PHYS 2AG Equations

Having the equations is a start. The next step is understanding what the equations mean, when to use them, and how to use them.

SI Prefixes

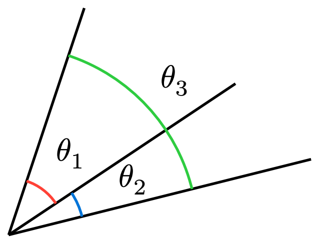
Name	Symbol	Factor
Quetta	Q	10^{30}
Ronna	R	10^{27}
Yotta	Y	10^{24}
Zetta	Z	10^{21}
Exa	E	10^{18}
Peta	P	10^{15}
Tera	T	10^{12}
Giga	G	10^{9}
Mega	M	10^{6}
Kilo	k	10^{3}
Hecto	h	10^{2}
Deka	da	10^{1}
-	-	10^{0}
Deci	d	10^{-1}
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Femto	f	10^{-15}
Atto	a	10^{-18}
Zepto	z	10^{-21}
Yocto	y	10^{-24}
Ronto	r	10^{-27}
Quecto	q	10^{-30}

Constants

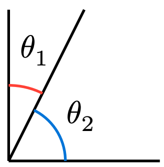
- $g = 9.8\ \frac{\mathrm{m}}{\mathrm{s}^2}$
- $G = 6.674 \times 10^{-11}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$
- $M_{\mathrm{Earth}} = 5.97 \times 10^{24}\ \ \mathrm{kg}$
- $R_{\mathrm{Earth}} = 6.38 \times 10^{6}\ \ \mathrm{m}$
- $1\ \mathrm{rev} = 2\pi\ \mathrm{rad} = 360°$

Geometry

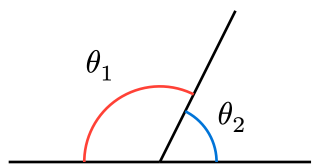
- Angle addition postulate
  - $\theta_1 + \theta_2 = \theta_3$



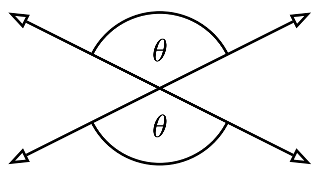
- Complementary angles
  - $\theta_1 + \theta_2 = 90°$



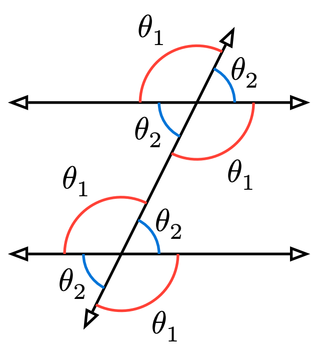
- Supplementary angles
  - $\theta_1 + \theta_2 = 180°$



- Vertical angles

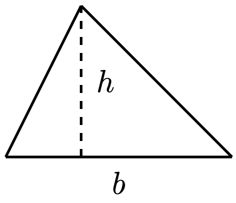


- Parallel lines cut by transversal

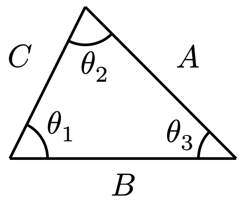


- Triangles

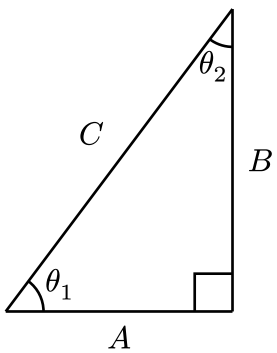
  - $A = \frac{1}{2}bh$



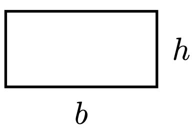
  - $\theta_1 + \theta_2 + \theta_3 = 180°$
  - $\frac{A}{\sin(\theta_1)} = \frac{B}{\sin(\theta_2)} = \frac{C}{\sin(\theta_3)}$ (law of sines)
  - $C^2 = A^2 + B^2 - 2AB\cos(\theta_3)$ (law of cosines)



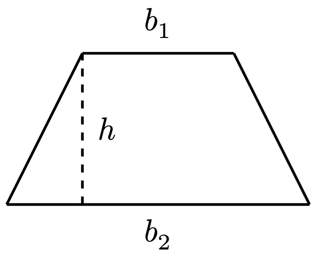
- Right triangles
  - $A^2 + B^2 = C^2$
  - $\theta_1 + \theta_2 = 90°$
  - $\sin(\theta_1) = \cos(\theta_2) = \frac{B}{C}$
  - $\cos(\theta_1) = \sin(\theta_2) = \frac{A}{C}$
  - $\tan(\theta_1) = \frac{1}{\tan(\theta_2)} = \frac{B}{A}$



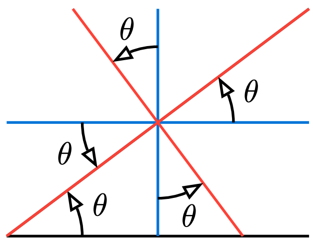
- Rectangles
  - $A = bh$
  - $P = 2b + 2h$



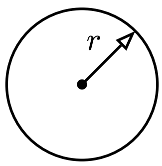
- Trapezoid
  - $A = \frac{1}{2}(b_1 + b_2)h$



- Inclined plane
  - Inclined planes are typically described by their angle above horizontal
  - The tilted coordinate axes are rotated that angle relative to the standard coordinate axes



- Circles
  - $C = 2\pi r$
  - $A = \pi r^2$



- Spheres
  - $A = 4\pi r^2$
  - $V = \frac{4}{3}\pi r^3$

Algebra

- Quadratic formula
  - $ax^2 + bx + c = 0$
  - $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Properties of logarithms
  - $\log_b(a) = p \iff b^p = a$
  - $\log_b(ac) = \log_b(a) + \log_b(c)$
  - $\log_b\left(\frac{a}{c}\right) = \log_b(a) - \log_b(c)$

- $\log_b(a^p) = p\log_b(a)$

- Sum and difference trig identities
  - $\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$
  - $\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$

Kinematics

- Note: x can be replaced with y depending on the direction being analyzed
- $v_{\text{avg}} = \frac{d}{\Delta t}$ (average speed)
- $v_{\text{avg,x}} = \frac{\Delta x}{\Delta t}$ (average velocity, x-component)
- $a_{\text{avg,x}} = \frac{\Delta v_x}{\Delta t}$
- If $a = \text{constant}$:
  - $v_x = v_{0x} + a_x t$
  - $x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$
  - $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$
- Knows and Unknowns
  - Point 1 ⟶ Point 2

x_0	y_0
x	y
v_{0x}	v_{0y}
v_x	v_y
a_x	a_y
t	

Forces

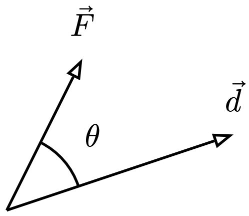
- Newton's laws of motion
  1. $\vec{a} = 0 \Longleftrightarrow \Sigma\vec{F} = 0$
  2. $\Sigma\vec{F} = m\vec{a}$
  3. $\vec{F}_{b \text{ on } a} = -\vec{F}_{a \text{ on } b}$
- Gravitational force (F_g)
  - $F_g = mg$
  - $F_g = G\frac{m_1 m_2}{r^2}$
  - Points straight down towards the center of the Earth
- Normal force (n, N, or F_n)
  - No general formula
  - Pushes perpendicular to surfaces in contact
- Applied force (F_{app})
  - Push or pull in a specified direction
- Tension (T or F_T)
  - No general formula
  - Pulls along rope/chain/cable
- Kinetic friction (f_k)
  - $f_k = \mu_k n$
  - Acts opposite relative motion of surfaces
- Static friction (f_s)
  - $f_s \leq \mu_s n$
  - $f_{s,\text{max}} = \mu_s n$
  - Direction and magnitude change based on the situation
  - Acts when there is no relative motion between surfaces
  - Tries to stop relative motion between surfaces
- Spring force (F_s)
  - $F_s = ks$
  - Points in the direction opposite the squish or stretch
- Pivot force (P)
  - No general formula
  - Usually broken into P_x and P_y
  - Force from pin that keeps attached object from translating away
- Free body diagram (FBD) reminders
  - Identify your system
  - Identify the direction of the acceleration

- Identify the direction of the velocity
- Choose a coordinate system
- Draw all external forces acting on the system
- Note the angle for any forces acting at an angle relative to the chosen axes

- Centripetal acceleration (a_c)
  - $a_c = \frac{v^2}{r} = r\omega^2$
  - Points towards the center of the circular path

Work and Energy

- Work (W)
  - Energy transferred via forces
  - $W = Fd\cos(\theta)$



- External heat (Q_{ext})
  - Thermal energy transferred from the environment to/from the system
- Power (P)
  - $P = \frac{\Delta E}{\Delta T}$
- Kinetic energy (K)
  - Energy of motion
  - $K = \frac{1}{2}mv^2$
- Kinetic energy (K_{rot})
  - Energy of rotational motion
  - $K = \frac{1}{2}I\omega^2$
- Gravitational potential energy (U_g)
  - Energy due to position in a gravitational field
  - $U_g = mgh$
  - Make sure to choose a reference level
- Elastic potential energy (U_s)
  - Energy due to squishing/stretching a spring from its natural length
  - $U_g = \frac{1}{2}ks^2$
- Mechanical energy (E_{mech})
  - Kinetic energy plus potential energy
  - $E_{\text{mech}} = K + U$
- Internal energy (U_{int})
  - $\Delta U_{\text{th}} = mc\Delta T$ (due to temperature)
  - $\Delta U_p = \pm mL$ (due to phase change)
    - All the substance must reach the phase change temperature before the transition occurs
    - Temperature remains constant until all the substance has undergone the phase change
- Sound energy (U_{sound})
  - Energy of sound waves
- Conservation of energy
  - $E_i + W_{\text{nc}} + Q_{\text{ext}} = E_f$
  - $K_i + U_{gi} + U_{si} + \dots + W_{\text{nc}} + Q_{\text{ext}} = K_f + U_{gf} + U_{sf} + \dots + \Delta U_{\text{int}}$

Momentum

- $\vec{p} = m\vec{v}$ (momentum)
- $\vec{J} = \vec{F}_{\text{net,avg}}\Delta t = \Delta\vec{p}$ (impulse-momentum theorem)
- Conservation of momentum
  - If $\vec{J} = 0$, then $p_i = p_f$
  - Applies to pretty much all collisions & explosions
- Collision types
  - Perfectly inelastic
    - "Sticky"
    - $v_{1f} = v_{2f} = \dots = v_f$
  - Inelastic

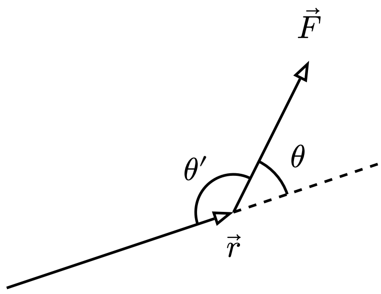
    - In between perfectly inelastic and perfectly elastic
  - Perfectly elastic
    - "Super Bouncy"
    - $K_i = K_f$

Rotational Kinematics

- Switching between linear (tangential) and angular quantities
  - Angles must be in radians
  - $s = r\theta$
  - $v_t = r\omega$
  - $a_t = r\alpha$
- $\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t}$
- $\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t}$
- If $\alpha =$ constant:
  - $\omega = \omega_0 + \alpha t$
  - $x = x_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
  - $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

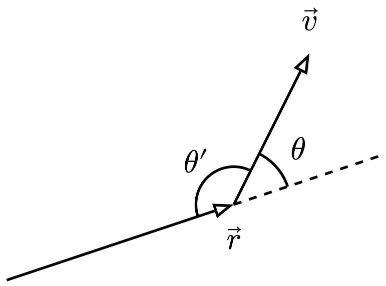
Rotational Dynamics

- Torque (τ)
  - $\tau = \pm F r \sin(\theta)$
  - $\sin(\theta) = \sin(\theta')$, so either angle can be used



- $\Sigma\tau = I\alpha$ (Newton's second law rotational)
- $I = \Sigma m r^2$
  - Moment of inertia formulas are derived by breaking objects up into pieces, then adding up mr^2 for each piece
  - Implies that moments of inertia can add together
- $I = I_{\text{cm}} + MD^2$ (parallel axis theorem)
- When rolling without slipping, $v_{\text{cm}} = r\omega$
- Rigid body diagram (RBD) reminders
  - Identify your system
  - Identify the direction of the acceleration or angular acceleration
  - Identify the direction of the velocity or angular velocity
  - Choose a coordinate system
  - Choose an axis of rotation (if it's not rotating)
  - Draw all external forces acting on the system at the location where they are acting
  - Note the angle for any forces acting at an angle relative to the chosen axes, or relative to a vector pointing from the pivot to where the force acts
  - Note how far each force is acting from the axis of rotation
- Effective mass (m_{eff})
  - $m_{\text{eff}} = \frac{I}{r^2}$
  - The effective mass of a pulley when analyzing a pulley system as a whole
  - Contributes to the total system mass
  - I is the moment of inertia of the pulley about its axle
  - r is the distance from the axle to where the string is attached to the pulley
- Angular momentum (L)

  - $L = mvr\sin(\theta)$ (point particle)
  - $\sin(\theta) = \sin(\theta')$, so either angle can be used

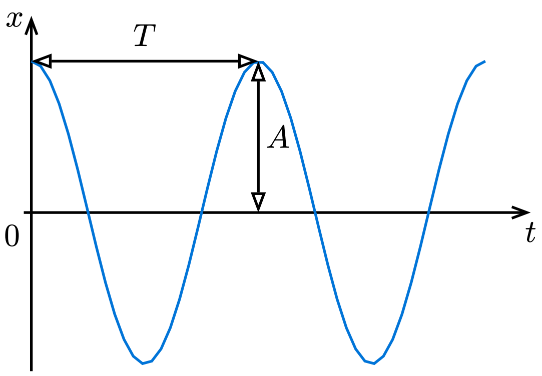


  - $L = I\omega$ (not universal, but works for most cases in the class)
  - $\tau_{\text{net,avg}}\Delta t = \Delta L$

Static Equilibrium

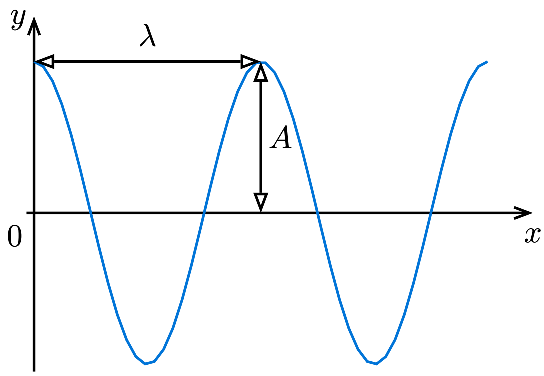
- If $\vec{a} = 0$ and $\alpha = 0$:
  - $\Sigma F_x = 0$
  - $\Sigma F_y = 0$
  - $\Sigma \tau = 0$
- Since it is not rotating, any axis of rotation can be chosen for the purpose of calculating torques

Simple Harmonic Motion



- Oscillation terms
  - Period (T)
  - Frequency (f)
  - Angular frequency (ω)
  - $T = \frac{1}{f}$ or $f = \frac{1}{T}$
  - $\omega = \frac{2\pi}{T} = 2\pi f$
  - Amplitude (A)
- Mass on a spring
  - $\omega = \sqrt{\frac{k}{m}}$
- Simple pendulum
  - $\omega = \sqrt{\frac{g}{L}}$
  - Assumes $\theta <\sim 15°$ so that $\theta \approx \sin(\theta)$
- Kinematics
  - $x(t) = A\cos(\omega t + \phi)$
  - $x_{\max} = A$
  - $v(t) = -A\omega\sin(\omega t + \phi)$
  - $v_{\max} = A\omega$
  - $a(t) = -A\omega^2\cos(\omega t + \phi)$
  - $a_{\max} = A\omega^2$

Waves



- $v = f\lambda$
  - Wave speed depends on properties of the medium, not on wavelength and frequency
  - $v = \sqrt{\frac{F_T}{\mu}}$ (speed of wave in a string)
  - $v = \left(331.5\ \frac{\text{m}}{\text{s}}\right) + \left(0.6\ \frac{\text{m/s}}{°\text{C}}\right)T$ (speed of sound in air)
- Wave function
  - $y(x, t) = A\cos(kx \pm \omega t)$
    - Plus when moving in the $-x$ direction
    - Minus when moving in the $+x$ direction

  - Focus on a single point, $x = 0$, over time
    - $y(0, t) = A\cos(\pm\omega t)$
  - Focus on the shape of the wave at a single point in time, $t = 0$
    - $y(x, 0) = A\cos(kx)$
  - Could add a phase angle ϕ inside the $\cos()$ if needed
    - $y(x, t) = A\cos(kx \pm \omega t + \phi)$
- Intensity (I)
  - $I = \frac{P}{A}$
  - Can be total power divided by the total area spread over
  - Can be power delivered divided by the area of a detector
- Sound intensity level (β)
  - $\beta = (10\text{ dB})\log\left(\frac{I}{I_0}\right)$
  - $I_0 = 10^{-12}\ \frac{\mathrm{W}}{\mathrm{m}^2}$ (threshold of hearing)
  - $\beta_2 - \beta_1 = (10\text{ dB})\log\left(\frac{I_2}{I_1}\right)$
- Doppler effect for sound
  - $f_o = f_s\frac{v \pm v_o}{v \mp v_s}$
  - v: speed of sound
  - o: observer
  - s: source
  - Use the upper sign if the observer/source is moving towards the other
  - Use the lower sign if the observer/source is moving away from the other

Fluids

- $\rho = \frac{m}{V}$ (density)
- $P = \frac{F}{A}$ (pressure)
- $P = P_0 + \rho g h$ (pressure at a depth in a fluid)
- Buoyant force (B or F_B)
  - $B = \rho_{\text{fluid}} V_{\text{displaced}} g$
  - Points opposite $\vec{g}$