Discrete Distribution Families
Bernoulli(p)
Binary trial (success/failure).
PMF: $P(X=x)=p^x(1-p)^{1-x}$, $x\in\{0,1\}$
Mean: p, Var: $p(1-p)$
R: `d/p/q/rbinom(n=1, p)`
Binomial(n,p)
successes in n Bernoulli trials.
PMF: $P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$
Mean: np, Var: $np(1-p)$
R: `dbinom, pbinom, qbinom, rbinom`
Geometric(p)
Failures before 1st success.
PMF: $P(X=x)=(1-p)^x p$, $x\geq 0$
Mean: $(1-p)/p$, Var: $(1-p)/p^2$
R: `dgeom, pgeom, qgeom, rgeom`
NegBin(k,p)
Failures before k-th success.
PMF: $P(X=x)=\binom{x+k-1}{x}p^k(1-p)^x$
Mean: $k(1-p)/p$, Var: $k(1-p)/p^2$
R: `dnbinom, pnbinom, qnbinom, rnbinom`
Poisson(λ)
Counts in interval, rate λ.
PMF: $P(X=x)=\frac{\lambda^x e^{-\lambda}}{x!}$
Mean: λ, Var: λ
R: `dpois, ppois, qpois, rpois`
Continuous Distribution Families
Uniform(a,b)
All values equally likely on $[a,b]$.
PDF: $f(x)=\frac{1}{b-a}$, $a\leq x\leq b$
Mean: $(a+b)/2$, Var: $(b-a)^2/12$
Skewness: 0 (symmetric)
R: `dunif, punif, qunif, runif`

Normal(μ,σ^2)
Bell-shaped, $-\infty<\mu<\infty, \sigma^2>0$
PDF: $f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/(2\sigma^2)}$
Mean: μ, Var: σ^2
Skewness: 0 (symmetric)
R: `dnorm, pnorm, qnorm, rnorm;`

Lognormal(μ,σ^2)
If $\ln X\sim N(\mu,\sigma^2)$.
PDF: $f(x)=\frac{1}{x\sigma\sqrt{2\pi}}e^{-(\ln x-\mu)^2/(2\sigma^2)}$
Mean: $e^{\mu+\sigma^2/2}$, Var: $(e^{\sigma^2}-1)e^{2\mu+\sigma^2}$
Skewness: $(e^{\sigma^2}+2)\sqrt{e^{\sigma^2}-1}$
R: `dlnorm, plnorm, qlnorm, rlnorm;`

Exponential(λ)
Time between Poisson events.
Positive RV, wait time, memoryless
(λ) is average rate
(β) is mean wait time
PDF: $f(x)=\lambda e^{-\lambda x}, x\geq 0$
Mean: $1/\lambda$, Var: $1/\lambda^2$
Skewness: 2
R: `dexp, pexp, qexp, rexp;`

Beta(α,β)
On $[0,1]$, Bayesian priors.
Uniform distribution special case
The Gamma function (Γ)is a generalization
of the factorial function to non-integer numbers.
PDF: $f(x)=\frac{1}{B(\alpha,\beta)}x^{\alpha-1}(1-x)^{\beta-1}$
Mean: $\alpha/(\alpha+\beta)$, Var: $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Skewness: $\frac{2(\beta-\alpha)\sqrt{\alpha+\beta+1}}{(\alpha+\beta+2)\sqrt{\alpha\beta}}$
R: `dbeta, pbeta, qbeta, rbeta;`

Weibull(k,λ)
Lifetimes, survival, reliability.
Exponential family (when k = 1), longer you wait
Survival Analysis
PDF: $f(x)=\frac{k}{\lambda}(x/\lambda)^{k-1}e^{-(x/\lambda)^k}$
Mean: $\lambda\Gamma(1+1/k)$, Var: $\lambda^2[\Gamma(1+2/k)-\Gamma(1+1/k)^2]$
Skewness: $\frac{\Gamma(1+3/k)\lambda^3-3\mu\sigma^2-\mu^3}{\sigma^3}$
R: `dweibull, pweibull, qweibull, rweibull;`

Gamma(α,θ)
Waiting time for α events.
non-negative numbers
α is shape parameter
θ is scale parameter
PDF: $f(x)=\frac{1}{\Gamma(\alpha)\theta^\alpha}x^{\alpha-1}e^{-x/\theta}$
Mean: $\alpha\theta$, Var: $\alpha\theta^2$
Skewness: $2/\sqrt{\alpha}$
R: `dgamma, pgamma, qgamma, rgamma;`

R: Computing Expected Value
Discrete RV
```
X <- 0:2
p <- c(0.2,0.5,0.3)
EV <- sum(X*p)
```

Continuous RV (numerical integration)
```
f <- function(x) 2*x
EV <- integrate(function(x) x*f(x), 0,1)$value
```

From sample (Monte Carlo)
```
samples <- rnorm(10000, mean=5, sd=2)
mean(samples)
```
Define the PDF function
```
f <- function(x)  2*x
prob <- integrate(f, lower = 0.5, upper =
0.75)$value # Integrate over [0.5, 0.75]
```
Conditional Distributions
Let X and Y be two random variables.

$$f_{X|Y}(x|y)=\begin{cases}\frac{P(X=x,Y=y)}{P(Y=y)}, & \text{discrete case}\\ \frac{f_{X,Y}(x,y)}{f_Y(y)}, & \text{continuous case, } f_Y(y)>0\end{cases}$$

Conditional dist is just a segment of marginal dist, then re-normalized to have an area under the curve equal to 1   If X and Y are independent, in continouse case,

$$f_{Y|X}(y)=f_Y(y)$$

This means conditional PDF of Y and X is marginal PDF of Y.
Random Sample
It is independent and identically distributed (iid). Each pair of observations are independent, and each observation comes from the same distribution.
MLE
Great way to find estimators. Applied on multi and univariate. Relies on random sample of n observations. Mean, is a estimator for univariate, multivariate, linear regression
Steps for MLE
1.Nature of variable (discrete or contin)
2.estimate the parameters of a theoretical distribution (eg λ in a Poisson distribution)
3.Choose distribution: Normal, Exponential, Poisson, Binomial, etc.)   4.Play with the parameters for that family of distributions to find the one that would be most likely given our data and choose the corresponding parametric estimates)   5.To obtain these estimates, we use the likelihood function of our observed random sample.)