

Carnot Cycle Exercise

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$$T_H = 490 \text{ K}$$

$$V_c = 1.90 \times 10^{-3} \text{ m}^3$$

1. Purpose

The goal of the exercise is to perform various calculations related to the Carnot cycle.

2. Given

- $T_H = 490$ K
- $T_C = 300$ K
- $P_c = 1.01 \times 10^5$ Pa
- $V_c = 1.90 \times 10^{-3}$ m³
- $Q_{a \rightarrow b} = 300$ J
- $\gamma = 1.40$
- d.o.f. = 5
- $C_v = \frac{5}{2}R$
- $C_p = \frac{7}{2}R$

3. Derivations

3.1. Temperature–Volume Relationship for Adiabatic Process

$$\begin{aligned}
 P_i V_i^\gamma &= P_f V_f^\gamma \\
 P_i V_i V_i^{\gamma-1} &= P_f V_f V_f^{\gamma-1} \\
 nRT_i V_i^{\gamma-1} &= nRT_f V_f^{\gamma-1} \\
 T_i V_i^{\gamma-1} &= T_f V_f^{\gamma-1}
 \end{aligned}$$

3.2. Work by Gas for Isothermal Process

$$\begin{aligned}
 W &= \int_{i \rightarrow f} P dV \\
 W &= \int_{V_i}^{V_f} \frac{nRT}{V} dV \\
 W &= nRT \ln(V) \Big|_{V_i}^{V_f} \\
 W &= nRT (\ln(V_f) - \ln(V_i)) \\
 W &= nRT \ln \left(\frac{V_f}{V_i} \right)
 \end{aligned}$$

3.3. Work by Gas for Adiabatic Process

$$\begin{aligned}
W &= \int_{i \rightarrow f} P dV \\
W &= \int_{V_i}^{V_f} (P_i V_i^\gamma) V^{-\gamma} dV \\
W &= \frac{P_i V_i^\gamma}{1 - \gamma} V^{1-\gamma} \Big|_{V_i}^{V_f} \\
W &= \frac{P_i V_i^\gamma}{1 - \gamma} (V_f^{1-\gamma} - V_i^{1-\gamma}) \\
W &= \frac{P_i V_i^\gamma V_f^{1-\gamma} - P_i V_i^\gamma V_i^{1-\gamma}}{1 - \gamma} \\
W &= \frac{P_f V_f^\gamma V_f^{1-\gamma} - P_i V_i^\gamma V_i^{1-\gamma}}{1 - \gamma} \\
W &= \frac{P_f V_f - P_i V_i}{1 - \gamma} \\
W &= \frac{P_i V_i - P_f V_f}{\gamma - 1}
\end{aligned}$$

4. Results

Table 1. Pressure, Volume, and Temperature for Key PointsNote: $T_a = T_b = T_H$ and $T_c = T_d = T_C$

Point	P (Pa)	V (m ³)	T (K)
a	1.46×10^6	2.14×10^{-4}	490
b	5.62×10^5	5.57×10^{-4}	490
c	1.01×10^5	1.90×10^{-3}	300
d	2.63×10^5	7.30×10^{-4}	300

Table 2. Heat Transferred, Change in Internal Energy, Work Done, and Change in Entropy for Key Processes

Process	Type	Q (J)	W (J)	ΔU (J)	ΔS (J/K)
$a \rightarrow b$	isothermal expansion	300	300	0	0.612
$b \rightarrow c$	adiabatic expansion	0	304	-304	0
$c \rightarrow d$	isothermal compression	-184	-184	0	-0.612
$d \rightarrow a$	adiabatic compression	0	-304	304	0

4.1. Moles of Gas (n)

$$\begin{aligned}
 P_c V_c &= n R T_c \\
 n &= \frac{P_c V_c}{R T_c} \\
 n &= 0.0770 \text{ mol}
 \end{aligned}$$

4.2. Pressure (P_b) and Volume (V_b) at b

$$\begin{aligned}
 T_b V_b^{\gamma-1} &= T_c V_c^{\gamma-1} \\
 V_b &= V_c \left(\frac{T_c}{T_b} \right)^{\frac{1}{\gamma-1}} \\
 V_b &= 5.57 \times 10^{-4} \text{ m}^3
 \end{aligned}$$

$$\begin{aligned}
 P_b V_b &= n R T_b \\
 P_b &= \frac{n R T_b}{V_b} \\
 P_b &= 5.62 \times 10^5 \text{ Pa}
 \end{aligned}$$

4.3. Pressure (P_a) and Volume (V_a) at a

$$\begin{aligned}
 \Delta U_{a \rightarrow b} &= Q_{a \rightarrow b} - W_{a \rightarrow b} \\
 0 &= Q_{a \rightarrow b} - n R T_H \ln \left(\frac{V_b}{V_a} \right) \\
 \ln \left(\frac{V_b}{V_a} \right) &= \frac{Q_{a \rightarrow b}}{n R T_H} \\
 \frac{V_b}{V_a} &= e^{Q_{a \rightarrow b} / (n R T_H)} \\
 V_a &= V_b e^{-Q_{a \rightarrow b} / (n R T_H)} \\
 V_a &= 2.14 \times 10^{-4} \text{ m}^3
 \end{aligned}$$

$$\begin{aligned}
 P_a V_a &= n R T_a \\
 P_a &= \frac{n R T_a}{V_a} \\
 P_a &= 1.46 \times 10^6 \text{ Pa}
 \end{aligned}$$

4.4. Pressure (P_d) and Volume (V_d) at d

$$\begin{aligned}
 T_d V_d^{\gamma-1} &= T_a V_a^{\gamma-1} \\
 V_d &= V_a \left(\frac{T_a}{T_d} \right)^{\frac{1}{\gamma-1}} \\
 V_d &= 7.30 \times 10^{-4} \text{ m}^3
 \end{aligned}$$

$$\begin{aligned}
 P_d V_d &= n R T_d \\
 P_d &= \frac{n R T_d}{V_d} \\
 P_d &= 2.63 \times 10^5 \text{ Pa}
 \end{aligned}$$

4.5. Process Variables $a \rightarrow b$

$$\Delta U_{a \rightarrow b} = 0$$

$$\begin{aligned}
 \Delta U_{a \rightarrow b} &= Q_{a \rightarrow b} - W_{a \rightarrow b} \\
 0 &= Q_{a \rightarrow b} - W_{a \rightarrow b} \\
 W_{a \rightarrow b} &= Q_{a \rightarrow b} \\
 W_{a \rightarrow b} &= 300 \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 \Delta S_{a \rightarrow b} &= \int_{a \rightarrow b} \frac{dQ}{T} \\
 \Delta S_{a \rightarrow b} &= \frac{1}{T_H} \int_{a \rightarrow b} dQ \\
 \Delta S_{a \rightarrow b} &= \frac{Q_{a \rightarrow b}}{T_H} \\
 \Delta S_{a \rightarrow b} &= 0.612 \text{ J/K}
 \end{aligned}$$

4.6. Process Variables $b \rightarrow c$

$$Q_{b \rightarrow c} = 0$$

$$\begin{aligned}
 W_{b \rightarrow c} &= \frac{P_b V_b - P_c V_c}{\gamma - 1} \\
 W_{b \rightarrow c} &= 304 \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 \Delta U_{b \rightarrow c} &= Q_{b \rightarrow c} - W_{b \rightarrow c} \\
 \Delta U_{b \rightarrow c} &= -W_{b \rightarrow c} \\
 \Delta U_{b \rightarrow c} &= -304 \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 \Delta S_{b \rightarrow c} &= \int_{b \rightarrow c} \frac{dQ}{T} \\
 \Delta S_{b \rightarrow c} &= 0
 \end{aligned}$$

4.7. Process Variables $c \rightarrow d$

$$\Delta U_{c \rightarrow d} = 0$$

$$W_{c \rightarrow d} = nRT_C \ln \left(\frac{V_d}{V_c} \right)$$

$$W_{c \rightarrow d} = -184 \text{ J}$$

$$\Delta U_{c \rightarrow d} = Q_{c \rightarrow d} - W_{c \rightarrow d}$$

$$0 = Q_{c \rightarrow d} - W_{c \rightarrow d}$$

$$Q_{c \rightarrow d} = W_{c \rightarrow d}$$

$$Q_{c \rightarrow d} = -184 \text{ J}$$

$$\Delta S_{c \rightarrow d} = \int_{c \rightarrow d} \frac{dQ}{T}$$

$$\Delta S_{c \rightarrow d} = \frac{1}{T_C} \int_{c \rightarrow d} dQ$$

$$\Delta S_{c \rightarrow d} = \frac{Q_{c \rightarrow d}}{T_C}$$

$$\Delta S_{c \rightarrow d} = -0.612 \text{ J/K}$$

4.8. Process Variables $d \rightarrow a$

$$Q_{d \rightarrow a} = 0$$

$$W_{d \rightarrow a} = \frac{P_d V_d - P_a V_a}{\gamma - 1}$$

$$W_{d \rightarrow a} = -304 \text{ J}$$

$$\Delta U_{d \rightarrow a} = Q_{d \rightarrow a} - W_{d \rightarrow a}$$

$$\Delta U_{d \rightarrow a} = -W_{d \rightarrow a}$$

$$\Delta U_{d \rightarrow a} = 304 \text{ J}$$

$$\Delta S_{d \rightarrow a} = \int_{d \rightarrow a} \frac{dQ}{T}$$

$$\Delta S_{d \rightarrow a} = 0$$

5. Conclusion

6. Citations

- [1] Karen Schnurbusch, *Physics 4B Lab Book*, Mt. San Antonio College, 2023, pp. 35-38.
- [2] Karen Schnurbusch, *Physics 4B Equations*, Mt. San Antonio College, 2023, pp. 1-3.