Carnot Cycle Exercise

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$$T_H = 490 \text{ K}$$

 $V_c = 1.90 \times 10^{-3} \text{ m}^3$

1. Purpose

The goal of the exercise is to perform various calculations related to the Carnot cycle.

2. Given

- $T_H = 490 \text{ K}$
- $T_C = 300 \text{ K}$
- $P_c = 1.01 \times 10^5 \text{ Pa}$
- $V_c = 1.90 \times 10^{-3} \text{ m}^3$
- $Q_{a\to b} = 300 \text{ J}$
- $\gamma = 1.40$
- d.o.f. = 5
- $C_v = \frac{5}{2}R$
- $C_p = \frac{7}{2}R$

3. Derivations

3.1. Pressure-Volume Relationship for Adiabatic Process

$$\Delta U = Q - W$$

$$\Delta U = -W$$

$$dU = -dW$$

$$nC_v dT = -PdV$$

$$ndT = -\frac{PdV}{C_v}$$

$$PV = nRT$$

$$PdV + VdP = nRdT$$

$$PdV + VdP = (C_p - C_v)ndT$$

$$PdV + VdP = -(C_p - C_v)\frac{PdV}{C_v}$$

$$PdV + VdP = \left(1 - \frac{C_p}{C_v}\right)PdV$$

$$VdP = -\frac{C_p}{C_v}PdV$$

$$VdP = -\gamma PdV$$

$$\frac{dP}{P} = -\gamma \frac{dV}{V}$$

$$\int \frac{dP}{P} = -\int \gamma \frac{dV}{V}$$

$$\ln(P) = -\gamma \ln(V) + \text{const.}$$

$$\ln(P) + \ln(V^{\gamma}) = \text{const.}$$

$$\ln(PV^{\gamma}) = \text{const.}$$

$$PV^{\gamma} = \text{const.}$$

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

3.2. Temperature-Volume Relationship for Adiabatic Process

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

$$P_i V_i V_i^{\gamma - 1} = P_f V_f V_f^{\gamma - 1}$$

$$nRT_i V_i^{\gamma - 1} = nRT_f V_f^{\gamma - 1}$$

$$T_i V_i^{\gamma - 1} = T_f V_f^{\gamma - 1}$$

3.3. Work by Gas for Isothermal Process

$$W = \int_{i \to f} P dV$$

$$W = \int_{V_i}^{V_f} \frac{nRT}{V} dV$$

$$W = nRT \ln(V)|_{V_i}^{V_f}$$

$$W = nRT(\ln(V_f) - \ln(V_i))$$

$$W = nRT \ln\left(\frac{V_f}{V_i}\right)$$

3.4. Work by Gas for Adiabatic Process

$$W = \int_{i \to f} P dV$$

$$W = \int_{V_i}^{V_f} (P_i V_i^{\gamma}) V^{-\gamma} dV$$

$$W = \frac{P_i V_i^{\gamma}}{1 - \gamma} V^{1-\gamma} |_{V_i}^{V_f}$$

$$W = \frac{P_i V_i^{\gamma}}{1 - \gamma} (V_f^{1-\gamma} - V_i^{1-\gamma})$$

$$W = \frac{P_i V_i^{\gamma} V_f^{1-\gamma} - P_i V_i^{\gamma} V_i^{1-\gamma}}{1 - \gamma}$$

$$W = \frac{P_f V_f^{\gamma} V_f^{1-\gamma} - P_i V_i^{\gamma} V_i^{1-\gamma}}{1 - \gamma}$$

$$W = \frac{P_f V_f - P_i V_i}{1 - \gamma}$$
$$W = \frac{P_i V_i - P_f V_f}{\gamma - 1}$$

4. Results

Table 1. Pressure, Volume, and Temperature for Key Points Note: $T_a = T_b = T_H$ and $T_c = T_d = T_C$

Point	P (Pa)	$V~(\mathrm{m}^3)$	T(K)
a b	1.46×10^6 5.62×10^5	$2.14 \times 10^{-4} \\ 5.57 \times 10^{-4}$	490 490
$_{ m d}^{ m c}$	$1.01 \times 10^5 \\ 2.63 \times 10^5$	1.90×10^{-3} 7.30×10^{-4}	300 300

Table 2. Heat Transferred, Change in Internal Energy, Work Done, and Change in Entropy for Key Processes

Process	Type	Q(J)	W (J)	ΔU (J)	$\Delta S \; (\mathrm{J/K})$
$a \rightarrow b$	isothermal expansion	300	300	0	0.612
$b \to c$	adiabatic expansion	0	304	-304	0
$c \to d$	isothermal compression	-184	-184	0	-0.612
$d \rightarrow a$	adiabatic compression	0	-304	304	0

4.1. Moles of Gas (n)

$$P_c V_c = nRT_c$$

$$n = \frac{P_c V_c}{RT_c}$$

$$n = 0.0770 \text{ mol}$$

4.2. Pressure (P_b) and Volume (V_b) at b

$$\begin{split} T_b V_b^{\gamma-1} &= T_c V_c^{\gamma-1} \\ V_b &= V_c \left(\frac{T_c}{T_b}\right)^{\frac{1}{\gamma-1}} \\ V_b &= 5.57 \times 10^{-4} \text{ m}^3 \end{split}$$

$$P_b V_b = nRT_b$$

Table 3. Pressure and Volume Points on Carnot Cycle PV Diagram

Section		P (Pa)	$V~(\mathrm{m}^3)$
\overline{a}		1.46×10^{6}	2.14×10^{-4}
$a \rightarrow b$	1	1.28×10^{6}	2.45×10^{-4}
$a \rightarrow b$	2	1.13×10^6	2.76×10^{-4}
$a \rightarrow b$	3	1.02×10^6	3.08×10^{-4}
$a \to b$	4	9.25×10^5	3.39×10^{-4}
$a \to b$	5	8.47×10^{5}	3.70×10^{-4}
$a \to b$	6	7.81×10^{5}	4.01×10^{-4}
$a \rightarrow b$	7	7.25×10^{5}	4.32×10^{-4}
$a \rightarrow b$	8	6.76×10^{5}	4.64×10^{-4}
$a \to b$	9	6.33×10^{5}	4.95×10^{-4}
$a \to b$	10	5.96×10^{5}	5.26×10^{-4}
b		5.62×10^5	5.57×10^{-4}
$b \to c$	1	4.26×10^{5}	6.79×10^{-4}
$b \to c$	2	3.38×10^{5}	8.01×10^{-4}
$b \to c$	3	2.77×10^{5}	9.23×10^{-4}
$b \to c$	4	2.33×10^{5}	1.05×10^{-3}
$b \to c$	5	2.00×10^{5}	1.17×10^{-3}
$b \to c$	6	1.74×10^{5}	1.29×10^{-3}
$b \to c$	7	1.53×10^{5}	1.41×10^{-3}
$b \to c$	8	1.36×10^{5}	1.53×10^{-3}
$b \to c$	9	1.22×10^{5}	1.66×10^{-3}
$b \to c$	10	1.11×10^{5}	1.78×10^{-3}
c		1.01×10^{5}	1.90×10^{-3}
$c \to d$	1	1.07×10^{5}	1.79×10^{-3}
$c \to d$	2	1.14×10^{5}	1.69×10^{-3}
$c \to d$	3	1.21×10^{5}	1.58×10^{-3}
$c \to d$	4	1.30×10^{5}	1.47×10^{-3}
$c \to d$	5	1.40×10^{5}	1.37×10^{-3}
$c \to d$	6	1.52×10^{5}	1.26×10^{-3}
$c \to d$	7	1.66×10^{5}	1.16×10^{-3}
$c \to d$	8	1.83×10^{5}	1.05×10^{-3}
$c \to d$	9	2.04×10^{5}	9.42×10^{-4}
$c \to d$	10	2.30×10^{5}	8.36×10^{-4}
d		2.63×10^{5}	7.30×10^{-4}
$d \rightarrow a$	1	2.89×10^{5}	6.83×10^{-4}
$d \rightarrow a$	2	3.19×10^{5}	6.36×10^{-4}
$d \rightarrow a$	3	3.55×10^{5}	5.89×10^{-4}
$d \rightarrow a$	4	3.99×10^5	5.42×10^{-4}
$d \rightarrow a$	5	4.52×10^5	4.95×10^{-4}
$d \rightarrow a$	6	5.20×10^5	4.48×10^{-4}
$d \rightarrow a$	7	6.07×10^5	4.01×10^{-4}
$d \rightarrow a$	8	7.22×10^{5}	3.55×10^{-4}
$d \rightarrow a$	9	8.81×10^5	3.08×10^{-4}
$d \rightarrow a$	10	1.11×10^{6}	2.61×10^{-4}

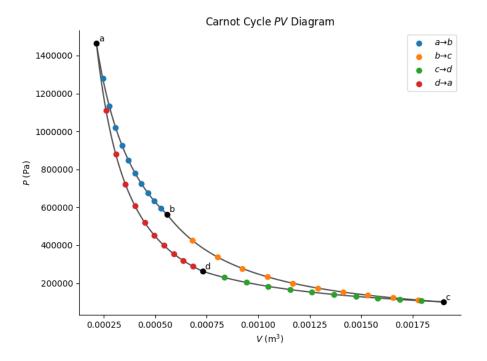


Figure 1. Carnot Cycle PV Diagram

$$P_b = \frac{nRT_b}{V_b}$$

$$P_b = 5.62 \times 10^5 \text{ Pa}$$

4.3. Pressure (P_a) and Volume (V_a) at a

$$\Delta U_{a \to b} = Q_{a \to b} - W_{a \to b}$$

$$0 = Q_{a \to b} - nRT_H \ln\left(\frac{V_b}{V_a}\right)$$

$$\ln\left(\frac{V_b}{V_a}\right) = \frac{Q_{a \to b}}{nRT_H}$$

$$\frac{V_b}{V_a} = e^{Q_{a \to b}/(nRT_H)}$$

$$V_a = V_b e^{-Q_{a \to b}/(nRT_H)}$$

$$V_a = 2.14 \times 10^{-4} \text{ m}^3$$

$$P_aV_a = nRT_a$$

$$P_a = \frac{nRT_a}{V_a}$$

$$P_a = 1.46 \times 10^6 \text{ Pa}$$

4.4. Pressure (P_d) and Volume (V_d) at d

$$T_d V_d^{\gamma - 1} = T_a V_a^{\gamma - 1}$$

$$V_d = V_a \left(\frac{T_a}{T_d}\right)^{\frac{1}{\gamma - 1}}$$

$$V_d = 7.30 \times 10^{-4} \text{ m}^3$$

$$P_dV_d = nRT_d$$

$$P_d = \frac{nRT_d}{V_d}$$

$$P_d = 2.63 \times 10^5 \text{ Pa}$$

4.5. Process Variables $a \rightarrow b$

$$\Delta U_{a \to b} = 0$$

$$\Delta U_{a \to b} = Q_{a \to b} - W_{a \to b}$$
$$0 = Q_{a \to b} - W_{a \to b}$$
$$W_{a \to b} = Q_{a \to b}$$
$$W_{a \to b} = 300 \text{ J}$$

$$\Delta S_{a \to b} = \int_{a \to b} \frac{dQ}{T}$$

$$\Delta S_{a \to b} = \frac{1}{T_H} \int_{a \to b} dQ$$

$$\Delta S_{a \to b} = \frac{Q_{a \to b}}{T_H}$$

$$\Delta S_{a \to b} = 0.612 \text{ J/K}$$

4.6. Process Variables $b \rightarrow c$

$$Q_{b\to c}=0$$

$$W_{b\rightarrow c} = \frac{P_b V_b - P_c V_c}{\gamma - 1}$$

$$W_{b\rightarrow c} = 304 \text{ J}$$

$$\Delta U_{b\to c} = Q_{b\to c} - W_{b\to c}$$
$$\Delta U_{b\to c} = -W_{b\to c}$$
$$\Delta U_{b\to c} = -304 \text{ J}$$

$$\Delta S_{b \to c} = \int_{b \to c} \frac{dQ}{T}$$
$$\Delta S_{b \to c} = 0$$

4.7. Process Variables $c \to d$

$$\Delta U_{c \to d} = 0$$

$$W_{c \to d} = nRT_C \ln \left(\frac{V_d}{V_c}\right)$$
$$W_{c \to d} = -184 \text{ J}$$

$$\Delta U_{c \to d} = Q_{c \to d} - W_{c \to d}$$
$$0 = Q_{c \to d} - W_{c \to d}$$
$$Q_{c \to d} = W_{c \to d}$$
$$Q_{c \to d} = -184 \text{ J}$$

$$\Delta S_{c \to d} = \int_{c \to d} \frac{dQ}{T}$$

$$\Delta S_{c \to d} = \frac{1}{T_C} \int_{c \to d} dQ$$

$$\Delta S_{c \to d} = \frac{Q_{c \to d}}{T_C}$$

$$\Delta S_{c \to d} = -0.612 \text{ J/K}$$

4.8. Process Variables $d \rightarrow a$

$$Q_{d\to a}=0$$

$$W_{d\rightarrow a} = \frac{P_d V_d - P_a V_a}{\gamma - 1}$$

$$W_{d\rightarrow a} = -304 \text{ J}$$

$$\Delta U_{d\rightarrow a} = Q_{d\rightarrow a} - W_{d\rightarrow a}$$

$$\Delta U_{d\rightarrow a} = -W_{d\rightarrow a}$$

$$\Delta U_{d\rightarrow a} = 304 \text{ J}$$

$$\Delta S_{d \to a} = \int_{d \to a} \frac{dQ}{T}$$
$$\Delta S_{d \to a} = 0$$

5. Conclusion

6. Citations

- [1] Karen Schnurbusch, *Physics 4B Lab Book*, Mt. San Antonio College, 2023, pp. 35-38.
- [2] Karen Schnurbusch, Physics 4B Equations, Mt. San Antonio College, 2023, pp. 1-3.