Study of Predator-Prey Dynamics

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Lotka-Volterra Model (LVM)

The Lotka-Volterra Model (LVM) models the dynamics between a predator species (y) and a prey species (x) over time (t).

$$\frac{dy}{dt} = -ay + bxy \quad \frac{dx}{dt} = +dx - cyx$$

The model depends on four parameters.

- a: decay rate of the predators
- b: proportionality for how predators grow due to eating prey
- c: proportionality for how prey decay due to being eaten by predators
- d: growth rate of the prey

Dividing the differential equations by each other yields $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{b}{c} \frac{y\left(x - \frac{a}{b}\right)}{x\left(\frac{d}{c} - y\right)}$$

Since this equation does not depend explicitly on time, it can be used to create phase portraits. The solutions swirl counter-clockwise around $x = \frac{a}{b}$ and $y = \frac{d}{c}$.

Types of Simulation Outcomes

Conditions for Good Modeling

```
parameters = {
    "breed_time": 3,
    "energy_gain": 4,
    "breed_energy": 15,
    "side_length": 80,
    "aspect_ratio": 9/8,
    "initial_fish": 500,
    "initial_sharks": 400,
    "steps": 500,
    "start_energy": 9,
    "use_basic_setup": True,
}

def get_initialization_parameters(params):
    """
    Return a dictionary with just the parameters needed to initialize the game array.
    """
    result = {}
    desired_keys = ["initial_fish", "initial_sharks", "breed_time", "breed_energy"]
    for key in desired_keys:
```

```
        result[key] = params[key]
    return result

def get_simulation_parameters(params):
    """
    Return a dictionary with just the parameters needed to run the simulation.
    """
    result = {}
    desired_keys = ["steps", "breed_time", "energy_gain", "breed_energy",
"start_energy"]
    for key in desired_keys:
        result[key] = params[key]
    return result
```

```
import wa_tor
import default_parameters
import matplotlib.pyplot as plt

# Specify the test values to use when testing each parameter
test_ranges = {
    "breed_time": range(1, 15 + 1),
    "energy_gain": range(2, 18 + 1),
    "breed_energy": range(default_parameters.parameters["start_energy"] + 1, 25 + 1),
    "side_length": range(40, 120 + 10, 10),
    "aspect_ratio": [i/8 for i in range(8, 16 + 1)],
    "initial_fish": range(200, 1000 + 50, 50),
    "initial_sharks": range(200, 1000 + 50, 50),
    "start_energy": range(1, default_parameters.parameters["breed_energy"] - 1),
}

def test_outcome_chances(target_param, test_values, trials, params=None):
    """
    Vary the target parameter to have the given test values.
    For each value, run the simulation for the specified number of trials and
calculate the chance of each of the possible outcomes.
    - Everything went extinct
    - Fish fill the board
    - Simulation could keep going

    Return a dictionary containing three lists of chances, one list for each outcome.
    Each list contains the chances found using each test value of the target
parameter.
    """
    # Set the parameters to the default if not specified
    if params is None:
        params = default_parameters.parameters.copy()

    overall_chances = {
        "everything_extinct": [],
        "fish_fill_board": [],
        "still_going": [],
```

```
    }

    for value in test_values:
        # Set the target parameter
        params[target_param] = value

        # Calculate the board dimensions based on side length and aspect ratio
        side_length = params["side_length"]
        other_side = int(side_length * params["aspect_ratio"])
        dims = [side_length, other_side]

        # Extract the needed parameters for later steps
        init_params = default_parameters.get_initialization_parameters(params)
        sim_params = default_parameters.get_simulation_parameters(params)
        # Keep track of the counts for the possible outcomes
        everything_extinct_count = 0
        fish_fill_count = 0

        for _ in range(trials):
            # Initialize the game array
            initial_game_array = wa_tor.create_empty_game_array(dims)
            if params["use_basic_setup"]:
                wa_tor.initialize_game_array_randomly(initial_game_array,
**init_params)
            else:
                wa_tor.initialize_game_array_circular(initial_game_array,
**init_params)

            # Run the simulation
            fish_counts, shark_counts =
wa_tor.run_simulation_minimal(initial_game_array, **sim_params)

            # Check whether fish filled the board or if sharks and fish both went
extinct
            # Update the counts for these events
            size = dims[0] * dims[1]
            if fish_counts[-1] + shark_counts[-1] < 0:
                everything_extinct_count += 1
            elif fish_counts[-1] == size:
                fish_fill_count += 1

        # Store the chances of each possible outcome
        still_going_count = trials - everything_extinct_count - fish_fill_count
        overall_chances["everything_extinct"].append(everything_extinct_count /
trials)
        overall_chances["fish_fill_board"].append(fish_fill_count / trials)
        overall_chances["still_going"].append(still_going_count / trials)

    return overall_chances

def plot_and_test_outcome_chances(fname, target_param, test_values, trials,
```

```
params=None):
    """
    Run the function test_outcome_chances() with the given arguments, then plot the
results.
    Save the figure at the given file name.
    """
    outcome_chances = test_outcome_chances(target_param, test_values, trials, params)

    fig, ax = plt.subplots()
    ax.plot(test_values, outcome_chances["everything_extinct"], "o", label="Both
Extinct")
    ax.plot(test_values, outcome_chances["fish_fill_board"], "^", label="Sharks
Extinct")
    ax.plot(test_values, outcome_chances["still_going"], ".", label="Neither Extinct")
    ax.set(xlabel=target_param, ylabel="Chance")
    ax.legend()
    fig.tight_layout()
    fig.savefig(fname)

def run_standard_test(target_parameter, use_basic_setup):
    """
    Run a standard test on the target parameter.
    Perform 25 trials with use_basic_setup optionally toggled.
    """
    trials = 25
    test_values = test_ranges[target_parameter]
    params = default_parameters.parameters.copy()
    params["use_basic_setup"] = use_basic_setup

    if use_basic_setup:
        fname = f"media/outcome_chances_{target_parameter}.svg"
    else:
        fname = f"media/outcome_chances_{target_parameter}_circular.svg"

    plot_and_test_outcome_chances(fname, target_parameter, test_values, trials,
params)
```

```
import measure_outcome_chances as tst

tst.run_standard_test("breed_time", True)
```

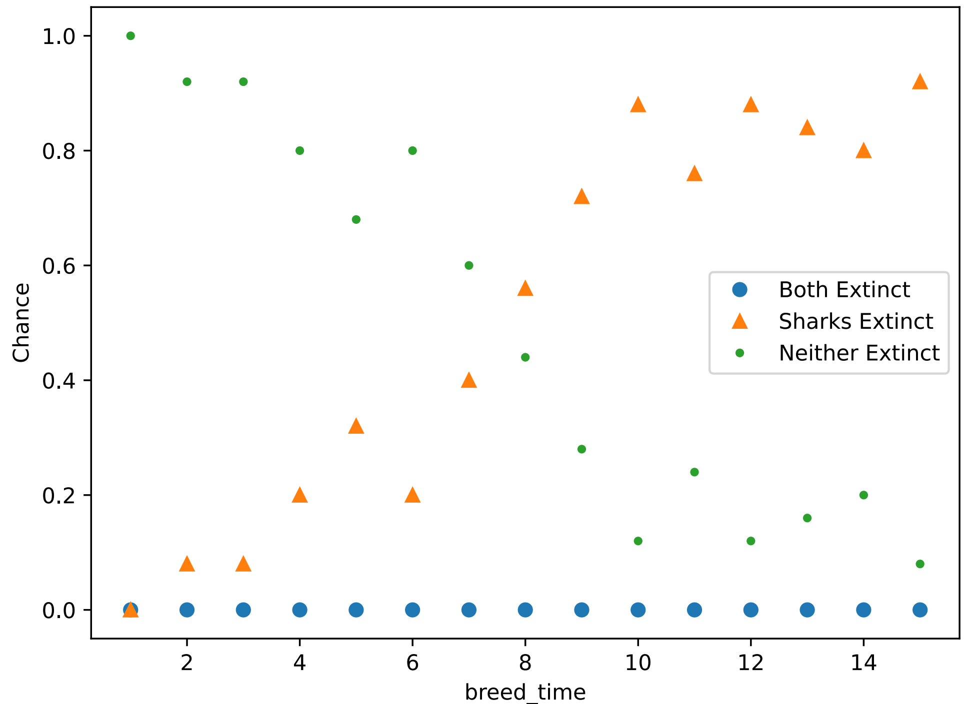

Figure 1: Outcome Chances vs `breed_time`

```
import measure_outcome_chances as tst

tst.run_standard_test("energy_gain", True)
```

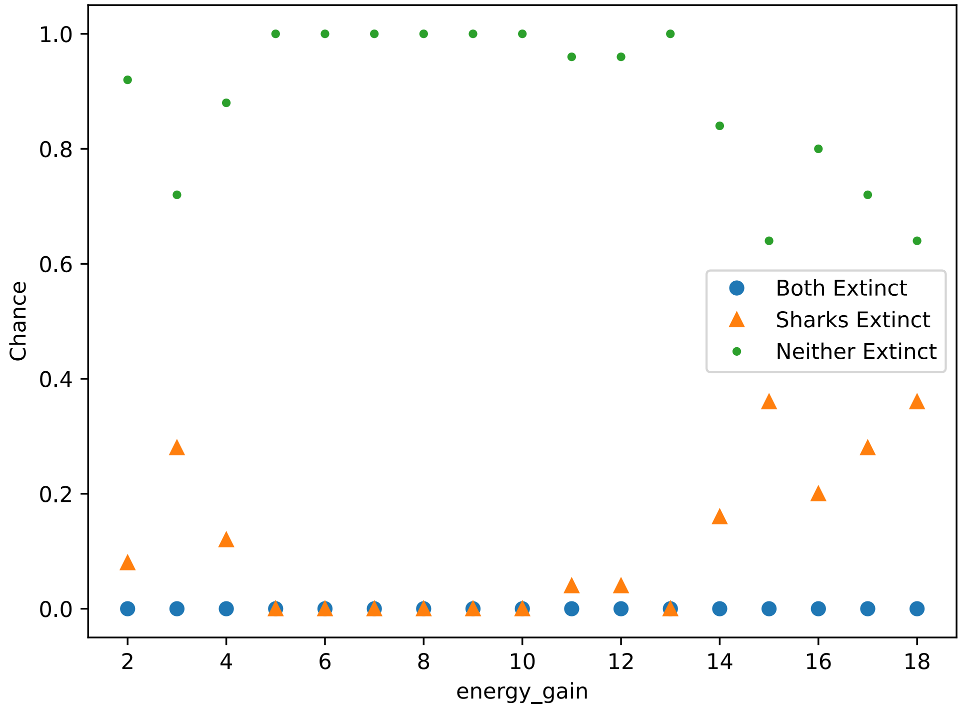

Figure 2: Outcome Chances vs `energy_gain`

```
import measure_outcome_chances as tst

tst.run_standard_test("breed_energy", True)
```

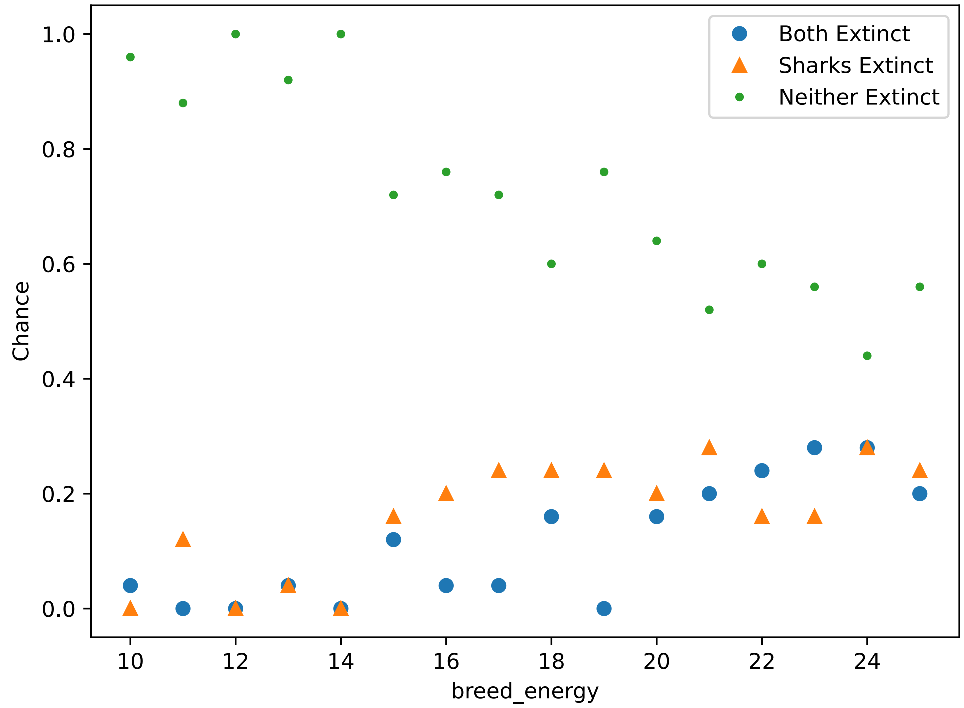

Figure 3: Outcome Chances vs `breed_energy`

```
import measure_outcome_chances as tst

tst.run_standard_test("side_length", True)
```

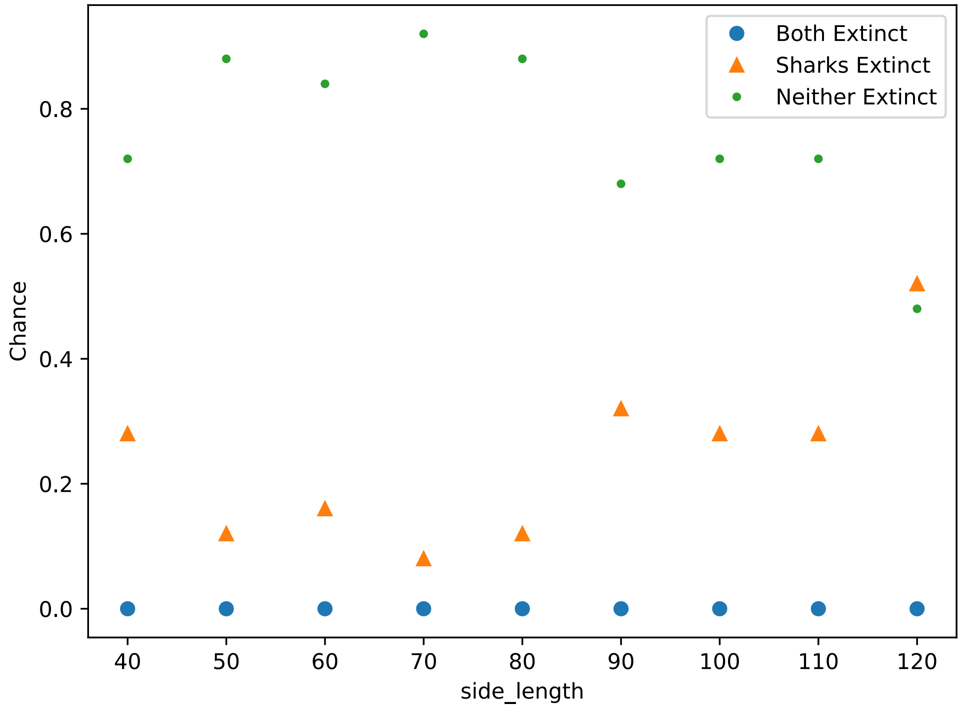

Figure 4: Outcome Chances vs `side_length`

```
import measure_outcome_chances as tst

tst.run_standard_test("aspect_ratio", True)
```

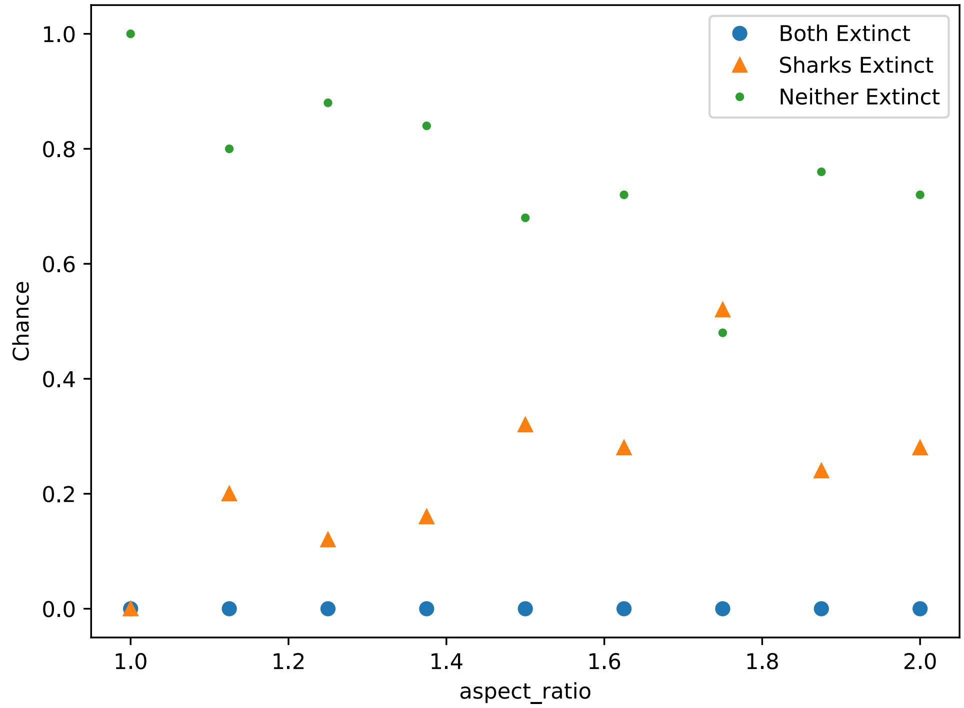

Figure 5: Outcome Chances vs `aspect_ratio`

```
import measure_outcome_chances as tst

tst.run_standard_test("initial_fish", True)
```

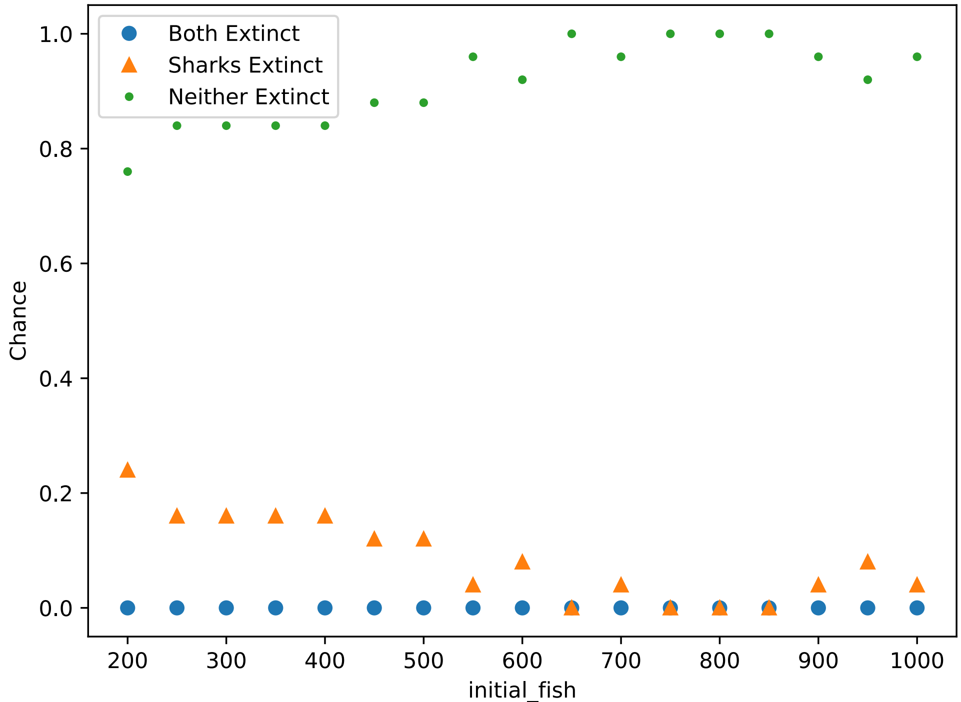

Figure 6: Outcome Chances vs `initial_fish`

```
import measure_outcome_chances as tst

tst.run_standard_test("initial_sharks", True)
```

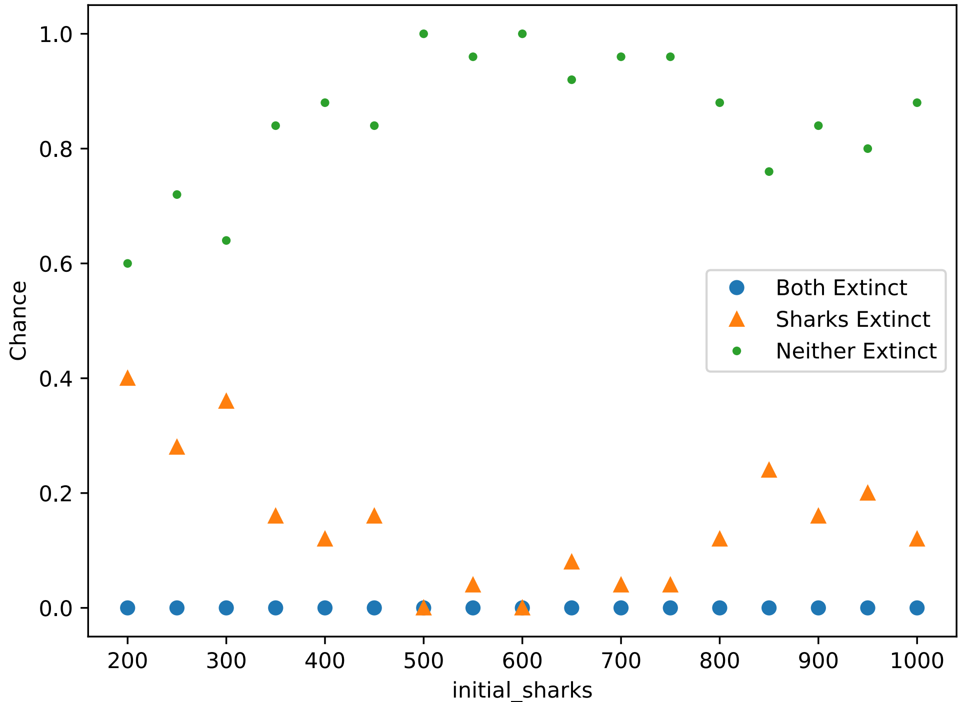

Figure 7: Outcome Chances vs `initial_sharks`

```
import measure_outcome_chances as tst

tst.run_standard_test("start_energy", True)
```

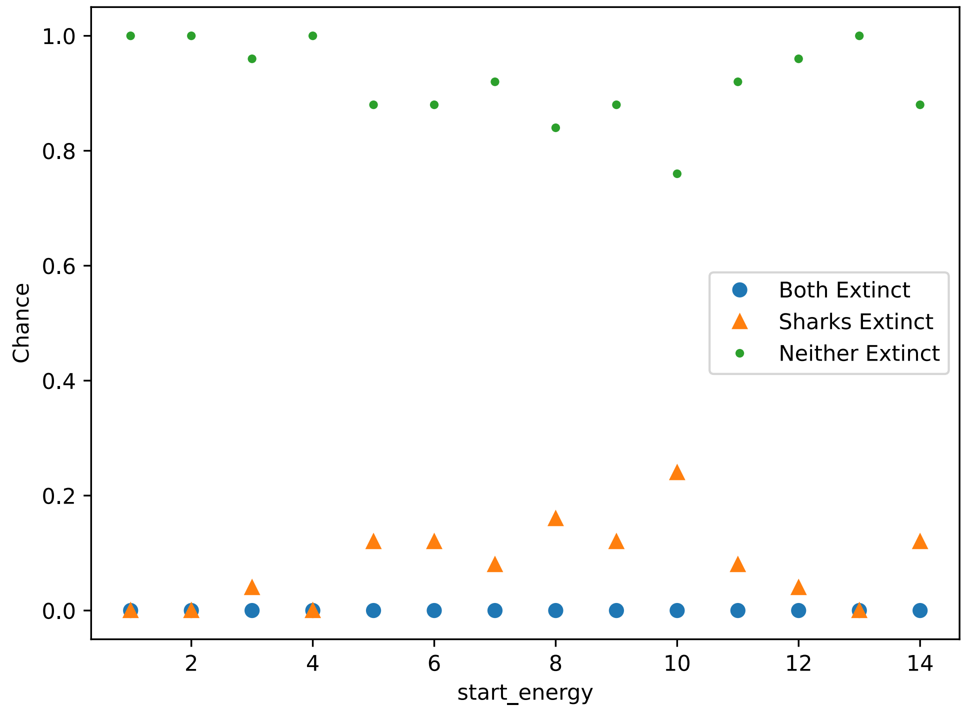

Figure 8: Outcome Chances vs `start_energy`

Main Simulation Parameters

`breed_time`

`energy_gain`

`breed_energy`

Circular Initialization

Extension