THDC INSTITUTE OF HYDROPOWER ENGINEERING AND TECHNOLOGY

Assignment – 01

Formal Language and Automata Theory

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CSE-B

Q1 → Consider the language S^+, where $S = \{a, b\}$. How many words does this language have of length 2? of length 3? of length n?

⇒ The language S^*, where $S = \{a, b\}$ includes all possible strings formed by conatening elements from of S, including the empty string.

- Length 2
  with two positions there are $2 * 2 = 2^2 = 4$
  4 possible words
- Length 3
  There are $2 \times 2 \times 2 = 2^3 = 8$
  8 ~~to~~ possible words
- Length n
  2^n possible words

Q2 Consider the language S^*, where $S = \{ab, ba\}$. write out all the words in S^* that have seven or fewer letters. Can any words in this language contain the substrings aaa or bbb?

⇒ No, any words in this language cannot contains the substrings aaa or bbb because the language S only contains 'ab' and 'ba'. Therefore, it is impossible to have consecutive idential letters.

Q3 Consider the language S^*, where $S = \{aa\ aba\ baa\}$. Show that the words aabaa, baaabaaa, & baaaaababaaa are all in the language. Can any word in this language be interpreted as a string of elements from S in two differents ways? Can any word in this language have an add total numbers of a's?

⇒ No word in this language can be intrepreted as a string of elements from S in two different ways because each word can ~~b~~only be parsed in one unique way based on the structure of the string "aababaa".

⇒ No word in this language can have an ~~oo~~ odd number of 'a's because ~~d~~ each element in S has an even number of a's. Concatenating any number of these elements will always result in a word ~~o~~ with an even number's of a's.

Q4 Show that if the concatenation of two words (neither Λ) in PALINDROME is also a word in PALINDROME then both words are powers of some other words; that is, if x and y and xy are all in PALINDROME then there is word z such that $x = z^n$ and $y = z^m$ for some integers n & m.

⇒ A palindrome reads the same forwards and backwards. If x and y are palindromes, then $x = x^R$ and $y = y^R$, where R denotes the reverse. If xy is also a palindrome, then $xy = (xy)^R = yx$.

→ Commutation

From $xy = yx$, it's deduced that x and y commute. A well-known result in formal language theory states that if two ~~words~~ words commute, they are powers of the same words.
Therefore, there exits a word z such that $x = z^n$ and $y = z^m$ for some integers n & m

Q5 Let $S = \{ab, bb\}$ and $T = \{ab, bb, bbb\}$

① Show that $S^* \neq T^*$; but that $S^* \subseteq T^*$

② Prove ~~d~~ in general that if $S \subset T$ then $S^* \subset T^*$. Find examples of S and T for which:

③ $S \subset T$ but $S \neq T$ and yet $S^* = T^*$

① Showing $S^* \neq T^*$ and $S^* \subset T^*$

$T^* = \{ \Lambda, ab, bb, bbb, abab,$

Since bbb is in T^* but n0-9 in S^* . S^*

→ Every string that can be formed by concatenating elements of S can also be formed by concatenating elements of T (Since $S \subseteq T$), Therefore . $S^* \subset T^*$

→ Proving if $S \subseteq T$ then $S^* \subseteq T^*$

- Let x be an arbitrary string S^*.
- By definition of kleene star, x can be written as a concatenation of elements from S : $x = s_1 s_2 \dots s_n$ where each $s_i \in S$.
- Since $S \subseteq T$, each s_i is also in T.
- Therefore, x is a concatenation of elements from T, which means. $x \in T^*$
- Since x was an arbitrary string S^*, this shows that every element of S^* is also in T^*, so $S^* \subseteq T^*$

Example

- $S = \{a\}, \{a, b\}$

Q6 Prove that for all sets S.

① $(S^+)^* = (S^*)^*$

② $(S^+)^+ = S^+$

⇒ S^+ → Represents the set of all possible strings formed by concatenating one or more strings from S.

$(S^+)^*$ - Represents the set of all possible strings formed by concatenating zero or more strings from S^+. This includes the empty string and any combination of strings from S^+.

$S^*)^*$ → Represents the set of all possible strings formed ting zero or more strings from S^*.

Since $S+$ is a subset of S^*, any string formed by concatenating elements from $S+$ can also be formed by concatenating elements from S^*. Therefore, $(S+)^*$ is a subset of (S^*). Conversely, since S is a subset of $S+$, any string formed by concatenating elements from S can ~~b~~ also ~~by~~ be formed by concatenating elements from $(S+)$.
Therefore, $(S)^*$ is a subset $(S+)^*$. Hence, $(S+)^* = (S^*)^*$

2 → Since $S+$ is already defined as all concatenations of one or more strings from S, concatenating strings from $S+$ again will still result in a strings than can be formed by concatenating one or more strings from S. Therefore, $(S+)+$ is equivalent to $S+$
Hence, $(S+)+ = S+$

→ By definition $(S^{**})^* = S^{***}$ is this bigger than S^* ;? ~~Is~~ Is it bigger than S.

S^{***} is not bigger than S^*. Both sets contains an infinite numbers of elements.

S^{***} is bigger than S, because it is made up of all ~~con~~ concatenations of elements of S.

Q8 Let $S = \{a, bb, bab, abaab\}$. Is abbabaabab in S^*
Is abaabbabbaabb ? Does any word in S^* have an total no. of b's ?

→ abbabaabab = (abab)(a)(abab)
Since abab is not in S, abbabaabab is not in S^*.

- abaabbabbaabb = (abaab)(bb)(a)(abaab)(bb)
Since abaab, bb, a are in S.
abaabbabbaabb is in S^*.

a has 0 b's

bb has 2 b's

bab has 2 b's

abaab has 2 b's.

Since, bb, bab, abaab has even no. of b's then no concatenation have odd no. of b's.

Q9 Consider the language S^* where $S = \{aa, ab, ba, bb\}$. Give another description of this language. Give an example of a set S such that S^* contains all possible strings of a's and b's that have length divisible by three.

⇒ $S = \{aaa, aab, aba, baa, abb, bab, bba, bbb\}$

This set consists of all possible strings of length 3 using the alphabet $\{a, b\}$.

When this set is used to form S^*, it will generate strings whose length is a multiple of 3.