

```
(%i18) grad(F, x):=[diff(F(x), x[1]), diff(F(x), x[2])];
```

```
(%o18) grad(F, x):=[ $\frac{d}{dx_1} F(x), \frac{d}{dx_2} F(x)$ ]
```

```
(%i25) fpx:[x=0, y=0];
```

```
  xhat:[x, y];
```

```
  yhat:[z];
```

```
  A:matrix([0, -1], [1, 0]);
```

```
  f(xhat, yhat):=[xhat[1]·yhat - xhat[1]^4, xhat[2]·yhat + xhat[1]·xhat[2]·yhat];
```

```
  B:-1;
```

```
  g(xhat, yhat):=[-(xhat[1]^2 + xhat[2]^2) + yhat^2 + sin(xhat[1]^3)];
```

```
(fpx) [x=0, y=0]
```

```
(xhat) [x, y]
```

```
(yhat) [z]
```

```
(A)  $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ 
```

```
(%o23) f(xhat, yhat):=[xhat1 yhat - xhat14, xhat2 yhat + xhat1
  xhat2 yhat]
```

```
(B) -1
```

```
(%o25) g(xhat, yhat):=[-(xhat12 + xhat22) + yhat2 + sin(xhat13)]
```

```
(%i12) M(F):=grad(F, xhat).(A.xhat + f(xhat, F(xhat))) - B.F(xhat) - g(xhat, F(xhat));
```

```
  Mp(pk, dkp1):=grad(dkp1, xhat).(A.xhat) - B.dkp1(xhat) + (grad(pk, xhat).(A.xhat
```

```
  dk(w, α, k):=sum(α[i]·w[1]^(k-i)·w[2]^i, i, 0, k);
```

```
  generate_equations(poly, k):=makelist(at(ratcoeff(poly, x^(k-i)·y^i), fpx), i, 0, k)
```

```
(%o9) M(F):=grad(F, xhat) . (A . xhat + f(xhat, F(xhat))) - B .
```

```
  F(xhat) - g(xhat, F(xhat))
```

```
(%o11) dk(w, α, k):=
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$$\sum_{i=0}^k \alpha_i w_1^{k-i} w_2^i$$

```
(%i16) p(w):=0$
for i:1 thru 3 step 1 do (
  define(d(w), dk(w, c, i+1)),
  define(Mpe(x, y), expand(taylor(Mp(p, d)[1], x, 0, i+2))),
  eqs:generate_equations(Mpe(x, y), i+1),
  cs:solve(eqs, makelist(c[j], j, 0, i+1))[1],
  define(p(w), at(d(w), cs) + p(w))
)$
p(xhat);
ratsimp(taylor(subst(x, y, M(p)), x, 0, 5));
```

$$(\%o15) \quad -y^4 + \frac{3y^3}{10} - 2x^2y^2 + \frac{3xy^2}{10} - y^2 + \frac{3x^2y}{5} - x^4 + \frac{2x^3}{5} - x^2$$

$$(\%o16) \quad \left[-\frac{18x^5}{5} \right]$$