Carnot Cycle Exercise

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$T_H = 490$ K
$V_c = 1.90 \times 10^{-3}$ m^3

1. Purpose

The goal of the exercise is to perform various calculations related to the Carnot cycle.

2. Given

- $T_H = 490$ K
- $T_C = 300$ K
- $P_c = 1.01 \times 10^5$ Pa
- $V_c = 1.90 \times 10^{-3}$ m^3
- $Q_{a \to b} = 300$ J
- $\gamma = 1.40$
- d.o.f. $= 5$
- $C_v = \frac{5}{2} R$
- $C_p = \frac{7}{2} R$

3. Derivations

3.1. Temperature–Volume Relationship for Adiabatic Process

$$\begin{aligned}
P_i V_i^\gamma &= P_f V_f^\gamma \\
P_i V_i V_i^{\gamma-1} &= P_f V_f V_f^{\gamma-1} \\
nRT_i V_i^{\gamma-1} &= nRT_f V_f^{\gamma-1} \\
T_i V_i^{\gamma-1} &= T_f V_f^{\gamma-1}
\end{aligned}$$

3.2. Work by Gas for Isothermal Process

$$\begin{aligned}
W &= \int_{i \to f} P dV \\
W &= \int_{V_i}^{V_f} \frac{nRT}{V} dV \\
W &= nRT \ln(V) \Big|_{V_i}^{V_f} \\
W &= nRT (\ln(V_f) - \ln(V_i)) \\
W &= nRT \ln\left(\frac{V_f}{V_i}\right)
\end{aligned}$$

3.3. Work by Gas for Adiabatic Process

$$W = \int_{i \to f} P dV$$
$$W = \int_{V_i}^{V_f} (P_i V_i^\gamma) V^{-\gamma} dV$$
$$W = \frac{P_i V_i^\gamma}{1-\gamma} V^{1-\gamma} \Big|_{V_i}^{V_f}$$
$$W = \frac{P_i V_i^\gamma}{1-\gamma} (V_f^{1-\gamma} - V_i^{1-\gamma})$$
$$W = \frac{P_i V_i^\gamma V_f^{1-\gamma} - P_i V_i^\gamma V_i^{1-\gamma}}{1-\gamma}$$
$$W = \frac{P_f V_f^\gamma V_f^{1-\gamma} - P_i V_i^\gamma V_i^{1-\gamma}}{1-\gamma}$$
$$W = \frac{P_f V_f - P_i V_i}{1-\gamma}$$
$$W = \frac{P_i V_i - P_f V_f}{\gamma - 1}$$

4. Results

Table 1. Pressure, Volume, and Temperature for Key Points
Note: $T_a = T_b = T_H$ and $T_c = T_d = T_C$

Point	P (Pa)	V (m^3)	T (K)
a	1.46×10^6	2.14×10^{-4}	490
b	5.62×10^5	5.57×10^{-4}	490
c	1.01×10^5	1.90×10^{-3}	300
d	2.63×10^5	7.30×10^{-4}	300

Table 2. Heat Transferred, Change in Internal Energy, Work Done, and Change in Entropy for Key Processes

Process	Type	Q (J)	W (J)	ΔU (J)	ΔS (J/K)
$a \to b$	isothermal expansion	300	300	0	0.612
$b \to c$	adiabatic expansion	0	304	−304	0
$c \to d$	isothermal compression	−184	−184	0	−0.612
$d \to a$	adiabatic compression	0			0

4.1. Moles of Gas (n)

$$\begin{aligned} P_c V_c &= nRT_c \\ n &= \frac{P_c V_c}{RT_c} \\ n &= 0.0770 \text{ mol} \end{aligned}$$

4.2. Pressure (P_b) and Volume (V_b) at b

$$\begin{aligned} T_b V_b^{\gamma-1} &= T_c V_c^{\gamma-1} \\ V_b &= V_c \left(\frac{T_c}{T_b}\right)^{\frac{1}{\gamma-1}} \\ V_b &= 5.57 \times 10^{-4} \text{ m}^3 \end{aligned}$$

$$\begin{aligned} P_b V_b &= nRT_b \\ P_b &= \frac{nRT_b}{V_b} \\ P_b &= 5.62 \times 10^{5} \text{ Pa} \end{aligned}$$

4.3. Pressure (P_a) and Volume (V_a) at a

$$\begin{aligned} \Delta U_{a\to b} &= Q_{a\to b} - W_{a\to b} \\ 0 &= Q_{a\to b} - nRT_H \ln\left(\frac{V_b}{V_a}\right) \\ \ln\left(\frac{V_b}{V_a}\right) &= \frac{Q_{a\to b}}{nRT_H} \\ \frac{V_b}{V_a} &= e^{Q_{a\to b}/(nRT_H)} \\ V_a &= V_b e^{-Q_{a\to b}/(nRT_H)} \\ V_a &= 2.14 \times 10^{-4} \text{ m}^3 \end{aligned}$$

$$\begin{aligned} P_a V_a &= nRT_a \\ P_a &= \frac{nRT_a}{V_a} \\ P_a &= 1.46 \times 10^{6} \text{ Pa} \end{aligned}$$

4.4. Pressure (P_d) and Volume (V_d) at d

$$\begin{aligned} T_d V_d^{\gamma-1} &= T_a V_a^{\gamma-1} \\ V_d &= V_a \left(\frac{T_a}{T_d}\right)^{\frac{1}{\gamma-1}} \\ V_d &= 7.30 \times 10^{-4} \text{ m}^3 \end{aligned}$$

$$\begin{aligned}
P_d V_d &= nRT_d \\
P_d &= \frac{nRT_d}{V_d} \\
P_d &= 2.63 \times 10^5 \text{ Pa}
\end{aligned}$$

4.5. Process Variables $a \to b$

$$\Delta U_{a\to b} = 0$$

$$\begin{aligned}
\Delta U_{a\to b} &= Q_{a\to b} - W_{a\to b} \\
0 &= Q_{a\to b} - W_{a\to b} \\
W_{a\to b} &= Q_{a\to b} \\
W_{a\to b} &= 300 \text{ J}
\end{aligned}$$

$$\begin{aligned}
\Delta S_{a\to b} &= \int\limits_{a\to b} \frac{dQ}{T} \\
\Delta S_{a\to b} &= \frac{1}{T_H} \int\limits_{a\to b} dQ \\
\Delta S_{a\to b} &= \frac{Q_{a\to b}}{T_H} \\
\Delta S_{a\to b} &= 0.612 \text{ J/K}
\end{aligned}$$

4.6. Process Variables $b \to c$

$$Q_{b\to c} = 0$$

$$\begin{aligned}
W_{b\to c} &= \frac{P_b V_b - P_c V_c}{\gamma - 1} \\
W_{b\to c} &= 304 \text{ J}
\end{aligned}$$

$$\begin{aligned}
\Delta U_{b\to c} &= Q_{b\to c} - W_{b\to c} \\
\Delta U_{b\to c} &= -W_{b\to c} \\
\Delta U_{b\to c} &= -304 \text{ J}
\end{aligned}$$

$$\begin{aligned}
\Delta S_{b\to c} &= \int\limits_{b\to c} \frac{dQ}{T} \\
\Delta S_{b\to c} &= 0
\end{aligned}$$

4.7. Process Variables $c \to d$

$$\Delta U_{c \to d} = 0$$

$$\begin{aligned} W_{c \to d} &= nRT_C \ln\left(\frac{V_d}{V_c}\right) \\ W_{c \to d} &= -184 \text{ J} \end{aligned}$$

$$\begin{aligned} \Delta U_{c \to d} &= Q_{c \to d} - W_{c \to d} \\ 0 &= Q_{c \to d} - W_{c \to d} \\ Q_{c \to d} &= W_{c \to d} \\ Q_{c \to d} &= -184 \text{ J} \end{aligned}$$

$$\begin{aligned} \Delta S_{c \to d} &= \int\limits_{c \to d} \frac{dQ}{T} \\ \Delta S_{c \to d} &= \frac{1}{T_C} \int\limits_{c \to d} dQ \\ \Delta S_{c \to d} &= \frac{Q_{c \to d}}{T_C} \\ \Delta S_{c \to d} &= -0.612 \text{ J/K} \end{aligned}$$

5. Conclusion

6. Citations

[1] Karen Schnurbusch, *Physics 4B Lab Book*, Mt. San Antonio College, 2023, pp. 35-38.
[2] Karen Schnurbusch, *Physics 4B Equations*, Mt. San Antonio College, 2023, pp. 1-3.