R Distribution Function Types

- **d*** : Density / PMF (probability mass or density function). Example: `dnorm(x, mean, sd)`
- **p*** : Cumulative distribution function (CDF). Example: `pnorm(q, mean, sd) = P(X` $\le q$)
- `q* : Quantile function (inverse CDF). Example: qnorm(p, mean, sd) = x s.t. P(X` $\le x) = p$
- `r* : Random number generation. Example: rnorm(n, mean, sd)`

Discrete Distribution Families

Bernoulli(p)

Binary trial (success/failure).
PMF: $P(X = x) = p^x(1-p)^{1-x}$, $x \in \{0, 1\}$
Mean: p, Var: $p(1-p)$
R: `d/p/q/rbinom(n=1, p)`

Binomial(n, p)

successes in n Bernoulli trials.
PMF: $P(X = x) = \binom{n}{x}p^x(1-p)^{n-x}$
Mean: np, Var: $np(1-p)$
R: `dbinom, pbinom, qbinom, rbinom`

Geometric(p)

Failures before 1st success.
PMF: $P(X = x) = (1-p)^x p$, $x \ge 0$
Mean: $(1-p)/p$, Var: $(1-p)/p^2$
R: `dgeom, pgeom, qgeom, rgeom`

NegBin(k, p)

Failures before k-th success.
PMF: $P(X = x) = \binom{x+k-1}{x}p^k(1-p)^x$
Mean: $k(1-p)/p$, Var: $k(1-p)/p^2$
R: `dnbinom, pnbinom, qnbinom, rnbinom`

Poisson(λ)

Counts in interval, rate λ.
PMF: $P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$
Mean: λ, Var: λ
R: `dpois, ppois, qpois, rpois`

Continuous Distribution Families

Uniform(a, b)

All values equally likely on $[a, b]$.
PDF: $f(x) = \frac{1}{b-a}$, $a \le x \le b$
Mean: $(a+b)/2$, Var: $(b-a)^2/12$
Skewness: 0 (symmetric)
R: `dunif, punif, qunif, runif`

Normal(μ, σ^2)

Bell-shaped,
$-\infty < \mu < \infty$
$\sigma^2 > 0$
PDF :f(x)= $\frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/(2\sigma^2)}$
Mean: μ, Var: σ^2
Skewness: 0 (symmetric)
R: `dnorm, pnorm, qnorm, rnorm;`

Lognormal(μ, σ^2)

If $\ln X \sim N(\mu, \sigma^2)$.
PDF: $f(x) = \frac{1}{x\sigma\sqrt{2\pi}}e^{-(\ln x-\mu)^2/(2\sigma^2)}$
Mean: $e^{\mu+\sigma^2/2}$, Var: $(e^{\sigma^2}-1)e^{2\mu+\sigma^2}$
Skewness: $(e^{\sigma^2}+2)\sqrt{e^{\sigma^2}-1}$
R: `dlnorm, plnorm, qlnorm, rlnorm;`

Exponential(λ)

Time between Poisson events.
Positive RV, wait time, memoryless
(λ) is average rate
(β) is mean wait time
PDF: $f(x) = \lambda e^{-\lambda x}, x \ge 0$
Mean: $1/\lambda$, Var: $1/\lambda^2$
Skewness: 2
R: `dexp, pexp, qexp, rexp;`

Beta(α, β)

On $[0, 1]$, Bayesian priors.
PDF: $f(x) = \frac{1}{B(\alpha,\beta)}x^{\alpha-1}(1-x)^{\beta-1}$
Mean: $\alpha/(\alpha+\beta)$, Var: $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Skewness: $\frac{2(\beta-\alpha)\sqrt{\alpha+\beta+1}}{(\alpha+\beta+2)\sqrt{\alpha\beta}}$
R: `dbeta, pbeta, qbeta, rbeta;`

Weibull(k, λ)

Lifetimes, survival, reliability.
PDF: $f(x) = \frac{k}{\lambda}(x/\lambda)^{k-1}e^{-(x/\lambda)^k}$
Mean: $\lambda\Gamma(1+1/k)$, Var: $\lambda^2[\Gamma(1+2/k) - \Gamma(1+1/k)^2]$
Skewness: $\frac{\Gamma(1+3/k)\lambda^3 - 3\mu\sigma^2 - \mu^3}{\sigma^3}$
R: `dweibull, pweibull, qweibull, rweibull;`

Gamma(α, θ)

Waiting time for α events.
PDF: $f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha}x^{\alpha-1}e^{-x/\theta}$
Mean: $\alpha\theta$, Var: $\alpha\theta^2$
Skewness: $2/\sqrt{\alpha}$
R: `dgamma, pgamma, qgamma, rgamma;`

R: Computing Expected Value

Discrete RV

```
X <- 0:2
p <- c(0.2,0.5,0.3)
EV <- sum(X*p)
```

Continuous RV (numerical integration)

```
f <- function(x) 2*x
EV <- integrate(function(x) x*f(x), 0,1)$value
```

From sample (Monte Carlo)

```
samples <- rnorm(10000, mean=5, sd=2)
mean(samples)
```

Define the PDF function

```
f <- function(x)  2*x
prob <- integrate(f, lower = 0.5, upper =
0.75)$value # Integrate over [0.5, 0.75]
```