# Contents



# 1 Miscellaneous

## 1.1 VS Code Config

```
"command": "clear; ./runner ${file} ${fileBasenameNoExtension} 0"
"command": "clear; ./runner ${file} ${fileBasenameNoExtension} 1"
"command": "clear; rm -r .cache"

runner:
#!/bin/bash
cp=$1; uf=$3; tmp='.cache'; bp="$tmp/$2"; ch="$tmp/${2}-hash"
li="======================================================================"
ti="${li}\nTime: %es"
rc() {
    mkdir -p "$tmp"
}
sch() {
    rc; echo -n "$(gh)" > "$ch"
}
gch() {
    rc
    if [[ -f "$ch" ]]; then cat $ch
    else echo -n 'NULL'; fi
}
gh() {
    sha256sum $cp
}
nr() {
    oh=$(gch); nh=$(gh)
    if [[ "$oh" == "$nh" ]]; then return 1
    else return 0; fi
}
cc() {
    g++ $cp -O2 -std=gnu++17 -Wall -Wextra -Wshadow -DLOCAL -o $bp
}
ma() {
    ulimit -s unlimited
    echo $li
    if nr; then
        if [[ -f "$bp" ]]; then rm "$bp"; fi
        sch; echo 'Compiling...'; echo $li; cc; echo $li
    fi
    if [[ -f "$bp" ]]; then
        echo 'Running...'; echo $li
        if (( $uf )); then command time -f "$ti" ./$bp < IN
        else command time -f "$ti" ./$bp; fi
        echo $li
    fi
}
ma
```

## 1.2 Day of Date

```
// 0-based
const vector<int> T = {
  0, 3, 2, 5, 0, 3,
  5, 1, 4, 6, 2, 4
}

int day(int d, int m, int y) {
  y -= (m < 3);
  return (y + y / 4 - y / 100 + y / 400 + T[m - 1] + d) % 7;
}
```

## 1.3 Number of Days since 1-1-1

```
int rdn(int d, int m, int y) {
  if(m < 3)
    --y, m += 12;
  return 365 * y + y / 4 - y / 100 + y / 400
         + (153 * m - 457) / 5 + d - 306;
}
```

### 1.4 Enumerate Subsets of a Bitmask

```
int x = 0;
do {
  // do stuff with the bitmask here
  x = (x + 1 + ~m) & m;
} while(x != 0);
```

### 1.5 Fast IO

```
int read() {
  char c;
  do {
    c = getchar_unlocked();
  } while(c < 33);
  int res = 0;
  int mul = 1;
  if(c == '-') {
    mul = -1;
    c = getchar_unlocked();
  }
  while('0' <= c && c <= '9') {
    res = res * 10 + c - '0';
    c = getchar_unlocked();
  }
  return res * mul;
}

void write(int x) {
  static char wbuf[10];
  if(x < 0) {
    putchar_unlocked('-');
    x = -x;
  }
  int idx = 0;
  while(x) {
    wbuf[idx++] = x % 10;
    x /= 10;
  }
  if(idx == 0)
    putchar_unlocked('0');
  for(int i = idx - 1; i >= 0; --i)
    putchar_unlocked(wbuf[i] + '0');
}

void write(const char* s) {
  while(*s) {
    putchar_unlocked(*s);
    ++s;
  }
}
```

### 1.6 Int to Roman

```
const string R[] = {
  "M", "CM", "D", "CD", "C", "XC", "L",
  "XL", "X", "IX", "V", "IV", "I"
};

const int N[] = {
  1000, 900, 500, 400, 100, 90,
  50, 40, 10, 9, 5, 4, 1
};

string to_roman(int x) {
  if(x == 0) {
    return "O";  // Not decimal 0!
  }
  string res = "";
  for(int i = 0; i < 13; ++i)
    while(x >= N[i])
      x -= N[i], res += R[i];
  return res;
}
```

### 1.7 Josephus Problem

```
ll josephus(ll n, ll k) { // O(k log n)
  if(n == 1)
    return 0;
  if(k == 1)
    return n - 1;
  if(k > n)
    return (josephus(n - 1, k) + k) % n;
  ll cnt = n / k;
  ll res = josephus(n - cnt, k);
  res -= n % k;
  if(res < 0)
    res += n;
  else
    res += res / (k - 1);
  return res;
}

int josephus(int n, int k) { // O(n)
  int res = 0;
  for(int i = 1; i <= n; ++i)
    res = (res + k) % i;
  return res + 1;
}
```

### 1.8 Random Primes

```
36671 74101 724729 825827 924997 1500005681 2010408371 2010405347
```

### 1.9 RNG

```
// RNG - rand_int(min, max), inclusive

mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());

template<class T>
T rand_int(T mn, T mx) {
  return uniform_int_distribution<T>(mn, mx)(rng);
}
```

## 2 Data Structures

### 2.1 2D Segment Tree

```
struct Segtree2D {
  struct Segtree {
    struct node {
      int l, r, val;
      node* lc, * rc;
      node(int _l, int _r, int _val = INF) : l(_l), r(_r), val(_val),
```

Proof by... forgor
Bina Nusantara University
Owmicron, ZeroScar, KerakTelor


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```
      lc(NULL), rc(NULL) {}
  };
  typedef node* pnode;

  pnode root;

  Segtree(int l, int r) {
    root = new node(l, r);
  }

  void update(pnode& nw, int x, int val) {
    int l = nw->l, r = nw->r, mid = (l + r) / 2;
    if(l == r)
      nw->val = val;
    else {
      assert(l <= x && x <= r);
      pnode& child = x <= mid ? nw->lc : nw->rc;
      if(!child)
        child = new node(x, x, val);
      else if(child->l <= x && x <= child->r)
        update(child, x, val);
      else {
        do {
          if(x <= mid)
            r = mid;
          else
            l = mid + 1;
          mid = (l + r) / 2;
        } while((x <= mid) == (child->l <= mid));
        pnode nxt = new node(l, r);
        if(child->l <= mid)
          nxt->lc = child;
        else
          nxt->rc = child;
        child = nxt;
        update(nxt, x, val);
      }
      nw->val = min(nw->lc ? nw->lc->val : INF,
                    nw->rc ? nw->rc->val : INF);
    }
  }

  int query(pnode& nw, int x1, int x2) {
    if(!nw)
      return INF;
    int& l = nw->l, &r = nw->r;
    if(r < x1 || x2 < l)
      return INF;
    if(x1 <= l && r <= x2)
      return nw->val;
    int ret = min(query(nw->lc, x1, x2),
                  query(nw->rc, x1, x2));
    return ret;
  }

  void update(int x, int val) {
    assert(root->l <= x && x <= root->r);
    update(root, x, val);
  }

  int query(int l, int r) {
    return query(root, l, r);
  }
};

struct node {
  int l, r;
  Segtree y;
  node* lc, * rc;
  node(int _l, int _r) : l(_l), r(_r), y(0, MAX),
    lc(NULL), rc(NULL) {}
};
typedef node* pnode;

pnode root;

Segtree2D(int l, int r) {
  root = new node(l, r);
}

void update(pnode& nw, int x, int y, int val) {
  int& l = nw->l, &r = nw->r, mid = (l + r) / 2;
  if(l == r)
    nw->y.update(y, val);
  else {
    if(x <= mid) {
      if(!nw->lc)
        nw->lc = new node(l, mid);
      update(nw->lc, x, y, val);
    } else {
      if(!nw->rc)
        nw->rc = new node(mid + 1, r);
      update(nw->rc, x, y, val);
    }
    val = min(nw->lc ? nw->lc->y.query(y, y) : INF,
              nw->rc ? nw->rc->y.query(y, y) : INF);
    nw->y.update(y, val);
  }
}

int query(pnode& nw, int x1, int x2, int y1, int y2) {
  if(!nw)
    return INF;
  int& l = nw->l, &r = nw->r;
  if(r < x1 || x2 < l)
    return INF;
  if(x1 <= l && r <= x2)
    return nw->y.query(y1, y2);
  int ret = min(query(nw->lc, x1, x2, y1, y2),
                query(nw->rc, x1, x2, y1, y2));
  return ret;
}

void update(int x, int y, int val) {
  assert(root->l <= x && x <= root->r);
  update(root, x, y, val);
}

int query(int x1, int x2, int y1, int y2) {
  return query(root, x1, x2, y1, y2);
}
};
```

## 2.2 Fenwick RU-RQ

```
void updtRL(int l, int r, ll val) {
  updt(BIT1, l, val), updt(BIT1, r + 1, -val);
  updt(BIT2, l, val * (l - 1)), updt(BIT2, r + 1, -val * r);
}
ll query(int k) {
  return que(BIT1, k) * k - que(BIT2, k);
}
```

## 2.3 Heavy-Light Decomposition

```
struct HLD {
  int n;
  vector<int> id, size, idx, up, root, st;
  vector<vector<int>> adj, chain;
  SegTree seg;
```

```
  HLD(const vector<vector<int>>& edges) :
    n(edges.size()), id(n, -1), size(n, -1), idx(n, -1),
    up(n, -1), adj(edges), seg(n) {
    precompute(0, -1);
    decompose(0, -1);
    int cnt = 0;
    st.resize(chain.size());
    for(int i = 0; i < (int) chain.size(); ++i) {
      st[i] = cnt;
      cnt += chain[i].size();
    }
  }

  void precompute(int pos, int dad) {
    size[pos] = 1;
    up[pos] = dad;
    for(auto& i : adj[pos]) {
      if(i != dad) {
        precompute(i, pos);
        size[pos] += size[i];
      }
    }
  }

  void decompose(int pos, int dad) {
    if(id[pos] == -1) {
      id[pos] = chain.size();
      root.push_back(pos);
      chain.emplace_back();
    }
    idx[pos] = chain[id[pos]].size();
    chain[id[pos]].push_back(pos);
    int mx = 0, heavy = -1;
    for(auto& i : adj[pos]) {
      if(i != dad && size[i] > mx) {
        mx = size[i];
        heavy = i;
      }
    }
    if(heavy != -1)
      id[heavy] = id[pos];
    for(auto& i : adj[pos]) {
      if(i != dad)
        decompose(i, pos);
    }
  }

  void update(int ch, int l, int r, int val) {
    seg.update(st[ch] + l, st[ch] + r, val);
  }

  int query(int ch, int l, int r, int val) {
    return seg.query(st[ch] + l, st[ch] + r, val);
  }
};

// how to move from u to v
while(1) {
  if(hld.id[u] == hld.id[v]) {
    if(hld.idx[u] > hld.idx[v])
      swap(u, v);
    hld.update(hld.id[u], hld.idx[u], hld.idx[v], w);
    // or hld.query(hld.id[u], hld.idx[u], hld.idx[v]);
    break;
  }
  if(hld.id[u] < hld.id[v])
    swap(u, v);
  hld.update(hld.id[u], 0, hld.idx[u], w);
  // or hld.query(hld.id[u], 0, hld.idx[u]);
  u = hld.up[hld.root[hld.id[u]]];
}
```

## 2.4 Li-Chao Tree

```
// max li-chao tree
// works for the range [0, MAX - 1]
// if min li-chao tree:
// replace every call to max() with min() and every > with <
// also replace -INF with INF

struct Func {
  ll m, c;
  ll operator()(ll x) {
    return x * m + c;
  }
};

const int MAX = 1e9 + 1;
const ll INF = 1e18;
const Func NIL = {0, -INF};

struct Node {
  Func f;
  Node* lc;
  Node* rc;

  Node() : f(NIL), lc(nullptr), rc(nullptr) {}
  Node(const Node& n) : f(n.f), lc(nullptr), rc(nullptr) {}
};

Node* root = new Node;

void insert(Func f, Node* cur = root, int l = 0, int r = MAX - 1) {
  int m = l + (r - l) / 2;
  bool left = f(l) > cur->f(l);
  bool mid = f(m) > cur->f(m);
  if(mid)
    swap(f, cur->f);
  if(l != r) {
    if(left != mid) {
      if(!cur->lc)
        cur->lc = new Node(*cur);
      insert(f, cur->lc, l, m);
    } else {
      if(!cur->rc)
        cur->rc = new Node(*cur);
      insert(f, cur->rc, m + 1, r);
    }
  }
}

ll query(ll x, Node* cur = root, int l = 0, int r = MAX - 1) {
  if(!cur)
    return -INF;
  if(l == r)
    return cur->f(x);
  int m = l + (r - l) / 2;
  if(x <= m)
    return max(cur->f(x), query(x, cur->lc, l, m));
  else
    return max(cur->f(x), query(x, cur->rc, m + 1, r));
}
```

## 2.5 Persistent Segment Tree

```
class PersistentSegtree {
private:
  int n, ptr, sz;
  struct P {
```

```
    int val = 0, l, r;
  };
  vector<P> node;
  vector<int> root;

  int newNode() {
    node.emplace_back();
    return ptr++;
  }
  int copyNode(int idx) {
    node.push_back(node[idx]);
    return ptr++;
  }
  int build(int l, int r) {
    int idx = newNode();
    if(l == r)
      return idx;
    node[idx].l = build(l, (l + r) / 2);
    node[idx].r = build((l + r) / 2 + 1, r);
    return idx;
  }
  int update(int idx, int l, int r, int x, int val) {
    idx = copyNode(idx);
    if(l == r) {
      node[idx].val += val;
      return idx;
    }
    int mid = (l + r) / 2;
    if(x <= mid)
      node[idx].l = update(node[idx].l, l, mid, x, val);
    else
      node[idx].r = update(node[idx].r, mid + 1, r, x, val);
    node[idx].val = node[node[idx].l].val + node[node[idx].r].val;
    return idx;
  }
  int query(int idxl, int idxr, int l, int r, int x, int y) {
    if(y < l || r < x)
      return 0;
    if(x <= l && r <= y)
      return node[idxr].val - node[idxl].val;
    int mid = (l + r) / 2;
    return query(node[idxl].l, node[idxr].l, l, mid, x, y)
          + query(node[idxl].r, node[idxr].r, mid + 1, r, x, y);
  }

public:
  PersistentSegtree(int _n) : n(_n), ptr(0) {
    sz = 30 * n;
    node.reserve(sz);
    root.push_back(build(1, n));
  }
  void update(int x, int val) {
    root.push_back(update(root.back(), 1, n, x, val));
  }
  int query(int l, int r, int x, int y) {
    return query(root[l - 1], root[r], 1, n, x, y);
  }
};
```

## 2.6 STL PBDS

```
// ost = ordered set
// omp = ordered map
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace __gnu_pbds;

template<class T>
using ost = tree<T, null_type, less<T>, rb_tree_tag,
      tree_order_statistics_node_update>;
template<class T, class U>
using omp = tree<T, U, less<T>, rb_tree_tag,
      tree_order_statistics_node_update>;
```

## 2.7 Treap

```
// Complexity: O(log N) for split and merge
//
// empty treap: Treap* tr = nullptr;
// insert v at x: [l, r] = split(tr, x), m = Treap(v), merge lmr
// delete at x: [l, r] = split(tr, x), [m, r] = split(r, 1), merge lr
// lazy prop: propagate every time a node is accessed

mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());

using Key = int;

struct Treap {
  Key val;
  Treap* left;
  Treap* right;
  int prio, sz;
  Treap() {}
  Treap(int _val);
};

int size(Treap* tr) {
  return tr ? tr->sz : 0;
}

void update(Treap* tr) {
  tr->sz = 1 + size(tr->left) + size(tr->right);
}

Treap::Treap(Key _val) :
  val(_val), left(nullptr), right(nullptr), prio(rng()) {
  update(this);
}

pair<Treap*, Treap*> split(Treap* tr, int sz) {
  if(!tr) return {nullptr, nullptr};
  int left_sz = size(tr->left);
  if(sz <= left_sz) {
    auto [left, mid] = split(tr->left, sz);
    tr->left = mid;
    update(tr);
    return {left, tr};
  } else {
    auto [mid, right] = split(tr->right, sz - left_sz - 1);
    tr->right = mid;
    update(tr);
    return {tr, right};
  }
}

Treap* merge(Treap* l, Treap* r) {
  if(!l)
    return r;
  if(!r)
    return l;
  if(l->prio < r->prio) {
    l->right = merge(l->right, r);
    update(l);
    return l;
  } else {
    r->left = merge(l, r->left);
    update(r);
    return r;
  }
```




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```
}
```

## 2.8 Unordered Map Custom Hash

```
struct custom_hash {
  static uint64_t splitmix64(uint64_t x) {
    x += 0x9e3779b97f4a7c15;
    x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
    x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
  }
  size_t operator()(uint64_t x) const {
    static const uint64_t FIXED_RANDOM =
      chrono::steady_clock::now().time_since_epoch().count();
    return splitmix64(x + FIXED_RANDOM);
  }
};

unordered_map<int, int, custom_hash> umap;
```

## 2.9 Mo's on Tree

$ST(u) \leq ST(v)$
$P = LCA(u, v)$
If $P = u$, query $[ST(u), ST(v)]$
Else query $[EN(u), ST(v)] + [ST(P), ST(P)]$

## 2.10 Link-Cut Tree

```
// Represents a forest of unrooted trees. You can add and remove edges
// (as long as the result is still a forest), and check whether two
// nodes are in the same tree.
// Complexity: log(n)

struct Node { // Splay tree. Root's pp contains tree's parent.
  Node* p = 0, * pp = 0, * c[2];
  int sz = 0;
  //
  Node() {
    c[0] = c[1] = 0;
    fix();
  }
  void fix() {
    sz = 1;
    if(c[0]) c[0]->p = this, sz += c[0]->sz;
    if(c[1]) c[1]->p = this, sz += c[1]->sz;
    // (+ update sum of subtree elements etc. if wanted)
  }
  int up() {
    return p ? p->c[1] == this : -1;
  }
  void rot(int i, int b) {
    int h = i ^ b;
    Node* x = c[i], * y = (b == 2 ? x : x->c[h]), * z = (b ? y : x);
    if(y->p = p) p->c[up()] = y;
    c[i] = z->c[i ^ 1];
    if(b < 2) x->c[h] = y->c[h ^ 1], z->c[h ^ 1] = b ? x : this;
    y->c[i ^ 1] = b ? this : x;
    fix();
    x->fix();
    y->fix();
    if(p) p->fix();
    swap(pp, y->pp);
  }
  // Splay this up to the root. Always finishes without flip set.
  void splay() {
    while(p) {
      int c1 = up(), c2 = p->up();
      if(c2 == -1) p->rot(c1, 2);
      else p->p->rot(c2, c1 != c2);
    }
  }
};
struct LinkCut {
  vector<Node> node;

  LinkCut(int N) : node(N + 1) {}
  void link(int u, int v) { // add an edge u --> v
    assert(!connected(u, v));
    access(&node[u]);
    access(&node[v]);
    node[u].c[0] = &node[v];
    node[v].p = &node[u];
    node[u].fix();
  }
  void cut(int u, int v) { // remove an edge u --> v
    assert(connected(u, v));
    Node* x = &node[v], * top = &node[u];
    access(top);
    top->c[0] = top->c[0]->p = 0;
    top->fix();
  }
  bool connected(int u, int v) { // are u, v in the same tree?
    return root(u) == root(v);
  }
  int root(int u) { // find the root id of node u
    Node* x = &node[u];
    access(x);
    for(; x->c[0]; x = x->c[0]);
    x->splay();
    return (int)((vector<Node>::iterator)x - node.begin());
  }
  // Move u to root aux tree. Return the root of the root aux tree.
  Node* access(Node* u) {
    u->splay();
    Node* last = u;
    if(Node*& x = u->c[1]) {
      x->pp = u;
      x->p = 0;
      x = 0;
      u->fix();
    }
    for(Node * pp; (pp = u->pp) && (last = pp);) {
      pp->splay();
      if(pp->c[1]) pp->c[1]->p = 0, pp->c[1]->pp = pp;
      pp->c[1] = u;
      u->p = pp;
      u->pp = 0;
      pp->fix();
      u->splay();
    }
    return last;
  }
  int depth(int u) {
    access(&node[u]);
    return node[u].sz - 1;
  }
  Node* lca(int u, int v) {
    access(&node[u]);
    return access(&node[v]);
  }
};
```

# 3 Dynamic Programming

## 3.1 DP Convex Hull

```
/* dp[i] = min k<i {dp[k] + x[i]*m[k]}
   Make sure gradient (m[i]) is either non-increasing if min,
   or non-decreasing if max. x[i] must be non-decreasing. just sort */
int y[N], m[N];
// while this is true, pop back from dq. a=new line, b=last, c=2nd last
bool cekx(int a, int b, int c) {
  // if not enough, change to cross mul
  // if cross mul, beware of negative denominator, and overflow
  return (double)(y[b] - y[a]) / (m[a] - m[b]) <= (double)(y[c] - y[b]) /
         (m[b] - m[c]);
}
```

### 3.2 DP DNC

```
void f(int rem, int l, int r, int optl, int optr) {
  if(l > r)
    return;
  int mid = l + r >> 1;
  int opt = MOD, optid = mid;
  for(int i = optl; i <= mid && i <= optr; ++i) {
    if(dp[rem - 1][i] + c[i][mid] < opt) {
      opt = dp[rem - 1][i] + c[i][mid];
      optid = i;
    }
  }
  dp[rem][mid] = opt;
  f(rem, l, mid - 1, optl, optid);
  f(rem, mid + 1, r, optid, optr);
  return;
}
rep(i, 1, n)dp[1][i] = c[0][i];
rep(i, 2, k)f(i, i, n, i, n);
```

### 3.3 DP Knuth-Yao

```
// opt[i+1][j] <= opt[i][j] <= opt[i][j+1]
// dp[i][j] = min{k} dp[i][k]+dp[k][j]+cost[i][j]
for(int k = 0; k <= n; k++) {
  for(int i = 0; i + k <= n; i++) {
    if(k < 2)
      dp[i][i + k] = 0, opt[i][i + k] = i;
    else {
      int sta = opt[i][i + k - 1];
      int end = opt[i + 1][i + k];
      for(int j = sta; j <= end; j++) {
        if(dp[i][j] + dp[j][i + k] + cost[i][i + k] < dp[i][i + k]) {
          dp[i][i + k] = dp[i][j] + dp[j][i + k] + cost[i][i + k];
          opt[i][i + k] = j;
        }
      }
    }
  }
}
```

## 4 Geometry

### 4.1 Geometry Template

```
/*
TABLE OF CONTENT
0. Basic Rule
    0.1. Everything is in double
    0.2. Every comparison use EPS
    0.3. Every degree in rad
1. General Double Operation
    1.1. const double EPS=1E-9
    1.2. const double PI=acos(-1.0)
    1.3. const double INFD=1E9
    1.3. between_d(double x,double l,double r)
        check whether x is between l and r inclusive with EPS
    1.4. same_d(double x,double y)
        check whether x=y with EPS
    1.5. dabs(double x)
        absolute value of x
2. Point
    2.1. struct point
        2.1.1. double x,y
            cartesian coordinate of the point
        2.1.2. point()
            default constructor
        2.1.3. point(double _x,double _y)
            constructor, set the point to (_x,_y)
        2.1.4. bool operator< (point other)
            regular pair<double,double> operator < with EPS
        2.1.5. bool operator== (point other)
            regular pair<double,double> operator == with EPS
    2.2. hypot(point P)
        length of hypotenuse of point P to (0,0)
    2.3. e_dist(point P1,point P2)
        euclidean distance from P1 to P2
    2.4. m_dist(point P1,point P2)
        manhattan distance from P1 to P2
    2.5. point rotate(point P,point O,double angle)
        rotate point P from the origin O by angle ccw
3. Vector
    3.1. struct vec
        3.1.1. double x,y
            x and y magnitude of the vector
        3.1.2. vec()
            default constructor
        3.1.3. vec(double _x,double _y)
            constructor, set the vector to (_x,_y)
        3.1.4. vec(point A,point B)
            constructor, set the vector to vector AB (A->B)
*/
/*General Double Operation*/

const double PI = acos(-1.0);
const double INFD = 1E9;
double between_d(double x, double l, double r) {
  return (min(l, r) <= x + EPS && x <= max(l, r) + EPS);
}
double same_d(double x, double y) {
  return between_d(x, y, y);
}
double dabs(double x) {
  if(x < EPS)
    return -x;
  return x;
}
/*Point*/
struct point {
  double x, y;
  point() {
    x = y = 0.0;
  }
  point(double _x, double _y) {
    x = _x;
    y = _y;
  }
  bool operator< (point other) {
    if(x < other.x + EPS)
      return true;
    if(x + EPS > other.x)
      return false;
    return y < other.y + EPS;
  }
  bool operator== (point other) {
```

```
    return same_d(x, other.x) && same_d(y, other.y);
  }
};
double e_dist(point P1, point P2) {
  return hypot(P1.x - P2.x, P1.y - P2.y);
}
double m_dist(point P1, point P2) {
  return dabs(P1.x - P2.x) + dabs(P1.y - P2.y);
}
double pointBetween(point P, point L, point R) {
  return (e_dist(L, P) + e_dist(P, R) == e_dist(L, R));
}
bool collinear(point P, point L,
               point R) { //newly added(luis), cek 3 poin segaris
  return P.x * (L.y - R.y) + L.x * (R.y - P.y) + R.x * (P.y - L.y) ==
         0; // bole gnti "dabs(x)<"EPS
}

/*Vector*/
struct vec {
  double x, y;
  vec() {
    x = y = 0.0;
  }
  vec(double _x, double _y) {
    x = _x;
    y = _y;
  }
  vec(point A) {
    x = A.x;
    y = A.y;
  }
  vec(point A, point B) {
    x = B.x - A.x;
    y = B.y - A.y;
  }
};
vec scale(vec v, double s) {
  return vec(v.x * s, v.y * s);
}
vec flip(vec v) {
  return vec(-v.x, -v.y);
}
double dot(vec u, vec v) {
  return (u.x * v.x + u.y * v.y);
}
double cross(vec u, vec v) {
  return (u.x * v.y - u.y * v.x);
}
double norm_sq(vec v) {
  return (v.x * v.x + v.y * v.y);
}
point translate(point P, vec v) {
  return point(P.x + v.x, P.y + v.y);
}
point rotate(point P, point O, double angle) {
  vec v(O);
  P = translate(P, flip(v));
  return translate(point(P.x * cos(angle) - P.y * sin(angle),
                         P.x * sin(angle) + P.y * cos(angle)), v);
}
point mid(point P, point Q) {
  return point((P.x + Q.x) / 2, (P.y + Q.y) / 2);
}
double angle(point A, point O, point B) {
  vec OA(O, A), OB(O, B);
  return acos(dot(OA, OB) / sqrt(norm_sq(OA) * norm_sq(OB)));
}
int orientation(point P, point Q, point R) {
  vec PQ(P, Q), PR(P, R);
  double c = cross(PQ, PR);
  if(c < -EPS)
    return -1;
  if(c > EPS)
    return 1;
  return 0;
}
/*Line*/
struct line {
  double a, b, c;
  line() {
    a = b = c = 0.0;
  }
  line(double _a, double _b, double _c) {
    a = _a;
    b = _b;
    c = _c;
  }
  line(point P1, point P2) {
    if(P1 < P2)
      swap(P1, P2);
    if(same_d(P1.x, P2.x))
      a = 1.0, b = 0.0, c = -P1.x;
    else
      a = -(P1.y - P2.y) / (P1.x - P2.x), b = 1.0, c = -(a * P1.x) - P1.y;
  }
  line(point P, double slope) {
    if(same_d(slope, INFD))
      a = 1.0, b = 0.0, c = -P.x;
    else
      a = -slope, b = 1.0, c = -(a * P.x) - P.y;
  }
  bool operator== (line other) {
    return same_d(a, other.a) && same_d(b, other.b) && same_d(c, other.c);
  }
  double slope() {
    if(same_d(b, 0.0))
      return INFD;
    return -(a / b);
  }
};
bool paralel(line L1, line L2) {
  return same_d(L1.a, L2.a) && same_d(L1.b, L2.b);
}
bool intersection(line L1, line L2, point& P) {
  if(paralel(L1, L2))
    return false;
  P.x = (L2.b * L1.c - L1.b * L2.c) / (L2.a * L1.b - L1.a * L2.b);
  if(same_d(L1.b, 0.0))
    P.y = -(L2.a * P.x + L2.c);
  else
    P.y = -(L1.a * P.x + L1.c);
  return true;
}
double pointToLine(point P, point A, point B, point& C) {
  vec AP(A, P), AB(A, B);
  double u = dot(AP, AB) / norm_sq(AB);
  C = translate(A, scale(AB, u));
  return e_dist(P, C);
}
double lineToLine(line L1, line L2) {
  if(!paralel(L1, L2))
    return 0.0;
  return dabs(L2.c - L1.c) / sqrt(L1.a * L1.a + L1.b * L1.b);
}
/*Line Segment*/
struct segment {
  point P, Q;
  line L;
  segment() {
    point T1;
    P = Q = T1;
```


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```
    line T2;
    L = T2;
  }
  segment(point _P, point _Q) {
    P = _P;
    Q = _Q;
    if(Q < P)
      swap(P, Q);
    line T(P, Q);
    L = T;
  }
  bool operator== (segment other) {
    return P == other.P && Q == other.Q;
  }
};
bool onSegment(point P, segment S) {
  if(orientation(S.P, S.Q, P) != 0)
    return false;
  return between_d(P.x, S.P.x, S.Q.x) && between_d(P.y, S.P.y, S.Q.y);
}
bool s_intersection(segment S1, segment S2) {
  double o1 = orientation(S1.P, S1.Q, S2.P);
  double o2 = orientation(S1.P, S1.Q, S2.Q);
  double o3 = orientation(S2.P, S2.Q, S1.P);
  double o4 = orientation(S2.P, S2.Q, S1.Q);
  if(o1 != o2 && o3 != o4)
    return true;
  if(o1 == 0 && onSegment(S2.P, S1))
    return true;
  if(o2 == 0 && onSegment(S2.Q, S1))
    return true;
  if(o3 == 0 && onSegment(S1.P, S2))
    return true;
  if(o4 == 0 && onSegment(S1.Q, S2))
    return true;
  return false;
}
double pointToSegment(point P, point A, point B, point& C) {
  vec AP(A, P), AB(A, B);
  double u = dot(AP, AB) / norm_sq(AB);
  if(u < EPS) {
    C = A;
    return e_dist(P, A);
  }
  if(u + EPS > 1.0) {
    C = B;
    return e_dist(P, B);
  }
  return pointToLine(P, A, B, C);
}
double segmentToSegment(segment S1, segment S2) {
  if(s_intersection(S1, S2))
    return 0.0;
  double ret = INFD;
  point dummy;
  ret = min(ret, pointToSegment(S1.P, S2.P, S2.Q, dummy));
  ret = min(ret, pointToSegment(S1.Q, S2.P, S2.Q, dummy));
  ret = min(ret, pointToSegment(S2.P, S1.P, S1.Q, dummy));
  ret = min(ret, pointToSegment(S2.Q, S1.P, S1.Q, dummy));
  return ret;
}
/*Circle*/
struct circle {
  point P;
  double r;
  circle() {
    point P1;
    P = P1;
    r = 0.0;
  }
  circle(point _P, double _r) {
    P = _P;
    r = _r;
  }
  circle(point P1, point P2) {
    P = mid(P1, P2);
    r = e_dist(P, P1);
  }
  circle(point P1, point P2, point P3) {
    vector<point> T;
    T.clear();
    T.pb(P1);
    T.pb(P2);
    T.pb(P3);
    sort(T.begin(), T.end());
    P1 = T[0];
    P2 = T[1];
    P3 = T[2];
    point M1, M2;
    M1 = mid(P1, P2);
    M2 = mid(P2, P3);
    point Q2, Q3;
    Q2 = rotate(P2, P1, PI / 2);
    Q3 = rotate(P3, P2, PI / 2);
    vec P1Q2(P1, Q2), P2Q3(P2, Q3);
    point M3, M4;
    M3 = translate(M1, P1Q2);
    M4 = translate(M2, P2Q3);
    line L1(M1, M3), L2(M2, M4);
    intersection(L1, L2, P);
    r = e_dist(P, P1);
  }
  bool operator==(circle other) {
    return (P == other.P && same_d(r, other.r));
  }
};
bool insideCircle(point P, circle C) {
  return e_dist(P, C.P) <= C.r + EPS;
}
bool c_intersection(circle C1, circle C2, point& P1, point& P2) {
  double d = e_dist(C1.P, C2.P);
  if(d > C1.r + C2.r) {
    return false;  //d+EPS kalo butuh
  }
  if(d < dabs(C1.r - C2.r) + EPS)
    return false;
  double x1 = C1.P.x, y1 = C1.P.y, r1 = C1.r, x2 = C2.P.x, y2 = C2.P.y, r2 = C2.r;
  double a = (r1 * r1 - r2 * r2 + d * d) / (2 * d), h = sqrt(r1 * r1 - a * a);
  point T(x1 + a * (x2 - x1) / d, y1 + a * (y2 - y1) / d);
  P1 = point(T.x - h * (y2 - y1) / d, T.y + h * (x2 - x1) / d);
  P2 = point(T.x + h * (y2 - y1) / d, T.y - h * (x2 - x1) / d);
  return true;
}
bool lc_intersection(line L, circle O, point& P1, point& P2) {
  double a = L.a, b = L.b, c = L.c, x = O.P.x, y = O.P.y, r = O.r;
  double A = a * a + b * b, B = 2 * a * b * y - 2 * a * c - 2 * b * b * x,
         C = b * b * x * x + b * b * y * y - 2 * b * c * y + c * c - b * b * r * r;
  double D = B * B - 4 * A * C;
  point T1, T2;
  if(same_d(b, 0.0)) {
    T1.x = c / a;
    if(dabs(x - T1.x) + EPS > r)
      return false;
    if(same_d(T1.x - r - x, 0.0) || same_d(T1.x + r - x, 0.0)) {
      P1 = P2 = point(T1.x, y);
      return true;
    }
    double dx = dabs(T1.x - x), dy = sqrt(r * r - dx * dx);
    P1 = point(T1.x, y - dy);
    P2 = point(T1.x, y + dy);
    return true;
  }
```

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```
  if(same_d(D, 0.0)) {
    T1.x = -B / (2 * A);
    T1.y = (c - a * T1.x) / b;
    P1 = P2 = T1;
    return true;
  }
  if(D < EPS)
    return false;
  D = sqrt(D);
  T1.x = (-B - D) / (2 * A);
  T1.y = (c - a * T1.x) / b;
  P1 = T1;
  T2.x = (-B + D) / (2 * A);
  T2.y = (c - a * T2.x) / b;
  P2 = T2;
  return true;
}
bool sc_intersection(segment S, circle C, point& P1, point& P2) {
  bool cek = lc_intersection(S.L, C, P1, P2);
  if(!cek)
    return false;
  double x1 = S.P.x, y1 = S.P.y, x2 = S.Q.x, y2 = S.Q.y;
  bool b1 = between_d(P1.x, x1, x2) && between_d(P1.y, y1, y2);
  bool b2 = between_d(P2.x, x1, x2) && between_d(P2.y, y1, y2);
  if(P1 == P2)
    return b1;
  if(b1 || b2) {
    if(!b1)
      P1 = P2;
    if(!b2)
      P2 = P1;
    return true;
  }
  return false;
}
/*Triangle*/
double t_perimeter(point A, point B, point C) {
  return e_dist(A, B) + e_dist(B, C) + e_dist(C, A);
}
double t_area(point A, point B, point C) {
  double s = t_perimeter(A, B, C) / 2;
  double ab = e_dist(A, B), bc = e_dist(B, C), ac = e_dist(C, A);
  return sqrt(s * (s - ab) * (s - bc) * (s - ac));
}
circle t_inCircle(point A, point B, point C) {
  vector<point> T;
  T.clear();
  T.pb(A);
  T.pb(B);
  T.pb(C);
  sort(T.begin(), T.end());
  A = T[0];
  B = T[1];
  C = T[2];
  double r = t_area(A, B, C) / (t_perimeter(A, B, C) / 2);
  double ratio = e_dist(A, B) / e_dist(A, C);
  vec BC(B, C);
  BC = scale(BC, ratio / (1 + ratio));
  point P;
  P = translate(B, BC);
  line AP1(A, P);
  ratio = e_dist(B, A) / e_dist(B, C);
  vec AC(A, C);
  AC = scale(AC, ratio / (1 + ratio));
  P = translate(A, AC);
  line BP2(B, P);
  intersection(AP1, BP2, P);
  return circle(P, r);
}
circle t_outCircle(point A, point B, point C) {
  return circle(A, B, C);
}
/*Polygon*/
struct polygon {
  vector<point> P;
  polygon() {
    P.clear();
  }
  polygon(vector<point>& _P) {
    P = _P;
  }
};
bool rayCast(point P, polygon& A) {
  point Q(P.x, 10000);
  line cast(P, Q);
  int cnt = 0;
  FOR(i, (int)(A.P.size()) - 1) {
    line temp(A.P[i], A.P[i + 1]);
    point I;
    bool B = intersection(cast, temp, I);
    if(!B)
      continue;
    else if(I == A.P[i] || I == A.P[i + 1])
      continue;
    else if(pointBetween(I, A.P[i], A.P[i + 1]) && pointBetween(I, P, Q))
      cnt++;
  }
  return cnt % 2 == 1;
}
// line segment p-q intersect with line A-B.
point lineIntersectSeg(point p, point q, point A, point B) {
  double a = B.y - A.y;
  double b = A.x - B.x;
  double c = B.x * A.y - A.x * B.y;
  double u = fabs(a * p.x + b * p.y + c);
  double v = fabs(a * q.x + b * q.y + c);
  return point((p.x * v + q.x * u) / (u + v), (p.y * v + q.y * u) / (u + v));
}
// cuts polygon Q along the line formed by point a -> point b
// (note: the last point must be the same as the first point)
vector<point> cutPolygon(point a, point b, const vector<point>& Q) {
  vector<point> P;
  for(int i = 0; i < (int)Q.size(); i++) {
    double left1 = cross(toVec(a, b), toVec(a, Q[i]));
    double left2 = 0;
    if(i != (int)Q.size() - 1)
      left2 = cross(toVec(a, b), toVec(a, Q[i + 1]));
    if(left1 > -EPS)
      P.push_back(Q[i]);
    if(left1 * left2 < -EPS)
      P.push_back(lineIntersectSeg(Q[i], Q[i + 1], a, b));
  }
  if(!P.empty() && !(P.back() == P.front()))
    P.push_back(P.front());
  return P;
}
circle minCoverCircle(polygon& A) {
  vector<point> p = A.P;
  point c;
  circle ret;
  double cr = 0.0;
  int i, j, k;
  c = p[0];
  for(i = 1; i < p.size(); i++) {
    if(e_dist(p[i], c) >= cr + EPS) {
      c = p[i], cr = 0;
      ret = circle(c, cr);
      for(j = 0; j < i; j++) {
        if(e_dist(p[j], c) >= cr + EPS) {
          c = mid(p[i], p[j]);
          cr = e_dist(p[i], c);
          ret = circle(c, cr);
```

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```
        for(k = 0; k < j; k++) {
          if(e_dist(p[k], c) >= cr + EPS) {
            ret = circle(p[i], p[j], p[k]);
            c = ret.P;
            cr = ret.r;
          }
        }
      }
    }
  }
  return ret;
}
/*Geometry Algorithm*/
double DP[110][110];
double minCostPolygonTriangulation(polygon& A) {
  if(A.P.size() < 3)
    return 0;
  FOR(i, A.P.size()) {
    for(int j = 0, k = i; k < A.P.size(); j++, k++) {
      if(k < j + 2)
        DP[j][k] = 0.0;
      else {
        DP[j][k] = INFD;
        REP(l, j + 1, k - 1) {
          double cost = e_dist(A.P[j], A.P[k]) + e_dist(A.P[k], A.P[l]) + e_dist(A.P[l↩
              ],
                       A.P[j]);
          DP[j][k] = min(DP[j][k], DP[j][l] + DP[l][k] + cost);
        }
      }
    }
  }
  return DP[0][A.P.size() - 1];
}
```

## 4.2 Convex Hull

```
typedef double TD;                 // for precision shits
namespace GEOM {
  typedef pair<TD, TD> Pt;         // vector and points
  const TD EPS = 1e-9;
  const TD maxD = 1e9;
  TD cross(Pt a, Pt b, Pt c) {     // right hand rule
    TD v1 = a.first - c.first;     // (a-c) X (b-c)
    TD v2 = a.second - c.second;
    TD u1 = b.first - c.first;
    TD u2 = b.second - c.second;
    return v1 * u2 - v2 * u1;
  }
  TD cross(Pt a, Pt b) {           // a X b
    return a.first * b.second - a.second * b.first;
  }
  TD dot(Pt a, Pt b, Pt c) {       // (a-c) . (b-c)
    TD v1 = a.first - c.first;
    TD v2 = a.second - c.second;
    TD u1 = b.first - c.first;
    TD u2 = b.second - c.second;
    return v1 * u1 + v2 * u2;
  }
  TD dot(Pt a, Pt b) {             // a . b
    return a.first * b.first + a.second * b.second;
  }
  TD dist(Pt a, Pt b) {
    return sqrt((a.first - b.first) * (a.first - b.first) +
                (a.second - b.second) * (a.second - b.second));
  }
  TD shoelaceX2(vector<Pt>& convHull) {
    TD ret = 0;
    for(int i = 0, n = convHull.size(); i < n; i++)
      ret += cross(convHull[i], convHull[(i + 1) % n]);
    return ret;
  }
  vector<Pt> createConvexHull(vector<Pt>& points) {
    sort(points.begin(), points.end());
    vector<Pt> ret;
    for(int i = 0; i < points.size(); i++) {
      while(ret.size() > 1 &&
              cross(points[i], ret[ret.size() - 1], ret[ret.size() - 2]) < -EPS)
        ret.pop_back();
      ret.push_back(points[i]);
    }
    for(int i = points.size() - 2, sz = ret.size(); i >= 0; i--) {
      while(ret.size() > sz &&
              cross(points[i], ret[ret.size() - 1], ret[ret.size() - 2]) < -EPS)
        ret.pop_back();
      if(i == 0)
        break;
      ret.push_back(points[i]);
    }
    return ret;
    bool isInside(Pt pv, vector<Pt>& x) { //using winding number
      int n = x.size(), wn = 0;
      x.push_back(x[0]);
      for(int i = 0; i < n; ++i) {
        if(((x[i + 1].first <= pv.first && x[i].first >= pv.first) ||
            (x[i + 1].first >= pv.first && x[i].first <= pv.first)) &&
            ((x[i + 1].second <= pv.second && x[i].second >= pv.second) ||
             (x[i + 1].second >= pv.second && x[i].second <= pv.second))) {
          if(cross(x[i], x[i + 1], pv) == 0) {
            x.pop_back();
            return true;
          }
        }
      }
      for(int i = 0; i < n; ++i) {
        if(x[i].second <= pv.second) {
          if(x[i + 1].second > pv.second && cross(x[i], x[i + 1], pv) > 0)
            ++wn;
        } else if(x[i + 1].second <= pv.second && cross(x[i], x[i + 1], pv) < 0)
          --wn;
      }
      x.pop_back();
      return wn != 0;
    }
  }
  bool isInside(Pt pv, vector<Pt>& x) { //using winding number
    int n = x.size(), wn = 0;
    x.push_back(x[0]);
    for(int i = 0; i < n; ++i) {
      if(((x[i + 1].first <= pv.first && x[i].first >= pv.first) ||
          (x[i + 1].first >= pv.first && x[i].first <= pv.first)) &&
          ((x[i + 1].second <= pv.second && x[i].second >= pv.second) ||
           (x[i + 1].second >= pv.second && x[i].second <= pv.second))) {
        if(cross(x[i], x[i + 1], pv) == 0) {
          x.pop_back();
          return true;
        }
      }
    }
    for(int i = 0; i < n; ++i) {
      if(x[i].second <= pv.second) {
        if(x[i + 1].second > pv.second && cross(x[i], x[i + 1], pv) > 0)
          ++wn;
      } else if(x[i + 1].second <= pv.second && cross(x[i], x[i + 1], pv) < 0)
        --wn;
    }
    x.pop_back();
    return wn != 0;
  }
}
```



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## 4.3 Closest Pair of Points

```
#define fi first
#define se second
typedef pair<int, int> pii;
struct Point {
  int x, y, id;
};
int compareX(const void* a, const void* b) {
  Point* p1 = (Point*)a, * p2 = (Point*)b;
  return (p1->x - p2->x);
}
int compareY(const void* a, const void* b) {
  Point* p1 = (Point*)a, * p2 = (Point*)b;
  return (p1->y - p2->y);
}
double dist(Point p1, Point p2) {
  return sqrt((double)(p1.x - p2.x) * (p1.x - p2.x) +
              (double)(p1.y - p2.y) * (p1.y - p2.y)
             );
}
pair<pii, double> bruteForce(Point P[], int n) {
  double min = 1e8;
  pii ret = pii(-1, -1);
  for(int i = 0; i < n; ++i)
    for(int j = i + 1; j < n; ++j)
      if(dist(P[i], P[j]) < min) {
        ret = pii(P[i].id, P[j].id);
        min = dist(P[i], P[j]);
      }
  return pair<pii, double> (ret, min);
}
pair<pii, double> getmin(pair<pii, double> x, pair<pii, double> y) {
  if(x.fi.fi == -1 && x.fi.se == -1)
    return y;
  if(y.fi.fi == -1 && y.fi.se == -1)
    return x;
  return (x.se < y.se) ? x : y;
}
pair<pii, double> stripClosest(Point strip[], int size, double d) {
  double min = d;
  pii ret = pii(-1, -1);
  qsort(strip, size, sizeof(Point), compareY);
  for(int i = 0; i < size; ++i)
    for(int j = i + 1; j < size && (strip[j].y - strip[i].y) < min; ++j)
      if(dist(strip[i], strip[j]) < min) {
        ret = pii(strip[i].id, strip[j].id);
        min = dist(strip[i], strip[j]);
      }
  return pair<pii, double>(ret, min);
}
pair<pii, double> closestUtil(Point P[], int n) {
  if(n <= 3)
    return bruteForce(P, n);
  int mid = n / 2;
  Point midPoint = P[mid];
  pair<pii, double> dl = closestUtil(P, mid);
  pair<pii, double> dr = closestUtil(P + mid, n - mid);
  pair<pii, double> d = getmin(dl, dr);
  Point strip[n];
  int j = 0;
  for(int i = 0; i < n; i++)
    if(abs(P[i].x - midPoint.x) < d.second)
      strip[j] = P[i], j++;
  return getmin(d, stripClosest(strip, j, d.second));
}
pair<pii, double> closest(Point P[], int n) {
  qsort(P, n, sizeof(Point), compareX);
  return closestUtil(P, n);
}
Point P[50005];
int main() {
  int n;
  scanf("%d", &n);
  for(int a = 0; a < n; a++) {
    scanf("%d%d", &P[a].x, &P[a].y);
    P[a].id = a;
  }
  pair<pii, double> hasil = closest(P, n);
  if(hasil.fi.fi > hasil.fi.se)
    swap(hasil.fi.fi, hasil.fi.se);
  printf("%d %d %.6lf\n", hasil.fi.fi, hasil.fi.se, hasil.se);
  return 0;
}
```

## 4.4 Smallest Enclosing Circle

```
// welzl's algo to find the 2d minimum enclosing circle of a set of points
// expected O(N)
// directions: remove duplicates and shuffle points, then call welzl(points)

struct Point {
  double x;
  double y;
};

struct Circle {
  double x, y, r;
  Circle() {}
  Circle(double _x, double _y, double _r): x(_x), y(_y), r(_r) {}
};

Circle trivial(const vector<Point>& r) {
  if(r.size() == 0)
    return Circle(0, 0, -1);
  else if(r.size() == 1)
    return Circle(r[0].x, r[0].y, 0);
  else if(r.size() == 2) {
    double cx = (r[0].x + r[1].x) / 2.0, cy = (r[0].y + r[1].y) / 2.0;
    double rad = hypot(r[0].x - r[1].x, r[0].y - r[1].y) / 2.0;
    return Circle(cx, cy, rad);
  } else {
    double x0 = r[0].x, x1 = r[1].x, x2 = r[2].x;
    double y0 = r[0].y, y1 = r[1].y, y2 = r[2].y;
    double d = (x0 - x2) * (y1 - y2) - (x1 - x2) * (y0 - y2);
    double cx = (((x0 - x2) * (x0 + x2) + (y0 - y2) * (y0 + y2)) / 2 *
                 (y1 - y2) - ((x1 - x2) * (x1 + x2) + (y1 - y2) * (y1 + y2)) / 2
                 * (y0 - y2)) / d;
    double cy = (((x1 - x2) * (x1 + x2) + (y1 - y2) * (y1 + y2)) / 2 *
                 (x0 - x2) - ((x0 - x2) * (x0 + x2) + (y0 - y2) * (y0 + y2)) / 2
                 * (x1 - x2)) / d;
    return Circle(cx, cy, hypot(x0 - cx, y0 - cy));
  }
}

// SHUFFLE THE POINTS FIRST!!!!!!
Circle welzl(const vector<Point>& p, int idx = 0, vector<Point> r = {}) {
  if(idx == (int) p.size() || r.size() == 3)
    return trivial(r);
  Circle d = welzl(p, idx + 1, r);
  if(hypot(p[idx].x - d.x, p[idx].y - d.y) > d.r) {
    r.push_back(p[idx]);
    d = welzl(p, idx + 1, r);
  }
  return d;
}
```

## 4.5 Sutherland-Hodgman Algorithm

```
// Complexity: linear time
// Ada 2 poligon, cari poligon intersectionnya
// poly_point = hasilnya, clipper = pemotongnya
#include<bits/stdc++.h>
using namespace std;

const double EPS = 1e-9;

struct point {
  double x, y;
  point(double _x, double _y): x(_x), y(_y) {}
};
struct vec {
  double x, y;
  vec(double _x, double _y): x(_x), y(_y) {}
};

point pivot(0, 0);
vec toVec(point a, point b) {
  return vec(b.x - a.x, b.y - a.y);
}
double dist(point a, point b) {
  return hypot(a.x - b.x, a.y - b.y);
}
double cross(vec a, vec b) {
  return a.x * b.y - a.y * b.x;
}
bool ccw(point p, point q, point r) {
  return cross(toVec(p, q), toVec(p, r)) > 0;
}
bool collinear(point p, point q, point r) {
  return fabs(cross(toVec(p, q), toVec(p, r))) < EPS;
}
bool lies(point a, point b, point c) {
  if((c.x >= min(a.x, b.x) && c.x <= max(a.x, b.x)) &&
      (c.y >= min(a.y, b.y) && c.y <= max(a.y, b.y)))
    return true;
  else
    return false;
}
bool anglecmp(point a, point b) {
  if(collinear(pivot, a, b))
    return dist(pivot, a) < dist(pivot, b);
  double d1x = a.x - pivot.x, d1y = a.y - pivot.y;
  double d2x = b.x - pivot.x, d2y = b.y - pivot.y;
  return (atan2(d1y, d1x) - atan2(d2y, d2x)) < 0;
}

point intersect(point s1, point e1, point s2, point e2) {
  double x1, x2, x3, x4, y1, y2, y3, y4;
  x1 = s1.x;
  y1 = s1.y;
  x2 = e1.x;
  y2 = e1.y;
  x3 = s2.x;
  y3 = s2.y;
  x4 = e2.x;
  y4 = e2.y;
  double num1 = (x1 * y2 - y1 * x2) * (x3 - x4) - (x1 - x2) * (x3 * y4 - y3 * x4);
  double num2 = (x1 * y2 - y1 * x2) * (y3 - y4) - (y1 - y2) * (x3 * y4 - y3 * x4);
  double den = (x1 - x2) * (y3 - y4) - (y1 - y2) * (x3 - x4);
  double new_x = num1 / den;
  double new_y = num2 / den;
  return point(new_x, new_y);
}

void clip(vector <point>& poly_points, point point1, point point2) {
  vector <point> new_points;
  new_points.clear();
  for(int i = 0; i < poly_points.size(); i++) {
    int k = (i + 1) % poly_points.size();
    double i_pos = ccw(point1, point2, poly_points[i]);
    double k_pos = ccw(point1, point2, poly_points[k]);
    //in in
    if(i_pos <= 0  && k_pos <= 0)
      new_points.push_back(poly_points[k]);
    //out in
    else if(i_pos > 0  && k_pos <= 0) {
      new_points.push_back(intersect(point1, point2, poly_points[i],
                                      poly_points[k]));
      new_points.push_back(poly_points[k]);
    }
    // in out
    else if(i_pos <= 0  && k_pos > 0) {
      new_points.push_back(intersect(point1, point2, poly_points[i],
                                      poly_points[k]));
    }
    //out out
    else {
    }
  }
  poly_points.clear();
  for(int i = 0; i < new_points.size(); i++)
    poly_points.push_back(new_points[i]);
}
double area(const vector <point>& P) {
  double result = 0.0;
  double x1, y1, x2, y2;
  for(int i = 0; i < P.size() - 1; i++) {
    x1 = P[i].x;
    y1 = P[i].y;
    x2 = P[i + 1].x;
    y2 = P[i + 1].y;
    result += (x1 * y2 - x2 * y1);
  }
  return fabs(result) / 2;
}
void suthHodgClip(vector <point>& poly_points, vector <point> clipper_points) {
  for(int i = 0; i < clipper_points.size(); i++) {
    int k = (i + 1) % clipper_points.size();
    clip(poly_points, clipper_points[i], clipper_points[k]);
  }
}
vector<point> sortku(vector<point> P) {
  int P0 = 0;
  int i;
  for(i = 1; i < 3; i++) {
    if(P[i].y < P[P0].y || (P[i].y == P[P0].y && P[i].x > P[P0].x))
      P0 = i;
  }
  point temp = P[0];
  P[0] = P[P0];
  P[P0] = temp;
  pivot = P[0];
  sort(++P.begin(), P.end(), anglecmp);
  reverse(++P.begin(), P.end());
  return P;
}
int main {
  clipper_points = sortku(clipper_points);
  suthHodgClip(poly_points, clipper_points);
}
```

## 4.6 Centroid of Polygon

$C_x = \frac{1}{6A} \sum_{i=0}^{n-1} (x_i + x_{i+1})(x_i \, y_{i+1} - x_{i+1} \, y_i)$
$C_y = \frac{1}{6A} \sum_{i=0}^{n-1} (y_i + y_{i+1})(x_i \, y_{i+1} - x_{i+1} \, y_i)$



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### 4.7 Pick Theorem

$A$: Area of a simply closed lattice polygon
$B$: Number of lattice points on the edges
$I$: Number of points in the interior
$A = I + \frac{B}{2} - 1$

## 5 Graphs

### 5.1 Articulation Point and Bridge

```
// gr -> adj list
// vector vis, low -> initialize to -1
// int timer -> initialize to 0
void dfs(int pos, int dad = -1) {
  vis[pos] = low[pos] = timer++;
  int kids = 0;
  for(auto& i : gr[pos]) {
    if(i == dad)
      continue;
    if(vis[i] >= 0)
      low[pos] = min(low[pos], vis[i]);
    else {
      dfs(i, pos);
      low[pos] = min(low[pos], low[i]);
      if(low[i] > vis[pos])
        is_bridge(pos, i)
        if(low[i] >= vis[pos] && dad >= 0)
          is_articulation_point(pos)
          ++kids;
    }
  }
  if(dad == -1 && kids > 1)
    is_articulation_point(pos)
  }
```

### 5.2 SCC and Strong Orientation

```
#define N 10020
vector<int> adj[N];
bool vis[N], ins[N];
int disc[N], low[N], gr[N];
stack<int> st;
int id, grid;
void scc(int cur, int par) {
  disc[cur] = low[cur] = ++id;
  vis[cur] = ins[cur] = 1;
  st.push(cur);
  for(int to : adj[cur]) {
    //if (to==par) continue; // ini untuk SO(scc undirected)
    if(!vis[to])
      scc(to, cur);
    if(ins[to])
      low[cur] = min(low[cur], low[to]);
  }
  if(low[cur] == disc[cur]) {
    grid++; // group id
    while(ins[cur]) {
      gr[st.tp] = grid;
      ins[st.tp] = 0;
      st.pop();
    }
  }
}
```

### 5.3 Centroid Decomposition

```
int build_cen(int nw) {
  com_cen(nw, 0); //fungsi untuk itung size subtree
  int siz = sz[nw] / 2;
  bool found = false;
  while(!found) {
    found = true;
    for(int i : v[nw]) {
      if(!rem[i] && sz[i] < sz[nw]) {
        if(sz[i] > siz) {
          found = false;
          nw = i;
          break;
        }
      }
    }
  }
  big
  rem[nw] = true;
  for(int i : v[nw])if(!rem[i])
      par_cen[build_cen(i)] = nw;
  return nw;
}
```

### 5.4 Dinic's Maximum Flow

```
// O(VE log(max_flow)) if scaling == 1
// O((V + E) sqrt(E)) if unit graph (turn scaling off)
// O((V + E) sqrt(V)) if bipartite matching (turn scaling off)
// indices are 0-based
const ll INF = 1e18;

struct Dinic {
  struct Edge {
    int v;
    ll cap, flow;
    Edge(int _v, ll _cap): v(_v), cap(_cap), flow(0) {}
  };

  int n;
  ll lim;
  vector<vector<int>> gr;
  vector<Edge> e;
  vector<int> idx, lv;

  bool has_path(int s, int t) {
    queue<int> q;
    q.push(s);
    lv.assign(n, -1);
    lv[s] = 0;
    while(!q.empty()) {
      int c = q.front();
      q.pop();
      if(c == t)
        break;
      for(auto& i : gr[c]) {
        ll cur_flow = e[i].cap - e[i].flow;
        if(lv[e[i].v] == -1 && cur_flow >= lim) {
          lv[e[i].v] = lv[c] + 1;
          q.push(e[i].v);
        }
      }
    }
    return lv[t] != -1;
  }

  ll get_flow(int s, int t, ll left) {
    if(!left || s == t)
      return left;
    while(idx[s] < (int) gr[s].size()) {
```


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```
      int i = gr[s][idx[s]];
      if(lv[e[i].v] == lv[s] + 1) {
        ll add = get_flow(e[i].v, t, min(left, e[i].cap - e[i].flow));
        if(add) {
          e[i].flow += add;
          e[i ^ 1].flow -= add;
          return add;
        }
      }
      ++idx[s];
    }
    return 0;
  }

  Dinic(int vertices, bool scaling = 1) : // toggle scaling here
    n(vertices), lim(scaling ? 1 << 30 : 1), gr(n) {}

  void add_edge(int from, int to, ll cap, bool directed = 1) {
    gr[from].push_back(e.size());
    e.emplace_back(to, cap);
    gr[to].push_back(e.size());
    e.emplace_back(from, directed ? 0 : cap);
  }

  ll get_max_flow(int s, int t) { // call this
    ll res = 0;
    while(lim) {  // scaling
      while(has_path(s, t)) {
        idx.assign(n, 0);
        while(ll add = get_flow(s, t, INF))
          res += add;
      }
      lim >>= 1;
    }
    return res;
  }
};
```

## 5.5 Minimum Cost Maximum Flow

```
using FlowT = ll;
using CostT = ll;

const FlowT F_INF = 1e18;
const CostT C_INF = 1e18;
const int MAX_V = 1e5 + 5;
const int MAX_E = 1e6 + 5;

namespace MCMF {
  int n, E;
  int adj[MAX_E], nxt[MAX_E], lst[MAX_V], frm[MAX_V], vis[MAX_V];
  FlowT cap[MAX_E], flw[MAX_E], totalFlow;
  CostT cst[MAX_E], dst[MAX_V], totalCost;

  void init(int _n) {
    n = _n;
    fill_n(lst, n, -1), E = 0;
  }
  void add(int u, int v, FlowT ca, CostT co) {
    adj[E] = v, cap[E] = ca, flw[E] = 0, cst[E] = +co;
    nxt[E] = lst[u], lst[u] = E++;
    adj[E] = u, cap[E] =  0, flw[E] = 0, cst[E] = -co;
    nxt[E] = lst[v], lst[v] = E++;
  }
  int spfa(int s, int t) {
    fill_n(dst, n, C_INF), dst[s] = 0;
    queue<int> que;
    que.push(s);
    while(que.size()) {
      int u = que.front();
      que.pop();
      for(int e = lst[u]; e != -1; e = nxt[e])
        if(flw[e] < cap[e]) {
          int v = adj[e];
          if(dst[v] > dst[u] + cst[e]) {
            dst[v] = dst[u] + cst[e];
            frm[v] = e;
            if(!vis[v]) {
              vis[v] = 1;
              que.push(v);
            }
          }
        }
      vis[u] = 0;
    }
    return dst[t] < C_INF;
  }
  pair<FlowT, CostT> solve(int s, int t) {
    totalCost = 0, totalFlow = 0;
    while(1) {
      if(!spfa(s, t))
        break;
      FlowT mn = F_INF;
      for(int v = t, e = frm[v]; v != s; v = adj[e ^ 1], e = frm[v])
        mn = min(mn, cap[e] - flw[e]);
      for(int v = t, e = frm[v]; v != s; v = adj[e ^ 1], e = frm[v]) {
        flw[e] += mn;
        flw[e ^ 1] -= mn;
      }
      totalFlow += mn;
      totalCost += mn * dst[t];
    }
    return {totalFlow, totalCost};
  }
};
```

## 5.6 Flows with Demands

```
let S0 be the source and T0 be the original sink
1. add 2 additional nodes, call them S1 and T1
2. connect S0 to nodes normally
3. connect nodes to T0 normally
4. for each edge(U, V), cap = original cap - demand
5. for each node N :
   1. add an edge(S1, N), cap = sum of inward demand to N
   2. add an edge(N, T1), cap = sum of outward demand from N
6. add an edge(T0, S0), cap = INF
7. the above is not a typo!
8. run max flow normally
9. for each edge(S1, V) and (U, T1), check if flow == cap
if step #9 fails, then it is not possible to satisfy the given demand
```

Mathematically, let $d(e)$ be the demand of edge $e$. Let $V$ be the set of every vertex in the graph.

- $c'(S_1, v) = \sum_{u \in V} d(u, v)$ for each edge $(s', v)$.
- $c'(v, T_1) = \sum_{v \in V} d(v, w)$ for each edge $(v, t')$.
- $c'(u, v) = c(u, v) - d(u, v)$ for each edge $(u, v)$ in the old network.
- $c'(T_0, S_0) = \infty$

## 5.7 Hungarian

```
template <typename TD> struct Hungarian {
  TD INF = 1e9;  //max_inf
  int n;
  vector<vector<TD> > adj; // cost[left][right]
  vector<TD> hl, hr, slk;
```

```
  vector<int> fl, fr, vl, vr, pre;
  deque<int> q;
  Hungarian(int _n) {
    n = _n;
    adj = vector<vector<TD> >(n, vector<TD> (n, 0));
  }
  int check(int i) {
    if(vl[i] = 1, fl[i] != -1)
      return q.push_back(fl[i]), vr[fl[i]] = 1;
    while(i != -1)
      swap(i, fr[fl[i] = pre[i]]);
    return 0;
  }
  void bfs(int s) {
    slk.assign(n, INF);
    vl.assign(n, 0);
    vr = vl;
    q.assign(vr[s] = 1, s);
    for(TD d;;) {
      for(; !q.empty(); q.pop_front()) {
        for(int i = 0, j = q.front(); i < n; i++) {
          if(d = hl[i] + hr[j] - adj[i][j], !vl[i] && d <= slk[i]) {
            if(pre[i] = j, d)
              slk[i] = d;
            else if(!check(i))
              return;
          }
        }
      }
      d = INF;
      for(int i = 0; i < n; i++) if(!vl[i] && d > slk[i])
          d = slk[i];
      for(int i = 0; i < n; i++) {
        if(vl[i])
          hl[i] += d;
        else
          slk[i] -= d;
        if(vr[i])
          hr[i] -= d;
      }
      for(int i = 0; i < n; i++) if(!vl[i] && !slk[i] && !check(i))
          return;
    }
  }
  TD solve() {
    fl.assign(n, -1);
    fr = fl;
    hl.assign(n, 0);
    hr = hl;
    pre.assign(n, 0);
    for(int i = 0; i < n; i++)
      hl[i] = *max_element(adj[i].begin(), adj[i].begin() + n);
    for(int i = 0; i < n; i++)
      bfs(i);
    TD ret = 0;
    for(int i = 0; i < n; i++) if(adj[i][fl[i]])
        ret += adj[i][fl[i]];
    return ret;
  }
}; //i wiLL be matched with fl[i]
```

## 5.8 Edmonds' Blossom

```
// Maximum matching on general graphs in O(V^2 E)
// Indices are 1-based
// Stolen from ko_osaga's cheatsheet
struct Blossom {
  vector<int> vis, dad, orig, match, aux;
  vector<vector<int>> conn;
  int t, N;
  queue<int> Q;

  void augment(int u, int v) {
    int pv = v;
    do {
      pv = dad[v];
      int nv = match[pv];
      match[v] = pv;
      match[pv] = v;
      v = nv;
    } while(u != pv);
  }

  int lca(int v, int w) {
    ++t;
    while(true) {
      if(v) {
        if(aux[v] == t)
          return v;
        aux[v] = t;
        v = orig[dad[match[v]]];
      }
      swap(v, w);
    }
  }

  void blossom(int v, int w, int a) {
    while(orig[v] != a) {
      dad[v] = w;
      w = match[v];
      if(vis[w] == 1) {
        Q.push(w);
        vis[w] = 0;
      }
      orig[v] = orig[w] = a;
      v = dad[w];
    }
  }

  bool bfs(int u) {
    fill(vis.begin(), vis.end(), -1);
    iota(orig.begin(), orig.end(), 0);
    Q = queue<int>();
    Q.push(u);
    vis[u] = 0;
    while(!Q.empty()) {
      int v = Q.front();
      Q.pop();
      for(int x : conn[v]) {
        if(vis[x] == -1) {
          dad[x] = v;
          vis[x] = 1;
          if(!match[x]) {
            augment(u, x);
            return 1;
          }
          Q.push(match[x]);
          vis[match[x]] = 0;
        } else if(vis[x] == 0 && orig[v] != orig[x]) {
          int a = lca(orig[v], orig[x]);
          blossom(x, v, a);
          blossom(v, x, a);
        }
      }
    }
    return false;
  }

  Blossom(int n) : // n = vertices
    vis(n + 1), dad(n + 1), orig(n + 1), match(n + 1),
    aux(n + 1), conn(n + 1), t(0), N(n) {
```

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```
    for(int i = 0; i <= n; ++i) {
      conn[i].clear();
      match[i] = aux[i] = dad[i] = 0;
    }
  }

  void add_edge(int u, int v) {
    conn[u].push_back(v);
    conn[v].push_back(u);
  }

  int solve() { // call this for answer
    int ans = 0;
    vector<int> V(N - 1);
    iota(V.begin(), V.end(), 1);
    shuffle(V.begin(), V.end(), mt19937(0x94949));
    for(auto x : V) {
      if(!match[x]) {
        for(auto y : conn[x]) {
          if(!match[y]) {
            match[x] = y, match[y] = x;
            ++ans;
            break;
          }
        }
      }
    }
    for(int i = 1; i <= N; ++i) {
      if(!match[i] && bfs(i))
        ++ans;
    }
    return ans;
  }
};
```

### 5.9 Eulerian Path or Cycle

```
// finds a eulerian path / cycle
// visits each edge only once
// properties:
// - cycle: degrees are even
// - path: degrees are even OR degrees are even except for 2 vertices
// how to use: g = adjacency list g[n] = connected to n, undirected
// if there is a vertex u with an odd degree, call dfs(u)
// else call on any vertex
// ans = path result

vector<set<int>> g;
vector<int> ans;

void dfs(int u) {
  while(g[u].size()) {
    int v = *g[u].begin();
    g[u].erase(v);
    g[v].erase(u);
    dfs(v);
  }
  ans.push_back(u);
}
```

### 5.10 Hierholzer's Algorithm

```
// Eulerian on Directed Graph
stack<int> path;
vector<int> euler;
inline void hierholzer() {
  path.push(0);
  int cur = 0;
  while(!path.empty()) {
    if(!adj[cur].empty()) {
      path.push(cur);
      int next = adj[cur].back();
      adj[cur].pob();
      cur = next;
    } else {
      euler.pb(cur);
      cur = path.top();
      path.pop();
    }
  }
  reverse(euler.begin(), euler.end());
}
```

### 5.11 2-SAT

```
struct TwoSAT {
  int n;
  vector<vector<int>> g, gr;
  vector<int> comp, topological_order, answer;
  vector<bool> vis;

  TwoSAT() {}
  TwoSAT(int _n) :
    n(_n), g(2 * n), gr(2 * n), comp(2 * n), answer(2 * n), vis(2 * n) {}

  void add_edge(int u, int v) {
    g[u].push_back(v);
    gr[v].push_back(u);
  }

  // For the following three functions
  // int x, bool val: if 'val' is true, we take the variable to be x.
  // Otherwise we take it to be x's complement.

  // At least one of them is true
  void add_clause_or(int i, bool f, int j, bool p) {
    add_edge(i + (f ? n : 0), j + (p ? 0 : n));
    add_edge(j + (p ? n : 0), i + (f ? 0 : n));
  }

  // Only one of them is true
  void add_clause_xor(int i, bool f, int j, bool p) {
    add_clause_or(i, f, j, p);
    add_clause_or(i, !f, j, !p);
  }

  // Both of them have the same value
  void add_clause_and(int i, bool f, int j, bool p) {
    add_clause_xor(i, !f, j, p);
  }

  // Topological sort
  void dfs(int u) {
    vis[u] = true;
    for(const auto& v : g[u])
      if(!vis[v])
        dfs(v);
    topological_order.push_back(u);
  }

  // Extracting strongly connected components
  void scc(int u, int id) {
    vis[u] = true;
    comp[u] = id;
    for(const auto& v : gr[u])
      if(!vis[v])
        scc(v, id);
  }
```



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```cpp
  bool satisfiable() {
    fill(vis.begin(), vis.end(), false);
    for(int i = 0; i < 2 * n; i++)
      if(!vis[i])
        dfs(i);
    fill(vis.begin(), vis.end(), false);
    reverse(topological_order.begin(), topological_order.end());
    int id = 0;
    for(const auto& v : topological_order)
      if(!vis[v])
        scc(v, id++);
    // Constructing the answer
    for(int i = 0; i < n; i++) {
      if(comp[i] == comp[i + n])
        return false;
      answer[i] = (comp[i] > comp[i + n] ? 1 : 0);
    }
    return true;
  }
};
```

# 6 Math

## 6.1 Extended Euclidean GCD

```cpp
// computes x and y such that ax + by = gcd(a, b) in O(log (min(a, b)))
// returns {gcd(a, b), x, y}
tuple<int, int, int> gcd(int a, int b) {
  if(b == 0) return {a, 1, 0};
  auto [d, x1, y1] = gcd(b, a % b);
  return {d, y1, x1 - y1* (a / b)};
}
```

## 6.2 Generalized CRT

```cpp
template<typename T>
T extended_euclid(T a, T b, T& x, T& y) {
  if(b == 0) {
    x = 1;
    y = 0;
    return a;
  }
  T xx, yy, gcd;
  gcd = extended_euclid(b, a % b, xx, yy);
  x = yy;
  y = xx - (yy * (a / b));
  return gcd;
}
template<typename T>
T MOD(T a, T b) {
  return (a % b + b) % b;
}
// return x, lcm. x = a % n && x = b % m
template<typename T>
pair<T, T> CRT(T a, T n, T b, T m) {
  T _n, _m;
  T gcd = extended_euclid(n, m, _n, _m);
  if(n == m) {
    if(a == b)
      return pair<T, T>(a, n);
    else
      return pair<T, T>(-1, -1);
  } else if(abs(a - b) % gcd != 0)
    return pair<T, T>(-1, -1);
  else {
    T lcm = m * n / gcd;
    T x = MOD(a + MOD(n * MOD(_n * ((b - a) / gcd), m / gcd), lcm), lcm);
    return pair<T, T>(x, lcm);
  }
}
```

## 6.3 Generalized Lucas Theorem

```cpp
/*Special Lucas : (n,k) % p^x
  fctp[n] =  Product of the integers less than or equal
  to n that are not divisible by p
  Precompute fctp*/
LL p
LL E(LL n, int m) {
  LL tot = 0;
  while(n != 0)
    tot += n / m, n /= m;
  return tot;
}
LL funct(LL n, LL base) {
  LL ans = fast(fctp[base], n / base, base) * fctp[n % base] % base;
  return ans;
}
LL F(LL n, LL base) {
  LL ans = 1;
  while(n != 0) {
    ans = (ans * funct(n, base)) % base;
    n /= p;
  }
  return ans;
}
LL special_lucas(LL n, LL r, LL base) {
  p = fprime(base);
  LL pow = E(n, p) - E(n - r, p) - E(r, p);
  LL TOP = fast(p, pow, base) * F(n, base) % base;
  LL BOT = F(r, base) * F(n - r, base) % base;
  return (TOP * fast(BOT, totien(base) - 1, base)) % base;
}
//End of Special Lucas
```

## 6.4 Linear Diophantine

```cpp
//FOR SOLVING MINIMUM ABS(X) + ABS(Y)
ll x, y, newX, newY, target = 0;
ll extGcd(ll a, ll b) {
  if(b == 0) {
    x = 1, y = 0;
    return a;
  }
  ll ret = extGcd(b, a % b);
  newX = y;
  newY = x - y * (a / b);
  x = newX;
  y = newY;
  return ret;
}
ll fix(ll sol, ll rt) {
  ll ret = 0;
  //CASE SOLUTION(X/Y) < TARGET
  if(sol < target)
    ret = -floor(abs(sol + target) / (double)rt);
  //CASE SOLUTION(X/Y) > TARGET
  if(sol > target)
    ret = ceil(abs(sol - target) / (double)rt);
  return ret;
}
ll work(ll a, ll b, ll c) {
  ll gcd = extGcd(a, b);
  ll solX = x * (c / gcd);
  ll solY = y * (c / gcd);
  a /= gcd;
```


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```
  b /= gcd;
  ll fi = abs(fix(solX, b));
  ll se = abs(fix(solY, a));
  ll lo = min(fi, se);
  ll hi = max(fi, se);
  ll ans = abs(solX) + abs(solY);
  for(ll i = lo; i <= hi; i++) {
    ans = min(ans, abs(solX + i * b) + abs(solY - i * a));
    ans = min(ans, abs(solX - i * b) + abs(solY + i * a));
  }
  return ans;
}
```

## 6.5 Modular Linear Equation

```
// finds all solutions to ax = b (mod n)
vi modular_linear_equation_solver(int a, int b, int n) {
  int x, y;
  vi ret;
  int g = extended_euclid(a, n, x, y);
  if(!(b % g)) {
    x = mod(x * (b / g), n);
    for(int i = 0; i < g; i++)
      ret.push_back(mod(x + i * (n / g), n));
  }
  return ret;
}
```

## 6.6 Miller-Rabin and Pollard's Rho

```
namespace MillerRabin {
  const vector<ll> primes = { // deterministic up to 2^64 - 1
    2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37
  };
  ll gcd(ll a, ll b) {
    return b ? gcd(b, a % b) : a;
  }
  ll powa(ll x, ll y, ll p) { // (x ^ y) % p
    if(!y)
      return 1;
    if(y & 1)
      return ((__int128) x * powa(x, y - 1, p)) % p;
    ll temp = powa(x, y >> 1, p);
    return ((__int128) temp * temp) % p;
  }
  bool miller_rabin(ll n, ll a, ll d, int s) {
    ll x = powa(a, d, n);
    if(x == 1 || x == n - 1)
      return 0;
    for(int i = 0; i < s; ++i) {
      x = ((__int128) x * x) % n;
      if(x == n - 1)
        return 0;
    }
    return 1;
  }
  bool is_prime(ll x) { // use this
    if(x < 2)
      return 0;
    int r = 0;
    ll d = x - 1;
    while((d & 1) == 0) {
      d >>= 1;
      ++r;
    }
    for(auto& i : primes) {
      if(x == i)
        return 1;
      if(miller_rabin(x, i, d, r))
        return 0;
    }
    return 1;
  }
}

namespace PollardRho {
  mt19937_64 generator(chrono::steady_clock::now()
                       .time_since_epoch().count());
  uniform_int_distribution<ll> rand_ll(0, LLONG_MAX);
  ll f(ll x, ll b, ll n) { // (x^2 + b) % n
    return (((__int128) x * x) % n + b) % n;
  }
  ll rho(ll n) {
    if(n % 2 == 0)
      return 2;
    ll b = rand_ll(generator);
    ll x = rand_ll(generator);
    ll y = x;
    while(1) {
      x = f(x, b, n);
      y = f(f(y, b, n), b, n);
      ll d = MillerRabin::gcd(abs(x - y), n);
      if(d != 1)
        return d;
    }
  }
  void pollard_rho(ll n, vector<ll>& res) {
    if(n == 1)
      return;
    if(MillerRabin::is_prime(n)) {
      res.push_back(n);
      return;
    }
    ll d = rho(n);
    pollard_rho(d, res);
    pollard_rho(n / d, res);
  }
  vector<ll> factorize(ll n, bool sorted = 1) { // use this
    vector<ll> res;
    pollard_rho(n, res);
    if(sorted)
      sort(res.begin(), res.end());
    return res;
  }
}
```

## 6.7 Berlekamp-Massey

```
#include <bits/stdc++.h>
using namespace std;
#define pb push_back
typedef long long ll;
#define SZ 233333
const int MOD = 1e9 + 7; //or any prime
ll qp(ll a, ll b) {
  ll x = 1;
  a %= MOD;
  while(b) {
    if(b & 1)
      x = x * a % MOD;
    a = a * a % MOD;
    b >>= 1;
  }
  return x;
}
namespace linear_seq {
  vector<int> BM(vector<int> x) {
    //ls: (shortest) relation sequence (after filling zeroes) so far
    //cur: current relation sequence
```


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```
    vector<int> ls, cur;
    //lf: the position of ls (t')
    //ld: delta of ls (v')
    int lf = -1, ld = -1;
    for(int i = 0; i < int(x.size()); ++i) {
      ll t = 0;
      //evaluate at position i
      for(int j = 0; j < int(cur.size()); ++j)
        t = (t + x[i - j - 1] * (ll)cur[j]) % MOD;
      if((t - x[i]) % MOD == 0) {
        continue;  //good so far
      }
      //first non-zero position
      if(!cur.size()) {
        cur.resize(i + 1);
        lf = i;
        ld = (t - x[i]) % MOD;
        continue;
      }
      //cur=cur-c/ld*(x[i]-t)
      ll k = -(x[i] - t) * qp(ld, MOD - 2) % MOD/*1/ld*/;
      vector<int> c(i - lf - 1); //add zeroes in front
      c.pb(k);
      for(int j = 0; j < int(ls.size()); ++j)
        c.pb(-ls[j]*k % MOD);
      if(c.size() < cur.size())
        c.resize(cur.size());
      for(int j = 0; j < int(cur.size()); ++j)
        c[j] = (c[j] + cur[j]) % MOD;
      //if cur is better than ls, change ls to cur
      if(i - lf + (int)ls.size() >= (int)cur.size())
        ls = cur, lf = i, ld = (t - x[i]) % MOD;
      cur = c;
    }
    for(int i = 0; i < int(cur.size()); ++i)
      cur[i] = (cur[i] % MOD + MOD) % MOD;
    return cur;
  }
  int m; //length of recurrence
//a: first terms
//h: relation
  ll a[SZ], h[SZ], t_[SZ], s[SZ], t[SZ];
//calculate p*q mod f
  void mull(ll* p, ll* q) {
    for(int i = 0; i < m + m; ++i)
      t_[i] = 0;
    for(int i = 0; i < m; ++i) if(p[i])
        for(int j = 0; j < m; ++j)
          t_[i + j] = (t_[i + j] + p[i] * q[j]) % MOD;
    for(int i = m + m - 1; i >= m; --i) if(t_[i])
        //miuns t_[i]x^{i-m}(x^m-\sum_{j=0}^{m-1} x^{m-j-1}h_j)
        for(int j = m - 1; ~j; --j)
          t_[i - j - 1] = (t_[i - j - 1] + t_[i] * h[j]) % MOD;
    for(int i = 0; i < m; ++i)
      p[i] = t_[i];
  }
  ll calc(ll K) {
    for(int i = m; ~i; --i)
      s[i] = t[i] = 0;
    //init
    s[0] = 1;
    if(m != 1)
      t[1] = 1;
    else
      t[0] = h[0];
    //binary-exponentiation
    while(K) {
      if(K & 1)
        mull(s, t);
      mull(t, t);
      K >>= 1;
    }
    ll su = 0;
    for(int i = 0; i < m; ++i)
      su = (su + s[i] * a[i]) % MOD;
    return (su % MOD + MOD) % MOD;
  }
  int work(vector<int> x, ll n) {
    if(n < int(x.size()))
      return x[n];
    vector<int> v = BM(x);
    m = v.size();
    if(!m)
      return 0;
    for(int i = 0; i < m; ++i)
      h[i] = v[i], a[i] = x[i];
    return calc(n);
  }
}
using linear_seq::work;

const vector<int> sequence = {
  0, 2, 2, 28, 60, 836, 2766
};
int main() {
  cout << work(sequence, 7) << '\n';
}
```

## 6.8 Fast Fourier Transform

```
using ld = double; // change to long double if reach 10^18
using cd = complex<ld>;
const ld PI = acos(-(ld)1);

void fft(vector<cd>& a, int sign = 1) {
  int n = a.size();
  ld theta = sign * 2 * PI / n;
  for(int i = 0, j = 1; j < n - 1; j++) {
    for(int k = n >> 1; k > (i ^= k); k >>= 1);
    if(j < i)
      swap(a[i], a[j]);
  }
  for(int m, mh = 1; (m = mh << 1) <= n; mh = m) {
    int irev = 0;
    for(int i = 0; i < n; i += m) {
      cd w = exp(cd(0, theta * irev));
      for(int k = n >> 2; k > (irev ^= k); k >>= 1);
      for(int j = i; j < mh + i; j++) {
        int k = j + mh;
        cd x = a[j] - a[k];
        a[j] += a[k];
        a[k] = w * x;
      }
    }
  }
  if(sign == -1) for(cd& i : a)
      i /= n;
}

vector<ll> multiply(vector<ll> const& a, vector<ll> const& b) {
  vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
  int n = 1;
  while(n < a.size() + b.size())
    n <<= 1;
  fa.resize(n);
  fb.resize(n);
  fft(fa);
  fft(fb);
  for(int i = 0; i < n; i++)
    fa[i] *= fb[i];
  fft(fa, -1);
```

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```
  vector<ll> res(n);
  for(int i = 0; i < n; i++)
    res[i] = round(fa[i].real());
  return res;
}
```

### 6.9 Number Theoretic Transform

```
namespace FFT {
  /* ----- Adjust the constants here ----- */
  const int LN = 24; //23
  const int N = 1 << LN;
  typedef long long LL; // 2**23 * 119 + 1. 998244353
// `MOD` must be of the form 2**`LN` * k + 1, where k odd.
  const LL MOD = 9223372036737335297; // 2**24 * 54975513881 + 1.
  const LL PRIMITIVE_ROOT = 3; // Primitive root modulo `MOD`.
  /* ----- End of constants ----- */
  LL root[N];
  inline LL power(LL x, LL y) {
    LL ret = 1;
    for(; y; y >>= 1) {
      if(y & 1)
        ret = (__int128) ret * x % MOD;
      x = (__int128) x * x % MOD;
    }
    return ret;
  }
  inline void init_fft() {
    const LL UNITY = power(PRIMITIVE_ROOT, MOD - 1 >> LN);
    root[0] = 1;
    for(int i = 1; i < N; i++)
      root[i] = (__int128) UNITY * root[i - 1] % MOD;
    return;
  }
// n = 2^k is the length of polynom
  inline void fft(int n, vector<LL>& a, bool invert) {
    for(int i = 1, j = 0; i < n; ++i) {
      int bit = n >> 1;
      for(; j >= bit; bit >>= 1)
        j -= bit;
      j += bit;
      if(i < j)
        swap(a[i], a[j]);
    }
    for(int len = 2; len <= n; len <<= 1) {
      LL wlen = (invert ? root[N - N / len] : root[N / len]);
      for(int i = 0; i < n; i += len) {
        LL w = 1;
        for(int j = 0; j<len >> 1; j++) {
          LL u = a[i + j];
          LL v = (__int128) a[i + j + len / 2] * w % MOD;
          a[i + j] = ((__int128) u + v) % MOD;
          a[i + j + len / 2] = ((__int128) u - v + MOD) % MOD;
          w = (__int128) w * wlen % MOD;
        }
      }
    }
    if(invert) {
      LL inv = power(n, MOD - 2);
      for(int i = 0; i < n; i++)
        a[i] = (__int128) a[i] * inv % MOD;
    }
    return;
  }
  inline vector<LL> multiply(vector<LL> a, vector<LL> b) {
    vector<LL> c;
    int len = 1 << 32 - __builtin_clz(a.size() + b.size() - 2);
    a.resize(len, 0);
    b.resize(len, 0);
    fft(len, a, false);
    fft(len, b, false);
    c.resize(len);
    for(int i = 0; i < len; ++i)
      c[i] = (__int128) a[i] * b[i] % MOD;
    fft(len, c, true);
    return c;
  }
//FFT::init_fft(); wajib di panggil init di awal
}
```

### 6.10 Gauss-Jordan

```
// Gauss-Jordan elimination with full pivoting.
//
// Uses:
//   (1) solving systems of linear equations (AX=B)
//   (2) inverting matrices (AX=I)
//   (3) computing determinants of square matrices
//
// Running time: O(n^3)
//
// INPUT:    a[][] = an nxn matrix
//           b[][] = an nxm matrix
//
// OUTPUT:   X      = an nxm matrix (stored in b[][])
//           A^{-1} = an nxn matrix (stored in a[][])
//           returns determinant of a[][]
const double EPS = 1e-10;

typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
T GaussJordan(VVT& a, VVT& b) {
  const int n = a.size();
  const int m = b[0].size();
  VI irow(n), icol(n), ipiv(n);
  T det = 1;
  for(int i = 0; i < n; i++) {
    int pj = -1, pk = -1;
    for(int j = 0; j < n; j++) if(!ipiv[j])
        for(int k = 0; k < n; k++) if(!ipiv[k])
            if(pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) {
              pj = j;
              pk = k;
            }
    if(fabs(a[pj][pk]) < EPS) {
      cerr << "Matrix is singular." << endl;
      exit(0);
    }
    ipiv[pk]++;
    swap(a[pj], a[pk]);
    swap(b[pj], b[pk]);
    if(pj != pk)
      det *= -1;
    irow[i] = pj;
    icol[i] = pk;
    T c = 1.0 / a[pk][pk];
    det *= a[pk][pk];
    a[pk][pk] = 1.0;
    for(int p = 0; p < n; p++)
      a[pk][p] *= c;
    for(int p = 0; p < m; p++)
      b[pk][p] *= c;
    for(int p = 0; p < n; p++) if(p != pk) {
        c = a[p][pk];
        a[p][pk] = 0;
        for(int q = 0; q < n; q++)
          a[p][q] -= a[pk][q] * c;
        for(int q = 0; q < m; q++)
```



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```
          b[p][q] -= b[pk][q] * c;
      }
  }
  for(int p = n - 1; p >= 0; p--) if(irow[p] != icol[p]) {
      for(int k = 0; k < n; k++)
        swap(a[k][irow[p]], a[k][icol[p]]);
    }
  return det;
}
int main() {
  const int n = 4;
  const int m = 2;
  double A[n][n] = { {1, 2, 3, 4}, {1, 0, 1, 0}, {5, 3, 2, 4}, {6, 1, 4, 6} };
  double B[n][m] = { {1, 2}, {4, 3}, {5, 6}, {8, 7} };
  VVT a(n), b(n);
  for(int i = 0; i < n; i++) {
    a[i] = VT(A[i], A[i] + n);
    b[i] = VT(B[i], B[i] + m);
  }
  double det = GaussJordan(a, b);
  // expected: 60
  cout << "Determinant: " << det << endl;
  // expected: -0.233333 0.166667 0.133333 0.0666667
  //           0.166667 0.166667 0.333333 -0.333333
  //           0.233333 0.833333 -0.133333 -0.0666667
  //           0.05 -0.75 -0.1 0.2
  cout << "Inverse: " << endl;
  for(int i = 0; i < n; i++) {
    for(int j = 0; j < n; j++)
      cout << a[i][j] << ' ';
    cout << endl;
  }
  // expected: 1.63333 1.3
  //           -0.166667 0.5
  //           2.36667 1.7
  //           -1.85 -1.35
  cout << "Solution: " << endl;
  for(int i = 0; i < n; i++) {
    for(int j = 0; j < m; j++)
      cout << b[i][j] << ' ';
    cout << endl;
  }
}
```

### 6.11 Derangement

```
der[0] = 1;
der[1] = 0;
for(int i = 2; i <= 10; ++i)
  der[i] = (ll)(i - 1) * (der[i - 1] + der[i - 2]);
```

### 6.12 Bernoulli Number

$\sum_{k=1}^{n} k^m = \frac{1}{m+1}\sum_{i=0}^{m}\binom{m+1}{i}B_i^+ n^{m+1-i} = m!\sum_{i=0}^{m}\frac{B_i^+ n^{m+1-i}}{i!(m+1-i)!}$

$B_n^+ = 1 - \sum_{i=0}^{n-1}\binom{n}{i}\frac{B_i^+}{n-i+1}, \quad B_0^+ = 1$

### 6.13 Forbenius Number

(X* Y) - (X + Y) and total count is(X - 1)* (Y - 1) / 2

### 6.14 Stars and Bars with Upper Bound

$P = (1 - X^{r_1+1})\ldots(1 - X^{r_n+1}) = \sum_i c_i X^{e_i}$

$Ans = \sum_i c_i\binom{N-e_i+n-1}{n-1}$

### 6.15 Arithmetic Sequences

$U_n = a + (n-1)a_1 + \frac{(n-1)(n-2)}{1\times 2}a_2 + \ldots + \frac{(n-1)(n-2)(n-3)\ldots}{1\times 2\times 3\times\ldots}a_r$

$S_n = n \times a + \frac{n(n-1)}{1\times 2}a_1 + \frac{n(n-1)(n-2)}{1\times 2\times 3}a_2 + \ldots + \frac{n(n-1)(n-2)(n-3)\ldots}{1\times 2\times 3\ldots}a_r$

### 6.16 FWHT

```
// Desc  : Transform a polynom to obtain a_i * b_j * x^(i XOR j) or combinations
// Time  : O(N log N) with N = 2^K
// OP  => c00 c01 c10 c11 | c00 c01 c10 c11 inv
// XOR =>  +1  +1  +1  -1 |  +1  +1  +1  -1 | div the inverse with size = n
// AND =>   1  +1   0   1 |   1  -1   0   1 | no comment
//  OR =>   1   0  +1   1 |   1   0  -1   1 | no comment
typedef vector<long long> vec;
void FWHT(vec& a) {
  int n = a.size();
  for(int lvl = 1; 2 * lvl <= n; lvl <<= 1) {
    for(int i = 0; i < n; i += 2 * lvl) {
      for(int j = 0; j < lvl; j++) { // do not forget to modulo
        long long u = a[i + j], v = a[i + lvl + j];
        a[i + j]       = u + v; // c00 * u + c01 * v
        a[i + lvl + j] = u - v; // c10 * u + c11 * v
      }
    }
  }
}  // you can convolve as usual
```

## 7 Strings

### 7.1 Aho-Corasick

```
const int K = 26;
struct Vertex {
  int next[K];
  bool leaf = 0;
  int p = -1, ans = 0;
  char pch;
  int link = -1, mlink = -1;
  //magic link, is the link to find the nearest leaf
  int go[K];
  Vertex(int p = -1, char ch = '$') : p(p), pch(ch) {
    fill(begin(next), end(next), -1);
    fill(begin(go), end(go), -1);
  }
};
vector<Vertex> t;
int add_string(string const& s) {
  int v = 0;
  for(char ch : s) {
    int c = ch - 'a';
    if(t[v].next[c] == -1) {
      t[v].next[c] = t.size();
      t.emplace_back(v, ch);
    }
    v = t[v].next[c];
  }
  t[v].leaf = 1;
  return v;
}
int go(int v, char ch);
int get_link(int v) {
  if(t[v].link == -1) {
    if(v == 0 || t[v].p == 0)
      t[v].link = 0;
    else
      t[v].link = go(get_link(t[v].p), t[v].pch);
  }
```

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```
  return t[v].link;
}
int get_mlink(int v) {
  if(t[v].mlink == -1) {
    if(v == 0 || t[v].p == 0)
      t[v].mlink = 0;
    else {
      t[v].mlink = go(get_link(t[v].p), t[v].pch);
      if(t[v].mlink && !t[t[v].mlink].leaf) {
        if(t[t[v].mlink].mlink == -1)
          get_mlink(t[v].mlink);
        t[v].mlink = t[t[v].mlink].mlink;
      }
    }
  }
  return t[v].mlink;
}
int go(int v, char ch) {
  int c = ch - 'a';
  if(t[v].go[c] == -1) {
    if(t[v].next[c] != -1)
      t[v].go[c] = t[v].next[c];
    else
      t[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
  }
  return t[v].go[c];
}
//t.pb(Vertex());
```

## 7.2 Eertree

```
/*
   Eertree - keep track of all palindromes and its occurences
   This code refers to problem Longest Palindromic Substring
https://www.spoj.com/problems/LPS/
*/
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;

struct node {
  int next[26];
  int sufflink;
  int len, cnt;
};

const int N = 1e5 + 69;
int n;
string s;
node tree[N];
int idx, suff;
int ans = 0;

void init_eertree() {
  idx = suff = 2;
  tree[1].len = -1, tree[1].sufflink = 1;
  tree[2].len = 0, tree[2].sufflink = 1;
}

bool add_letter(int x) {
  int cur = suff, curlen = 0;
  int nw = s[x] - 'a';
  while(1) {
    curlen = tree[cur].len;
    if(x - curlen - 1 >= 0 && s[x - curlen - 1] == s[x])
      break;
    cur = tree[cur].sufflink;
  }
  if(tree[cur].next[nw]) {
    suff = tree[cur].next[nw];
    return 0;
  }
  tree[cur].next[nw] = suff = ++idx;
  tree[idx].len = tree[cur].len + 2;
  ans = max(ans, tree[idx].len);
  if(tree[idx].len == 1) {
    tree[idx].sufflink = 2;
    tree[idx].cnt = 1;
    return 1;
  }
  while(1) {
    cur = tree[cur].sufflink;
    curlen = tree[cur].len;
    if(x - curlen - 1 >= 0 && s[x - curlen - 1] == s[x]) {
      tree[idx].sufflink = tree[cur].next[nw];
      break;
    }
  }
  tree[idx].cnt = tree[tree[idx].sufflink].cnt + 1;
  return 1;
}

int main() {
  ios::sync_with_stdio(0);
  cin.tie(0);
  cin >> n >> s;
  init_eertree();
  for(int i = 0; i < n; i++)
    add_letter(i);
  cout << ans << '\n';
  return 0;
}
```

## 7.3 Manacher's Algorithm

```
// Computes lps array. lps[i] means the longest palindromic substring centered at i (↩
    when i is even, it is between characters. when it is odd, it is on characters)lps↩
    [0] = 0; lps[1] = 1;
REP(i, 2, 2 * str.size()) {
  int l = i / 2 - lps[i] / 2;
  int r = (i - 1) / 2 + lps[i] / 2;
  while(1) { // widen
    if(l == 0 || r + 1 == str.size())
      break;
    if(str[l - 1] != str[r + 1])
      break;
    l--, r++;
  }
  lps[i] = r - l + 1;
  // jump
  if(lps[i] > 2) {
    int j = i - 1, k = i + 1; // while lps[j] inside lps[i]
    while(lps[j] - j < lps[i] - i)
      lps[k++] = lps[j--];
    lps[k] = lps[i] - (i - j); // set lps[k] to edge of lps[i]
    i = k - 1; // jump to mirror, which is k
  }
}
```

## 7.4 Suffix Array

```
struct SuffixArray {
  string s;
  vector<int> p, c, lcp;
  int n;
  SuffixArray(string _s) : s(_s) {
    s += '$';
    n = (int)s.size();
    p.resize(n);
```



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```
    c.resize(n);
    {
      // calculate for k = 0
      vector<pair<char, int>> v(n);
      for(int i = 0; i < n; ++i)
        v[i] = make_pair(s[i], i);
      sort(all(v));
      for(int i = 0; i < n; ++i)
        p[i] = v[i].se;
      c[p[0]] = 0;
      for(int i = 1; i < n; ++i)
        c[p[i]] = c[p[i - 1]] + (v[i].fi != v[i - 1].fi);
    }
    const auto countingSort = [](vector<int>& p, vector<int>& c) -> void {
      int n = (int)p.size();
      vector<int> cnt(n), pos(n);
      for(auto& i : c)
        ++cnt[i];
      for(int i = 1; i < n; ++i)
        pos[i] = pos[i - 1] + cnt[i - 1];
      vector<int> pNew(n);
      for(auto& i : p)
        pNew[pos[c[i]]++] = i;
      p = pNew;
    };
    for(int k = 0; (1 << k) < n; ++k) {
      for(int i = 0; i < n; ++i) {  // transition k -> k + 1
        // shift p[i] by 2^k to the left, so that second elements are
        // sorted
        p[i] = (p[i] + n - (1 << k)) % n;
      }
      countingSort(p, c);
      vector<int> cNew(n);
      for(int i = 1; i < n; ++i) {
        pair<int, int> cur = make_pair(
                              c[p[i]], c[(p[i] + (1 << k)) % n]
                            );
        pair<int, int> pre = make_pair(
                              c[p[i - 1]], c[(p[i - 1] + (1 << k)) % n]
                            );
        cNew[p[i]] = cNew[p[i - 1]] + (cur != pre);
      }
      c = cNew;
    }
    // lcp[i]: longest common prefix of s[p[i]] and s[p[i-1]]
    lcp.resize(n); // iterate from the longest suffix
    for(int i = 0, k = 0; i + 1 < n; ++i) {
      int pi = c[i]; // rank of suffix [i..]
      int j = p[pi - 1];
      for(; s[i + k] == s[j + k]; ++k)
        ;
      lcp[pi] = k;
      k = max(k - 1, 0);
    }
  }
};
```

## 7.5 Suffix Automaton

```
struct state {
  int len, link;
  map<char, int>next; //use array if TLE
};

const int MAXLEN = 100005;
state st[MAXLEN * 2];
int sz, last;

void sa_init() {
  sz = last = 0;
  st[0].len = 0;
  st[0].link = -1;
  st[0].next.clear();
  ++sz;
}
void sa_extend(char c) {
  int cur = sz++;
  st[cur].len = st[last].len + 1;
  st[cur].next.clear();
  int p;
  for(p = last; p != -1 && !st[p].next.count(c); p = st[p].link)
    st[p].next[c] = cur;
  if(p == -1)
    st[cur].link = 0;
  else {
    int q = st[p].next[c];
    if(st[p].len + 1 == st[q].len)
      st[cur].link = q;
    else {
      int clone = sz++;
      st[clone].len = st[p].len + 1;
      st[clone].next = st[q].next;
      st[clone].link = st[q].link;
      for(; p != -1 && st[p].next[c] == q; p = st[p].link)
        st[p].next[c] = clone;
      st[q].link = st[cur].link = clone;
    }
  }
  last = cur;
}
// forwarding
for(int i = 0; i < m; i++) {
  while(cur >= 0 && st[cur].next.count(pa[i]) == 0) {
    cur = st[cur].link;
    if(cur != -1)
      len = st[cur].len;
  }
  if(st[cur].next.count(pa[i])) {
    len++;
    cur = st[cur].next[pa[i]];
  } else
    len = cur = 0;
}
// shortening abc -> bc
if(l == m) {
  l--;
  if(l <= st[st[cur].link].len)
    cur = st[cur].link;
}
// finding lowest and highest length
int lo = st[st[cur].link].len + 1;
int hi = st[cur].len;
//Finding number of distinct substrings
//answer = distsub(0)
LL d[MAXLEN * 2];
LL distsub(int ver) {
  LL  tp = 1;
  if(d[ver])
    return d[ver];
  for(map<char, int>::iterator it = st[ver].next.begin();
      it != st[ver].next.end(); it++)
    tp += distsub(it->second);
  d[ver] = tp;
  return d[ver];
}
//Total Length of all distinct substrings
//call distsub first before call lesub
LL ans[MAXLEN * 2];
LL lesub(int ver) {
  LL tp = 0;
  if(ans[ver])
```


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```
    return ans[ver];
  for(map<char, int>::iterator it = st[ver].next.begin();
      it != st[ver].next.end(); it++)
    tp += lesub(it->second) + d[it->second];
  ans[ver] = tp;
  return ans[ver];
}
//find the k-th lexicographical substring
void kthsub(int ver, int K, string& ret) {
  for(map<char, int>::iterator it = st[ver].next.begin();
      it != st[ver].next.end(); it++) {
    int v = it->second;
    if(K <= d[v]) {
      K--;
      if(K == 0) {
        ret.push_back(it->first);
        return;
      } else {
        ret.push_back(it->first);
        kthsub(v, K, ret);
        return;
      }
    } else
      K -= d[v];
  }
}
// Smallest Cyclic Shift to obtain lexicographical smallest of All possible
//in int main do this
int main() {
  string S;
  sa_init();
  cin >> S; //input
  tp = 0;
  t = S.length();
  S += S;
  for(int a = 0; a < S.size(); a++)
    sa_extend(S[a]);
  minshift(0);
}
//the function
int tp, t;
void minshift(int ver) {
  for(map<char, int>::iterator it = st[ver].next.begin();
      it != st[ver].next.end(); it++) {
    tp++;
    if(tp == t) {
      cout << st[ver].len - t + 1 << endl;
      break;
    }
    minshift(it->second);
    break;
  }
}
//end of function
// LONGEST COMMON SUBSTRING OF TWO STRINGS
string lcs(string s, string t) {
  sa_init();
  for(int i = 0; i < (int)s.length(); ++i)
    sa_extend(s[i]);
  int v = 0,  l = 0,
      best = 0,  bestpos = 0;
  for(int i = 0; i < (int)t.length(); ++i) {
    while(v && ! st[v].next.count(t[i])) {
      v = st[v].link;
      l = st[v].length;
    }
    if(st[v].next.count(t[i])) {
      v = st[v].next[t[i]];
      ++l;
    }
    if(l > best)
      best = l,  bestpos = i;
  }
  return t.substr(bestpos - best + 1, best);
}
```

### 7.6 KMP

```
auto get_kmp = [&](string S, string T) -> vector<int> {
  // S is the text, T is the pattern // ababa aba -> expected: {1 2 3 2 3}
  int N = S.size(), M = T.size();
  vector<int> lps(M), kmp(N);
  for(int i = 1, j = 0; i < M;) {
    if(T[i] == T[j])
      lps[i++] = ++j;
    else {
      if(j)
        j = lps[j - 1];
      else
        lps[i++] = 0;
    }
  }
  for(int i = 0, j = 0; i < N;) {
    if(S[i] == T[j]) {
      kmp[i++] = ++j;
      if(j == M)
        j = lps[j - 1];
    } else {
      if(j)
        j = lps[j - 1];
      else
        kmp[i++] = 0;
    }
  }
  return kmp;
};
```

## 8 OEIS

### 8.1 A000108 (Catalan)

```
Catalan numbers
f(n) = nCk(2n,n) / (n+1) = nCk(2n,n) - nCk(2n,n+1) = f(n-1) * 2*(2*n-1) / (n+1)
1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900,
2674440, 9694845, 35357670, 129644790, 477638700, 1767263190, 6564120420,
24466267020, 91482563640, 343059613650, 1289904147324, 4861946401452,
18367353072152, 69533550916004, 263747951750360, 1002242216651368,
3814986502092304
```

### 8.2 A018819

```
Binary partition function: number of partitions of n into powers of 2
f(2m+1) = f(2m);  f(2m) = f(2m-1) + f(m)
1, 1, 2, 2, 4, 4, 6, 6, 10, 10, 14, 14, 20, 20, 26, 26, 36, 36, 46, 46, 60,
60, 74, 74, 94, 94, 114, 114, 140, 140, 166, 166, 202, 202, 238, 238, 284,
284, 330, 330, 390, 390, 450, 450, 524, 524, 598, 598, 692, 692, 786, 786,
900, 900, 1014, 1014, 1154, 1154, 1294, 1294
```

### 8.3 A092098

```
3-Portolan numbers: number of regions formed by n-secting the angles of
an equilateral triangle.
long long solve(long long n) {
    long long res = (n % 2 == 1 ? 3*n*n - 3*n + 1 : 3*n*n - 6*n + 6);
    const int bats = n/2 - 1;
    for (long long i=1; i<=bats; i++) for (long long j=1; j<=bats; j++) {
        long long num = i * (n-j) * n;
```


Proof by... forgor
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```
        long long denum = (n-i) * j + i * (n-j);
        res -= 6 * (num % denum == 0 && num / denum <= bats);
    } return res;
}
1, 6, 19, 30, 61, 78, 127, 150, 217, 246, 331, 366, 469, 510, 625, 678, 817,
870, 1027, 1080, 1261, 1326, 1519, 1566, 1801, 1878, 2107, 2190, 2437, 2520,
2791, 2886, 3169, 3270, 3559, 3678, 3997, 4110, 4447, 4548, 4921, 5034, 5419,
5550, 5899, 6078, 6487
```

### 8.4 A000127

```
Maximal number of regions obtained by joining n points around a circle by
straight lines
f(n) = (n^4 - 6*n^3 + 23*n^2 - 18*n + 24) / 24
1, 2, 4, 8, 16, 31, 57, 99, 163, 256, 386, 562, 794, 1093, 1471, 1941, 2517,
3214, 4048, 5036, 6196, 7547, 9109, 10903, 12951, 15276, 17902, 20854, 24158,
27841, 31931, 36457, 41449, 46938, 52956, 59536, 66712, 74519, 82993, 92171,
102091, 112792, 124314
```

### 8.5 A001534

```
Number of graphs with n nodes and n edges.
0, 0, 1, 2, 6, 21, 65, 221, 771, 2769, 10250, 39243, 154658, 628635, 2632420,
11353457, 50411413, 230341716, 1082481189, 5228952960, 25945377057,
132140242356, 690238318754
```