Resistance Lab

Vincent Edwards, Ali Mortada, Andrew Bringas

Mt. San Antonio College, Physics 4B, CRN 42240
April 26, 2023

1. Purpose

For part 1, the goal was to examine how material, cross-sectional area, and length impact resistance. For each coil of wire, resistance was determined in three ways: based on measurements of voltage and current, based on the resistance readings from the multimeter, and based on the dimensions of the wires and the resistivity of the materials. The results of the three methods of determining resistance were compared.

For part 2, the goal was to examine voltage, current, and resistance for two resistors in series and two resistors in parallel. For each nichrome resistor, resistance was determined in two ways: based on measurements of voltage and current, and based on the resistance readings from the multimeter. The results of these two methods were compared. The readings from the multimeter were accepted as the true resistance values, which were then used to calculated the theoretical resistance of the resistors in series and the resistors in parallel. These theoretical resistances were compared to experimental values found by setting up the circuits and measuring voltage and current. In addition, measurements of voltage and current were made at different parts of the series and parallel circuits in order to see if the matched what theory predicts.

2. Results

Table 1 contains the properties of the resistance coils used in part 1. ρ is the resistivity of the material. L is the length of the coil of wire. D is the diameter of the wire. The values for ρ, L, and D were assumed to be exact, without uncertainty. In addition, note that Nickel-Silver is a copper alloy consisting of copper, nickel, and zinc or manganese. The exact composition of the alloy used in the experiment is unknown. Thus, the resistivity was approximated to be 44×10^{-8} Ω m, the value for Manganin (Cu 84%, Mn 12%, Ni 4%), an alloy of similar composition.

Table 1. Part 1 Resistance Coils

Coil	Material	ρ (Ω m)	L (cm)	D (cm)
1	Nickel-Silver	44×10^{-8}	40	0.0254
2	Nickel-Silver	44×10^{-8}	80	0.0254
3	Nickel-Silver	44×10^{-8}	120	0.0254
4	Nickel-Silver	44×10^{-8}	160	0.0254
5	Nickel-Silver	44×10^{-8}	200	0.0254
6	Nickel-Silver	44×10^{-8}	200	0.0320
7	Copper	1.72×10^{-8}	2000	0.0254

Table 2 contains the measurements made during part 1. V is the voltage across the resistance coil. I is the current through the resistance coil. $(R_{\mathrm{m}} + r)$ is the resistance, measured using the multimeter, of the resistance coil and the wires connecting it to the multimeter. r is the resistance, measured using the multimeter to be 0.1 ± 0.1 Ω, of just the wires used for connecting the multimeter to the resistance coil.

Table 2. Part 1 Measurements
Note: r, the resistance of the two wires connected to the multimeter, was measured to be 0.1 ± 0.1 Ω.

Coil	V (V)	I (mA)	$R_{\mathrm{m}} + r$ (Ω)
1	0.214 ± 0.001	55.07 ± 0.05	4.0 ± 0.1
2	0.400 ± 0.001	49.72 ± 0.01	8.3 ± 0.1
3	0.536 ± 0.001	44.53 ± 0.02	12.0 ± 0.1
4	0.662 ± 0.001	41.70 ± 0.01	15.9 ± 0.1
5	0.772 ± 0.001	38.19 ± 0.01	20.3 ± 0.1
6	0.538 ± 0.001	45.44 ± 0.01	11.7 ± 0.1
7	0.357 ± 0.001	51.31 ± 0.01	7.3 ± 0.1

Table 3 contains the two experimental values for resistance in the coil ($R_{V/I}$ and R_{m}) and the theoretical value (R_{th}). $R_{V/I}$ is the experimental resistance determined using the measured values for V and I. R_{m} is the experimental resistance determined using the readings from the multimeter. R_{th} is the theoretical resistance determined using the dimensions of the wires (L and D) and the resistivity of the material (ρ).

Table 3. Part 1 Experimental and Theoretical Resistances

Coil	$R_{V/I}$ (Ω)	R_{m} (Ω)	R_{th} (Ω)
1	3.89 ± 0.02	3.9 ± 0.1	3.47
2	8.05 ± 0.02	8.2 ± 0.1	6.95
3	12.04 ± 0.02	11.9 ± 0.1	10.42
4	15.88 ± 0.02	15.8 ± 0.1	13.89
5	20.21 ± 0.03	20.2 ± 0.1	17.37
6	11.84 ± 0.02	11.6 ± 0.1	10.94
7	6.96 ± 0.02	7.2 ± 0.1	6.79

Figure 1 plots resistance ($R_{V/I}$, R_{m}, and R_{th}) versus length (L) for the first five resistance coils used in part 1. Those coils were chosen for the graph as they all had the same material and diameter. Since the graphs seemed linear (and that's what theory predicts) a linear fit was applied.

Table 4 contains the resistances measured using a multimeter of the two nichrome resistors used in part 2. $(R + r)$ is the resistance, measured using the multimeter, of the nichrome resistor and the wires connecting it to the multimeter. r is the resistance, measured using the multimeter to be 0.9 ± 0.1 Ω, of just the wires used for connecting the multimeter to the nichrome resistor. R is the calculated resistance of the nichrome resistor based on these measurements. These resistance values, $R_1 = 11.4 \pm 0.1$ Ω and $R_2 = 29.7 \pm 0.1$ Ω, will be referred to as the true values.

Table 5 has the theoretical resistances for part 2. R_{1+2} is the theoretical equivalent resistance for the two resistors in series. $R_{1||2}$ is the theoretical equivalent resistance for the two resistors in parallel.

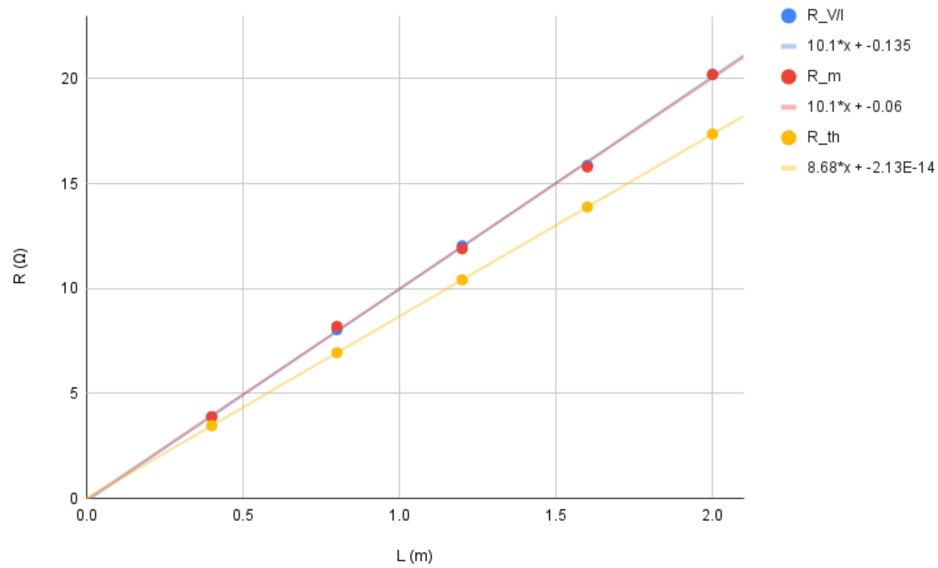

Figure 1. Resistance vs Length (Constant ρ and A)

Table 4. Part 2 True Resistances
Note: r, the resistance of the two wires connected to the multimeter, was measured to be 0.9 ± 0.1 Ω.

	$R + r$ (Ω)	R (Ω)
1	12.3 ± 0.1	11.4 ± 0.1
2	30.6 ± 0.1	29.7 ± 0.1

Table 5. Part 2 Theoretical Resistances

	Resistance (Ω)
R_{1+2}	41.1 ± 0.2
$R_{1\|\|2}$	8.24 ± 0.07

Table 6 contains the values for the circuit with just R_1 connected to the power supply. V is the voltage across the nichrome resistor. I is the current through the nichrome resistor. $R_{V/I}$ is the experimental resistance determined using the measured values for V and I.

Table 7 contains the values for the circuit with just R_2 connected to the power supply. V is the voltage across the nichrome resistor. I is the current through the nichrome resistor. $R_{V/I}$ is the experimental resistance determined using the measured values for V and I.

Table 8 contains the values for the circuit with R_1 and R_2 connected to the power supply in series. V is the voltage across both nichrome resistors. I is the current through both nichrome resistors. $R_{V/I}$ is the experimental resistance determined using

Table 6. Part 2 R_1 Experimental Values

V (V)	I (mA)	$R_{V/I}$ (Ω)
0.000 ± 0.001	0.00 ± 0.01	undefined
0.744 ± 0.001	66.8 ± 0.1	11.14 ± 0.02
1.491 ± 0.001	134.0 ± 0.1	11.13 ± 0.01
2.257 ± 0.001	202.8 ± 0.1	11.129 ± 0.007
3.000 ± 0.001	269.6 ± 0.1	11.128 ± 0.006

Table 7. Part 2 R_2 Experimental Values

V (V)	I (mA)	$R_{V/I}$ (Ω)
0.000 ± 0.001	0.00 ± 0.01	undefined
0.746 ± 0.001	24.96 ± 0.01	29.89 ± 0.04
1.500 ± 0.001	50.16 ± 0.01	29.9 ± 0.02
2.255 ± 0.001	75.4 ± 0.1	29.91 ± 0.04
2.998 ± 0.001	100.3 ± 0.1	29.89 ± 0.03

the measured values for V and I.

Table 8. Part 2 R_{1+2} Experimental Values

V (V)	I (mA)	$R_{V/I}$ (Ω)
0.000 ± 0.001	0.00 ± 0.01	undefined
0.757 ± 0.001	18.37 ± 0.01	41.21 ± 0.06
1.496 ± 0.001	36.29 ± 0.01	41.22 ± 0.03
2.254 ± 0.001	54.69 ± 0.01	41.21 ± 0.02
3.001 ± 0.001	72.8 ± 0.1	41.22 ± 0.06

Table 9 contains the values for the circuit with R_1 and R_2 connected to the power supply in parallel. V is the voltage across both nichrome resistors. I is the current through both nichrome resistors. $R_{V/I}$ is the experimental resistance determined using the measured values for V and I.

Table 9. Part 2 $R_{1||2}$ Experimental Values

V (V)	I (mA)	$R_{V/I}$ (Ω)
0.000 ± 0.001	0.00 ± 0.01	undefined
0.749 ± 0.001	91.4 ± 0.1	8.19 ± 0.01
1.502 ± 0.001	183.5 ± 0.1	8.185 ± 0.007
2.252 ± 0.001	275.5 ± 0.1	8.174 ± 0.005
3.000 ± 0.001	367.6 ± 0.1	8.161 ± 0.004

Table 10 contains the voltage and current measurements made of the series circuit at 4 V. V_1 is the voltage across resistor 1. V_2 is the voltage across resistor 2. V_{1+2} is the voltage across both resistors. I_1 is the current between the resistors, after coming from the power supply and through resistor 1. I_2 is the current after resistor 2, right before reaching the negative terminal of the power supply. I_{1+2} is the current next to the positive terminal of the power supply, right before reaching the first resistor.

Table 10. Part 2 Series Circuit at 4 V

	V (V)	I (mA)
R_1	1.084 ± 0.001	96.5 ± 0.1
R_2	2.915 ± 0.001	97.1 ± 0.1
R_{1+2}	4.002 ± 0.001	97.1 ± 0.1

Table 11 contains the voltage and current measurements made of the parallel circuit at 4 V. V_1 is the voltage across resistor 1. V_2 is the voltage across resistor 2. $V_{1||2}$ is the voltage across both resistors. I_1 is the current through resistor 1. I_2 is the current through resistor 2. $I_{1||2}$ is the current next to the positive terminal of the power supply, right before reaching the junction connected to the two resistors.

Table 11. Part 2 Parallel Circuit at 4 V

	V (V)	I (mA)
R_1	3.968 ± 0.001	410.4 ± 0.1
R_2	3.927 ± 0.001	155.6 ± 0.1
$R_{1\|\|2}$	4.002 ± 0.001	491.7 ± 0.1

3. Uncertainty

All the voltage, current, and resistance measurements in the experiment were made using a multimeter. If there were fluctuations in a particular reading with the multimeter (voltage, current, or resistance), then the uncertainty was taken to be the magnitude of the fluctuations, as that gives an upper bound for how much the value could have varied. If the reading with the multimeter (voltage, current, or resistance) was stable and did not fluctuate, then the uncertainty was taken to be the smallest increment of measure, since that was the limiting precision of the equipment.

In parts 1 and 2, $R_{V/I}$ was calculated using equation 1.

$$R_{V/I} = \frac{V}{I} \tag{1}$$

The uncertainty of $R_{V/I}$ is given by equation 2.

$$\begin{aligned} \Delta R_{V/I} &= \left[\left(\frac{\partial R_{V/I}}{\partial V} \Delta V \right)^2 + \left(\frac{\partial R_{V/I}}{\partial I} \Delta I \right)^2 \right]^{\frac{1}{2}} \\ &= \frac{V}{I} \left[\left(\frac{\Delta V}{V} \right)^2 + \left(\frac{\Delta I}{I} \right)^2 \right]^{\frac{1}{2}} \end{aligned} \tag{2}$$

In part 1, R_m was calculated using equation 3. Note that the combined value $(R_\mathrm{m} + r)$ was measured, as was the value for r. But R_m was not measured, hence why it is being calculated as a difference of two values.

$$R_\mathrm{m} = (R_\mathrm{m} + r) - r \tag{3}$$

The uncertainty of R_m is given by equation 4.

$$\begin{aligned}\Delta R_\mathrm{m} &= \left[\left(\frac{\partial R_\mathrm{m}}{\partial(R_\mathrm{m}+r)}\Delta(R_\mathrm{m}+r)\right)^2 + \left(\frac{\partial R_\mathrm{m}}{\partial r}\Delta r\right)^2\right]^{\frac{1}{2}} \\ &= \left[(\Delta(R_\mathrm{m}+r))^2 + (\Delta r)^2\right]^{\frac{1}{2}}\end{aligned} \tag{4}$$

Similarly in part 2, the true resistance values R_1 and R_2, referred to as just R, were calculated using equation 5. Note that the combined value $(R + r)$ was measured, as was the value for r. But R was not measured, hence why it is being calculated as a difference of two values.

$$R = (R + r) - r \tag{5}$$

The uncertainty of R is given by equation 6.

$$\begin{aligned}\Delta R &= \left[\left(\frac{\partial R}{\partial(R+r)}\Delta(R+r)\right)^2 + \left(\frac{\partial R}{\partial r}\Delta r\right)^2\right]^{\frac{1}{2}} \\ &= \left[(\Delta(R+r))^2 + (\Delta r)^2\right]^{\frac{1}{2}}\end{aligned} \tag{6}$$

In part 1, R_th was calculated using equation 7.

$$\begin{aligned}R_\mathrm{th} &= \frac{\rho L}{A} \\ &= \frac{\rho L}{\pi(D/2)^2} \\ &= \frac{4\rho L}{\pi D^2}\end{aligned} \tag{7}$$

Note that the values for ρ, L, and D were assumed to be exact, without uncertainty. Thus, R_th would have no uncertainty as well.

In part 2, R_{1+2}, the theoretical resistance for the two resistors in series, was calculated using equation 8.

$$R_{1+2} = R_1 + R_2 \tag{8}$$

The uncertainty of R_{1+2} is given by equation 9.

$$\begin{aligned}\Delta R_{1+2} &= \left[\left(\frac{\partial R_{1+2}}{\partial R_1}\Delta R_1\right)^2 + \left(\frac{\partial R_{1+2}}{\partial R_2}\Delta R_2\right)^2\right]^{\frac{1}{2}} \\ &= \left[(\Delta R_1)^2 + (\Delta R_2)^2\right]^{\frac{1}{2}}\end{aligned} \tag{9}$$

In part 2, $R_{1||2}$, the theoretical resistance for the two resistors in parallel, was calculated using equation 10.

$$R_{1||2} = \left(R_1^{-1} + R_2^{-1}\right)^{-1} \tag{10}$$

The uncertainty of $R_{1||2}$ is given by equation 11.

$$\begin{aligned} \Delta R_{1||2} &= \left[\left(\frac{\partial R_{1||2}}{\partial R_1}\Delta R_1\right)^2 + \left(\frac{\partial R_{1||2}}{\partial R_2}\Delta R_2\right)^2\right]^{\frac{1}{2}} \\ &= \left[\left(\left[R_1^{-1} + R_2^{-1}\right]^{-2}\frac{\Delta R_1}{R_1^2}\right)^2 + \left(\left[R_1^{-1} + R_2^{-1}\right]^{-2}\frac{\Delta R_2}{R_2^2}\right)^2\right]^{\frac{1}{2}} \end{aligned} \tag{11}$$

4. Conclusion

In part 1, the resistances of 7 different resistance coils were determined using three different techniques. Descriptions of the coils are in table 1, and the calculated resistance values are in table 3. The experimental resistances $R_{V/I}$ and R_m had similar values for all the coils. Coils 1, 4, and 5 had resistance values that overlapped within uncertainty, while the rest of the coils (2, 3, 6, and 7) had values that were still fairly close and almost within uncertainty. While the experimental resistances were similar, their values were all higher than the corresponding theoretical resistances R_th. One possible explanation is that when measuring voltage and resistance, the multimeter was connected to the resistance coils through a small bit of copper wire. This copper wire section was not included when measuring r, but it was included when measuring $(R_\mathrm{m} + r)$. As a result, this would cause R_m to be higher than expected. This extra resistance would also have increased the measured value for V, thereby causing $R_{V/I}$ to be higher than expected. Another possibility is that the temperature of the room did not match the reference temperature of 20 ℃. The table the resistivity values were taken from used a reference temperature of 20 ℃. If the room were warmer than 20 ℃, then the actual resistivity values would have been slightly higher than the ρ values used for calculations. This would cause R_th to be lower than expected.

Resistance coils 1 through 5 were all made of the same material and had the same cross-sectional area, but they had different lengths. Figure 1 plots resistance ($R_{V/I}$, R_m, and R_th) versus length (L). The graphs are linear with y-intercepts close to zero. Thus, the data indicates that the resistance of a wire/coil is proportional to its length.

Resistance coils 5 and 6 were made of the same material and had the same length, but they had different cross-sectional areas. The experimental resistances for coil 5 ($R_{V/I} = 20.21 \pm 0.03\ \Omega$, $R_\mathrm{m} = 20.2 \pm 0.1\ \Omega$) were higher than the experimental resistances for coil 6 ($R_{V/I} = 11.84 \pm 0.02\ \Omega$, $R_\mathrm{m} = 11.6 \pm 0.1\ \Omega$). The cross-sectional area of coil 5 ($A = \frac{1}{4}\pi(0.0254\ \mathrm{cm})^2 = 5.07 \times 10^{-4}\ \mathrm{cm}^2$) was lower than the cross-sectional area of coil 6 ($A = \frac{1}{4}\pi(0.0320\ \mathrm{cm})^2 = 8.04 \times 10^{-4}\ \mathrm{cm}^2$). Thus, the data suggests that as cross-sectional area increases, resistances decreases.

Resistance coil 7, made out of copper, was 10 times longer than coil 5, made out of nickel-silver, with the same cross-sectional area. Since it has already been shown that resistance is proportional to length, then dividing the experimental resistance values for coil 7 by 10 would allow us to compare the resistances of two wires of identical dimensions but different materials. Thus, our hypothetical 200 cm long copper resistance coil of diameter 0.0254 cm would have experimental resistances $R_{V/I} = 0.696 \pm 0.002\ \Omega$ and $R_\mathrm{m} = 0.72 \pm 0.01\ \Omega$. For comparison, coil 5 had experimental resistances $R_{V/I} = 20.21 \pm 0.03\ \Omega$ and $R_\mathrm{m} = 20.2 \pm 0.1\ \Omega$. The resistance of coil 5 was much higher than the resistance of the hypothetical copper wire with identical length and diameter. This supports the idea that resistance depends on material, with some materials having more resistance for a given wire shape than others.

In part 2, the resistance of two nichrome coils were measured, and those measurements were used to determine the resistance of four different circuits. The true values of both coils' resistances are in table 4, the experimental values of the first coil are in table 6, and the experimental values of the second coil are in table 7. For both coils, the experimental resistances $R_{V/I}$ were within uncertainty of each other, but were not within uncertainty of their respective coil's true resistance. However, the experimental resistances were fairly close to the true values, with a difference of around 0.3 Ω for R_1 and 0.2 Ω for R_2. (Did we have extra or less wires when measuring these experimental values? May explain why values didn't match up). After measuring the resistance of the first two circuits containing only one of the nichrome coils at a time, both coils were connected in series, then in parallel. The theoretical resistances of these two circuits are in table 5, the experimental equivalent resistance values of the series circuit are in table 8, and the experimental equivalent resistance values of the parallel circuit are in table 9. Similar to the first two circuits, the experimental resistances $R_{V/I}$ of the series circuit were all within uncertainty of each other, while the experimental resistances $R_{V/I}$ of the parallel circuit were not all within uncertainty of each other, although they were very close. The experimental resistances of the series circuit were within uncertainty of the theoretical resistance R_{1+2}, while the experimental resistances of the parallel circuit did not fall within uncertainty of the theoretical resistance $R_{1||2}$, although they were fairly close, being around 0.03 Ω less than the theoretical resistance.

(Continue part 2 analysis questions 6-11).

5. Citations

[1] Karen Schnurbusch, *Physics 4B Lab Book*, Mt. San Antonio College, 2023, pp. 65-70.
[2] Karen Schnurbusch, *Physics 4B Equations*, Mt. San Antonio College, 2023, pp. 4, 10.