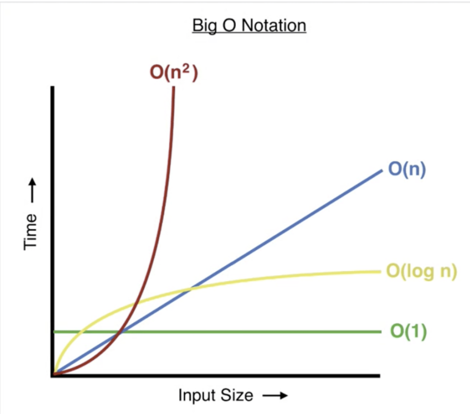
algorithm: a well-defined computational procedure designed to solve a problem. It consists of a sequence of precise steps that take some inputs, process them systematically, and produce corresponding outputs. **data structure**: is a way to organize and manage data, allowing us to write more efficient code in terms of both time and space.

0.1 Big O Notation

Definition: Describes asymptotic behavior of algorithms as input size n grows. **Common Classes**:

- $O(1)$: Constant - Runtime independent of n.
- $O(\log n)$: Logarithmic - Doubling n adds constant time.
- $O(\sqrt{n})$: sub-linear time complexity.
- $O(n)$: Linear - Doubling n doubles time.
- $O(n \log n)$: Linearithmic - Roughly $O(n)$ when n doubles.
- $O(n^2)$: Quadratic - Doubling n multiplies time by 4.
- $O(n^k)$: Polynomial - Time multiplied by 2^k.
- $O(2^n)$: Exponential - Doubling n multiplies time by 2^k.



0.2 Hash Tables

Properties: Keys must be **hashable** (immutable). **Operations**:

- $O(1)$ `insert`, `delete`, `lookup` (average).

Hash Function: Deterministic: same input → same output. **Collisions**: Handled by **chaining** (list per bucket). **Python Dict**:

- Keys must be **hashable**.
- Values can be any type.
- Lists cannot be keys (mutable).

```python
# Getting a value with a default (avoids KeyError)
salary = my_dict.get('salary', 'N/A')
# Loop through key-value pairs (most common and efficient)
for key, value in my_dict.items():
    print(f"Key: {key} -> Value: {value}")
#Binary Search
def search_sorted(data, key):
    low = 0
    high = len(data) - 1
    while (low <= high):
        mid = (high + low)//2
        if data[mid] == key:
            return True
        if key < data[mid]:
            high = mid - 1
        else:
            low = mid + 1
    return False
```

0.3 Graphs

For some graphs, DFS is equivalent to BFS.
Represent a **2D image** having M rows and N columns using a graph
Space Complexity is MN^2 in Adjacency Matrix
Space Complexity is MN in Adjacency List
Definition: $G = (V, E)$ where V=vertices, E=edges. **Types**:

- **Undirected**: edges have no direction (e.g., Twitter followers).
- **Directed**: edges **bidirectional** (e.g., Facebook friends).
- **Weighted**: edges have numerical values.
- **Unweighted**: edges represent presence/absence only.

Adjacency List: Array of lists: $Adj[u]$ is a list of neighbors of u.

- Space: $O(V + E)$.
- Good for **sparse graphs** ($E \ll V^2$).
- Lookup time: $O(V)$ worst case.

Adjacency Matrix: A matrix where $A[i][j] = 1$ if edge exists.

- Space: $O(V^2)$.
- Lookup time: $O(1)$.
- Good for **dense graphs**.

0.4 Breadth-First Search (BFS)

Purpose: Explores neighbors before going deeper. Finds shortest paths in unweighted graphs. **Data Structure**: Uses **Queue** (FIFO). **Complexity**: $O(V + E)$. **Implementation**:

1. Initialize stack with start node.
2. Mark start as **visited**.
3. While stack not empty:
4. - Pop node. #stack.pop(0) in Queue
5. - Process node.
6. - Enqueue unvisited neighbors.
7. - Mark neighbors as **visited**.

```python
# --- Option 1: Built-in BFS traversal ---
print("BFS traversal using networkx.bfs_tree:")
bfs_tree = nx.bfs_tree(G, source='A')
print(list(bfs_tree.nodes())) # nodes in BFS order
# OR equivalently:
print("\nUsing networkx.bfs_edges:")
bfs_edges = list(nx.bfs_edges(G, source='A'))
print("BFS edges:", bfs_edges)
# --- Option 2: Custom BFS implementation ---
def bfs_custom(G, start):
    visited = set()
    queue = deque([start])
    while queue:
        node = queue.popleft()
        if node not in visited:
            print(node, end=" ")
            visited.add(node)
            for neighbor in G.neighbors(node):
                if neighbor not in visited:
                    queue.append(neighbor)
    return visited
```

0.5 Depth-First Search (DFS)

Purpose: Explores as deep as possible before backtracking. **Data Structure**: Uses **Stack** (LIFO). **Complexity**: $O(V + E)$. **Implementation**:

1. Initialize stack with start node.
2. Mark start as **visited**.
3. While stack not empty:
4. - Pop node. #stack.pop() in Queue
5. - Process node.
6. - Push unvisited neighbors.
7. - Mark neighbors as **visited**.

```python
# --- Option 2: Custom DFS using recursion ---
def dfs_recursive(G, node, visited=None):
    if visited is None:
        visited = set()
    visited.add(node)
    print(node, end=" ")
    for neighbor in G.neighbors(node):
        if neighbor not in visited:
            dfs_recursive(G, neighbor, visited)
    return visited
print("\nCustom DFS (recursive):")
dfs_recursive(G, 'A')
# --- Option 3: Iterative DFS using a stack ---
def dfs_iterative(G, start):
    visited = set()
    stack = [start]
    while stack:
        node = stack.pop()
        if node not in visited:
            print(node, end=" ")
            visited.add(node)
            # Add neighbors to stack
            stack.extend(reversed(list(G.neighbors(node))))
    return visited
```

0.6 Sparse Matrix format (Scipy)

This is "Compressed Sparse Row" format.
It means we store an adjacency list per row.
Word counts: We might represent a document by the words in it, but only a small fraction of all words would appear in a given document.
Ratings: We might represent an Amazon item by the user ratings, but only a small fraction of all users have rated a given item.

CSR and CSC are efficient ways to store matrices with mostly zero entries, saving memory and speeding up relevant operations.

```python
x = scipy.sparse.random(5, 5, density=0.2, format="csr",
    random_state=321)
```

0.7 Trees

Trees are vital for organizing data **hierarchically** (parent-child relationships). **Modeling Real-World Systems**:

- **File Systems** (folders/subfolders).
- **Organization Charts** (manager-employee).
- **XML/HTML Documents** (nested tags).

Efficiency & Algorithms:

- **Binary Search Trees (BSTs)**: Fast $O(\log n)$ average time for lookup/insertion/deletion.
- **Balanced Trees** (AVL/Red-Black): Consistently fast performance.
- **Heaps**: Used for Priority Queues and `heapsort`.

Advanced Structures & Applications:

- **Tries**: Autocomplete, spell checking.
- **Syntax Trees**: Compilers/interpreters.
- **B-Trees / B+ Trees**: Database indexing, file systems.
- **Decision Trees**: Machine learning models (random forests).

0.8 Tree Terminology

Node: Fundamental part, stores data. **Root**: The first node in a tree (no parent). **Edge**: A link connecting two nodes. **Parent**: Immediate predecessor (a node has only one parent). **Child**: Immediate successor (a node can have ≥ 0 children). **Leaf Node**: A node with zero children. **Height**: Maximum number of edges from the **root** to a **leaf node**. **Subtree**: A portion of a tree starting from any node and including all its descendants. **Balanced Tree**: A tree where the height of the left and right subtrees of any node are nearly equal.

0.9 NetworkX Code Examples

Create Graphs:

- `G = nx.Graph()` (undirected)
- `G = nx.DiGraph()` (directed)

Add Nodes & Edges:

- `G.add_node("A")`
- `G.add_edge("A", "B")`
- `G.add_edges_from([("A", "B"), ("B", "C"), ("B", "D")])`

0.10 Time Measurement

Code Profiling:

- `timeit`: Python magic for timing small code snippets.
- `line_profiler`: Per-line timing.

Memory Profiling:

- `sys.getsizeof()`: Object size.
- `df.memory_usage()`: Pandas DataFrame memory.
- `np.sum(array.nbytes)`: NumPy arrays memory.

0.11 Tips & Best Practices

Visualization:

- `nx.draw(G, with_labels=True)`

General Tips:

- Use **set** for membership lookup.
- Use **dict** for fast key-value lookups.
- Prioritize coherent **data structure** operations based on needs.
- Consider **space-time tradeoffs**.
- BFS is better for optimizing **shortest paths** in unweighted graphs.
- **Hash table operations** are $O(1)$ avg., not worst-case.
- Prefer **sparse matrices** for graphs with $E \ll V^2$.