0.5\text{kg},\ m_2 = 1.0\text{kg},\ m_3 = 1.5\text{kg}$$
$$k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = 1000\text{N/m} \tag{1}$$

a) Use the coordinates $x_1,\ x_2,\ x_3$ and formulate the equations of motion for the system.
b) Calculate the natural frequencies and mode shapes for the free vibrations.
c) Calculate the component $a_{22} = a_{22}(\omega)$ of the admittance matrix. Plot this function in a diagram. Find frequencies ω^a satisfying: $a_{22}(\omega^a) = 0$ (anti-resonances).
d) Suppose that the masses are at rest at the initial moment ($t = 0$), and that mass m_1 is given the initial velocity $\dot{x}_1(0) = 10\text{m/s}$. Calculate the subsequent motion of the mechanical system. Plot the motion in a diagram for the time interval $0 \le t \le 1.0s$.
e) Consider the constrained system where the x_2 - coordinate is held fixed, i.e. $x_2 = 0$. Calculate the natural frequencies for this (constrained) system. Compare this result with the result of subtask c) above.

Use some math code (Matlab, Mathcad, Maple, …) to solve the problem.



Solution:

Name:..

Problem 3:1

A mechanical system is defined by its mass-matrix $\underline{M}$, its stiffness-matrix $\underline{K}$ and its damping matrix $\underline{C}$. *The transfer function* (admittance) of the system is given by (see Lecture Notes p.226)

$$\underline{A}(s)=\sum_{k=1}^{n}\left(\frac{\underline{R}_k}{s-s_k}+\frac{\underline{R}_k^*}{s-s_k^*}\right) \qquad (1)$$

where

$$\begin{aligned}
&\underline{R}_k=\frac{\overline{x}_k\,\overline{x}_k^T}{\alpha_k},\ \text{"the residual matrix"}\\
&(\underline{M}\,s_k^2+\underline{C}\,s_k+\underline{K})\overline{x}_k=\overline{0}\\
&\alpha_k=\overline{x}_k^T\,\underline{C}\,\overline{x}_k+2s_k\,\overline{x}_k^T\,\underline{M}\,\overline{x}_k\\
&k=1,\ldots,n
\end{aligned} \qquad (2)$$

Show that if the system is *diagonalizable* and $\overline{x}_k$ are *real modes* with modal relative dampings $0\le\varsigma_k<1$ then

$$\underline{A}(s)=\sum_{k=1}^{n}\left(\frac{\overline{x}_k\,\overline{x}_k^T}{2i\,\mu_k\,\omega_{d,k}(s-s_k)}-\frac{\overline{x}_k\,\overline{x}_k^T}{2i\,\mu_i\,\omega_{d,k}(s-s_k^*)}\right) \qquad (3)$$

where μ_k are modal masses and $\omega_{d,k}$ are the damped natural frequencies for the modes of the system and $i^2=-1$.

Solution:

Name:..

Problem 3:2

Consider the following model of a mechanical system consisting of three masses connected with springs according to the figure below.

$$m_1 = 0.5\text{kg},\ m_2 = 1.0\text{kg},\ m_3 = 1.5\text{kg} \\ k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = 1000\text{N/m} \tag{1}$$

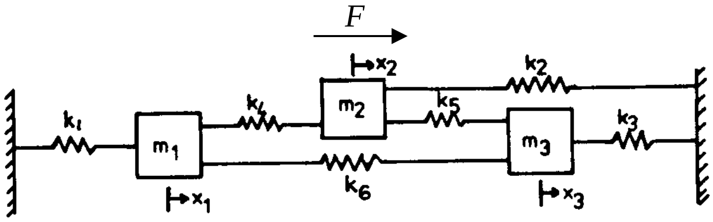
Introduce a Rayleigh damping to the system according to

$$\underline{C} = 0.005\underline{K} \tag{2}$$

where $\underline{C}$ is the damping matrix and $\underline{K}$ is the stiffness matrix of the mechanical system.

a) Use coordinates x_1, x_2, x_3 and formulate the equations of motion for the system.
b) Calculate the modal relative dampings, the damped natural frequencies and the corresponding modes shapes.
c) Show that the mode shapes are the same for the damped and the un-damped cases.
d) Calculate the component $F_{22} = F_{22}(\omega)$ of the frequency response matrix. Plot this function in a diagram, both amplitude and phase.
e) Apply an external harmonic load $F = F_0 \sin \omega t$ to the second mass (m_2) in the system and calculate the forced motion with $F_0 = 1000\text{N}$, $\omega = 30\text{rad/s}$: We assume that the system is starting from rest in the equilibrium position at $t = 0$. Show the motions in a diagram.

Use some math code (Matlab, Mathcad, Maple, …) to solve the problem.



Solution:

Name:..

Problem 3:3 (optional)

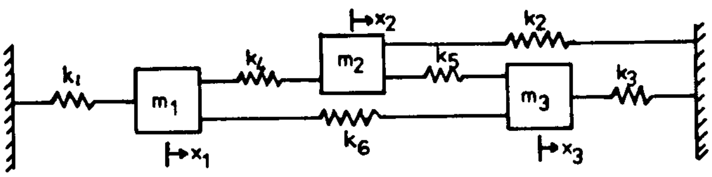
Consider the following model of a mechanical system consisting of three masses connected with springs according to the figure below.

$$\begin{aligned} &m_1 = 0.5\text{kg},\ m_2 = 1.0\text{kg},\ m_3 = 1.5\text{kg} \\ &k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = 1000\text{N/m} \end{aligned} \tag{1}$$

Introduce a viscous damping mechanism in the first spring (with stiffness k_1) with damping constant $c_1 = 0.05k_1$ and assume that for the other springs $c_2 = c_3 = c_4 = c_5 = c_6 = 0$.

a) Use coordinates $x_1,\ x_2,\ x_3$ and formulate the equations of motion for the system.
b) Is this problem diagonalizable?
c) Calculate the modal relative dampings, the damped natural frequencies and the corresponding modes shapes. Compare with the classical normal modes.
d) Calculate the component $F_{22} = F_{22}(\omega)$ of the frequency response matrix. Plot this function in a diagram, both amplitude and phase.

Use some math code (Matlab, Mathcad, Maple, …) to solve the problem.



Solution: