# **PHYS 2AG Equations**

Having the equations is a start. The next step is understanding what the equations mean, when to use them, and how to use them.

### **SI Prefixes**

Name	Symbol	Factor
Quetta	Q	$10^{30}$
Ronna	R	$10^{27}$
Yotta	Y	$10^{24}$
Zetta	Z	$10^{21}$
Exa	E	$10^{18}$
Peta	P	$10^{15}$
Tera	T	$10^{12}$
Giga	G	$10^{9}$
Mega	M	$10^{6}$
Kilo	k	$10^{3}$
Hecto	h	$10^{2}$
Deka	da	$10^{1}$
-	-	$10^{0}$
Deci	d	$10^{-1}$
Centi	c	$10^{-2}$
Milli	m	$10^{-3}$
Micro	μ	$10^{-6}$
Nano	n	$10^{-9}$
Pico	p	$10^{-12}$
Femto	f	$10^{-15}$
Atto	a	$10^{-18}$
Zepto	Z	$10^{-21}$
Yocto	у	$10^{-24}$
Ronto	r	$10^{-27}$
Quecto	q	$10^{-30}$

#### **Constants**

• 
$$g = 9.8 \frac{\text{m}}{\text{s}^2}$$

• 
$$G = 6.674 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

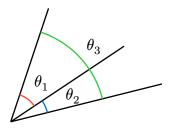
• 
$$M_{\rm Earth}=5.97\times 10^{24}\,{\rm kg}$$

• 
$$R_{\rm Earth} = 6.38 \times 10^6 \ {\rm m}$$

• 1 rev = 
$$2\pi \text{ rad} = 360^{\circ}$$

### **Geometry**

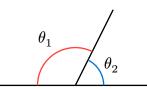
- Angle addition postulate
  - $\bullet \ \theta_1 + \theta_2 = \theta_3$



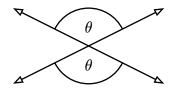
- Complementary angles
  - $\bullet \ \theta_1 + \theta_2 = 90^\circ$



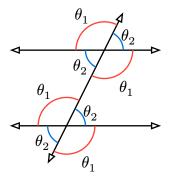
- Supplementary angles
  - $\bullet \ \theta_1 + \theta_2 = 180^{\circ}$



Vertical angles

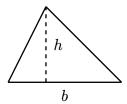


• Parallel lines cut by transversal



• Triangles

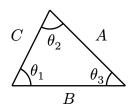
$$A = \frac{1}{2}bh$$



$$\bullet \ \theta_1 + \theta_2 + \theta_3 = 180^{\circ}$$

• 
$$\frac{A}{\sin(\theta_1)} = \frac{B}{\sin(\theta_2)} = \frac{C}{\sin(\theta_3)}$$
 (law of sines)

• 
$$C^2 = A^2 + B^2 - 2AB\cos(\theta_3)$$
 (law of cosines)



• Right triangles

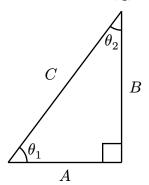
$$A^2 + B^2 = C^2$$

$$\bullet \ \theta_1 + \theta_2 = 90^{\circ}$$

$$\cdot \sin(\theta_1) = \cos(\theta_2) = \frac{B}{C}$$

$$\quad \bullet \ \cos(\theta_1) = \sin(\theta_2) = \tfrac{A}{C}$$

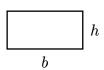
• 
$$\tan(\theta_1) = \frac{1}{\tan(\theta_2)} = \frac{B}{A}$$



Rectangles

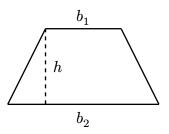
• 
$$A = bh$$

$$P = 2b + 2h$$

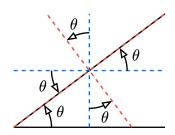


• Trapezoid

$$\quad \bullet \ A = \tfrac{1}{2}(b_1 + b_2)h$$



- Inclined plane
  - Inclined planes are typically described by their angle above horizontal
  - The tilted coordinate axes are rotated that angle relative to the standard coordinate axes



- Circles
  - $C = 2\pi r$
  - $A = \pi r^2$



- Spheres
  - $\quad \bullet \ A = 4\pi r^2$
  - $V = \frac{4}{3}\pi r^3$

## Algebra

· Quadratic formula

$$ax^2 + bx + c = 0$$

- $x = \frac{-b \pm \sqrt{b^2 4ac}}{2a}$
- Properties of logarithms

$$\log_b(a) = p \iff b^p = a$$

$$\log_b(ac) = \log_b(a) + \log_b(c)$$

• 
$$\log_b\left(\frac{a}{c}\right) = \log_b(a) - \log_b(c)$$

- $\log_b(a^p) = p \log_b(a)$
- Sum and difference trig identities
  - $\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$
  - $cos(A \pm B) = cos(A) cos(B) \mp sin(A) sin(B)$

### **Kinematics**

- Note: x can be replaced with y depending on the direction being analyzed
- $v_{\text{avg}} = \frac{d}{\Delta t}$  (average speed)
- $v_{\mathrm{avg,x}} = \frac{\Delta x}{\Delta t}$  (average velocity, x-component)
- $a_{\text{avg,x}} = \frac{\Delta v_x}{\Delta t}$
- If a = constant:
  - $v_x = v_{0x} + a_x t$
  - $x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$
  - $v_x^2 = v_{0x}^2 + 2a_x(x x_0)$
- Knows and Unknowns
  - Point  $1 \rightarrow Point 2$

$$egin{array}{cccc} x_0 & y_0 & y_0$$

#### **Forces**

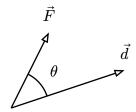
- Newton's laws of motion
  - 1.  $\vec{a} = 0 \iff \Sigma \vec{F} = 0$
  - 2.  $\Sigma \vec{F} = m\vec{a}$
  - 3.  $\vec{F}_{b \text{ on } a} = -\vec{F}_{a \text{ on } b}$
- Gravitational force  $(F_q)$ 
  - $F_q = mg$
  - $F_q = G \frac{m_1 m_2}{r^2}$
  - Points straight down towards the center of the Earth

- Normal force  $(n, N, \text{ or } F_n)$ 
  - No general formula
  - Pushes perpendicular to surfaces in contact
- Applied force  $(F_{app})$ 
  - ▶ Push or pull in a specified direction
- Tension (T or  $F_T$ )
  - No general formula
  - ▶ Pulls along rope/chain/cable
- Kinetic friction  $(f_k)$ 
  - $f_k = \mu_k n$
  - Acts opposite relative motion of surfaces
- Static friction  $(f_s)$ 
  - $f_s \leq \mu_s n$
  - $f_{s,\max} = \mu_s n$
  - Direction and magnitude change based on the situation
  - Acts when there is no relative motion between surfaces
  - Tries to stop relative motion between surfaces
- Spring force  $(F_s)$ 
  - $F_s = ks$
  - Points in the direction opposite the squish or stretch
- Pivot force (P)
  - No general formula
  - Usually broken into  $P_x$  and  $P_y$
  - Force from pin that keeps attached object from translating away
- Free body diagram (FBD) reminders
  - Identify your system
  - Identify the direction of the acceleration

- Identify the direction of the velocity
- ▶ Choose a coordinate system
- Draw all external forces acting on the system
- Note the angle for any forces acting at an angle relative to the chosen axes
- Centripetal acceleration  $(a_c)$ 
  - $\quad \bullet \ a_c = \tfrac{v^2}{r} = r \omega^2$
  - Points towards the center of the circular path

## Work and Energy

- Work (*W*)
  - Energy transferred via forces
  - $W = Fd\cos(\theta)$



- External heat  $(Q_{\text{ext}})$ 
  - Thermal energy transferred from the environment to/from the system
- Power (*P*)
  - $P = \frac{\Delta E}{\Delta T}$
- Kinetic energy (K)
  - ► Energy of motion
  - $K = \frac{1}{2}mv^2$
- Kinetic energy  $(K_{\mathrm{rot}})$ 
  - ► Energy of rotational motion
  - $K = \frac{1}{2}I\omega^2$
- Gravitational potential energy  $(U_a)$ 
  - Energy due to position in a gravitational field
  - $\bullet \ U_g = mgh$
  - ▶ Make sure to choose a reference level

- Elastic potential energy  $(U_s)$ 
  - Energy due to squishing/stretching a spring from its natural length
  - $U_q = \frac{1}{2}ks^2$
- Mechanical energy  $(E_{\mathrm{mech}})$ 
  - Kinetic energy plus potential energy
  - $E_{\text{mech}} = K + U$
- Internal energy  $(U_{int})$ 
  - $\Delta U_{\rm th} = mc\Delta T$  (due to temperature)
  - $\Delta U_p = \pm mL$  (due to phase change)
    - All the substance must reach the phase change temperature before the transition occurs
    - Temperature remains constant until all the substance has undergone the phase change
- Sound energy ( $U_{\rm sound}$ )
  - Energy of sound waves
- Conservation of energy
  - $E_i + W_{nc} + Q_{ext} = E_f$
  - $\begin{array}{c} {\color{red} \blacktriangleright} \; K_i + U_{gi} + U_{si} + \ldots + W_{\rm nc} + Q_{\rm ext} = \\ \\ K_f + U_{gf} + U_{sf} + \ldots + \Delta U_{\rm int} \end{array}$

#### Momentum

- $\vec{p} = m\vec{v}$  (momentum)
- $ec{J}=ec{F}_{
  m net,avg}\Delta t=\Deltaec{p}$  (impulsemomentum theorem)
- Conservation of momentum
  - If  $\vec{J} = 0$ , then  $p_i = p_f$
  - Applies to pretty much all collisions & explosions
- Collision types
  - Perfectly inelastic
    - "Sticky"
    - $-v_{1f} = v_{2f} = \dots = v_f$
  - ▶ Inelastic

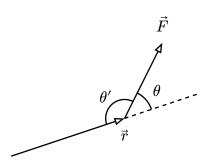
- In between perfectly inelastic and perfectly elastic
- Perfectly elastic
  - "Super Bouncy"
  - $K_i = K_f$

### **Rotational Kinematics**

- Switching between linear (tangential) and angular quantities
  - ► Angles must be in radians
  - $s = r\theta$
  - $v_t = r\omega$
  - $a_t = r\alpha$
- $\omega_{\text{avg}} = \frac{\Delta \theta}{\Delta t}$
- $\alpha_{\text{avg}} = \frac{\Delta \omega}{\Delta t}$
- If  $\alpha = \text{constant}$ :
  - $\omega = \omega_0 + \alpha t$
  - $x = x_0 + \omega_0 t + \frac{1}{2} \alpha t^2$
  - $\bullet \ \omega^2 = \omega_0^2 + 2\alpha(\theta \theta_0)$

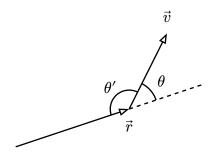
## **Rotational Dynamics**

- Torque  $(\tau)$ 
  - $\quad \boldsymbol{\tau} = \pm Fr\sin(\theta)$
  - $\sin(\theta) = \sin(\theta')$ , so either angle can be used



- $\Sigma \tau = I \alpha$  (Newton's second law rotational)
- $I = \Sigma mr^2$

- Moment of inertia formulas are derived by breaking objects up into pieces, then adding up  $mr^2$  for each piece
- Implies that moments of inertia can add together
- +  $I=I_{\mathrm{cm}}+MD^2$  (parallel axis theorem)
- When rolling without slipping,  $v_{\mathrm{cm}} = r \omega$
- Rigid body diagram (RBD) reminders
  - ▶ Identify your system
  - Identify the direction of the acceleration or angular acceleration
  - Identify the direction of the velocity or angular velocity
  - Choose a coordinate system
  - Choose an axis of rotation (if it's not rotating)
  - Draw all external forces acting on the system at the location where they are acting
  - Note the angle for any forces acting at an angle relative to the chosen axes, or relative to a vector pointing from the pivot to where the force acts
  - Note how far each force is acting from the axis of rotation
- Angular momentum (*L*)
  - $L = mvr \sin(\theta)$  (point particle)
  - $\sin(\theta) = \sin(\theta')$ , so either angle can be used

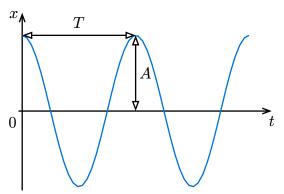


- $L = I\omega$  (not universal, but works for most cases in the class)
- $au_{
  m net,avg} \Delta t = \Delta L$

## Static Equilibrium

- If  $\vec{a} = 0$  and  $\alpha = 0$ :
  - $\Sigma F_x = 0$
  - $\Sigma F_y = 0$
  - $\Sigma \tau = 0$
- Since it is not rotating, any axis of rotation can be chosen for the purpose of calculating torques

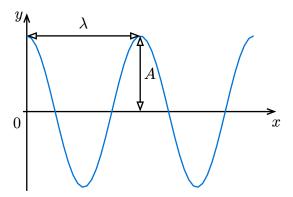
## **Simple Harmonic Motion**



- Oscillation terms
  - ightharpoonup Period (T)
  - Frequency (f)
  - Angular frequency  $(\omega)$
  - $T = \frac{1}{f}$  or  $f = \frac{1}{T}$
  - $\bullet \ \omega = \tfrac{2\pi}{T} = 2\pi f$
  - ightharpoonup Amplitude (A)
- Mass on a spring
  - $\omega = \sqrt{\frac{k}{m}}$
- Simple pendulum
  - $\omega = \sqrt{\frac{g}{L}}$
  - Assumes  $\theta < \sim 15^\circ$  so that  $\theta \approx \sin(\theta)$
- Kinematics
  - $x(t) = A\cos(\omega t + \phi)$
  - $x_{\text{max}} = A$

- $v(t) = -A\omega\sin(\omega t + \phi)$
- $v_{\mathrm{max}} = A\omega$
- $a(t) = -A\omega^2 \cos(\omega t + \phi)$
- $a_{\rm max} = A\omega^2$

### Waves



- $v = f\lambda$ 
  - Wave speed depends on properties of the medium, not on wavelength and frequency
  - $v = \sqrt{\frac{F_T}{\mu}}$  (speed of wave in a string)
  - $v = \left(331.5 \, \frac{\text{m}}{\text{s}}\right) + \left(0.6 \, \frac{\text{m/s}}{\text{°C}}\right) T$  (speed of sound in air)
- Wave function
  - $y(x,t) = A\cos(kx \pm \omega t)$ 
    - Plus when moving in the -x direction
    - Minus when moving in the +x direction
  - Focus on a single point, x = 0, over time
    - $y(0,t) = A\cos(\pm \omega t)$
  - Focus on the shape of the wave at a single point in time, t = 0
    - $-y(x,0) = A\cos(kx)$
  - Could add a phase angle φ inside the cos() if needed
    - $y(x,t) = A\cos(kx \pm \omega t + \phi)$

- Intensity (I)
  - $I = \frac{P}{A}$
  - Can be total power divided by the total area spread over
  - Can be power delivered divided by the area of a detector
- Sound intensity level ( $\beta$ )
  - $\beta = (10 \text{ dB}) \log \left(\frac{I}{I_0}\right)$
  - $I_0 = 10^{-12} \frac{\text{W}}{\text{m}^2}$  (threshold of hearing)
  - +  $\beta_2 \beta_1 = (10 \text{ dB}) \log \left(\frac{I_2}{I_1}\right)$
- Doppler effect for sound
  - $f_o = f_s \frac{v \pm v_o}{v \mp v_s}$
  - ► v: speed of sound
  - *▶ o*: observer
  - ► s: source
  - Use the upper sign if the observer/ source is moving towards the other
  - Use the lower sign if the observer/ source is moving away from the other

#### **Fluids**

- $\rho = \frac{m}{V}$  (density)
- $P = \frac{F}{A}$  (pressure)
- $P=P_0+\rho gh$  (pressure at a depth in a fluid)
- Buoyant force (B or  $F_B$ )
  - $B = \rho_{\mathrm{fluid}} V_{\mathrm{displaced}} g$
  - Points opposite  $\vec{g}$