

Carnot Cycle Exercise

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March 27, 2023

$$T_H = 490 \text{ K}$$

$$V_c = 1.90 \times 10^{-3} \text{ m}^3$$

1. Purpose

The goal of the exercise is to perform various calculations related to the Carnot cycle.

2. Given

- $T_H = 490 \text{ K}$
- $T_C = 300 \text{ K}$
- $P_c = 1.01 \times 10^5 \text{ Pa}$
- $V_c = 1.90 \times 10^{-3} \text{ m}^3$
- $Q_{a \rightarrow b} = 300 \text{ J}$
- $\gamma = 1.40$
- d.o.f. = 5
- $C_v = \frac{5}{2}R$
- $C_p = \frac{7}{2}R$

3. Derivations

3.1. Temperature–Volume Relationship for Adiabatic Process

$$\begin{aligned}
 P_i V_i^\gamma &= P_f V_f^\gamma \\
 P_i V_i V_i^{\gamma-1} &= P_f V_f V_f^{\gamma-1} \\
 nRT_i V_i^{\gamma-1} &= nRT_f V_f^{\gamma-1} \\
 T_i V_i^{\gamma-1} &= T_f V_f^{\gamma-1}
 \end{aligned}$$

3.2. Work by Gas for Isothermal Process

$$\begin{aligned}
 W &= \int_{V=V_i}^{V=V_f} P dV \\
 W &= \int_{V_i}^{V_f} \frac{nRT}{V} dV \\
 W &= nRT \ln(V) \Big|_{V_i}^{V_f} \\
 W &= nRT (\ln(V_f) - \ln(V_i)) \\
 W &= nRT \ln \left(\frac{V_f}{V_i} \right)
 \end{aligned}$$

Table 1. Pressure, Volume, and Temperature for Key Points
Note: $T_a = T_b = T_H$ and $T_c = T_d = T_C$

Point	P (Pa)	V (m ³)	T (K)
a	1.46×10^6	2.14×10^{-4}	490
b	5.62×10^5	5.57×10^{-4}	490
c	1.01×10^5	1.90×10^{-3}	300
d	2.63×10^5	7.30×10^{-4}	300

Table 2. Heat Transferred, Change in Internal Energy, Work Done, and Change in Entropy for Key Processes

Process	Type	Q (J)	W (J)	ΔU (J)	ΔS (J/K)
$a \rightarrow b$	isothermal expansion	300	300	0	0.612
$b \rightarrow c$	adiabatic expansion	0	304	-304	0
$c \rightarrow d$	isothermal compression			0	
$d \rightarrow a$	adiabatic compression	0			0

4. Results

4.1. Moles of Gas (n)

$$\begin{aligned}
 P_c V_c &= n R T_c \\
 n &= \frac{P_c V_c}{R T_c} \\
 n &= 0.0770 \text{ mol}
 \end{aligned}$$

4.2. Pressure (P_b) and Volume (V_b) at b

$$\begin{aligned}
 T_b V_b^{\gamma-1} &= T_c V_c^{\gamma-1} \\
 V_b &= V_c \left(\frac{T_c}{T_b} \right)^{\frac{1}{\gamma-1}} \\
 V_b &= 5.57 \times 10^{-4} \text{ m}^3
 \end{aligned}$$

$$\begin{aligned}
 P_b V_b &= n R T_b \\
 P_b &= \frac{n R T_b}{V_b} \\
 P_b &= 5.62 \times 10^5 \text{ Pa}
 \end{aligned}$$

4.3. Pressure (P_a) and Volume (V_a) at a

$$\Delta U_{a \rightarrow b} = Q_{a \rightarrow b} - W_{a \rightarrow b}$$

$$\begin{aligned}
0 &= Q_{a \rightarrow b} - nRT_H \ln \left(\frac{V_b}{V_a} \right) \\
\ln \left(\frac{V_b}{V_a} \right) &= \frac{Q_{a \rightarrow b}}{nRT_H} \\
\frac{V_b}{V_a} &= e^{Q_{a \rightarrow b}/(nRT_H)} \\
V_a &= V_b e^{-Q_{a \rightarrow b}/(nRT_H)} \\
V_a &= 2.14 \times 10^{-4} \text{ m}^3
\end{aligned}$$

$$\begin{aligned}
P_a V_a &= nRT_a \\
P_a &= \frac{nRT_a}{V_a} \\
P_a &= 1.46 \times 10^6 \text{ Pa}
\end{aligned}$$

4.4. Pressure (P_d) and Volume (V_d) at d

$$\begin{aligned}
T_d V_d^{\gamma-1} &= T_a V_a^{\gamma-1} \\
V_d &= V_a \left(\frac{T_a}{T_d} \right)^{\frac{1}{\gamma-1}} \\
V_d &= 7.30 \times 10^{-4} \text{ m}^3
\end{aligned}$$

$$\begin{aligned}
P_d V_d &= nRT_d \\
P_d &= \frac{nRT_d}{V_d} \\
P_d &= 2.63 \times 10^5 \text{ Pa}
\end{aligned}$$

4.5. Process Variables $a \rightarrow b$

$$\Delta U_{a \rightarrow b} = 0$$

$$\begin{aligned}
\Delta U_{a \rightarrow b} &= Q_{a \rightarrow b} - W_{a \rightarrow b} \\
0 &= Q_{a \rightarrow b} - W_{a \rightarrow b} \\
W_{a \rightarrow b} &= Q_{a \rightarrow b} \\
W_{a \rightarrow b} &= 300 \text{ J}
\end{aligned}$$

$$\Delta S_{a \rightarrow b} = \int_{a \rightarrow b} \frac{dQ}{T}$$

$$\Delta S_{a \rightarrow b} = \frac{1}{T_H} \int_{a \rightarrow b} dQ$$

$$\Delta S_{a \rightarrow b} = \frac{Q_{a \rightarrow b}}{T_H}$$

$$\Delta S_{a \rightarrow b} = 0.612 \text{ J/K}$$

4.6. Process Variables $b \rightarrow c$

$$Q_{b \rightarrow c} = 0$$

$$W_{b \rightarrow c} = \frac{P_b V_b - P_c V_c}{\gamma - 1}$$

$$W_{b \rightarrow c} = 304 \text{ J}$$

$$\Delta U_{b \rightarrow c} = Q_{b \rightarrow c} - W_{b \rightarrow c}$$

$$\Delta U_{b \rightarrow c} = -W_{b \rightarrow c}$$

$$\Delta U_{b \rightarrow c} = -304 \text{ J}$$

$$\Delta S_{b \rightarrow c} = \int_{b \rightarrow c} \frac{dQ}{T}$$

$$\Delta S_{b \rightarrow c} = 0$$

5. Conclusion

6. Citations

- [1] Karen Schnurbusch, *Physics 4B Lab Book*, Mt. San Antonio College, 2023, pp. 35-38.
- [2] Karen Schnurbusch, *Physics 4B Equations*, Mt. San Antonio College, 2023, pp. 1-3.