Deep Reinforcement Learning

Eligibility Traces

February 10, 2025

Vinay Kumar | Amberg

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Problem Statement

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How do we find good G_t?

- Should the agent rely on immediate reward ?

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Technische Hochschule
Amberg-Weiden

How do we find good G_t?

- Should the agent rely on immediate reward ?
- Or should agent consider long-term reward ?

Brief Recall

TD Methods

- After taking action a_t, observe reward r_{t+1} and next state s_{t+1}.

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- One-step TD target (G_t) and TD error (δ_t):

$$G_t = r_{t+1} + \gamma V(s_{t+1})$$

$$\delta_t = G_t - V(s_t) = r_{t+1} + \gamma V(s_{t+1}) - V(s_t)$$

where γ is the discount factor.

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  where γ is the discount factor.
- Update the value of the current state s_t:

$$V(s_t) \leftarrow V(s_t) + \alpha \delta_t$$

  where α is the learning rate.

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- Play an episode until the end of terminal state (e.g., game over or goal reached).

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  where γ is the discount factor.
- For each state s_t visited in the episode, update its value:

$$V(s_t) \leftarrow V(s_t) + \alpha(G_t - V(s_t))$$

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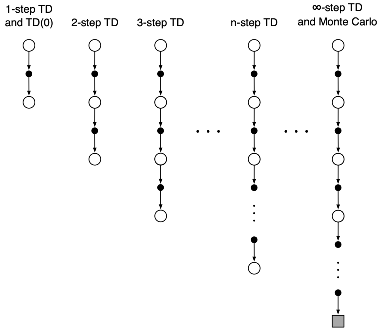
Brief Recall

n-step TD

- **Idea**: Look Farther into future when do TD backup (1, 2, 3,. . . , n steps)



https://bjpcjp.github.io/pdfs/math/nstep-bootstrap-RL.pdf

Brief Recall

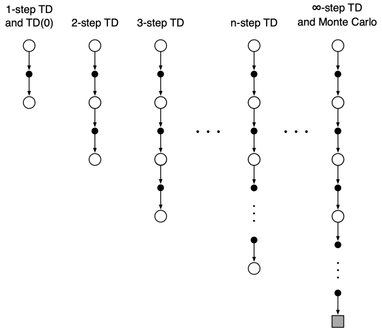
n-step TD

- **Idea**: Look Farther into future when do TD backup (1, 2, 3,. . . , n steps)
- The n-step return $G_t^{(n)}$ is given by:

$$G_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n V(S_{t+n})$$

Where:

  - $R_{t+1}, R_{t+2}, \ldots, R_{t+n}$: rewards at each step
  - γ: discount factor
  - $V(S_{t+n})$: value of state at time $t + n$



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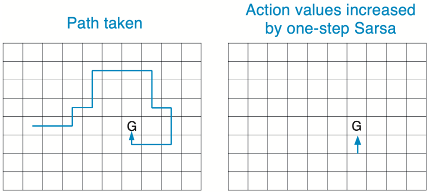
How can we learn most from the episode?



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Intuition behind

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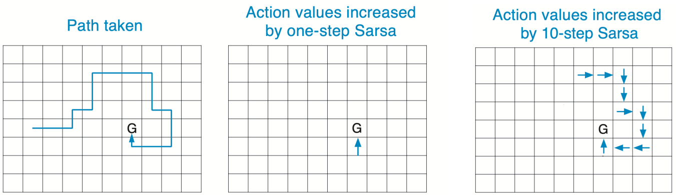

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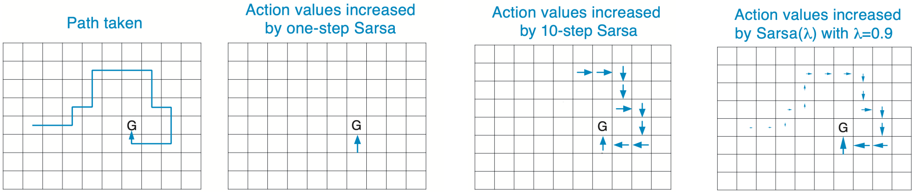

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Eligibility Tracing

- A bridge from TD to Monte Carlo methods

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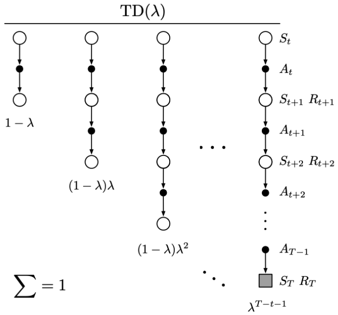
- A bridge from TD to Monte Carlo methods
- A temporary record of which states/actions are eligible for learning updates

Key Concepts
The Forward View

- The idea is to create an average of our n-step



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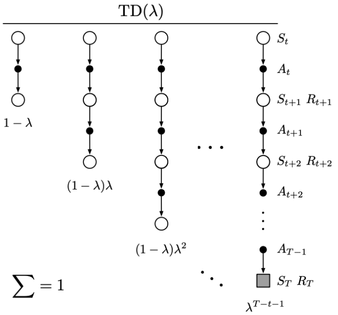
- The idea is to create an average of our n-step
- Weighting of past experiences in eligibility traces is controlled by λ(lambda)



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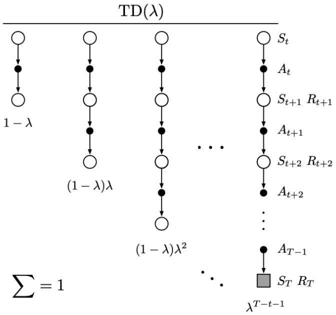

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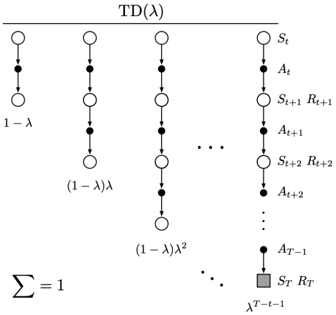

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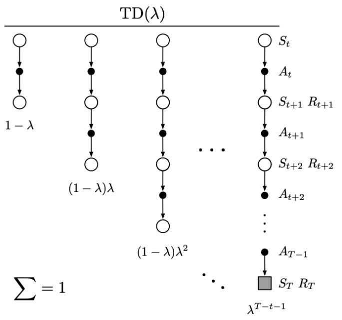
- Each state has a trace that keeps track how much credit that state should receive for future updates.



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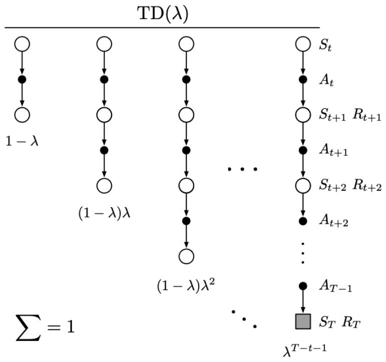
How weighting n sequence works ?

TD(λ)

- The one-step return is the largest weight, given by 1-λ



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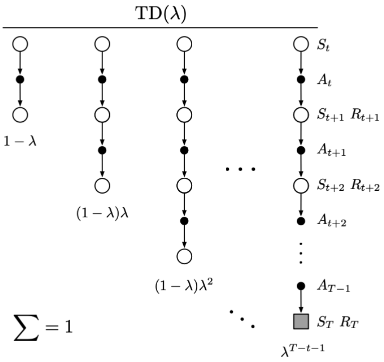

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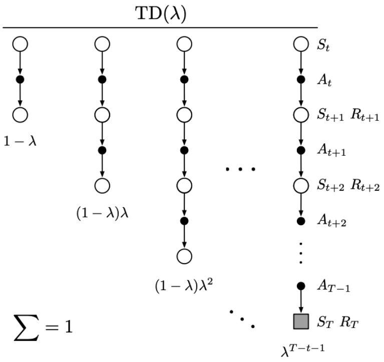

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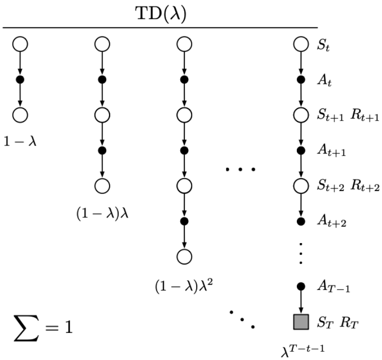

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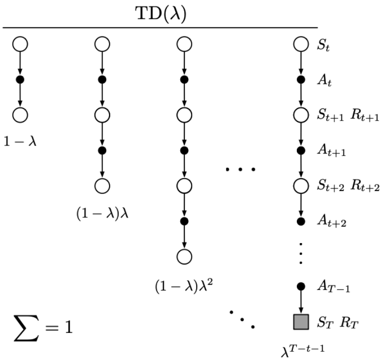

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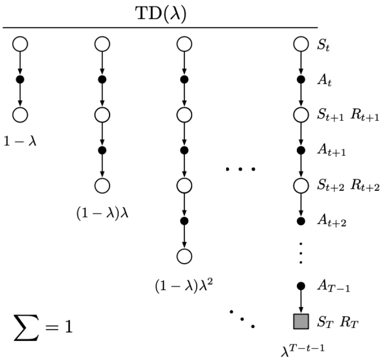
- If (λ =1) ?
- Overall update reduces to last component, i.e Monte Carlo



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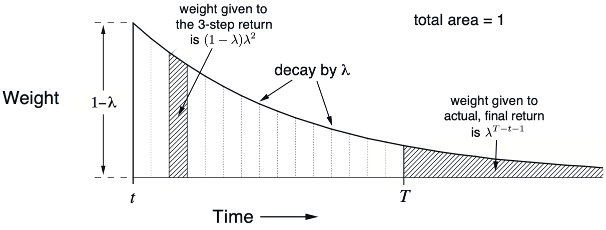
Working and showing



Weighting given in the λ-return to each of the n-step returns.

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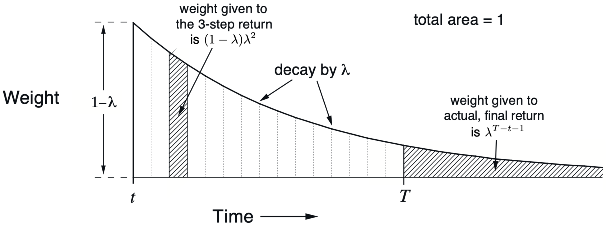

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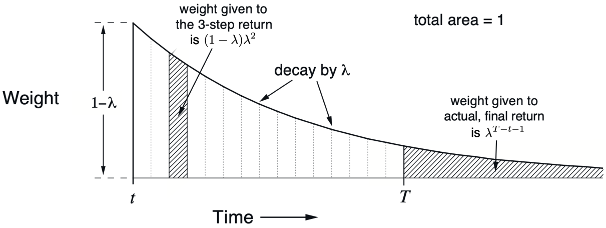

Weighting given in the λ-return to each of the n-step returns.

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- Known as Off-line λ-return algorithm

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The Forward View

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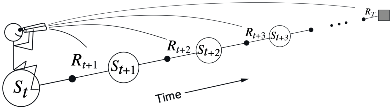

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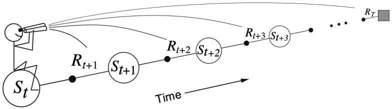

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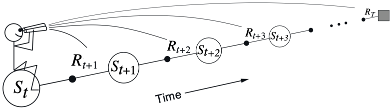

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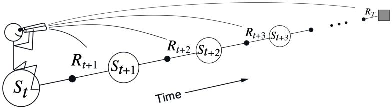

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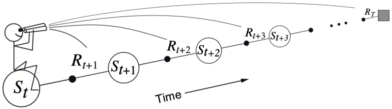
The Forward View

- Previously we discussed forward view (theoretical perspective)
- What we do is, we look ahead n steps for future rewards, and then we update our return
- **Problem arises when:**
- Cannot update until future rewards are known
- Somehow complex to implement because the update of each state depends on later events or rewards that are not available at current time



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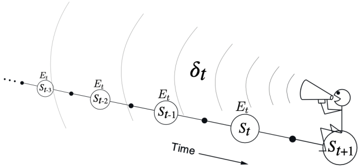
The Backward View

- Additional short term memory variable associated with each state



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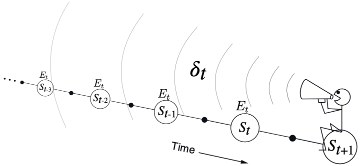
The Backward View

- Additional short term memory variable associated with each state
- Updates the value function continuously



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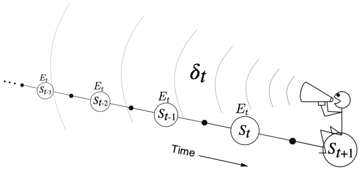
- When a reward is received, not just the current state, but also the past states get some credit



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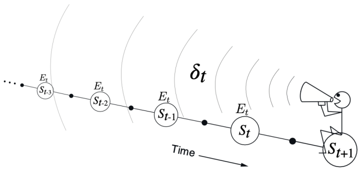
The Backward View

- Additional short term memory variable associated with each state
- Updates the value function continuously
- When a reward is received, not just the current state, but also the past states get some credit
- Recently visited states get more credit, while older states get less credit over time.



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Pseudo Code

TD(λ)

Semi-gradient TD(λ) for estimating $\hat{v} \approx v_\pi$

Input: the policy π to be evaluated
Input: a differentiable function $\hat{v} : \mathcal{S}^+ \times \mathbb{R}^d \to \mathbb{R}$ such that $\hat{v}(\text{terminal},\cdot) = 0$
Algorithm parameters: step size $\alpha > 0$, trace decay rate $\lambda \in [0, 1]$
Initialize value-function weights $\mathbf{w}$ arbitrarily (e.g., $\mathbf{w} = \mathbf{0}$)

Loop for each episode:
    Initialize S
    $\mathbf{z} \leftarrow \mathbf{0}$ (a d-dimensional vector)
    Loop for each step of episode:
    | Choose $A \sim \pi(\cdot|S)$
    | Take action A, observe R, S'
    | $\mathbf{z} \leftarrow \gamma\lambda\mathbf{z} + \nabla\hat{v}(S,\mathbf{w})$
    | $\delta \leftarrow R + \gamma\hat{v}(S',\mathbf{w}) - \hat{v}(S,\mathbf{w})$
    | $\mathbf{w} \leftarrow \mathbf{w} + \alpha\delta\mathbf{z}$
    | $S \leftarrow S'$
    until S' is terminal

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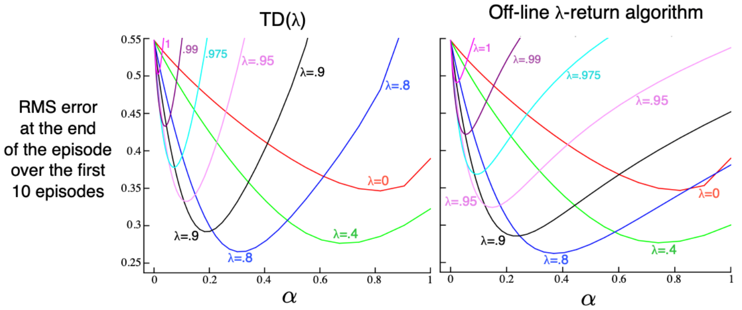
Comparison

Performance Evaluation



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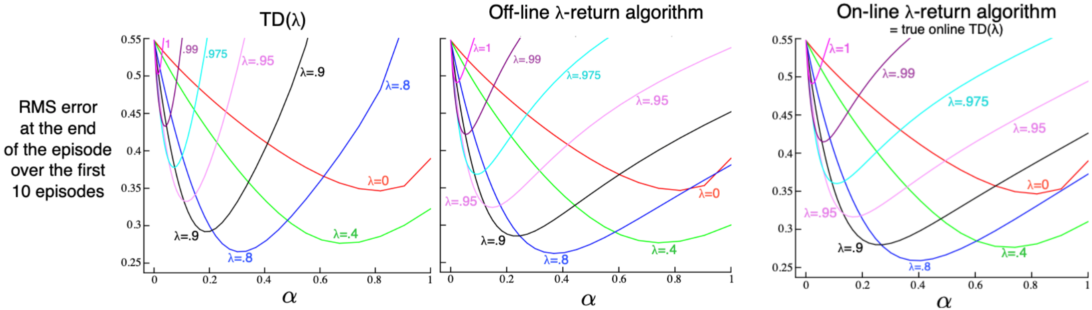

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Improvement

Generalized Advantage Estimation (GAE)

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- The temporal difference (TD) error at each timestep is:

$$\delta_t^V = r_t + \gamma V(s_{t+1}) - V(s_t)$$

- The total return G_t used in advantage calculation:

$$G_t = \delta_t + \gamma \delta_{t+1} + \cdots + \gamma^{n-1} \delta_{t+n-1}$$

which represents discounted sum of future rewards.

Improvement

The idea of eligibility traces

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Generalized Advantage Estimation (GAE)

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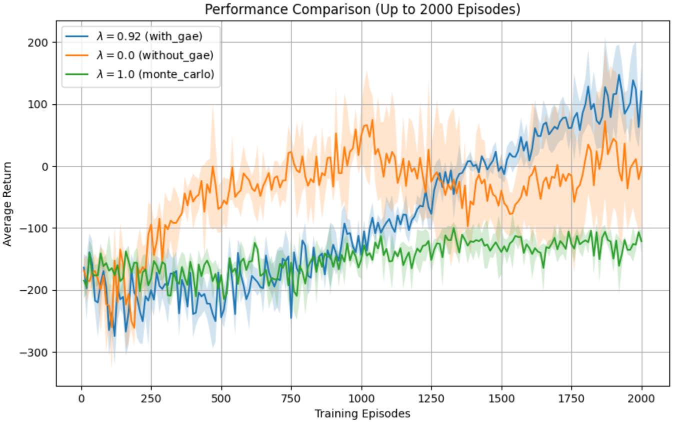
- Better decisions by looking at both short and long-term outcomes
- GAE sums over multiple TD errors from multiple time steps using eligibility traces
- Estimate the advantage function, which measures how much better an action is compared to the average action in a given state
- Works particularly well with deep neural networks and replay buffers.

Comparison on Lunar-Lander



Conclusion

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- GAE inherits eligibility traces, that provide better stability
- Improved performance in delayed reward scenarios
- Usage in real world scenarios like Robotics control, Game AI which helps deciding what to do in situations where each choice affects the next
- Combining GAE with advance models like PPO, make stable learning in many environments particularly real-world robotics applications.

End !

Thanks for paying attention !

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