

# Carnot Cycle Exercise

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$$T_H = 490 \text{ K}$$

$$V_c = 1.90 \times 10^{-3} \text{ m}^3$$

## 1. Purpose

The goal of the exercise is to perform various calculations related to the Carnot cycle.

## 2. Given

- $T_H = 490 \text{ K}$
- $T_C = 300 \text{ K}$
- $P_c = 1.01 \times 10^5 \text{ Pa}$
- $V_c = 1.90 \times 10^{-3} \text{ m}^3$
- $Q_{a \rightarrow b} = 300 \text{ J}$
- $\gamma = 1.40$
- d.o.f. = 5
- $C_v = \frac{5}{2}R$
- $C_p = \frac{7}{2}R$

## 3. Derivations

### 3.1. Pressure-Volume Relationship for Adiabatic Process

$$\Delta U = Q - W$$

$$\Delta U = -W$$

$$dU = -dW$$

$$nC_v dT = -PdV$$

$$ndT = -\frac{PdV}{C_v}$$

$$PV = nRT$$

$$PdV + VdP = nRdT$$

$$PdV + VdP = (C_p - C_v)ndT$$

$$PdV + VdP = -(C_p - C_v)\frac{PdV}{C_v}$$

$$PdV + VdP = \left(1 - \frac{C_p}{C_v}\right)PdV$$

$$VdP = -\frac{C_p}{C_v}PdV$$

$$VdP = -\gamma PdV$$

$$\frac{dP}{P} = -\gamma \frac{dV}{V}$$

$$\int \frac{dP}{P} = -\int \gamma \frac{dV}{V}$$

$$\begin{aligned}
\ln(P) &= -\gamma \ln(V) + \text{const.} \\
\ln(P) + \ln(V^\gamma) &= \text{const.} \\
\ln(PV^\gamma) &= \text{const.} \\
PV^\gamma &= \text{const.} \\
P_i V_i^\gamma &= P_f V_f^\gamma
\end{aligned}$$

### 3.2. Temperature–Volume Relationship for Adiabatic Process

$$\begin{aligned}
P_i V_i^\gamma &= P_f V_f^\gamma \\
P_i V_i V_i^{\gamma-1} &= P_f V_f V_f^{\gamma-1} \\
nRT_i V_i^{\gamma-1} &= nRT_f V_f^{\gamma-1} \\
T_i V_i^{\gamma-1} &= T_f V_f^{\gamma-1}
\end{aligned}$$

### 3.3. Work by Gas for Isothermal Process

$$\begin{aligned}
W &= \int_{i \rightarrow f} P dV \\
W &= \int_{V_i}^{V_f} \frac{nRT}{V} dV \\
W &= nRT \ln(V) \Big|_{V_i}^{V_f} \\
W &= nRT (\ln(V_f) - \ln(V_i)) \\
W &= nRT \ln \left( \frac{V_f}{V_i} \right)
\end{aligned}$$

### 3.4. Work by Gas for Adiabatic Process

$$\begin{aligned}
W &= \int_{i \rightarrow f} P dV \\
W &= \int_{V_i}^{V_f} (P_i V_i^\gamma) V^{-\gamma} dV \\
W &= \frac{P_i V_i^\gamma}{1-\gamma} V^{1-\gamma} \Big|_{V_i}^{V_f} \\
W &= \frac{P_i V_i^\gamma}{1-\gamma} (V_f^{1-\gamma} - V_i^{1-\gamma}) \\
W &= \frac{P_i V_i^\gamma V_f^{1-\gamma} - P_i V_i^\gamma V_i^{1-\gamma}}{1-\gamma} \\
W &= \frac{P_f V_f^\gamma V_f^{1-\gamma} - P_i V_i^\gamma V_i^{1-\gamma}}{1-\gamma}
\end{aligned}$$

$$W = \frac{P_f V_f - P_i V_i}{1 - \gamma}$$

$$W = \frac{P_i V_i - P_f V_f}{\gamma - 1}$$

## 4. Results

**Table 1.** Pressure, Volume, and Temperature for Key Points

Note:  $T_a = T_b = T_H$  and  $T_c = T_d = T_C$

Point	$P$ (Pa)	$V$ (m <sup>3</sup> )	$T$ (K)
a	$1.46 \times 10^6$	$2.14 \times 10^{-4}$	490
b	$5.62 \times 10^5$	$5.57 \times 10^{-4}$	490
c	$1.01 \times 10^5$	$1.90 \times 10^{-3}$	300
d	$2.63 \times 10^5$	$7.30 \times 10^{-4}$	300

**Table 2.** Heat Transferred, Change in Internal Energy, Work Done, and Change in Entropy for Key Processes

Process	Type	$Q$ (J)	$W$ (J)	$\Delta U$ (J)	$\Delta S$ (J/K)
$a \rightarrow b$	isothermal expansion	300	300	0	0.612
$b \rightarrow c$	adiabatic expansion	0	304	-304	0
$c \rightarrow d$	isothermal compression	-184	-184	0	-0.612
$d \rightarrow a$	adiabatic compression	0	-304	304	0

### 4.1. Moles of Gas ( $n$ )

$$P_c V_c = n R T_c$$

$$n = \frac{P_c V_c}{R T_c}$$

$$n = 0.0770 \text{ mol}$$

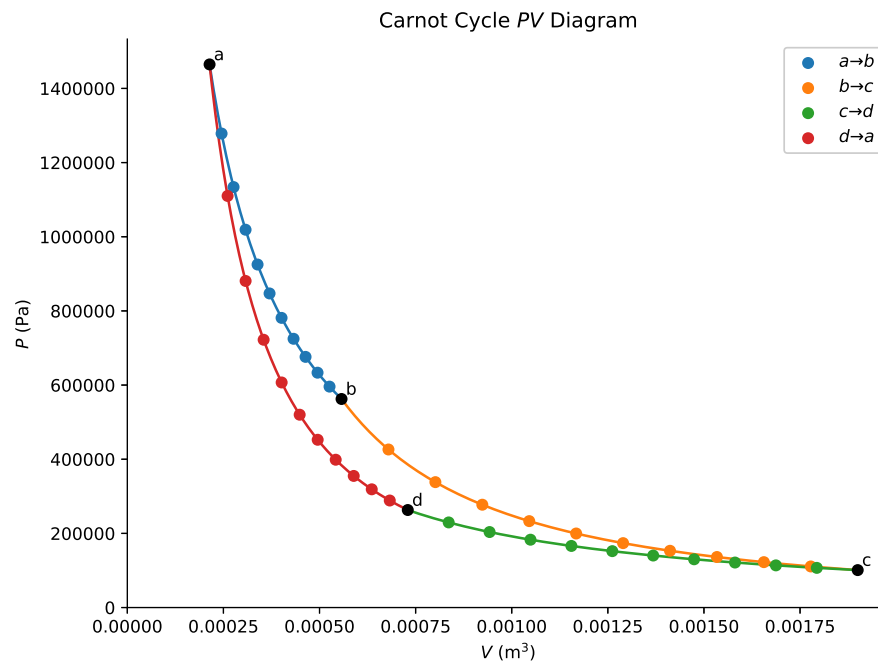
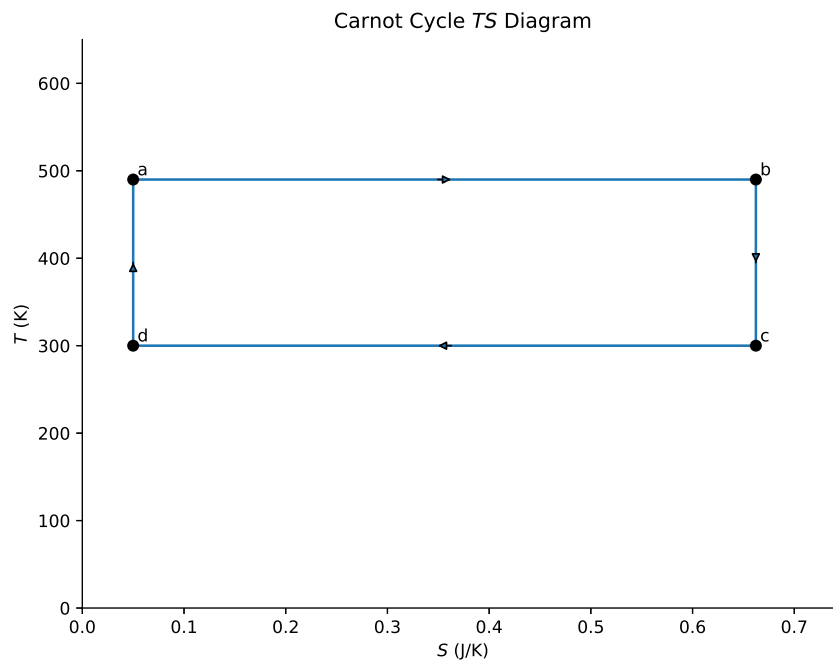
### 4.2. Pressure ( $P_b$ ) and Volume ( $V_b$ ) at $b$

$$T_b V_b^{\gamma-1} = T_c V_c^{\gamma-1}$$

$$V_b = V_c \left( \frac{T_c}{T_b} \right)^{\frac{1}{\gamma-1}}$$

$$V_b = 5.57 \times 10^{-4} \text{ m}^3$$

$$P_b V_b = n R T_b$$

**Figure 1.** Carnot Cycle  $PV$  Diagram**Figure 2.** Carnot Cycle  $TS$  Diagram

**Table 3.** Pressure and Volume Points on Carnot Cycle  $PV$  Diagram

Section		$P$ (Pa)	$V$ (m <sup>3</sup> )
$a$		$1.46 \times 10^6$	$2.14 \times 10^{-4}$
$a \rightarrow b$	1	$1.28 \times 10^6$	$2.45 \times 10^{-4}$
$a \rightarrow b$	2	$1.13 \times 10^6$	$2.76 \times 10^{-4}$
$a \rightarrow b$	3	$1.02 \times 10^6$	$3.08 \times 10^{-4}$
$a \rightarrow b$	4	$9.25 \times 10^5$	$3.39 \times 10^{-4}$
$a \rightarrow b$	5	$8.47 \times 10^5$	$3.70 \times 10^{-4}$
$a \rightarrow b$	6	$7.81 \times 10^5$	$4.01 \times 10^{-4}$
$a \rightarrow b$	7	$7.25 \times 10^5$	$4.32 \times 10^{-4}$
$a \rightarrow b$	8	$6.76 \times 10^5$	$4.64 \times 10^{-4}$
$a \rightarrow b$	9	$6.33 \times 10^5$	$4.95 \times 10^{-4}$
$a \rightarrow b$	10	$5.96 \times 10^5$	$5.26 \times 10^{-4}$
$b$		$5.62 \times 10^5$	$5.57 \times 10^{-4}$
$b \rightarrow c$	1	$4.26 \times 10^5$	$6.79 \times 10^{-4}$
$b \rightarrow c$	2	$3.38 \times 10^5$	$8.01 \times 10^{-4}$
$b \rightarrow c$	3	$2.77 \times 10^5$	$9.23 \times 10^{-4}$
$b \rightarrow c$	4	$2.33 \times 10^5$	$1.05 \times 10^{-3}$
$b \rightarrow c$	5	$2.00 \times 10^5$	$1.17 \times 10^{-3}$
$b \rightarrow c$	6	$1.74 \times 10^5$	$1.29 \times 10^{-3}$
$b \rightarrow c$	7	$1.53 \times 10^5$	$1.41 \times 10^{-3}$
$b \rightarrow c$	8	$1.36 \times 10^5$	$1.53 \times 10^{-3}$
$b \rightarrow c$	9	$1.22 \times 10^5$	$1.66 \times 10^{-3}$
$b \rightarrow c$	10	$1.11 \times 10^5$	$1.78 \times 10^{-3}$
$c$		$1.01 \times 10^5$	$1.90 \times 10^{-3}$
$c \rightarrow d$	1	$1.07 \times 10^5$	$1.79 \times 10^{-3}$
$c \rightarrow d$	2	$1.14 \times 10^5$	$1.69 \times 10^{-3}$
$c \rightarrow d$	3	$1.21 \times 10^5$	$1.58 \times 10^{-3}$
$c \rightarrow d$	4	$1.30 \times 10^5$	$1.47 \times 10^{-3}$
$c \rightarrow d$	5	$1.40 \times 10^5$	$1.37 \times 10^{-3}$
$c \rightarrow d$	6	$1.52 \times 10^5$	$1.26 \times 10^{-3}$
$c \rightarrow d$	7	$1.66 \times 10^5$	$1.16 \times 10^{-3}$
$c \rightarrow d$	8	$1.83 \times 10^5$	$1.05 \times 10^{-3}$
$c \rightarrow d$	9	$2.04 \times 10^5$	$9.42 \times 10^{-4}$
$c \rightarrow d$	10	$2.30 \times 10^5$	$8.36 \times 10^{-4}$
$d$		$2.63 \times 10^5$	$7.30 \times 10^{-4}$
$d \rightarrow a$	1	$2.89 \times 10^5$	$6.83 \times 10^{-4}$
$d \rightarrow a$	2	$3.19 \times 10^5$	$6.36 \times 10^{-4}$
$d \rightarrow a$	3	$3.55 \times 10^5$	$5.89 \times 10^{-4}$
$d \rightarrow a$	4	$3.99 \times 10^5$	$5.42 \times 10^{-4}$
$d \rightarrow a$	5	$4.52 \times 10^5$	$4.95 \times 10^{-4}$
$d \rightarrow a$	6	$5.20 \times 10^5$	$4.48 \times 10^{-4}$
$d \rightarrow a$	7	$6.07 \times 10^5$	$4.01 \times 10^{-4}$
$d \rightarrow a$	8	$7.22 \times 10^5$	$3.55 \times 10^{-4}$
$d \rightarrow a$	9	$8.81 \times 10^5$	$3.08 \times 10^{-4}$
$d \rightarrow a$	10	$1.11 \times 10^6$	$2.61 \times 10^{-4}$

$$P_b = \frac{nRT_b}{V_b}$$

$$P_b = 5.62 \times 10^5 \text{ Pa}$$

4.3. Pressure ( $P_a$ ) and Volume ( $V_a$ ) at a

$$\Delta U_{a \rightarrow b} = Q_{a \rightarrow b} - W_{a \rightarrow b}$$

$$0 = Q_{a \rightarrow b} - nRT_H \ln \left( \frac{V_b}{V_a} \right)$$

$$\ln \left( \frac{V_b}{V_a} \right) = \frac{Q_{a \rightarrow b}}{nRT_H}$$

$$\frac{V_b}{V_a} = e^{Q_{a \rightarrow b}/(nRT_H)}$$

$$V_a = V_b e^{-Q_{a \rightarrow b}/(nRT_H)}$$

$$V_a = 2.14 \times 10^{-4} \text{ m}^3$$

$$P_a V_a = nRT_a$$

$$P_a = \frac{nRT_a}{V_a}$$

$$P_a = 1.46 \times 10^6 \text{ Pa}$$

4.4. Pressure ( $P_d$ ) and Volume ( $V_d$ ) at d

$$T_d V_d^{\gamma-1} = T_a V_a^{\gamma-1}$$

$$V_d = V_a \left( \frac{T_a}{T_d} \right)^{\frac{1}{\gamma-1}}$$

$$V_d = 7.30 \times 10^{-4} \text{ m}^3$$

$$P_d V_d = nRT_d$$

$$P_d = \frac{nRT_d}{V_d}$$

$$P_d = 2.63 \times 10^5 \text{ Pa}$$

4.5. Process Variables  $a \rightarrow b$

$$\Delta U_{a \rightarrow b} = 0$$

$$\Delta U_{a \rightarrow b} = Q_{a \rightarrow b} - W_{a \rightarrow b}$$

$$0 = Q_{a \rightarrow b} - W_{a \rightarrow b}$$

$$W_{a \rightarrow b} = Q_{a \rightarrow b}$$

$$W_{a \rightarrow b} = 300 \text{ J}$$

$$\begin{aligned}\Delta S_{a \rightarrow b} &= \int_{a \rightarrow b} \frac{dQ}{T} \\ \Delta S_{a \rightarrow b} &= \frac{1}{T_H} \int_{a \rightarrow b} dQ \\ \Delta S_{a \rightarrow b} &= \frac{Q_{a \rightarrow b}}{T_H} \\ \Delta S_{a \rightarrow b} &= 0.612 \text{ J/K}\end{aligned}$$

4.6. Process Variables  $b \rightarrow c$ 

$$Q_{b \rightarrow c} = 0$$

$$\begin{aligned}W_{b \rightarrow c} &= \frac{P_b V_b - P_c V_c}{\gamma - 1} \\ W_{b \rightarrow c} &= 304 \text{ J}\end{aligned}$$

$$\begin{aligned}\Delta U_{b \rightarrow c} &= Q_{b \rightarrow c} - W_{b \rightarrow c} \\ \Delta U_{b \rightarrow c} &= -W_{b \rightarrow c} \\ \Delta U_{b \rightarrow c} &= -304 \text{ J}\end{aligned}$$

$$\begin{aligned}\Delta S_{b \rightarrow c} &= \int_{b \rightarrow c} \frac{dQ}{T} \\ \Delta S_{b \rightarrow c} &= 0\end{aligned}$$

4.7. Process Variables  $c \rightarrow d$ 

$$\Delta U_{c \rightarrow d} = 0$$

$$\begin{aligned}W_{c \rightarrow d} &= nRT_C \ln \left( \frac{V_d}{V_c} \right) \\ W_{c \rightarrow d} &= -184 \text{ J}\end{aligned}$$

$$\begin{aligned}\Delta U_{c \rightarrow d} &= Q_{c \rightarrow d} - W_{c \rightarrow d} \\ 0 &= Q_{c \rightarrow d} - W_{c \rightarrow d} \\ Q_{c \rightarrow d} &= W_{c \rightarrow d} \\ Q_{c \rightarrow d} &= -184 \text{ J}\end{aligned}$$



$$\begin{aligned}\Delta S_{c \rightarrow d} &= \int_{c \rightarrow d} \frac{dQ}{T} \\ \Delta S_{c \rightarrow d} &= \frac{1}{T_C} \int_{c \rightarrow d} dQ \\ \Delta S_{c \rightarrow d} &= \frac{Q_{c \rightarrow d}}{T_C} \\ \Delta S_{c \rightarrow d} &= -0.612 \text{ J/K}\end{aligned}$$

#### 4.8. Process Variables $d \rightarrow a$

$$Q_{d \rightarrow a} = 0$$

$$\begin{aligned}W_{d \rightarrow a} &= \frac{P_d V_d - P_a V_a}{\gamma - 1} \\ W_{d \rightarrow a} &= -304 \text{ J}\end{aligned}$$

$$\begin{aligned}\Delta U_{d \rightarrow a} &= Q_{d \rightarrow a} - W_{d \rightarrow a} \\ \Delta U_{d \rightarrow a} &= -W_{d \rightarrow a} \\ \Delta U_{d \rightarrow a} &= 304 \text{ J}\end{aligned}$$

$$\begin{aligned}\Delta S_{d \rightarrow a} &= \int_{d \rightarrow a} \frac{dQ}{T} \\ \Delta S_{d \rightarrow a} &= 0\end{aligned}$$

## 5. Conclusion

## 6. Citations

- [1] Karen Schnurbusch, *Physics 4B Lab Book*, Mt. San Antonio College, 2023, pp. 35-38.
- [2] Karen Schnurbusch, *Physics 4B Equations*, Mt. San Antonio College, 2023, pp. 1-3.