Circuits Lab

Vincent Edwards

Mt. San Antonio College, Physics 4B, CRN 42240
May 8, 2023

$R_2 = 40\ \Omega$

1. Purpose

The goal of the exercise was to use Kirchhoff's rules to analyze 7 different circuits and calculate current, voltage, and power for each circuit element. For the first 4 circuits, the voltage and current calculations were compared to results obtained using an online circuit simulator.

2. Results

The following sections contain the theoretical voltage, current, and power for each circuit element for each of the 7 circuits. In addition, there is an annotated circuit diagram for each circuit. In each case, the voltage across a resistor was calculated with $V = IR$, and the power dissipated by a circuit element with $P = VI$. $R_2 = 40\ \Omega$ for each circuit.

2.1. Circuit 1

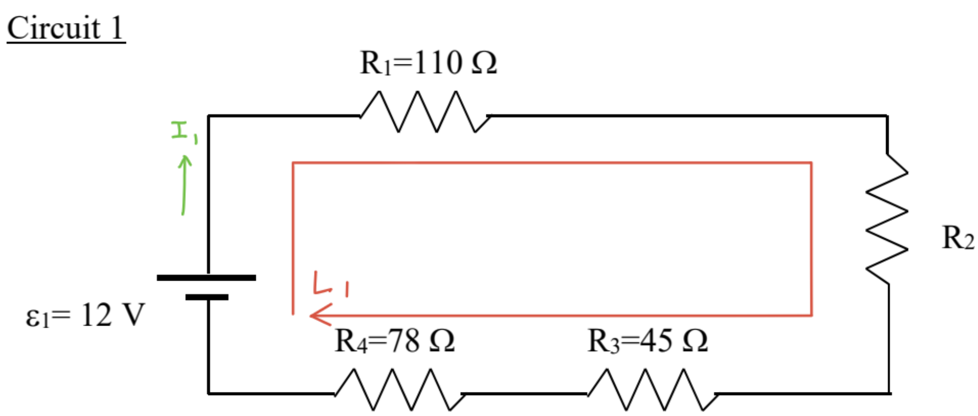

Figure 1. Circuit 1

Table 1. Circuit 1 V, I, and P

	V (V)	I (A)	P (W)
$\mathcal{E}_1$	12.0	0.0440	0.527
R_1	4.84	0.0440	0.213
R_2	1.76	0.0440	0.0773
R_3	1.98	0.0440	0.0869
R_4	3.43	0.0440	0.151

$$\mathrm{L}_1 : \mathcal{E}_1 - I_1R_1 - I_1R_2 - I_1R_3 - I_1R_4 = 0$$

$$I_1 = \frac{\mathcal{E}_1}{R_1 + R_2 + R_3 + R_4}$$

$$I_1 = 0.0440\ \mathrm{A}$$

$$\begin{aligned}
\sum P_{\text{supplied}} &= \sum P_{\text{dissipated}} \\
P_{\mathcal{E}_1} &= P_1 + P_2 + P_3 + P_4 \\
0.527\ \text{W} &\stackrel{\checkmark}{=} 0.527\ \text{W}
\end{aligned}$$

2.2. Circuit 2

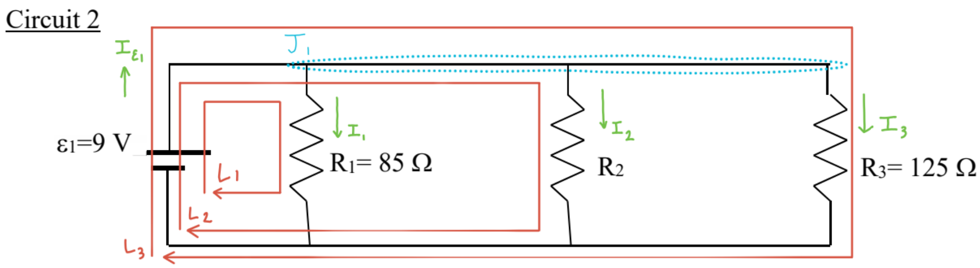

Figure 2. Circuit 2

Table 2. Circuit 2 V, I, and P

	V (V)	I (A)	P (W)
$\mathcal{E}_1$	9.00	0.403	3.63
R_1	9.00	0.106	0.953
R_2	9.00	0.225	2.02
R_3	9.00	0.0720	0.648

$$\begin{aligned} \mathrm{J}_1: \qquad I_{\mathcal{E}_1} &= I_1 + I_2 + I_3 \\ I_1 + I_2 + I_3 - I_{\mathcal{E}_1} &= 0 \end{aligned}$$

$$\begin{aligned} \mathrm{L}_1: \qquad \mathcal{E}_1 - I_1 R_1 &= 0 \\ I_1 R_1 &= \mathcal{E}_1 \end{aligned}$$

$$\begin{aligned} \mathrm{L}_2: \qquad \mathcal{E}_1 - I_2 R_2 &= 0 \\ I_2 R_2 &= \mathcal{E}_1 \end{aligned}$$

$$\begin{aligned} \mathrm{L}_3: \qquad \mathcal{E}_1 - I_3 R_3 &= 0 \\ I_3 R_3 &= \mathcal{E}_1 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & 0 & R_3 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_{\mathcal{E}_1} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathcal{E}_1 \\ \mathcal{E}_1 \\ \mathcal{E}_1 \end{bmatrix}$$

$$\begin{aligned} I_1 &= 0.106 \text{ A} \\ I_2 &= 0.225 \text{ A} \\ I_3 &= 0.0720 \text{ A} \\ I_{\mathcal{E}_1} &= 0.403 \text{ A} \end{aligned}$$

$$\begin{aligned} \sum P_{\text{supplied}} &= \sum P_{\text{dissipated}} \\ P_{\mathcal{E}_1} &= P_1 + P_2 + P_3 \\ 3.63 \text{ W} &\stackrel{\checkmark}{=} 3.63 \text{ W} \end{aligned}$$

2.3. Circuit 3

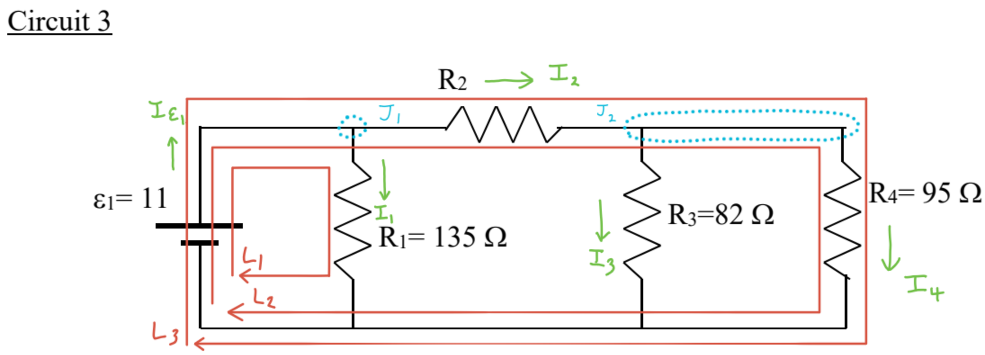

Figure 3. Circuit 3

Table 3. Circuit 3 V, I, and P

	V (V)	I (A)	P (W)
$\mathcal{E}_1$	11.0	0.212	2.34
R_1	11.0	0.0815	0.896
R_2	5.24	0.131	0.686
R_3	5.76	0.0703	0.405
R_4	5.76	0.0607	0.350

$$\begin{aligned} \mathrm{J}_1: \quad I_{\mathcal{E}_1} &= I_1 + I_2 \\ I_1 + I_2 - I_{\mathcal{E}_1} &= 0 \end{aligned}$$

$$\begin{aligned} \mathrm{J}_2: \quad I_2 &= I_3 + I_4 \\ I_2 - I_3 - I_4 &= 0 \end{aligned}$$

$$\begin{aligned} \mathrm{L}_1: \quad \mathcal{E}_1 - I_1 R_1 &= 0 \\ I_1 R_1 &= \mathcal{E}_1 \end{aligned}$$

$$\begin{aligned} \mathrm{L}_2: \mathcal{E}_1 - I_2 R_2 - I_3 R_3 &= 0 \\ I_2 R_2 + I_3 R_3 &= \mathcal{E}_1 \end{aligned}$$

$$\begin{aligned} \mathrm{L}_3: \mathcal{E}_1 - I_2 R_2 - I_4 R_4 &= 0 \\ I_2 R_2 + I_4 R_4 &= \mathcal{E}_1 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & -1 & 0 \\ R_1 & 0 & 0 & 0 & 0 \\ 0 & R_2 & R_3 & 0 & 0 \\ 0 & R_2 & 0 & R_4 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_{\mathcal{E}_1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mathcal{E}_1 \\ \mathcal{E}_1 \\ \mathcal{E}_1 \end{bmatrix}$$

$$\begin{aligned} I_1 &= 0.0815 \text{ A} \\ I_2 &= 0.131 \text{ A} \\ I_3 &= 0.0703 \text{ A} \\ I_4 &= 0.0607 \text{ A} \\ I_{\mathcal{E}_1} &= 0.212 \text{ A} \end{aligned}$$

$$\begin{aligned} \sum P_{\text{supplied}} &= \sum P_{\text{dissipated}} \\ P_{\mathcal{E}_1} &= P_1 + P_2 + P_3 + P_4 \\ 2.34 \text{ W} &\stackrel{\checkmark}{=} 2.34 \text{ W} \end{aligned}$$

2.4. Circuit 4

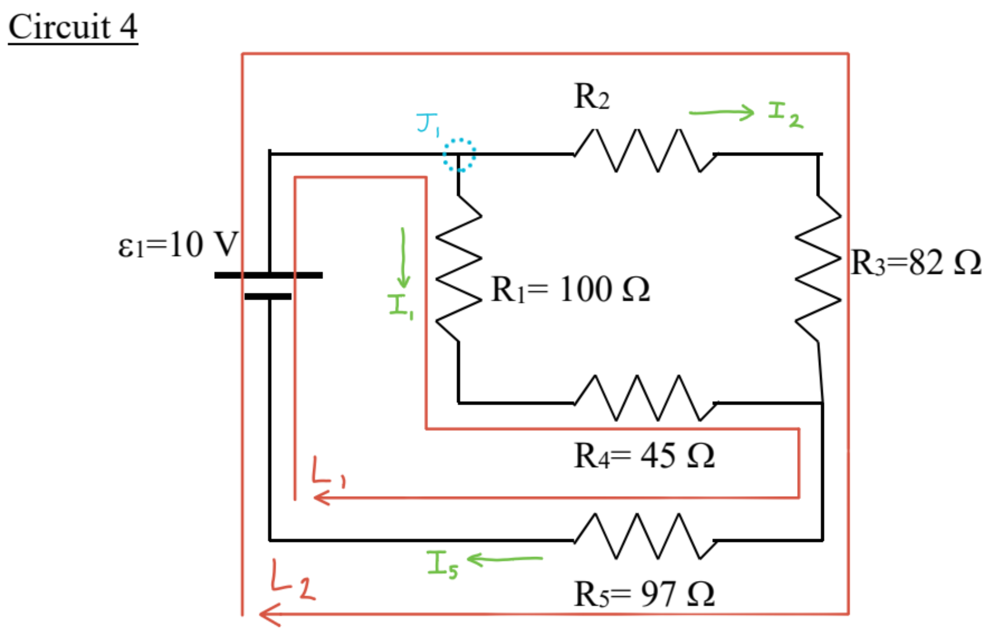

Figure 4. Circuit 4

Table 4. Circuit 4 V, I, and P

	V (V)	I (A)	P (W)
$\mathcal{E}_1$	10.0	0.0613	0.613
R_1	2.80	0.0280	0.0783
R_2	1.33	0.0333	0.0443
R_3	2.73	0.0333	0.0907
R_4	1.26	0.0280	0.0353
R_5	5.94	0.0613	0.364

$$\begin{aligned} \mathrm{J}_1: \quad & I_5 = I_1 + I_2 \\ & I_1 + I_2 - I_5 = 0 \end{aligned}$$

$$\begin{aligned} \mathrm{L}_1: \mathcal{E}_1 - I_1R_1 - I_1R_4 - I_5R_5 &= 0 \\ I_1(R_1 + R_4) + I_5R_5 &= \mathcal{E}_1 \end{aligned}$$

$$\begin{aligned} \mathrm{L}_2: \mathcal{E}_1 - I_2R_2 - I_2R_3 - I_5R_5 &= 0 \\ I_2(R_2 + R_3) + I_5R_5 &= \mathcal{E}_1 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ R_1 + R_4 & 0 & R_5 \\ 0 & R_2 + R_3 & R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_5 \end{bmatrix} = \begin{bmatrix} 0 \\ \mathcal{E}_1 \\ \mathcal{E}_1 \end{bmatrix}$$

$$I_1 = 0.0280 \text{ A}$$
$$I_2 = 0.0333 \text{ A}$$
$$I_5 = 0.0613 \text{ A}$$

$$\begin{aligned} \sum P_{\text{supplied}} &= \sum P_{\text{dissipated}} \\ P_{\mathcal{E}_1} &= P_1 + P_2 + P_3 + P_4 + P_5 \\ 0.613 \text{ W} &\stackrel{\checkmark}{=} 0.613 \text{ W} \end{aligned}$$

2.5. Circuit 5

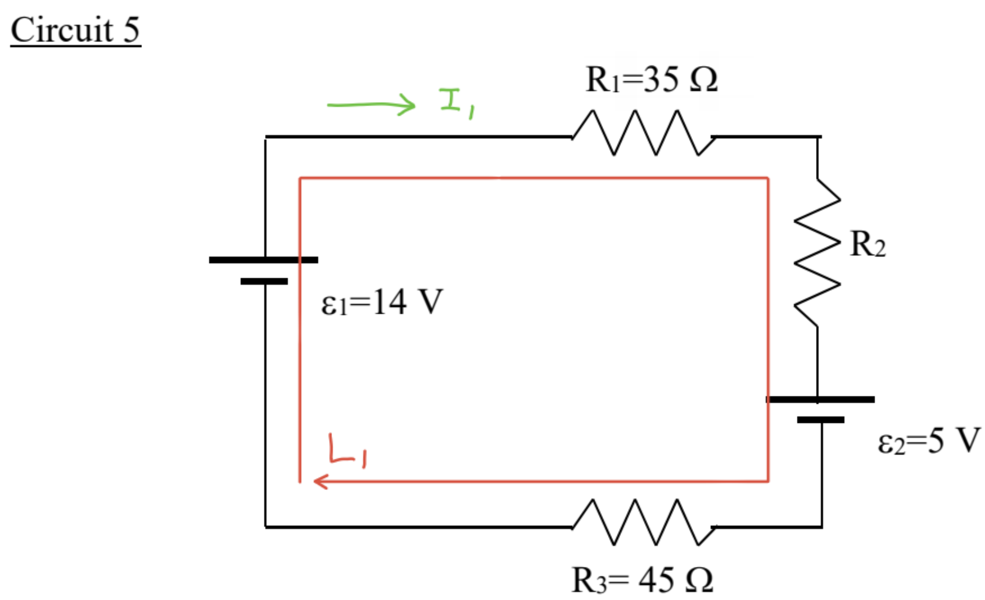

Figure 5. Circuit 5

Table 5. Circuit 5 V, I, and P

	V (V)	I (A)	P (W)
$\mathcal{E}_1$	14.0	0.0750	1.05
$\mathcal{E}_2$	5.00	0.0750	0.375
R_1	2.62	0.0750	0.197
R_2	3.00	0.0750	0.225
R_3	3.38	0.0750	0.253

$$\begin{aligned} \mathrm{L}_1 : \mathcal{E}_1 - I_1R_1 - I_1R_2 - \mathcal{E}_2 - I_1R_3 = 0 \\ I_1 &= \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2 + R_3} \\ I_1 &= 0.0750 \text{ A} \end{aligned}$$

$$\begin{aligned} \sum P_{\text{supplied}} &= \sum P_{\text{dissipated}} \\ P_{\mathcal{E}_1} &= P_1 + P_2 + P_3 + P_{\mathcal{E}_2} \\ 1.05 \text{ W} &\stackrel{\checkmark}{=} 1.05 \text{ W} \end{aligned}$$

2.6. Circuit 6

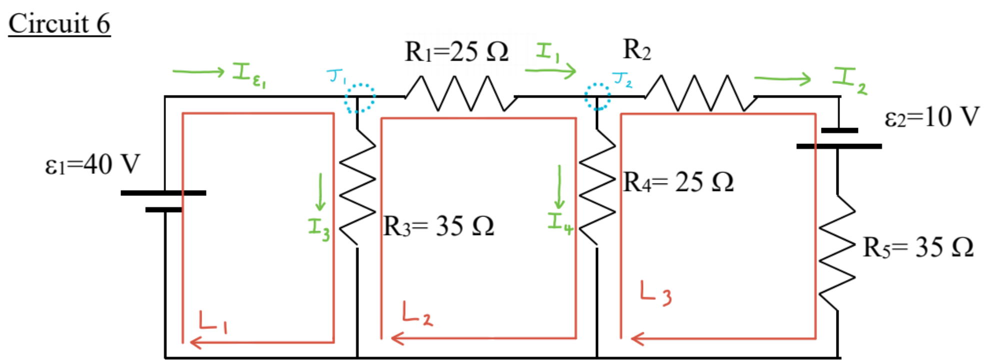

Figure 6. Circuit 6

Table 6. Circuit 6 V, I, and P

	V (V)	I (A)	P (W)
$\mathcal{E}_1$	40.0	2.11	84.6
$\mathcal{E}_2$	10.0	0.343	3.43
R_1	24.3	0.971	23.6
R_2	13.7	0.343	4.70
R_3	40.0	1.14	45.7
R_4	15.7	0.629	9.88
R_5	12.0	0.343	4.11

$$\begin{aligned}
\mathrm{J}_1: \quad && I_{\mathcal{E}_1} &= I_1 + I_3 \\
&& I_1 + I_3 - I_{\mathcal{E}_1} &= 0
\end{aligned}$$

$$\begin{aligned}
\mathrm{J}_2: \quad && I_1 &= I_2 + I_4 \\
&& I_1 - I_2 - I_4 &= 0
\end{aligned}$$

$$\begin{aligned}
\mathrm{L}_1: \quad && \mathcal{E}_1 - I_3 R_3 &= 0 \\
&& I_3 R_3 &= \mathcal{E}_1
\end{aligned}$$

$$\begin{aligned}
\mathrm{L}_2: \quad && -I_1 R_1 - I_4 R_4 + I_3 R_3 &= 0 \\
&& I_1 R_1 - I_3 R_3 + I_4 R_4 &= 0
\end{aligned}$$

$$\begin{aligned}
\mathrm{L}_3: \mathcal{E}_2 - I_2 R_2 - I_2 R_5 + I_4 R_4 &= 0 \\
I_2(R_2 + R_5) - I_4 R_4 &= \mathcal{E}_2
\end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & R_3 & 0 & 0 \\ R_1 & 0 & -R_3 & R_4 & 0 \\ 0 & R_2 + R_5 & 0 & -R_4 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_{\mathcal{E}_1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mathcal{E}_1 \\ 0 \\ \mathcal{E}_2 \end{bmatrix}$$

$$\begin{aligned}
I_1 &= 0.971 \text{ A} \\
I_2 &= 0.343 \text{ A} \\
I_3 &= 1.14 \text{ A} \\
I_4 &= 0.629 \text{ A} \\
I_{\mathcal{E}_1} &= 2.11 \text{ A}
\end{aligned}$$

$$\begin{aligned}
\sum P_{\text{supplied}} &= \sum P_{\text{dissipated}} \\
P_{\mathcal{E}_1} + P_{\mathcal{E}_2} &= P_1 + P_2 + P_3 + P_4 + P_5 \\
88.0 \text{ W} &\stackrel{\checkmark}{=} 88.0 \text{ W}
\end{aligned}$$

2.7. Circuit 7

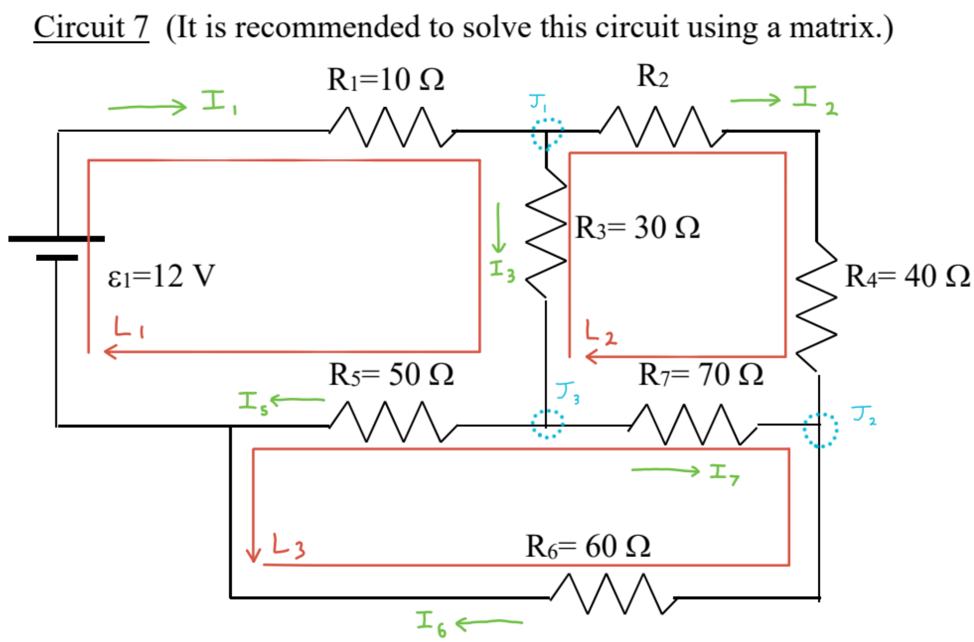

Figure 7. Circuit 7

Table 7. Circuit 7 V, I, and P

	V (V)	I (A)	P (W)
$\mathcal{E}_1$	12.0	0.200	2.40
R_1	2.00	0.200	0.399
R_2	2.58	0.0646	0.167
R_3	4.05	0.135	0.547
R_4	2.58	0.0646	0.167
R_5	5.95	0.119	0.709
R_6	4.83	0.0806	0.390
R_7	1.12	0.0160	0.0179

$$\begin{aligned}
\mathrm{J}_1: && I_1 &= I_2 + I_3 \\
&& I_1 - I_2 - I_3 &= 0 \\
\\
\mathrm{J}_2: && I_2 + I_7 &= I_6 \\
&& I_2 - I_6 + I_7 &= 0 \\
\\
\mathrm{J}_3: && I_3 &= I_5 + I_7 \\
&& I_3 - I_5 - I_7 &= 0 \\
\\
\mathrm{L}_1: && \mathcal{E}_1 - I_1R_1 - I_3R_3 - I_5R_5 &= 0 \\
&& I_1R_1 + I_3R_3 + I_5R_5 &= \mathcal{E}_1 \\
\\
\mathrm{L}_2: && -I_2R_2 - I_2R_4 + I_7R_7 + I_3R_3 &= 0 \\
&& I_2(R_2 + R_4) - I_3R_3 - I_7R_7 &= 0 \\
\\
\mathrm{L}_3: && -I_5R_5 + I_6R_6 + I_7R_7 &= 0 \\
&& I_5R_5 - I_6R_6 - I_7R_7 &= \mathcal{E}_2
\end{aligned}$$

$$\begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 & 0 & -1 \\ R_1 & 0 & R_3 & R_5 & 0 & 0 \\ 0 & R_2 + R_4 & -R_3 & 0 & 0 & -R_7 \\ 0 & 0 & 0 & R_5 & -R_6 & -R_7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \mathcal{E}_1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned}
I_1 &= 0.200\ \mathrm{A} \\
I_2 &= 0.0646\ \mathrm{A} \\
I_3 &= 0.135\ \mathrm{A} \\
I_5 &= 0.119\ \mathrm{A} \\
I_6 &= 0.0806\ \mathrm{A} \\
I_7 &= 0.0160\ \mathrm{A}
\end{aligned}$$

$$
\begin{aligned}
\sum P_{\text{supplied}} &= \sum P_{\text{dissipated}} \\
P_{\mathcal{E}_1} &= P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 \\
2.40\ \text{W} &\stackrel{\checkmark}{=} 2.40\ \text{W}
\end{aligned}
$$

3. Simulations

Circuits 1 through 4 were simulated using the Circuit Simulator Applet (located at https://www.falstad.com/circuit/). All the voltage and current values match the calculations to 3 significant figures (Yay!).

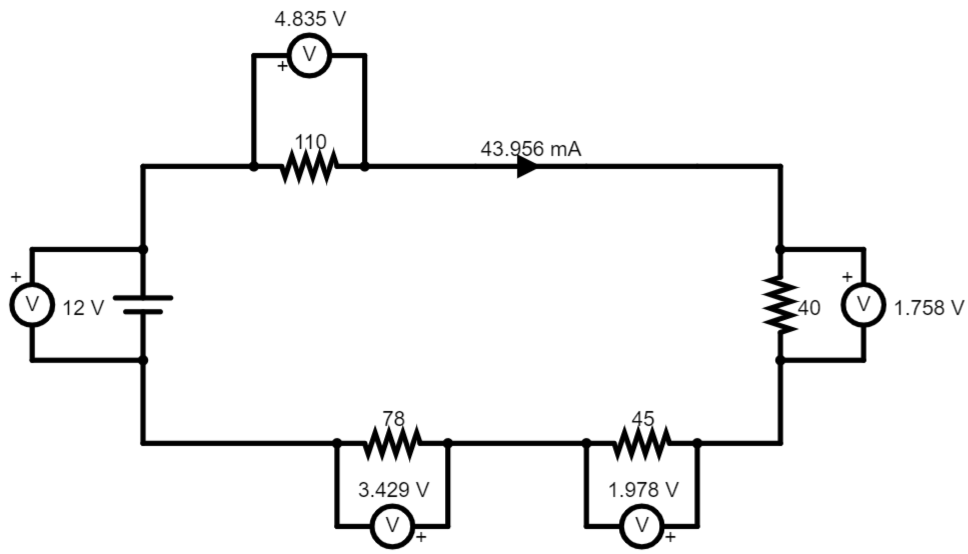

Figure 8. Circuit 1 Simulation

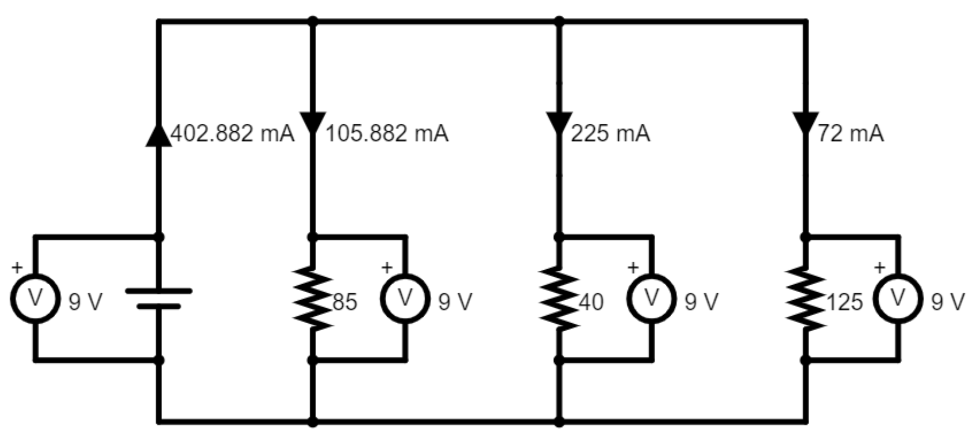

Figure 9. Circuit 2 Simulation

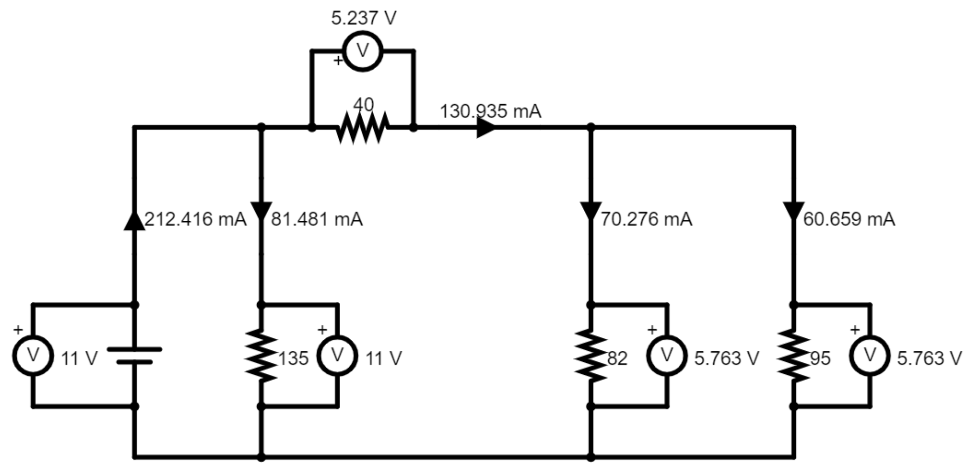

Figure 10. Circuit 3 Simulation

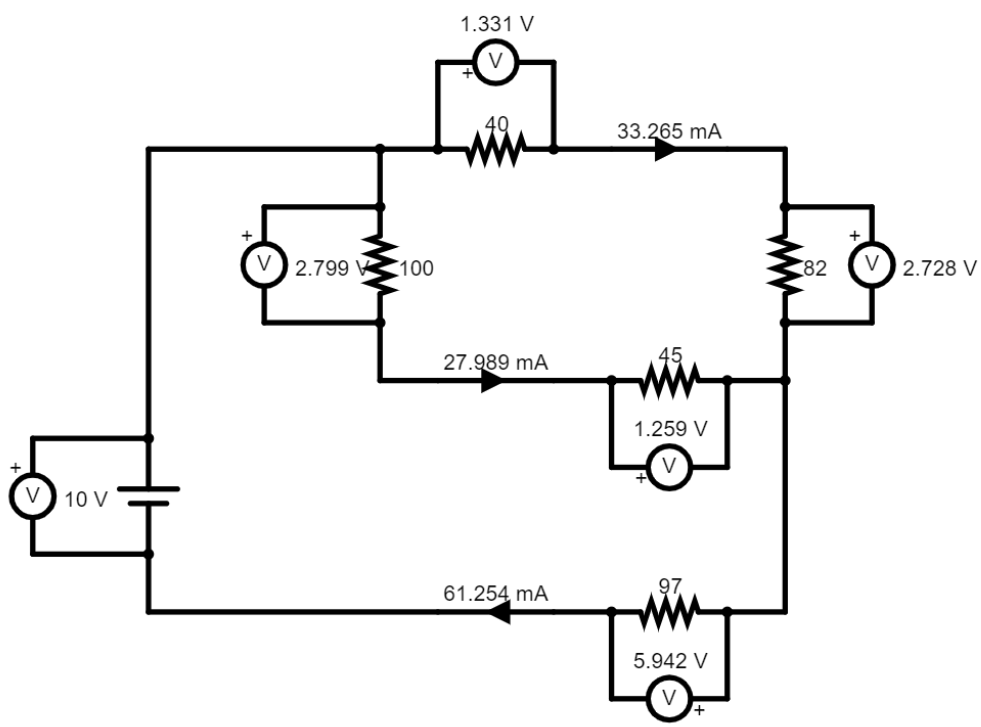

Figure 11. Circuit 4 Simulation

4. Conclusion

The goal of the exercise was to use Kirchhoff's rules to analyze 7 different circuits and calculate current, voltage, and power for each circuit element. Kirchhoff's junction rule states that the sum of the current into a junction equals the sum of the current out of the junction. This rule is a result of conservation of charge.

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

Kirchhoff's loop rule states that sum of all the voltages on a closed loop of circuit must equal 0. This rule is a result of conservation of energy.

$$\sum \Delta V = 0$$

Since the voltages of the power supplies and the resistances for each resistor were known, the main unknowns were the currents through the series sections of the circuits. Components connected in series have the same current through them due to conservation of charge (the rate of charge entering the section needs to equal the rate of charge exiting). As such, a current label was added to each of these sections on the circuit diagram. Kirchhoff's rules could be used to set up systems of equations to solve for those unknown currents. The junction rule needs to be applied 1 time less than the total number of junctions in the circuit. The loop rule needs to be applied the same number of times as internal spaces in the circuit diagram. This yields the same number of equations as unknowns.

Once the currents were known, the voltage across each resistor could be calculated using Ohm's law,

$$V = IR$$

and the power supplied/dissipated by each component could be calculated as the product of current and voltage.

$$P = VI$$

In line with conservation of energy, the total power supplied should equal the total power dissipated.

$$\sum P_{\text{supplied}} = \sum P_{\text{dissipated}}$$

This check was used on all the circuits to make sure the results seemed plausible. All the circuits passed this check to three significant figures. Note that the unrounded values for power were used for these checks in order to avoid rounding error.

For the first 4 circuits, the voltage and current calculations were compared to results obtained using an online circuit simulator. All the voltage and current values matched the calculations to 3 significant figures. This supports the idea that Kirchhoff's rules were applied correctly.

5. Citations

[1] Karen Schnurbusch, *Physics 4B Lab Book*, Mt. San Antonio College, 2023, pp. 71-74.
[2] Karen Schnurbusch, *Physics 4B Equations*, Mt. San Antonio College, 2023, pp. 4, 5.
[3] Paul Falstad, *Circuit Simulator Applet*, https://www.falstad.com/circuit/ Accessed 26 April 2023.