Carnot Cycle Exercise

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$$T_H = 490 \text{ K}$$

 $V_c = 1.90 \times 10^{-3} \text{ m}^3$

1. Purpose

The goal of the exercise is to perform various calculations related to the Carnot cycle.

2. Given

- $T_H = 490 \text{ K}$
- $T_C = 300 \text{ K}$
- $P_c = 1.01 \times 10^5 \text{ Pa}$
- $V_c = 1.90 \times 10^{-3} \text{ m}^3$
- $Q_{a \to b} = 300 \text{ J}$
- $\gamma = 1.40$

3. Derivations

3.1. Temperature-Volume Relationship for Adiabatic Process

Given:

- PV = nRT
- $P_i V_i^{\gamma} = P_f V_f^{\gamma}$

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

$$P_i V_i V_i^{\gamma - 1} = P_f V_f V_f^{\gamma - 1}$$

$$nRT_i V_i^{\gamma - 1} = nRT_f V_f^{\gamma - 1}$$

$$T_i V_i^{\gamma - 1} = T_f V_f^{\gamma - 1}$$

3.2. Work by Gas for Isothermal Process

Given:

- PV = nRT
- $W = \int P dV$

$$W = \int_{V=V_i}^{V=V_f} P dV$$

$$W = \int_{V_i}^{V_f} \frac{nRT}{V} dV$$

$$W = nRT \ln(V)|_{V_i}^{V_f}$$

$$W = nRT(\ln(V_f) - \ln(V_i))$$
$$W = nRT \ln\left(\frac{V_f}{V_i}\right)$$

4. Results

4.1. Moles of Gas(n)

$$P_cV_c = nRT_c$$

$$P_cV_c = nRT_C$$

$$n = \frac{P_cV_c}{RT_C}$$

$$n = 0.0770 \text{ mol}$$

4.2. Pressure (P_b) and Volume (V_b) at b

$$\begin{split} T_b V_b^{\gamma-1} &= T_c V_c^{\gamma-1} \\ V_b &= V_c \left(\frac{T_c}{T_b}\right)^{\frac{1}{\gamma-1}} \\ V_b &= V_c \left(\frac{T_C}{T_H}\right)^{\frac{1}{\gamma-1}} \\ V_b &= 5.57 \times 10^{-4} \text{ m}^3 \end{split}$$

$$P_bV_b = nRT_b$$

$$P_bV_b = nRT_H$$

$$P_b = \frac{nRT_H}{V_b}$$

$$P_b = 5.63 \times 10^5 \text{ Pa}$$

Table 1. Pressure, Volume, and Temperature for Key Points

Point	P (Pa)	$V~(\mathrm{m}^3)$	T(K)
a b c	5.63×10^5 1.01×10^5	5.57×10^{-4} 1.90×10^{-3}	490 490 300 300

5. Conclusion

6. Citations

- [1] Karen Schnurbusch, Physics 4B Lab Book, Mt. San Antonio College, 2023, pp. 35-38.
- [2] Karen Schnurbusch, Physics 4B Equations, Mt. San Antonio College, 2023, pp. 1-3.