$$\int_0^a x^2\sqrt{a^2-x^2}\,dx \qquad \begin{aligned} x &= a\cdot \text{sen}\,\theta \rightarrow x^2 = a^2\cdot\text{sen}^2\theta \\ dx &= a\cdot\cos\theta\cdot d\theta \end{aligned}$$

$$\theta \in [0, \pi/2]$$

se $x=0 \rightarrow 0 = a\cdot\text{sen}\,\theta \rightarrow \theta = 0$

se $x=a \rightarrow a = a\cdot\text{sen}\,\theta \rightarrow \theta = \pi/2$

$$\rightarrow \int_0^{\pi/2} (a^2\cdot\text{sen}^2\theta)\sqrt{a^2(1-\text{sen}^2\theta)}\cdot a\cdot\cos\theta\cdot d\theta$$

$$= \int_0^{\pi/2} a^4\cdot\text{sen}^2\theta\cdot\cos^2\theta\cdot d\theta = a^4\int_0^{\pi/2}\text{sen}^2\theta\cdot\cos^2\theta\cdot d\theta$$

$$* \ \sin(2\theta) = 2\cdot\sin\theta\cdot\cos\theta \longrightarrow \frac{\sin^2(2\theta)}{4} = \sin^2\theta\cdot\cos^2\theta$$

$$= a^4 \int_0^{\pi/2} sen^2\theta \cos^2\theta \, d\theta = a^4 \int_0^{\pi/2} \frac{\sin^2(2\theta)}{4} d\theta \qquad \begin{aligned} u &= 2\theta \\ du &= 2\,d\theta \end{aligned}$$

$$= \frac{a^4}{8} \int_0^{\pi} \sin^2(u)\, du \qquad * \sin^2(u) = \frac{1-\cos(2u)}{2}$$

$$= \frac{a^4}{8} \int_0^{\pi} \frac{1-\cos(2u)}{2} du$$

$$= \frac{a^4}{16} \left(u - \frac{1}{2} \sin(2u) \Big|_0^{\pi} \right)$$

$$= \frac{a^4}{16} \left(\left(\pi - \frac{1}{2}\sin(2\pi)\right) - \left(0 - \frac{1}{2}\sin(2 \cdot 0)\right) \right)$$

$$= \frac{a^4}{16} \pi \quad //$$