VECTORS

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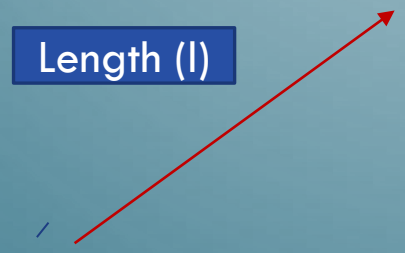
What is a vector ?

Vector is a quantity which has both the magnitude and direction.



Types of vectors

- 1) Unit Vector : Vector having magnitude equals to 1.
- 2) Co-initial Vectors : Vectors having same initial point are called as co-initial vectors.
- 3) Collinear Vectors : Vectors parallel to each other are called collinear vectors.
- 4)Equal Vectors: Vectors having equal magnitude and same direction.
- 5) Negative Vectors: Vectors having equal magnitude but exactly in opposite direction.
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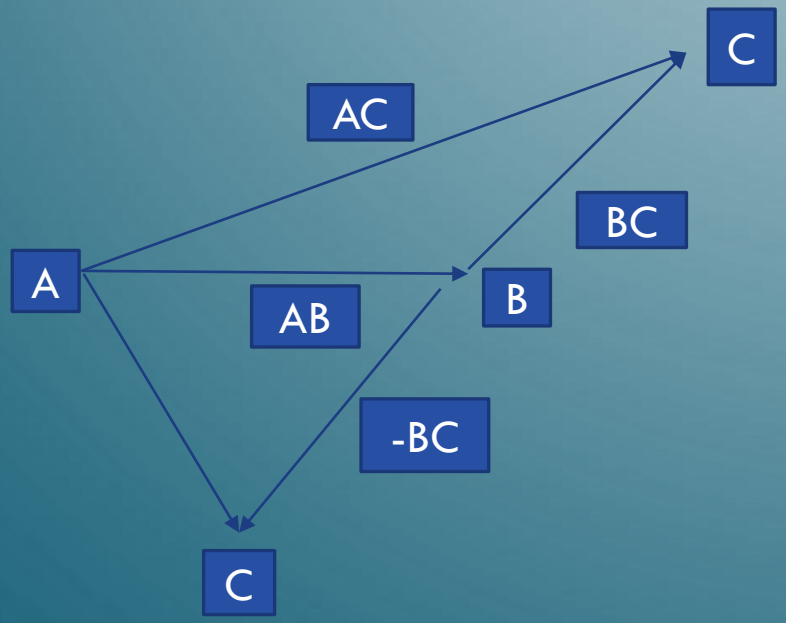
Addition of 2 Vectors

- $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$



Multiplying a Vector by a Scalar

- $\vec{a}$ ➔ 
- $2\vec{a}$ ➔ 
- $-0.5\vec{a}$ ➔

Dot Product of 2 Vectors

- Dot Product of $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$ and $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}$

$$\mathbf{v} \bullet \mathbf{w} = a_1a_2 + b_1b_2$$

OR

$$\mathbf{v} \bullet \mathbf{w} = \|\mathbf{v}\|\|\mathbf{w}\|\cos\theta$$

- Angle between two vectors (θ is the smallest non-negative angle between the two vectors)

$$\cos\theta = \frac{\mathbf{v} \bullet \mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|} \quad \text{and} \quad \theta = \cos^{-1}\left(\frac{\mathbf{v} \bullet \mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|}\right)$$

Cross Product of 2 vectors

$$\mathbf{A} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$$

$$\mathbf{B} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$$

$$\mathbf{A} \times \mathbf{B} = \det\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}(a_2b_3 - a_3b_2) + \mathbf{j}(a_3b_1 - a_1b_3) + \mathbf{k}(a_1b_2 - a_2b_1)$$

Example:

$\mathbf{A}=\mathbf{i}-\mathbf{j}$

$\mathbf{B}=\mathbf{i}+\mathbf{k}$

$\mathbf{A} \times \mathbf{B} = \mathbf{i}(-1-0) + \mathbf{j}(0-1) + \mathbf{k}(0-(-1)) = -\mathbf{i}-\mathbf{j}+\mathbf{k}$

Equation of Straight Line

- $Y = mX + C$

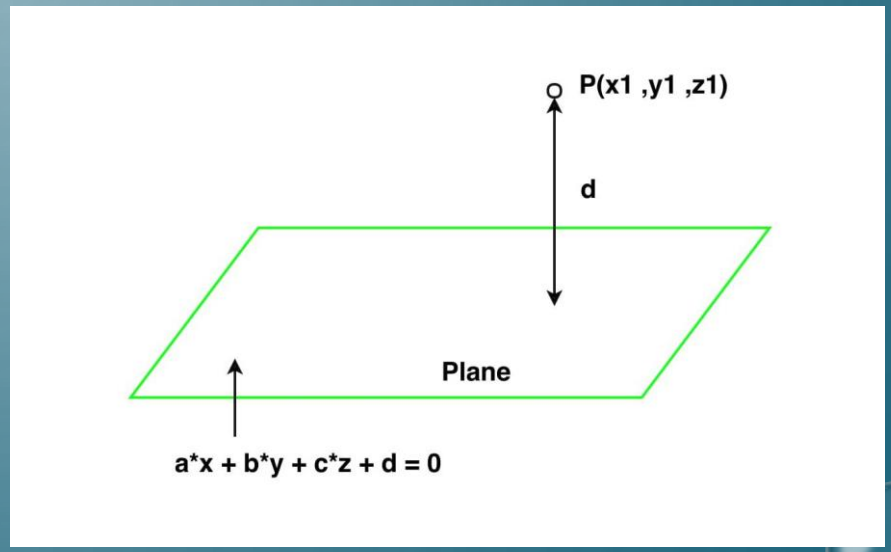
m → Slope

C → Intercept of line on Y axis.

Distance b/w a point from plane

The distance D between a point $P_0(x_0, y_0, z_0)$ and the plane $ax + by + cz + d = 0$ is

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$



Equation of a plane

- IF ANY PLANE P PASSES THROUGH THE POINT (X1,Y1,Z1) AND THE VECTOR NORMAL TO IT IS AI^+BJ^+CK^

- THEN ITS EQUATION IS GIVEN BY,

$$P: A(X-X1)+B(Y-Y1)+C(Z-Z1)=0$$