Carnot Cycle Exercise

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$T_H = 490$ K
$V_c = 1.90 \times 10^{-3}$ m^3

1. Purpose

The goal of the exercise is to perform various calculations related to the Carnot cycle.

2. Given

- $T_H = 490$ K
- $T_C = 300$ K
- $P_c = 1.01 \times 10^5$ Pa
- $V_c = 1.90 \times 10^{-3}$ m^3
- $Q_{a \to b} = 300$ J
- $\gamma = 1.40$
- d.o.f. $= 5$
- $C_v = \frac{5}{2}R$
- $C_p = \frac{7}{2}R$

3. Derivations

3.1. Temperature–Volume Relationship for Adiabatic Process

$$
\begin{aligned}
P_i V_i^\gamma &= P_f V_f^\gamma \\
P_i V_i V_i^{\gamma-1} &= P_f V_f V_f^{\gamma-1} \\
nRT_i V_i^{\gamma-1} &= nRT_f V_f^{\gamma-1} \\
T_i V_i^{\gamma-1} &= T_f V_f^{\gamma-1}
\end{aligned}
$$

3.2. Work by Gas for Isothermal Process

$$
\begin{aligned}
W &= \int_{V=V_i}^{V=V_f} P dV \\
W &= \int_{V_i}^{V_f} \frac{nRT}{V} dV \\
W &= nRT \ln(V) \Big|_{V_i}^{V_f} \\
W &= nRT(\ln(V_f) - \ln(V_i)) \\
W &= nRT \ln\left(\frac{V_f}{V_i}\right)
\end{aligned}
$$

Table 1. Pressure, Volume, and Temperature for Key Points
Note: $T_a = T_b = T_H$ and $T_c = T_d = T_C$

Point	P (Pa)	V (m^3)	T (K)
a	1.46×10^6	2.14×10^{-4}	490
b	5.62×10^5	5.57×10^{-4}	490
c	1.01×10^5	1.90×10^{-3}	300
d	2.63×10^5	7.30×10^{-4}	300

Table 2. Heat Transferred, Change in Internal Energy, Work Done, and Change in Entropy for Key Processes

Process	Type	Q (J)	W (J)	ΔU (J)	ΔS (J/K)
$a \to b$	isothermal expansion	300	300	0	0.612
$b \to c$	adiabatic expansion	0			0
$c \to d$	isothermal compression			0	
$d \to a$	adiabatic compression	0			0

4. Results

4.1. Moles of Gas (n)

$$\begin{aligned} P_c V_c &= nRT_c \\ n &= \frac{P_c V_c}{RT_c} \\ n &= 0.0770 \text{ mol} \end{aligned}$$

4.2. Pressure (P_b) and Volume (V_b) at b

$$\begin{aligned} T_b V_b^{\gamma-1} &= T_c V_c^{\gamma-1} \\ V_b &= V_c \left(\frac{T_c}{T_b}\right)^{\frac{1}{\gamma-1}} \\ V_b &= 5.57 \times 10^{-4} \text{ m}^3 \end{aligned}$$

$$\begin{aligned} P_b V_b &= nRT_b \\ P_b &= \frac{nRT_b}{V_b} \\ P_b &= 5.62 \times 10^5 \text{ Pa} \end{aligned}$$

4.3. Pressure (P_a) and Volume (V_a) at a

$$\Delta U_{a\to b} = Q_{a\to b} - W_{a\to b}$$

$$0 = Q_{a\to b} - nRT_H \ln\left(\frac{V_b}{V_a}\right)$$
$$\ln\left(\frac{V_b}{V_a}\right) = \frac{Q_{a\to b}}{nRT_H}$$
$$\frac{V_b}{V_a} = e^{Q_{a\to b}/(nRT_H)}$$
$$V_a = V_b e^{-Q_{a\to b}/(nRT_H)}$$
$$V_a = 2.14 \times 10^{-4}\ \mathrm{m}^3$$

$$P_a V_a = nRT_a$$
$$P_a = \frac{nRT_a}{V_a}$$
$$P_a = 1.46 \times 10^{6}\ \mathrm{Pa}$$

4.4. Pressure (P_d) and Volume (V_d) at d

$$T_d V_d^{\gamma-1} = T_a V_a^{\gamma-1}$$
$$V_d = V_a \left(\frac{T_a}{T_d}\right)^{\frac{1}{\gamma-1}}$$
$$V_d = 7.30 \times 10^{-4}\ \mathrm{m}^3$$

$$P_d V_d = nRT_d$$
$$P_d = \frac{nRT_d}{V_d}$$
$$P_d = 2.63 \times 10^{5}\ \mathrm{Pa}$$

4.5. Process Variables $a \to b$

$$\Delta U_{a\to b} = 0$$

$$\Delta U_{a\to b} = Q_{a\to b} - W_{a\to b}$$
$$0 = Q_{a\to b} - W_{a\to b}$$
$$W_{a\to b} = Q_{a\to b}$$
$$W_{a\to b} = 300\ \mathrm{J}$$

$$\Delta S_{a\to b} = \int\limits_{a\to b} \frac{dQ}{T}$$

$$\Delta S_{a\to b} = \frac{1}{T_H}\int\limits_{a\to b} dQ$$
$$\Delta S_{a\to b} = \frac{Q_{a\to b}}{T_H}$$
$$\Delta S_{a\to b} = 0.612\ \mathrm{J/K}$$

5. Conclusion

6. Citations

[1] Karen Schnurbusch, *Physics 4B Lab Book*, Mt. San Antonio College, 2023, pp. 35-38.
[2] Karen Schnurbusch, *Physics 4B Equations*, Mt. San Antonio College, 2023, pp. 1-3.