Determining IMU Attitude with Respect to NED Frame

# Overview

When the IMU is stationary:

- Accelerometers measure the specific force, which is approximately opposite to gravity, i.e., -g.

- Gyroscopes measure the Earth’s rotation rate vector ω\_ie, expressed in the body frame.

Given:

- The location (latitude, longitude), which allows you to compute the local gravity vector and the Earth rotation rate in the NED frame.

- The two vectors measured in the body frame (acceleration and rotation rate).

- The same two vectors known in the NED frame.

You can compute the rotation matrix (or attitude quaternion) from body to NED using a two-vector attitude determination method (Wahba’s problem).

# Step-by-Step Method

# 1. Define the reference vectors in NED

Let’s define the known vectors in the NED frame:

- Gravity (down direction):

g\_NED = [0, 0, g] where g ≈ 9.81 m/s²

- Earth rotation rate in NED:

ω\_ie,NED = ω\_E \* [cos(φ), 0, -sin(φ)]

where φ is the latitude, and ω\_E = 7.2921159 × 10⁻⁵ rad/s.

# 2. Measure the vectors in the body frame

From accelerometer (assuming static IMU):

a\_body = -g\_body

From gyroscope:

ω\_ie,body

These give the same physical vectors as in NED, but expressed in the body frame.

# 3. Solve Wahba’s problem

You now have:

- Two vectors in the body frame: v1\_B, v2\_B

- Two corresponding vectors in the NED frame: v1\_N, v2\_N

Using these pairs, you can solve for the rotation matrix C\_B^N (attitude matrix from body to NED), or compute a quaternion.

Methods:

- Davenport’s q-method

- SVD (Singular Value Decomposition)

- TRIAD method (if only two vectors are available and non-collinear)

TRIAD method example:

C\_B^N = [t1^B t2^B t3^B][t1^N t2^N t3^N]^T

Where t1, t2, t3 are orthonormal triads built from the input vectors.

# Assumptions and Notes

- IMU is stationary: no linear acceleration, only gravity affects the accelerometer.

- Magnetometer is not used in this approach (purely inertial).

- Gyro biases should ideally be small or corrected.

- If magnetometer is available, you could include it for full heading resolution, especially if at the poles (where Earth rate is vertical).

# Conclusion

1. Compute Earth rotation rate and gravity in the NED frame using latitude.

2. Measure the same vectors using the IMU (in body frame).

3. Solve for the rotation matrix or quaternion that aligns the body-frame vectors to the NED-frame vectors (e.g., via TRIAD or Wahba’s problem).

4. This gives you the attitude (orientation) of the IMU with respect to the local NED frame.