

PHYS 2AG Equations

Having the equations is a start. The next step is understanding what the equations mean, when to use them, and how to use them.

SI Prefixes

Name	Symbol	Factor
Quetta	Q	10^{30}
Ronna	R	10^{27}
Yotta	Y	10^{24}
Zetta	Z	10^{21}
Exa	E	10^{18}
Peta	P	10^{15}
Tera	T	10^{12}
Giga	G	10^9
Mega	M	10^6
Kilo	k	10^3
Hecto	h	10^2
Deka	da	10^1
-	-	10^0
Deci	d	10^{-1}
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Femto	f	10^{-15}
Atto	a	10^{-18}
Zepto	z	10^{-21}
Yocto	y	10^{-24}
Ronto	r	10^{-27}
Quecto	q	10^{-30}

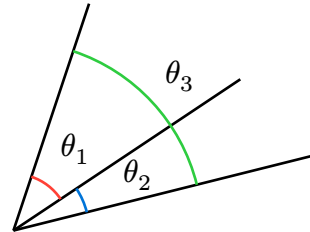
Constants

- $g = 9.8 \frac{\text{m}}{\text{s}^2}$
- $G = 6.674 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$
- $M_{\text{Earth}} = 5.97 \times 10^{24} \text{ kg}$
- $R_{\text{Earth}} = 6.38 \times 10^6 \text{ m}$
- $1 \text{ rev} = 2\pi \text{ rad} = 360^\circ$

Geometry

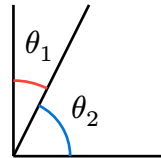
- Angle addition postulate

▸ $\theta_1 + \theta_2 = \theta_3$



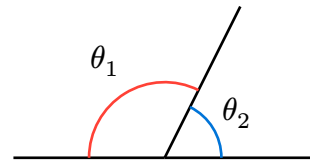
- Complementary angles

▸ $\theta_1 + \theta_2 = 90^\circ$

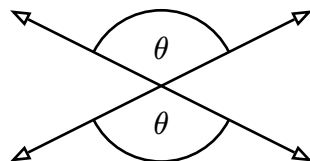


- Supplementary angles

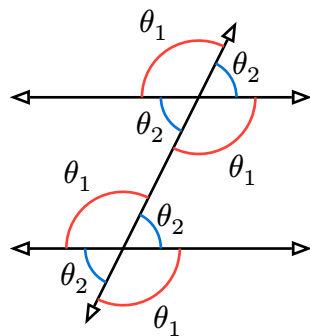
▸ $\theta_1 + \theta_2 = 180^\circ$



- Vertical angles

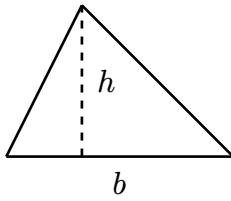


- Parallel lines cut by transversal



- Triangles

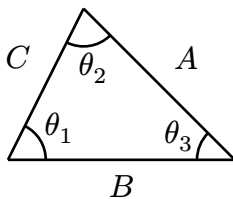
- $A = \frac{1}{2}bh$



- $\theta_1 + \theta_2 + \theta_3 = 180^\circ$

- $\frac{A}{\sin(\theta_1)} = \frac{B}{\sin(\theta_2)} = \frac{C}{\sin(\theta_3)}$ (law of sines)

- $C^2 = A^2 + B^2 - 2AB \cos(\theta_3)$ (law of cosines)



- Right triangles

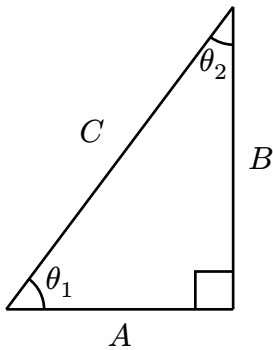
- $A^2 + B^2 = C^2$

- $\theta_1 + \theta_2 = 90^\circ$

- $\sin(\theta_1) = \cos(\theta_2) = \frac{B}{C}$

- $\cos(\theta_1) = \sin(\theta_2) = \frac{A}{C}$

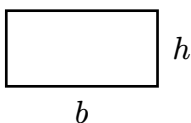
- $\tan(\theta_1) = \frac{1}{\tan(\theta_2)} = \frac{B}{A}$



- Rectangles

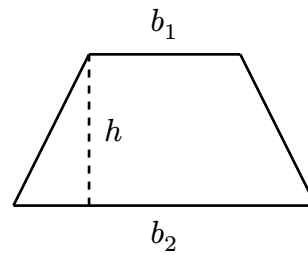
- $A = bh$

- $P = 2b + 2h$



- Trapezoid

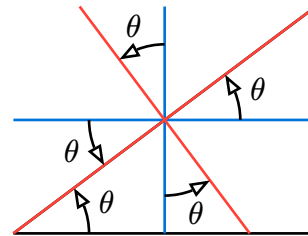
- $A = \frac{1}{2}(b_1 + b_2)h$



- Inclined plane

- Inclined planes are typically described by their angle above horizontal

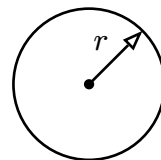
- The tilted coordinate axes are rotated that angle relative to the standard coordinate axes



- Circles

- $C = 2\pi r$

- $A = \pi r^2$



- Spheres

- $A = 4\pi r^2$

- $V = \frac{4}{3}\pi r^3$

Algebra

- Quadratic formula

- $ax^2 + bx + c = 0$

- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- Properties of logarithms

- $\log_b(a) = p \iff b^p = a$

- $\log_b(ac) = \log_b(a) + \log_b(c)$

- $\log_b\left(\frac{a}{c}\right) = \log_b(a) - \log_b(c)$

- $\log_b(a^p) = p \log_b(a)$
- Sum and difference trig identities
 - $\sin(A \pm B) = \sin(A) \cos(B) \pm \cos(A) \sin(B)$
 - $\cos(A \pm B) = \cos(A) \cos(B) \mp \sin(A) \sin(B)$

Kinematics

- Note: x can be replaced with y depending on the direction being analyzed
- $v_{\text{avg}} = \frac{d}{\Delta t}$ (average speed)
- $v_{\text{avg},x} = \frac{\Delta x}{\Delta t}$ (average velocity, x-component)
- $a_{\text{avg},x} = \frac{\Delta v_x}{\Delta t}$
- If a = constant:
 - $v_x = v_{0x} + a_x t$
 - $x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$
 - $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$
- Knows and Unknowns
 - Point 1 → Point 2

x_0	y_0
x	y
v_{0x}	v_{0y}
v_x	v_y
a_x	a_y
	t

Forces

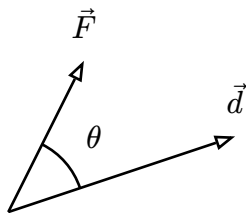
- Newton's laws of motion
 1. $\vec{a} = 0 \iff \Sigma \vec{F} = 0$
 2. $\Sigma \vec{F} = m\vec{a}$
 3. $\vec{F}_{b \text{ on } a} = -\vec{F}_{a \text{ on } b}$
- Gravitational force (F_g)
 - $F_g = mg$
 - $F_g = G \frac{m_1 m_2}{r^2}$
 - Points straight down towards the center of the Earth

- Normal force (n , N , or F_n)
 - No general formula
 - Pushes perpendicular to surfaces in contact
- Applied force (F_{app})
 - Push or pull in a specified direction
- Tension (T or F_T)
 - No general formula
 - Pulls along rope/chain/cable
- Kinetic friction (f_k)
 - $f_k = \mu_k n$
 - Acts opposite relative motion of surfaces
- Static friction (f_s)
 - $f_s \leq \mu_s n$
 - $f_{s,\text{max}} = \mu_s n$
 - Direction and magnitude change based on the situation
 - Acts when there is no relative motion between surfaces
 - Tries to stop relative motion between surfaces
- Spring force (F_s)
 - $F_s = ks$
 - Points in the direction opposite the squish or stretch
- Pivot force (P)
 - No general formula
 - Usually broken into P_x and P_y
 - Force from pin that keeps attached object from translating away
- Free body diagram (FBD) reminders
 - Identify your system
 - Identify the direction of the acceleration

- Identify the direction of the velocity
- Choose a coordinate system
- Draw all external forces acting on the system
- Note the angle for any forces acting at an angle relative to the chosen axes
- Centripetal acceleration (a_c)
 - $a_c = \frac{v^2}{r} = r\omega^2$
 - Points towards the center of the circular path

Work and Energy

- Work (W)
 - Energy transferred via forces
 - $W = Fd \cos(\theta)$



- External heat (Q_{ext})
 - Thermal energy transferred from the environment to/from the system
- Power (P)
 - $P = \frac{\Delta E}{\Delta T}$
- Kinetic energy (K)
 - Energy of motion
 - $K = \frac{1}{2}mv^2$
- Kinetic energy (K_{rot})
 - Energy of rotational motion
 - $K = \frac{1}{2}I\omega^2$
- Gravitational potential energy (U_g)
 - Energy due to position in a gravitational field
 - $U_g = mgh$
 - Make sure to choose a reference level

- Elastic potential energy (U_s)
 - Energy due to squishing/stretching a spring from its natural length
 - $U_g = \frac{1}{2}ks^2$
- Mechanical energy (E_{mech})
 - Kinetic energy plus potential energy
 - $E_{\text{mech}} = K + U$
- Internal energy (U_{int})
 - $\Delta U_{\text{th}} = mc\Delta T$ (due to temperature)
 - $\Delta U_p = \pm mL$ (due to phase change)
 - All the substance must reach the phase change temperature before the transition occurs
 - Temperature remains constant until all the substance has undergone the phase change
- Sound energy (U_{sound})
 - Energy of sound waves
- Conservation of energy
 - $E_i + W_{\text{nc}} + Q_{\text{ext}} = E_f$
 - $K_i + U_{gi} + U_{si} + \dots + W_{\text{nc}} + Q_{\text{ext}} = K_f + U_{gf} + U_{sf} + \dots + \Delta U_{\text{int}}$

Momentum

- $\vec{p} = m\vec{v}$ (momentum)
- $\vec{J} = \vec{F}_{\text{net,avg}}\Delta t = \Delta\vec{p}$ (impulse-momentum theorem)
- Conservation of momentum
 - If $\vec{J} = 0$, then $p_i = p_f$
 - Applies to pretty much all collisions & explosions
- Collision types
 - Perfectly inelastic
 - “Sticky”
 - $v_{1f} = v_{2f} = \dots = v_f$
 - Inelastic

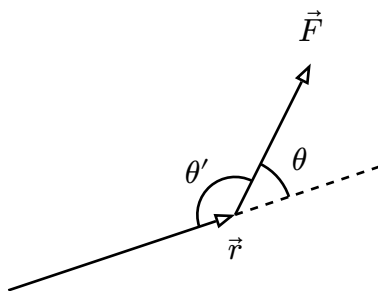
- In between perfectly inelastic and perfectly elastic
- ▶ Perfectly elastic
 - “Super Bouncy”
 - $K_i = K_f$

Rotational Kinematics

- Switching between linear (tangential) and angular quantities
 - ▶ Angles must be in radians
 - ▶ $s = r\theta$
 - ▶ $v_t = r\omega$
 - ▶ $a_t = r\alpha$
- $\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t}$
- $\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t}$
- If $\alpha = \text{constant}$:
 - ▶ $\omega = \omega_0 + \alpha t$
 - ▶ $x = x_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
 - ▶ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Rotational Dynamics

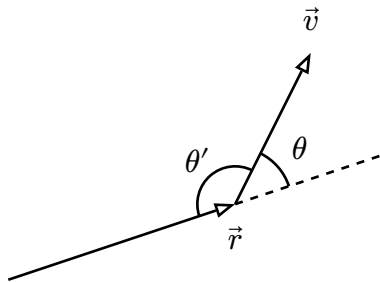
- Torque (τ)
 - ▶ $\tau = \pm Fr \sin(\theta)$
 - ▶ $\sin(\theta) = \sin(\theta')$, so either angle can be used



- $\Sigma\tau = I\alpha$ (Newton's second law rotational)
- $I = \Sigma mr^2$

- ▶ Moment of inertia formulas are derived by breaking objects up into pieces, then adding up mr^2 for each piece
- ▶ Implies that moments of inertia can add together
- $I = I_{\text{cm}} + MD^2$ (parallel axis theorem)
- When rolling without slipping, $v_{\text{cm}} = r\omega$
- Rigid body diagram (RBD) reminders
 - ▶ Identify your system
 - ▶ Identify the direction of the acceleration or angular acceleration
 - ▶ Identify the direction of the velocity or angular velocity
 - ▶ Choose a coordinate system
 - ▶ Choose an axis of rotation (if it's not rotating)
 - ▶ Draw all external forces acting on the system at the location where they are acting
 - ▶ Note the angle for any forces acting at an angle relative to the chosen axes, or relative to a vector pointing from the pivot to where the force acts
 - ▶ Note how far each force is acting from the axis of rotation
- Effective mass (m_{eff})
 - ▶ $m_{\text{eff}} = \frac{I}{r^2}$
 - ▶ The effective mass of a pulley when analyzing a pulley system as a whole
 - ▶ Contributes to the total system mass
 - ▶ I is the moment of inertia of the pulley about its axle
 - ▶ r is the distance from the axle to where the string is attached to the pulley
- Angular momentum (L)

- $L = mvr \sin(\theta)$ (point particle)
- $\sin(\theta) = \sin(\theta')$, so either angle can be used

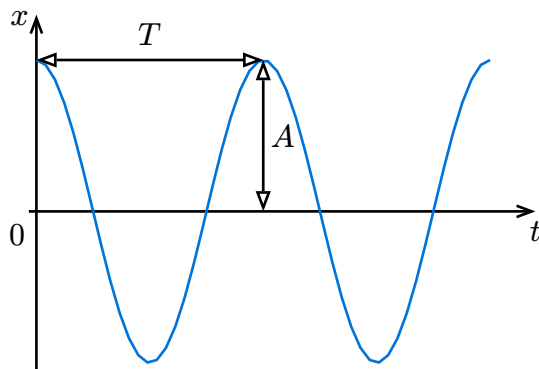


- $L = I\omega$ (not universal, but works for most cases in the class)
- $\tau_{\text{net,avg}} \Delta t = \Delta L$

Static Equilibrium

- If $\vec{a} = 0$ and $\alpha = 0$:
 - $\Sigma F_x = 0$
 - $\Sigma F_y = 0$
 - $\Sigma \tau = 0$
- Since it is not rotating, any axis of rotation can be chosen for the purpose of calculating torques

Simple Harmonic Motion



- Oscillation terms
 - Period (T)
 - Frequency (f)
 - Angular frequency (ω)
 - $T = \frac{1}{f}$ or $f = \frac{1}{T}$
 - $\omega = \frac{2\pi}{T} = 2\pi f$

- Amplitude (A)

- Mass on a spring

- $\omega = \sqrt{\frac{k}{m}}$

- Simple pendulum

- $\omega = \sqrt{\frac{g}{L}}$

- Assumes $\theta < \sim 15^\circ$ so that $\theta \approx \sin(\theta)$

- Kinematics

- $x(t) = A \cos(\omega t + \phi)$

- $x_{\text{max}} = A$

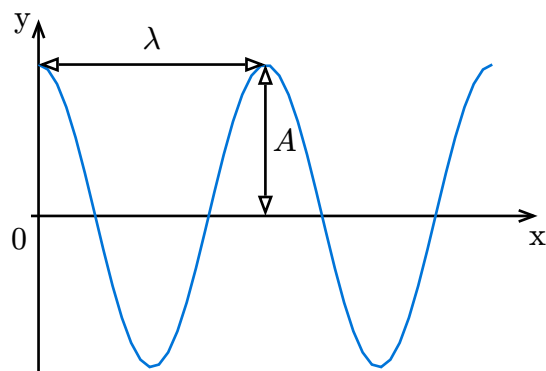
- $v(t) = -A\omega \sin(\omega t + \phi)$

- $v_{\text{max}} = A\omega$

- $a(t) = -A\omega^2 \cos(\omega t + \phi)$

- $a_{\text{max}} = A\omega^2$

Waves



- $v = f\lambda$
 - Wave speed depends on properties of the medium, not on wavelength and frequency
 - $v = \sqrt{\frac{F_T}{\mu}}$ (speed of wave in a string)
 - $v = \left(331.5 \frac{\text{m}}{\text{s}}\right) + \left(0.6 \frac{\text{m/s}}{^\circ\text{C}}\right)T$ (speed of sound in air)

- Wave function

- $y(x, t) = A \cos(kx \pm \omega t)$

- Plus when moving in the $-x$ direction

- Minus when moving in the $+x$ direction

- Focus on a single point, $x = 0$, over time
 - $y(0, t) = A \cos(\pm \omega t)$
- Focus on the shape of the wave at a single point in time, $t = 0$
 - $y(x, 0) = A \cos(kx)$
- Could add a phase angle ϕ inside the $\cos()$ if needed
 - $y(x, t) = A \cos(kx \pm \omega t + \phi)$
- Intensity (I)
 - $I = \frac{P}{A}$
 - Can be total power divided by the total area spread over
 - Can be power delivered divided by the area of a detector
- Sound intensity level (β)
 - $\beta = (10 \text{ dB}) \log\left(\frac{I}{I_0}\right)$
 - $I_0 = 10^{-12} \frac{\text{W}}{\text{m}^2}$ (threshold of hearing)
 - $\beta_2 - \beta_1 = (10 \text{ dB}) \log\left(\frac{I_2}{I_1}\right)$
- Doppler effect for sound
 - $f_o = f_s \frac{v \pm v_o}{v \mp v_s}$
 - v : speed of sound
 - o : observer
 - s : source
 - Use the upper sign if the observer/source is moving towards the other
 - Use the lower sign if the observer/source is moving away from the other

Fluids

- $\rho = \frac{m}{V}$ (density)
- $P = \frac{F}{A}$ (pressure)
- $P = P_0 + \rho gh$ (pressure at a depth in a fluid)
- Buoyant force (B or F_B)
 - $B = \rho_{\text{fluid}} V_{\text{displaced}} g$