

# PHYS 2AG Equations

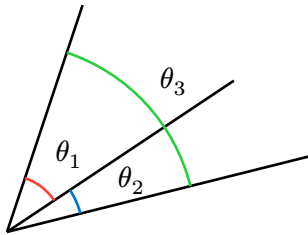
Having the equations is a start. The next step is understanding what the equations mean, when to use them, and how to use them.

## Constants

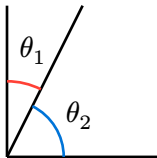
- $g = 9.8 \frac{\text{m}}{\text{s}^2}$
- $G = 6.674 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$

## Geometry

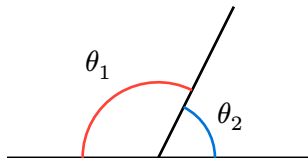
- Angle addition postulate
  - $\theta_1 + \theta_2 = \theta_3$



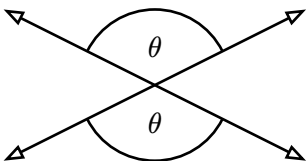
- Complementary angles
  - $\theta_1 + \theta_2 = 90^\circ$



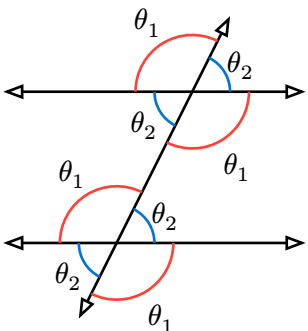
- Supplementary angles
  - $\theta_1 + \theta_2 = 180^\circ$



- Vertical angles

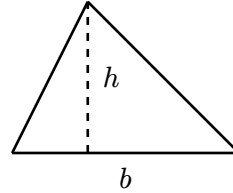


- Parallel lines cut by transversal

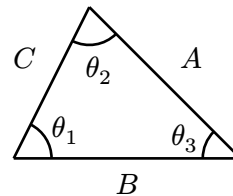


- Triangles

▸  $A = \frac{1}{2}bh$

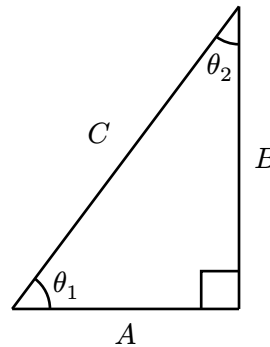


- $\theta_1 + \theta_2 + \theta_3 = 180^\circ$
- $\frac{A}{\sin(\theta_1)} = \frac{B}{\sin(\theta_2)} = \frac{C}{\sin(\theta_3)}$  (law of sines)
- $C^2 = A^2 + B^2 - 2AB \cos(\theta_3)$  (law of cosines)



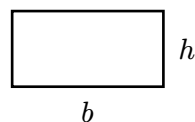
- Right triangles

- $A^2 + B^2 = C^2$
- $\theta_1 + \theta_2 = 90^\circ$
- $\sin(\theta_1) = \cos(\theta_2) = \frac{B}{C}$
- $\cos(\theta_1) = \sin(\theta_2) = \frac{A}{C}$
- $\tan(\theta_1) = \frac{1}{\tan(\theta_2)} = \frac{B}{A}$



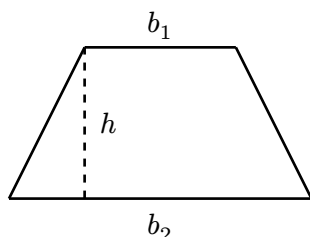
- Rectangles

- $A = bh$
- $P = 2b + 2h$

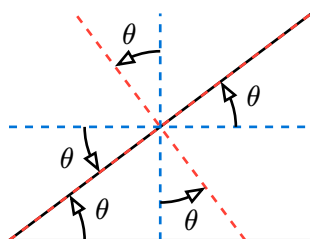


- Trapezoid

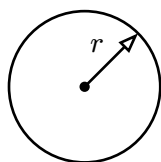
▸  $A = \frac{1}{2}(b_1 + b_2)h$



- Inclined plane
  - Inclined planes are typically described by their angle above horizontal
  - The tilted coordinate axes are rotated that angle relative to the standard coordinate axes



- Circles
  - $C = 2\pi r$
  - $A = \pi r^2$



- Spheres
  - $A = 4\pi r^2$
  - $V = \frac{4}{3}\pi r^3$

## Algebra

- Quadratic formula
  - $ax^2 + bx + c = 0$
  - $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Properties of logarithms
  - $\log_b(a) = p \iff b^p = a$
  - $\log_b(ac) = \log_b(a) + \log_b(c)$
  - $\log_b\left(\frac{a}{c}\right) = \log_b(a) - \log_b(c)$
  - $\log_b(a^p) = p \log_b(a)$
- Sum and difference trig identities
  - $\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$
  - $\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$

## Kinematics

- Note:  $x$  can be replaced with  $y$  depending on the direction being analyzed
- $v_{\text{avg}} = \frac{d}{\Delta t}$  (average speed)
- $v_{\text{avg},x} = \frac{\Delta x}{\Delta t}$  (average velocity, x-component)
- $a_{\text{avg},x} = \frac{\Delta v_x}{\Delta t}$
- If  $a = \text{constant}$ :
  - $v_x = v_{0x} + a_x t$
  - $x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$
  - $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$
- Knows and Unknowns
 

Point 1 $\rightarrow$ Point 2	
$x_0$	$y_0$
$x$	$y$
$v_{0x}$	$v_{0y}$
$v_x$	$v_y$
$a_x$	$a_y$
$t$	

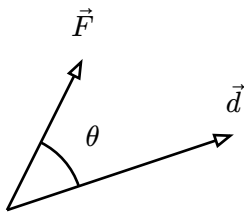
## Forces

- Newton's laws of motion
  1.  $\vec{a} = 0 \iff \Sigma \vec{F} = 0$
  2.  $\Sigma \vec{F} = m\vec{a}$
  3.  $\vec{F}_{b \text{ on } a} = -\vec{F}_{a \text{ on } b}$
- Gravitational force ( $F_g$ )
  - $F_g = mg$
  - $F_g = G \frac{m_1 m_2}{r^2}$
  - Points straight down towards the center of the Earth
- Normal force ( $n$ ,  $N$ , or  $F_n$ )
  - No general formula
  - Pushes perpendicular to surfaces in contact
- Applied force ( $F_{\text{app}}$ )
  - Push or pull in a specified direction
- Tension ( $T$  or  $F_T$ )
  - No general formula
  - Pulls along rope/chain/cable
- Kinetic friction ( $f_k$ )
  - $f_k = \mu_k n$
  - Acts opposite relative motion of surfaces
- Static friction ( $f_s$ )
  - $f_s \leq \mu_s n$
  - $f_{s,\text{max}} = \mu_s n$
  - Direction and magnitude change based on the situation
  - Acts when there is no relative motion between surfaces

- Tries to stop relative motion between surfaces
- Spring force ( $F_s$ )
  - $F_s = ks$
  - Points in the direction opposite the squish or stretch
- Pivot force ( $P$ )
  - No general formula
  - Usually broken into  $P_x$  and  $P_y$
  - Force from pin that keeps attached object from translating away
- Free body diagram (FBD) reminders
  - Identify your system
  - Identify the direction of the acceleration
  - Identify the direction of the velocity
  - Choose a coordinate system
  - Draw all external forces acting on the system
  - Note the angle for any forces acting at an angle relative to the chosen axes
- Centripetal acceleration ( $a_c$ )
  - $a_c = \frac{v^2}{r} = r\omega^2$
  - Points towards the center of the circular path
- Energy due to position in a gravitational field
  - $U_g = mgh$
  - Make sure to choose a reference level
- Elastic potential energy ( $U_s$ )
  - Energy due to squishing/stretching a spring from its natural length
  - $U_g = \frac{1}{2}ks^2$
- Internal energy ( $U_{\text{int}}$ )
  - $\Delta U_{\text{th}} = mc\Delta T$  (due to temperature)
  - $\Delta U_p = \pm mL$  (due to phase change)
    - All the substance must reach the phase change temperature before the transition occurs
    - Temperature remains constant until all the substance has undergone the phase change
- Sound energy ( $U_{\text{sound}}$ )
  - Energy of sound waves
- Conservation of energy
  - $E_i + W_{\text{nc}} + Q_{\text{ext}} = E_f$
  - $K_i + U_{gi} + U_{si} + \dots + W_{\text{nc}} + Q_{\text{ext}} = K_f + U_{gf} + U_{sf} + \dots + \Delta U_{\text{int}}$

## Work and Energy

- Work ( $W$ )
  - Energy transferred via forces
  - $W = Fd \cos(\theta)$



- External heat ( $Q_{\text{ext}}$ )
  - Thermal energy transferred from the environment to/from the system
- Power ( $P$ )
  - $P = \frac{\Delta E}{\Delta T}$
- Kinetic energy ( $K$ )
  - Energy of motion
  - $K = \frac{1}{2}mv^2$
- Kinetic energy ( $K_{\text{rot}}$ )
  - Energy of rotational motion
  - $K = \frac{1}{2}I\omega^2$
- Gravitational potential energy ( $U_g$ )

## Momentum

- $\vec{p} = m\vec{v}$  (momentum)
- $\vec{J} = \vec{F}_{\text{net,avg}}\Delta t = \Delta\vec{p}$  (impulse-momentum theorem)
- Conservation of momentum
  - If  $\vec{F}_{\text{net}} = 0$ , then  $p_i = p_f$
  - Applies to pretty much all collisions & explosions
- Collision types
  - Perfectly inelastic
    - “Sticky”
    - $v_{1f} = v_{2f} = \dots = v_f$
  - Inelastic
    - In between perfectly inelastic and perfectly elastic
  - Perfectly elastic
    - “Bouncy”
    - $K_i = K_f$

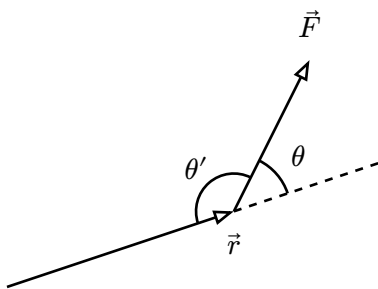
## Rotational Kinematics

- Switching between linear (tangential) and angular quantities
  - Angles must be in radians
  - $s = r\theta$
  - $v_t = r\omega$

- $a_t = r\alpha$
- $\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t}$
- $\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t}$
- If  $\alpha = \text{constant}$ :
  - $\omega = \omega_0 + \alpha t$
  - $x = x_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
  - $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

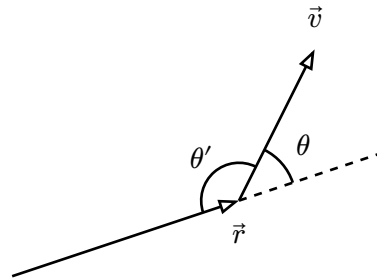
## Rotational Dynamics

- Torque ( $\tau$ )
  - $\tau = \pm Fr \sin(\theta)$
  - $\sin(\theta) = \sin(\theta')$ , so either angle can be used



- $\Sigma\tau = I\alpha$  (Newton's second law rotational)
- $I = \Sigma mr^2$ 
  - Moment of inertia formulas are derived by breaking objects up into pieces, then adding up  $mr^2$  for each piece
  - Implies that moments of inertia can add together
- $I = I_{\text{cm}} + MD^2$  (parallel axis theorem)
- When rolling without slipping,  $v_{\text{cm}} = r\omega$
- Rigid body diagram (RBD) reminders
  - Identify your system
  - Identify the direction of the acceleration or angular acceleration
  - Identify the direction of the velocity or angular velocity
  - Choose a coordinate system
  - Choose an axis of rotation (if it's not rotating)
  - Draw all external forces acting on the system at the location where they are acting
  - Note the angle for any forces acting at an angle relative to the chosen axes, or relative to a vector pointing from the pivot to where the force acts
  - Note how far each force is acting from the axis of rotation

- Angular momentum ( $L$ )
  - $L = mvr \sin(\theta)$  (point particle)
  - $\sin(\theta) = \sin(\theta')$ , so either angle can be used

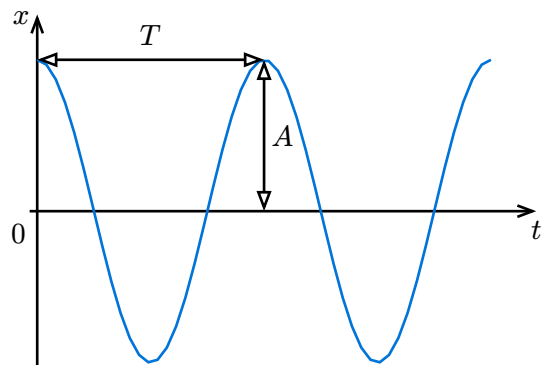


- $L = I\omega$  (not universal, but works for most cases in the class)
- $\tau_{\text{net,avg}}\Delta t = \Delta L$

## Static Equilibrium

- If  $\vec{a} = 0$  and  $\alpha = 0$ :
  - $\Sigma F_x = 0$
  - $\Sigma F_y = 0$
  - $\Sigma\tau = 0$
- Since it is not rotating, any axis of rotation can be chosen for the purpose of calculating torques

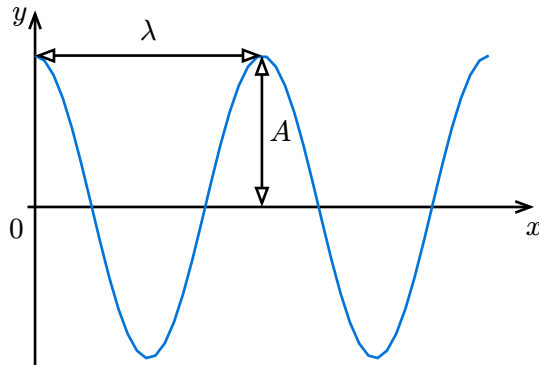
## Simple Harmonic Motion



- Oscillation terms
  - Period ( $T$ )
  - Frequency ( $f$ )
  - Angular frequency ( $\omega$ )
  - $T = \frac{1}{f}$  or  $f = \frac{1}{T}$
  - $\omega = \frac{2\pi}{T} = 2\pi f$
  - Amplitude ( $A$ )
- Mass on a spring
  - $\omega = \sqrt{\frac{k}{m}}$
- Simple pendulum
  - $\omega = \sqrt{\frac{g}{L}}$

- Assumes  $\theta < \sim 15^\circ$  so that  $\theta \approx \sin(\theta)$
- Kinematics
  - $x(t) = A \cos(\omega t + \phi)$
  - $x_{\max} = A$
  - $v(t) = -A\omega \sin(\omega t + \phi)$
  - $v_{\max} = A\omega$
  - $a(t) = -A\omega^2 \cos(\omega t + \phi)$
  - $a_{\max} = A\omega^2$

## Waves



- $v = f\lambda$ 
  - Wave speed depends on properties of the medium, not on wavelength and frequency
  - $v = \sqrt{\frac{F_T}{\mu}}$  (speed of wave in a string)
  - $v = (331.5 \frac{\text{m}}{\text{s}}) + (0.6 \frac{\text{m/s}}{^\circ\text{C}})T$  (speed of sound in air)
- Wave function
  - $y(x, t) = A \cos(kx \pm \omega t)$ 
    - Plus when moving in the  $-x$  direction
    - Minus when moving in the  $+x$  direction
  - Focus on a single point,  $x = 0$ , over time
    - $y(0, t) = A \cos(\pm \omega t)$
  - Focus on the shape of the wave at a single point in time,  $t = 0$ 
    - $y(x, 0) = A \cos(kx)$
  - Could add a phase angle  $\phi$  inside the  $\cos()$  if needed
    - $y(x, t) = A \cos(kx \pm \omega t + \phi)$
- Intensity ( $I$ )
  - $I = \frac{P}{A}$
  - Can be total power divided by the total area spread over
  - Can be power delivered divided by the area of a detector
- Sound intensity level ( $\beta$ )
  - $\beta = (10 \text{ dB}) \log\left(\frac{I}{I_0}\right)$
  - $I_0 = 10^{-12} \frac{\text{W}}{\text{m}^2}$  (threshold of hearing)

- $\beta_2 - \beta_1 = (10 \text{ dB}) \log\left(\frac{I_2}{I_1}\right)$
- Doppler effect for sound
  - $f_o = f_s \frac{v \pm v_o}{v \mp v_s}$
  - $v$ : speed of sound
  - $o$ : observer
  - $s$ : source
  - Use the upper sign if the observer/source is moving towards the other
  - Use the lower sign if the observer/source is moving away from the other

## Fluids

- $\rho = \frac{m}{V}$  (density)
- $P = \frac{F}{A}$  (pressure)
- $P = P_0 + \rho gh$  (pressure at a depth in a fluid)
- Buoyant force ( $B$  or  $F_B$ )
  - $B = \rho_{\text{fluid}} V_{\text{displaced}} g$
  - Points opposite  $\vec{g}$

## SI Prefixes

Name	Symbol	Factor
Quetta	Q	$10^{30}$
Ronna	R	$10^{27}$
Yotta	Y	$10^{24}$
Zetta	Z	$10^{21}$
Exa	E	$10^{18}$
Peta	P	$10^{15}$
Tera	T	$10^{12}$
Giga	G	$10^9$
Mega	M	$10^6$
Kilo	k	$10^3$
Hecto	h	$10^2$
Deka	da	$10^1$
-	-	$10^0$
Deci	d	$10^{-1}$
Centi	c	$10^{-2}$
Milli	m	$10^{-3}$
Micro	$\mu$	$10^{-6}$
Nano	n	$10^{-9}$
Pico	p	$10^{-12}$
Femto	f	$10^{-15}$
Atto	a	$10^{-18}$
Zepto	z	$10^{-21}$

Name	Symbol	Factor
Yocto	y	$10^{-24}$
Ronto	r	$10^{-27}$
Quecto	q	$10^{-30}$