Realistic Projectiles

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Contents

1. Quadratic Drag Equation

In introductory physics, projectiles are typically modeled as experiencing negligible air drag. For this project, projectiles were modeled as experiencing *quadratic drag*.

$$\frac{d^2\vec{r}}{dt^2} = \vec{g} - kv^2\hat{v}$$

The terms in this equation are as follows:

$$\begin{aligned} \vec{r} &= \begin{pmatrix} x \\ y \end{pmatrix} && \text{(position)} \\ \vec{v} &= \frac{d\vec{r}}{dt} && \text{(velocity)} \\ \vec{g} &= \begin{pmatrix} 0 \\ -g \end{pmatrix} && \text{(gravitation acceleration)} \\ k &= \text{"constant"} && \text{(drag constant)} \end{aligned}$$

The y axis points straight up, and the x axis points horizontally along the plane of motion of the projectile. This keeps the motion in 2 dimensions. Projectiles were started on the ground at $(x, y) = (0, 0)$.

To focus on scale-independent features of the motion, units of distance and time were used such that $g = 1$ and $k = 1$. This makes the terminal speed $v_\infty = 1$.

2. Runge-Kutta Four (RK4) Method

3. Interdependence of Horizontal and Vertical Motion

4. Trajectory Shapes

5. Firing Range

5.a. As a Function of Firing Angle

5.b. As a Function of Initial Speed

6. Extension