Circuits Lab

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$R_2 = 40\ \Omega$

1. Purpose

The goal of the exercise was to use Kirchhoff's rules to analyze 7 different circuits and calculate current, voltage, and power for each circuit element. For the first 4 circuits, the voltage and current calculations were compared to results obtained using an online circuit simulator.

2. Results

The following sections contain the theoretical voltage, current, and power for each circuit element for each of the 7 circuits. In addition, there is an annotated circuit diagram for each circuit. In each case, the voltage across a resistor was calculated with $V = IR$, and the power dissipated by a circuit element with $P = VI$.

2.1. Circuit 1

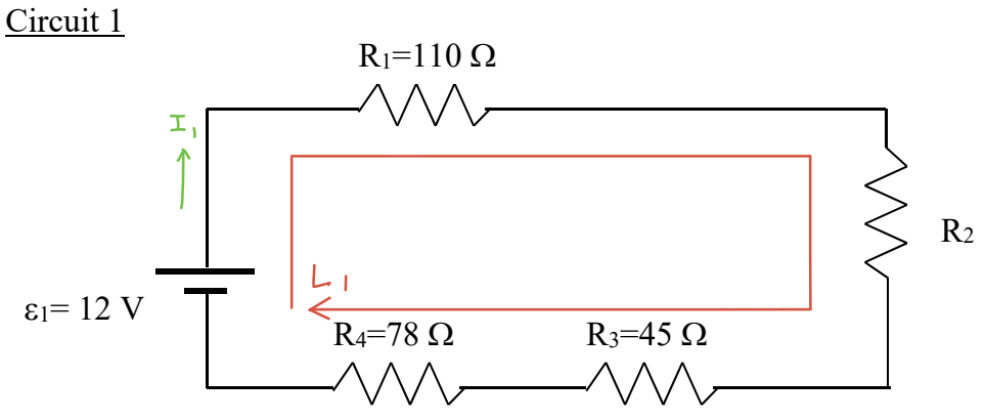

Figure 1. Circuit 1

Table 1. Circuit 1 V, I, and P

	V (V)	I (A)	P (W)
$\mathcal{E}_1$	12.0	0.0440	0.527
R_1	4.84	0.0440	0.213
R_2	1.76	0.0440	0.0773
R_3	1.98	0.0440	0.0869
R_4	3.43	0.0440	0.151

$$\mathrm{L}_1 : \mathcal{E}_1 - I_1 R_1 - I_1 R_2 - I_1 R_3 - I_1 R_4 = 0$$

$$I_1 = \frac{\mathcal{E}_1}{R_1 + R_2 + R_3 + R_4}$$

$$I_1 = 0.0440 \text{ A}$$

2.2. Circuit 2

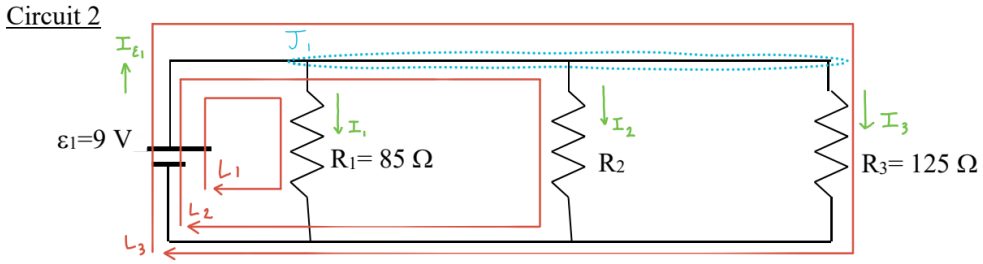

Figure 2. Circuit 2

Table 2. Circuit 2 V, I, and P

	V (V)	I (A)	P (W)
$\mathcal{E}_1$	9.00	0.403	3.63
R_1	9.00	0.106	0.953
R_2	9.00	0.225	2.02
R_3	9.00	0.0720	0.648

$$\begin{aligned} \text{J}_1: \quad I_{\mathcal{E}_1} &= I_1 + I_2 + I_3 \\ I_1 + I_2 + I_3 - I_{\mathcal{E}_1} &= 0 \end{aligned}$$

$$\begin{aligned} \text{L}_1: \quad \mathcal{E}_1 - I_1 R_1 &= 0 \\ I_1 R_1 &= \mathcal{E}_1 \end{aligned}$$

$$\begin{aligned} \text{L}_2: \quad \mathcal{E}_1 - I_2 R_2 &= 0 \\ I_2 R_2 &= \mathcal{E}_1 \end{aligned}$$

$$\begin{aligned} \text{L}_3: \quad \mathcal{E}_1 - I_3 R_3 &= 0 \\ I_3 R_3 &= \mathcal{E}_1 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & 0 & R_3 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_{\mathcal{E}_1} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathcal{E}_1 \\ \mathcal{E}_1 \\ \mathcal{E}_1 \end{bmatrix}$$

$$I_1 = 0.106 \text{ A}$$
$$I_2 = 0.225 \text{ A}$$
$$I_3 = 0.0720 \text{ A}$$
$$I_{\mathcal{E}_1} = 0.403 \text{ A}$$

2.3. Circuit 3

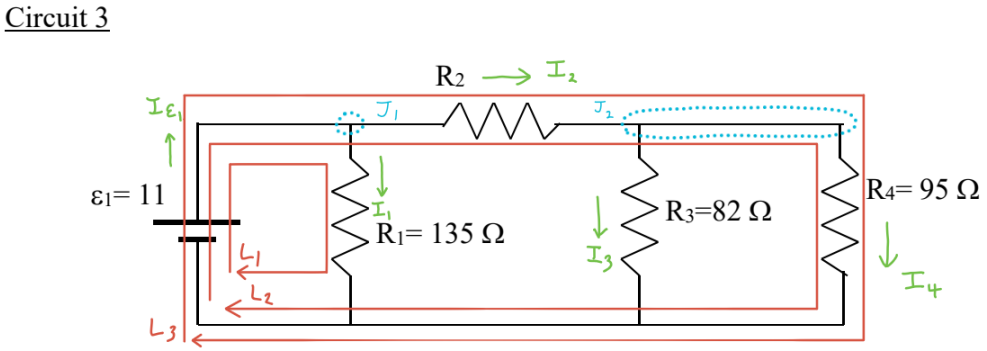

Figure 3. Circuit 3

Table 3. Circuit 3 V, I, and P

	V (V)	I (A)	P (W)
$\mathcal{E}_1$	11.0	0.212	2.34
R_1	11.0	0.0815	0.896
R_2	5.24	0.131	0.686
R_3	5.76	0.0703	0.405
R_4	5.76	0.0607	0.350

$$\begin{aligned} \text{J}_1: \quad I_{\mathcal{E}_1} &= I_1 + I_2 \\ I_1 + I_2 - I_{\mathcal{E}_1} &= 0 \end{aligned}$$

$$\begin{aligned} \text{J}_2: \quad I_2 &= I_3 + I_4 \\ I_2 - I_3 - I_4 &= 0 \end{aligned}$$

$$\begin{aligned} \text{L}_1: \quad \mathcal{E}_1 - I_1 R_1 &= 0 \\ I_1 R_1 &= \mathcal{E}_1 \end{aligned}$$

$$\begin{aligned} \text{L}_2: \mathcal{E}_1 - I_2 R_2 - I_3 R_3 &= 0 \\ I_2 R_2 + I_3 R_3 &= \mathcal{E}_1 \end{aligned}$$

$$\begin{aligned} \text{L}_3: \mathcal{E}_1 - I_2 R_2 - I_4 R_4 &= 0 \\ I_2 R_2 + I_4 R_4 &= \mathcal{E}_1 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & -1 & 0 \\ R_1 & 0 & 0 & 0 & 0 \\ 0 & R_2 & R_3 & 0 & 0 \\ 0 & R_2 & 0 & R_4 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_{\mathcal{E}_1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mathcal{E}_1 \\ \mathcal{E}_1 \\ \mathcal{E}_1 \end{bmatrix}$$

$$\begin{aligned} I_1 &= 0.0815 \text{ A} \\ I_2 &= 0.131 \text{ A} \\ I_3 &= 0.0703 \text{ A} \\ I_4 &= 0.0607 \text{ A} \\ I_{\mathcal{E}_1} &= 0.212 \text{ A} \end{aligned}$$

2.4. Circuit 4

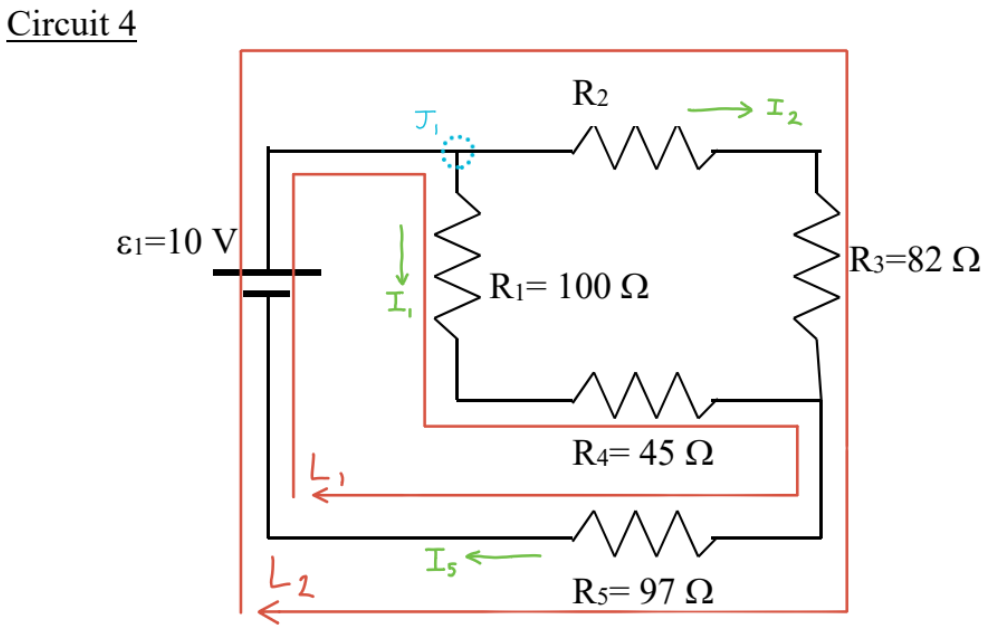

Figure 4. Circuit 4

Table 4. Circuit 4 V, I, and P

	V (V)	I (A)	P (W)
$\mathcal{E}_1$	10.0	0.0613	0.613
R_1	2.80	0.0280	0.0783
R_2	1.33	0.0333	0.0443
R_3	2.73	0.0333	0.0907
R_4	1.26	0.0280	0.0353
R_5	5.94	0.0613	0.364

$$\text{J}_1: \quad I_5 = I_1 + I_2$$
$$I_1 + I_2 - I_5 = 0$$

$$\text{L}_1: \mathcal{E}_1 - I_1R_1 - I_1R_4 - I_5R_5 = 0$$
$$I_1(R_1 + R_4) + I_5R_5 = \mathcal{E}_1$$

$$\text{L}_2: \mathcal{E}_1 - I_2R_2 - I_2R_3 - I_5R_5 = 0$$
$$I_2(R_2 + R_3) + I_5R_5 = \mathcal{E}_1$$

$$\begin{bmatrix} 1 & 1 & -1 \\ R_1 + R_4 & 0 & R_5 \\ 0 & R_2 + R_3 & R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_5 \end{bmatrix} = \begin{bmatrix} 0 \\ \mathcal{E}_1 \\ \mathcal{E}_1 \end{bmatrix}$$

$$I_1 = 0.0280 \text{ A}$$
$$I_2 = 0.0333 \text{ A}$$
$$I_5 = 0.0613 \text{ A}$$

2.5. Circuit 5

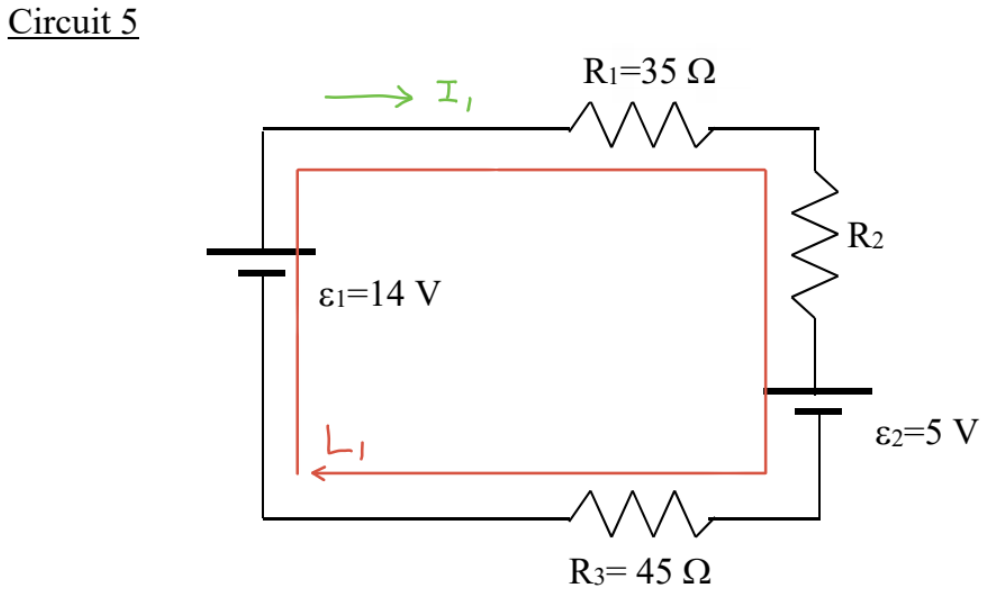

Figure 5. Circuit 5

Table 5. Circuit 5 V, I, and P

	V (V)	I (A)	P (W)
$\mathcal{E}_1$	14.0	0.0750	1.05
$\mathcal{E}_2$	5.00	0.0750	0.375
R_1	2.62	0.0750	0.197
R_2	3.00	0.0750	0.225
R_3	3.38	0.0750	0.253

$$\mathrm{L}_1 : \mathcal{E}_1 - I_1 R_1 - I_1 R_2 - \mathcal{E}_2 - I_1 R_3 = 0$$

$$I_1 = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2 + R_3}$$

$$I_1 = 0.0750 \text{ A}$$

2.6. Circuit 6

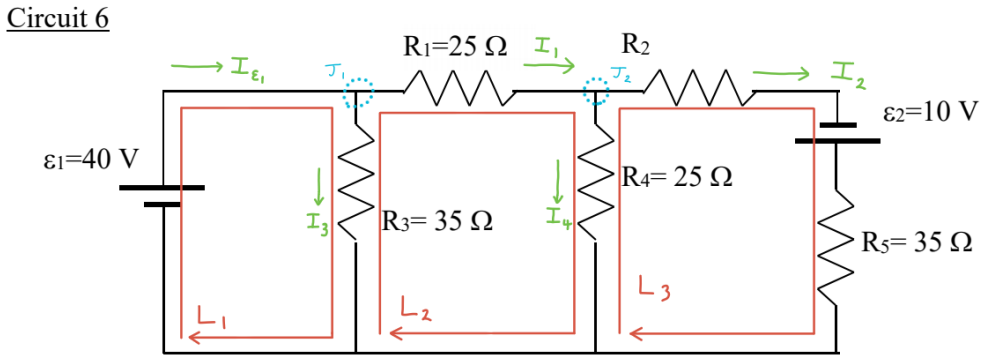

Figure 6. Circuit 6

Table 6. Circuit 6 V, I, and P

	V (V)	I (A)	P (W)
$\mathcal{E}_1$	40.0	2.11	84.6
$\mathcal{E}_2$	10.0	0.343	3.43
R_1	24.3	0.971	23.6
R_2	13.7	0.343	4.70
R_3	40.0	1.14	45.7
R_4	15.7	0.629	9.88
R_5	12.0	0.343	4.11

$$\begin{aligned}
\text{J}_1: \qquad I_{\mathcal{E}_1} &= I_1 + I_3 \\
I_1 + I_3 - I_{\mathcal{E}_1} &= 0
\end{aligned}$$

$$\begin{aligned}
\text{J}_2: \qquad I_1 &= I_2 + I_4 \\
I_1 - I_2 - I_4 &= 0
\end{aligned}$$

$$\begin{aligned}
\text{L}_1: \qquad \mathcal{E}_1 - I_3 R_3 &= 0 \\
I_3 R_3 &= \mathcal{E}_1
\end{aligned}$$

$$\begin{aligned}
\text{L}_2: \qquad -I_1 R_1 - I_4 R_4 + I_3 R_3 &= 0 \\
I_1 R_1 - I_3 R_3 + I_4 R_4 &= 0
\end{aligned}$$

$$\begin{aligned}
\text{L}_3: \mathcal{E}_2 - I_2 R_2 - I_2 R_5 + I_4 R_4 &= 0 \\
I_2(R_2 + R_5) - I_4 R_4 &= \mathcal{E}_2
\end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & R_3 & 0 & 0 \\ R_1 & 0 & -R_3 & R_4 & 0 \\ 0 & R_2 + R_5 & 0 & -R_4 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_{\mathcal{E}_1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mathcal{E}_1 \\ 0 \\ \mathcal{E}_2 \end{bmatrix}$$

$$\begin{aligned}
I_1 &= 0.971 \text{ A} \\
I_2 &= 0.343 \text{ A} \\
I_3 &= 1.14 \text{ A} \\
I_4 &= 0.629 \text{ A} \\
I_{\mathcal{E}_1} &= 2.11 \text{ A}
\end{aligned}$$

2.7. Circuit 7

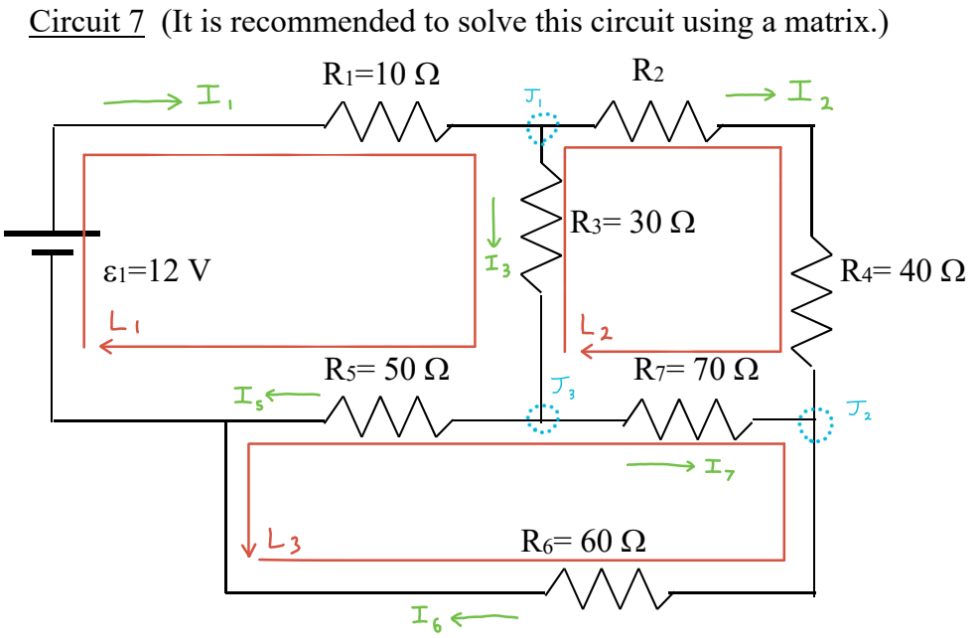

Figure 7. Circuit 7

Table 7. Circuit 7 V, I, and P

	V (V)	I (A)	P (W)
$\mathcal{E}_1$	12.0	0.200	2.40
R_1	2.00	0.200	0.399
R_2	2.58	0.0646	0.167
R_3	4.05	0.135	0.547
R_4	2.58	0.0646	0.167
R_5	5.95	0.119	0.709
R_6	4.83	0.0806	0.390
R_7	1.12	0.0160	0.0179

$$\begin{aligned}
\text{J}_1: \qquad I_1 &= I_2 + I_3 \\
I_1 - I_2 - I_3 &= 0
\end{aligned}$$

$$\begin{aligned}
\text{J}_2: \qquad I_2 + I_7 &= I_6 \\
I_2 - I_6 + I_7 &= 0
\end{aligned}$$

$$\begin{aligned}
\text{J}_3: \qquad I_3 &= I_5 + I_7 \\
I_3 - I_5 - I_7 &= 0
\end{aligned}$$

$$\begin{aligned}
\text{L}_1: \qquad \mathcal{E}_1 - I_1R_1 - I_3R_3 - I_5R_5 &= 0 \\
I_1R_1 + I_3R_3 + I_5R_5 &= \mathcal{E}_1
\end{aligned}$$

$$\begin{aligned}
\text{L}_2: -I_2R_2 - I_2R_4 + I_7R_7 + I_3R_3 &= 0 \\
I_2(R_2 + R_4) - I_3R_3 - I_7R_7 &= 0
\end{aligned}$$

$$\begin{aligned}
\text{L}_3: \qquad -I_5R_5 + I_6R_6 + I_7R_7 &= 0 \\
I_5R_5 - I_6R_6 - I_7R_7 &= \mathcal{E}_2
\end{aligned}$$

$$\begin{bmatrix}
1 & -1 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & -1 & 0 & -1 \\
R_1 & 0 & R_3 & R_5 & 0 & 0 \\
0 & R_2 + R_4 & -R_3 & 0 & 0 & -R_7 \\
0 & 0 & 0 & R_5 & -R_6 & -R_7
\end{bmatrix}
\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ \mathcal{E}_1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned}
I_1 &= 0.200 \text{ A} \\
I_2 &= 0.0646 \text{ A} \\
I_3 &= 0.135 \text{ A} \\
I_5 &= 0.119 \text{ A} \\
I_6 &= 0.0806 \text{ A} \\
I_7 &= 0.0160 \text{ A}
\end{aligned}$$

3. Simulations

Circuits 1 through 4 were simulated using the Circuit Simulator Applet (located at https://www.falstad.com/circuit/). All the voltage and current values match the calculations to 3 significant figures (Yay!).

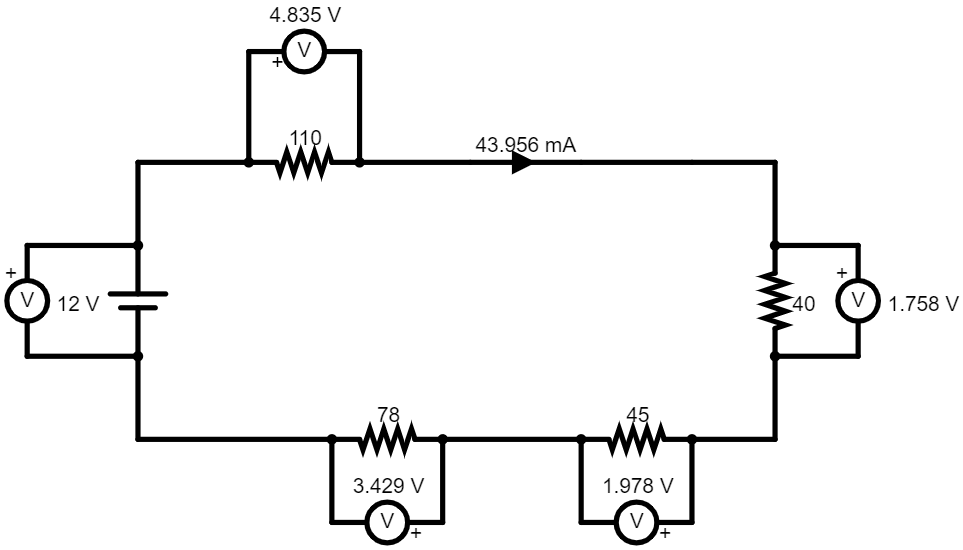

Figure 8. Circuit 1 Simulation

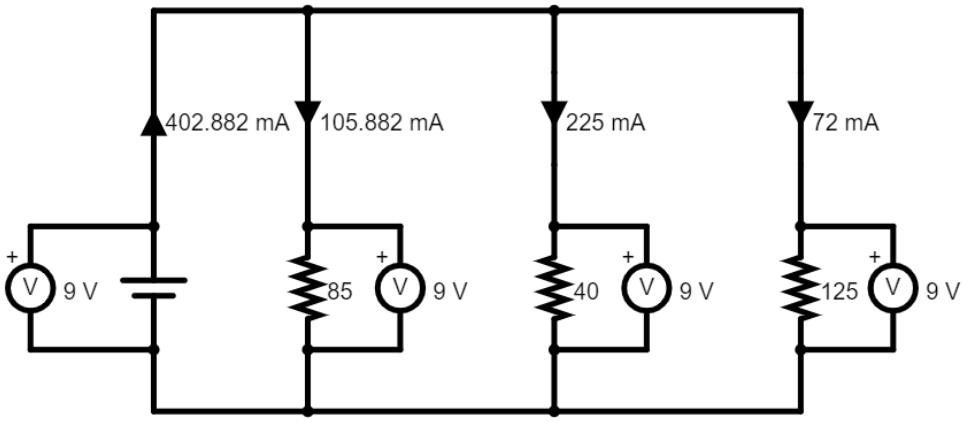

Figure 9. Circuit 2 Simulation

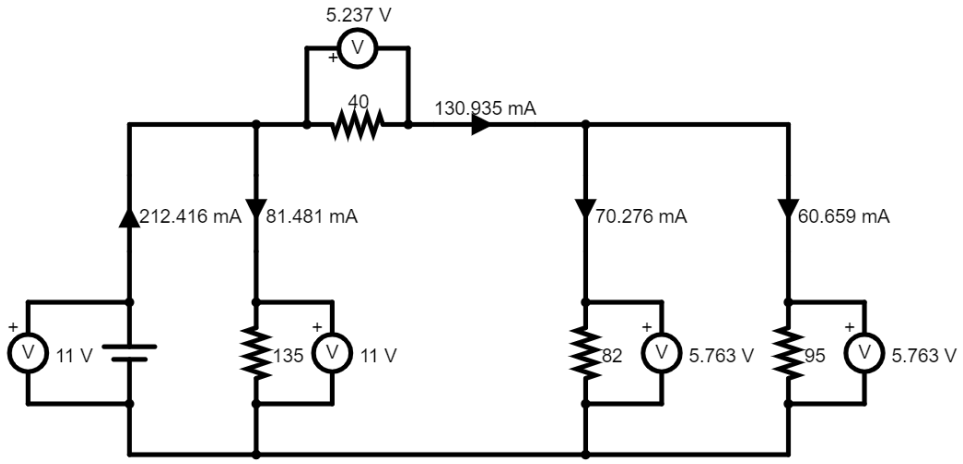

Figure 10. Circuit 3 Simulation

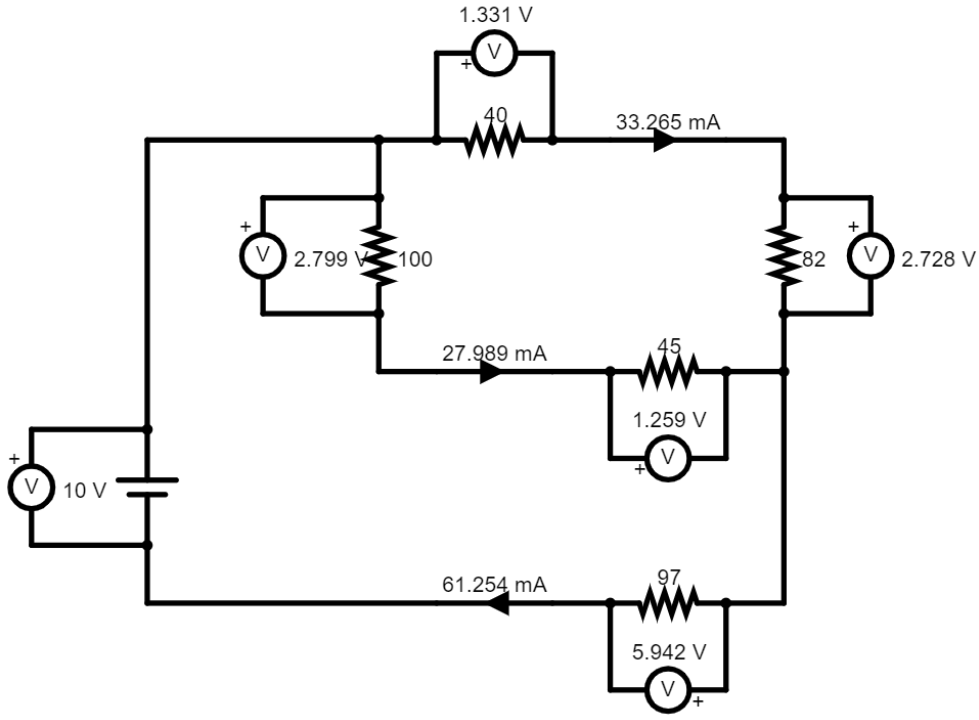

Figure 11. Circuit 4 Simulation

4. Conclusion

5. Citations

[1] Karen Schnurbusch, *Physics 4B Lab Book*, Mt. San Antonio College, 2023, pp. 71-74.
[2] Karen Schnurbusch, *Physics 4B Equations*, Mt. San Antonio College, 2023, pp. 4, 5.
[3] Paul Falstad, *Circuit Simulator Applet*, https://www.falstad.com/circuit/ Accessed 26 April 2023.