

```
In [1]: #Program-1(a): Rank of the matrix
import numpy as np
A=np.array([[0,2,3,4],[2,3,5,4],[4,8,13,12]])
print("A=",A)
rank=np.linalg.matrix_rank(A)
print("\n the rank of the given matrix=",rank)
```

```
A= [[ 0  2  3  4]
     [ 2  3  5  4]
     [ 4  8 13 12]]
```

the rank of the given matrix= 2

```
In [2]: # program-1(b): Rank of the matrix
import numpy as np
A=np.array([[1,2,4,3],[2,4,6,8],[4,8,12,16],[1,2,3,4]])
print("A=",A)
rank=np.linalg.matrix_rank(A)
print("\n the rank of the given matrix=",rank)
```

```
A= [[ 1  2  4  3]
     [ 2  4  6  8]
     [ 4  8 12 16]
     [ 1  2  3  4]]
```

the rank of the given matrix= 2

```
In [3]: #Program-2(a):Test the consistency of the given system of equations
#x+2y-z=1 ; 2x+y+4z=2 ; 3x+3y+4z=1
import numpy as np
A=np.matrix([[1,2,-1],[2,1,4],[3,3,4]])
B=np.matrix([[1],[2],[1]])
AB=np.concatenate((A,B),axis=1)
rA=np.linalg.matrix_rank(A)
rAB=np.linalg.matrix_rank(AB)
n=A.shape[1]
if(rA==rAB):
    print("The system of equations is consistent")
    if(rA==n):
        print("The system has unique solution")
        print(np.linalg.solve(A,B))
    else:
        print("The system has infinitely many solutions")
else:
    print("The system of equations is inconsistent")
```

The system of equations is consistent

The system has unique solution

```
[[ 7.]
 [-4.]
 [-2.]]
```

```
In [4]: #Program-2(b):Test the consistency of the given system of equations  
#5x+y+3z=20 ; 2x+5y+2z=18 ; 3x+2y+z=14  
import numpy as np  
A=np.matrix([[5,1,3],[2,5,2],[3,2,1]])  
B=np.matrix([[20],[18],[14]])  
AB=np.concatenate((A,B),axis=1)  
rA=np.linalg.matrix_rank(A)  
rAB=np.linalg.matrix_rank(AB)  
n=A.shape[1]  
if(rA==rAB):  
    print("The system of equations is consistent")  
    if(rA==n):  
        print("The system has unique solution")  
        print(np.linalg.solve(A,B))  
    else:  
        print("The system has infinitely many solutions")  
else:  
    print("The system of equations is inconsistent")
```

The system of equations is consistent

The system has unique solution

```
[[3.]  
 [2.]  
 [1.]]
```

```

In [5]: #Program-3(a): Solution of system of Linear equations by Gauss Siedel Method
#20x+y-2z=12 ; 3x+20y-z=-18 ; 2x-3y+20z=25
f1 = lambda x,y,z :(17-y+2*z)/20
f2 = lambda x,y,z :(-18-3*x+z)/20
f3 = lambda x,y,z :(25-2*x+3*y)/20
x0=0
y0=0
z0=0
count=1
e=float(input('Enter tolerable error:'))
print('\ncount\tx\ty\tz\n')
condition = True
while condition:
    x1 = f1(x0, y0, z0)
    y1 = f2(x1, y0, z0)
    z1 = f3(x1, y1, z0)
    print('%d\t%.4f\t%.4f\t%.4f\n'%(count,x1,y1,z1))
    e1 = abs(x0-x1);
    e2 = abs(y0-y1);
    e3 = abs(z0-z1);
    count+=1
    x0=x1
    y0=y1
    z0=z1
    condition = e1>e and e2>e and e3>e
print('\nsolution : x=%.3f,y=%.3f and z=%.3f\n'%(x1,y1,z1,))

```

Enter tolerable error:0.01

count	x	y	z
1	0.8500	-1.0275	1.0109
2	1.0025	-0.9998	0.9998
3	1.0000	-1.0000	1.0000

solution : x=1.000,y=-1.000 and z=1.000

```

In [6]: #Program-3(b): Solution of system of linear equations by Gauss Siedel Method
#5x+2y+z=12 ; x+4y+2z=15 ; x+2y+5z=20
f1 = lambda x,y,z :(12-2*y-z)/5
f2 = lambda x,y,z :(15-x-z)/4
f3 = lambda x,y,z :(20-x-2*y)/5
x0=0
y0=0
z0=0
count=1
e=float(input('Enter tolerable error:'))
print('\ncount\tx\ty\tz\n')
condition = True
while condition:
    x1 = f1(x0, y0, z0)
    y1 = f2(x1, y0, z0)
    z1 = f3(x1, y1, z0)
    print('%d\t%.4f\t%.4f\t%.4f\n'%(count,x1,y1,z1))
    e1 = abs(x0-x1);
    e2 = abs(y0-y1);
    e3 = abs(z0-z1);
    count+=1
    x0=x1
    y0=y1
    z0=z1
    condition = e1>e and e2>e and e3>e
print('\nsolution : x=%.3f,y=%.3f and z=%.3f\n'%(x1,y1,z1,))

```

Enter tolerable error:0.01

count	x	y	z
1	2.4000	3.1500	2.2600
2	0.6880	3.0130	2.6572
3	0.6634	2.9199	2.6994
4	0.6922	2.9021	2.7007

solution : x=0.692,y=2.902 and z=2.701

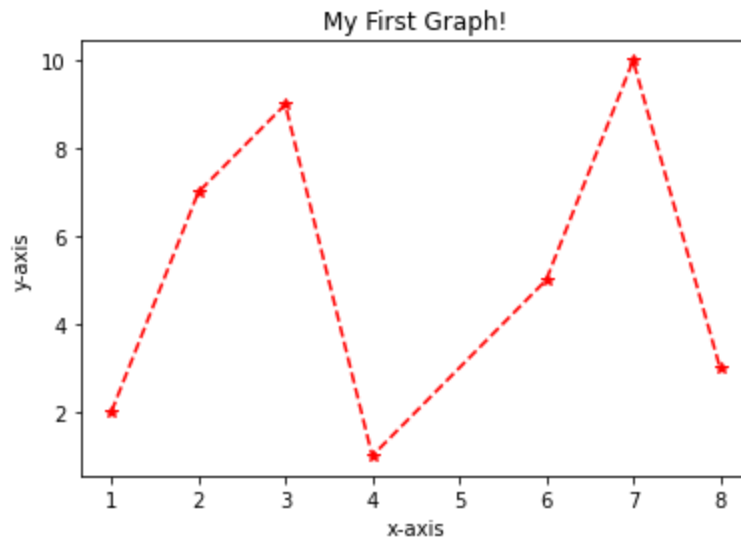
```
In [7]: #Program-4(a):Finding the largest eigen value and corresponding eigen vector by
#Given A=[[2,0,1],[0,2,0],[1,0,2]]
import numpy as np
x=np.array([1,0,0])
a=np.array([[2,0,1],[0,2,0],[1,0,2]])
def normalize(x):
    f=abs(x).max()
    xn=x/f
    return f,xn
for i in range(40):
    x=np.dot(a,x)
    lambda1,x=normalize(x)
print('Eigenvalue:',lambda1)
print('Eigenvector:',x)
```

Eigenvalue: 3.0  
Eigenvector: [1. 0. 1.]

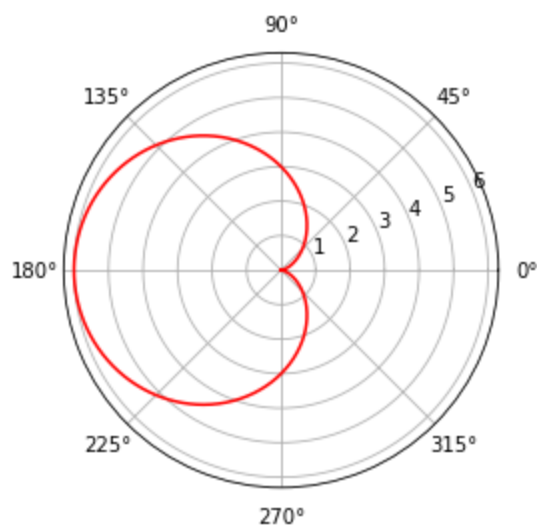
```
In [8]: #Program-4(b):Finding the largest eigen value and corresponding eigen vector by
#Given A=[[6,-2,2],[-2,3,-1],[2,-1,3]] , X=[1,1,1]
import numpy as np
x=np.array([1,1,1])
a=np.array([[6,-2,2],[-2,3,-1],[2,-1,3]])
def normalize(x):
    f=abs(x).max()
    xn=x/f
    return f,xn
for i in range(100):
    x=np.dot(a,x)
    lambda1,x=normalize(x)
print('Eigenvalue:',lambda1)
print('Eigenvector:',x)
```

Eigenvalue: 8.0  
Eigenvector: [ 1. -0.5 0.5]

```
In [4]: #Program-5(a): Python program for plotting a line
import numpy as np
import matplotlib.pyplot as plt
x= [1,2,3,4,6,7,8]
y= [2,7,9,1,5,10,3]
plt.plot(x,y,'r--*')
plt.xlabel('x-axis')
plt.ylabel('y-axis')
plt.title('My First Graph!')
plt.show()
```



```
In [10]: #Program-5(b):python program to plot cardioid  $r=3(1+\cos \theta)$ 
from pylab import*
theta=linspace(0,2*np.pi,1000)
r1=3-3*cos(theta)
polar(theta,r1,'r')
show()
```



```
In [11]: #Program-6(a): Finding the angle between two polar curves r1=4*(1+cos(t)) and r2=5*(1-cos(t))
from sympy import *
r,t=symbols('r,t')
r1=4*(1+cos(t));
r2=5*(1-cos(t));
dr1=diff(r1,t)
dr2=diff(r2,t)
t1=r1/dr1
t2=r2/dr2
q=solve (r1-r2,t)
w1=t1.subs({t:float (q[1])})
w2=t2.subs({t:float (q[1])})
y1=atan(w1)
y2=atan(w2)
w=abs(y1-y2)
print('Angle between curves in radian is %0.3f'%(w))
```

Angle between curves in radian is 1.571

```
In [3]: #Program-6(b): Finding the radius of curvature for r=asin(nt) at t=pi/2 and n=1
from sympy import *
t,r,a,n=symbols('t,r,a,n')
r=a*sin(n*t)
r1=Derivative(r,t).doit()
r2=Derivative(r1,t).doit()
rho=(r**2+r1**2)**1.5/(r**2+2*r1**2-r*r2)
rho1=rho.subs(t,pi/2)
rho1=rho1.subs(n,1)
print("The radius of curvature is")
display(simplify (rho1))
```

The radius of curvature is

$$\frac{(a^2)^{1.5}}{2a^2}$$

```
In [13]: #Program-7(a): Finding partial derivatives and jacobian of functions
#Prove that if u=e^x(x*cos(y)-y*sin(y)) then uxx+uyy=0
from sympy import *
x,y=symbols('x,y')
u=exp(x)*(x*cos(y)-y*sin(y))
display(u)
dux=diff(u,x)
duy=diff(u,y)
uxx=diff(dux,x)
uyy=diff(duy,y)
w=uxx+uyy
w1=simplify(w)
print('Ans:',float(w1))
```

$$(x \cos(y) - y \sin(y)) e^x$$

Ans: 0.0

```
In [14]: #Program-7(b):If  $u=xy/z$  , $v=yz/x$  , $w=zx/y$  then find the jacobian of  $u,v,w$  w.r.t  $x,y,z$ 
from sympy import *
x,y,z=symbols('x,y,z')
u=x*y/z
v=y*z/x
w=z*x/y
dux=diff(u,x)
duy=diff(u,y)
duz=diff(u,z)
dvx=diff(v,x)
dvy=diff(v,y)
dvz=diff(v,z)
dwx=diff(w,x)
dwy=diff(w,y)
dwz=diff(w,z)
J=Matrix([[dux,duy,duz],[dvx,dvy,dvz],[dwx,dwy,dwz]])
print("The jacobian matrix is")
display(J)
Jac=det(J).doit()
print('J=',Jac)
```

The jacobian matrix is

$$\begin{bmatrix} \frac{y}{z} & \frac{x}{z} & -\frac{xy}{z^2} \\ -\frac{yz}{x^2} & \frac{z}{x} & \frac{y}{x} \\ \frac{z}{y} & -\frac{xz}{y^2} & \frac{x}{y} \end{bmatrix}$$

J= 4

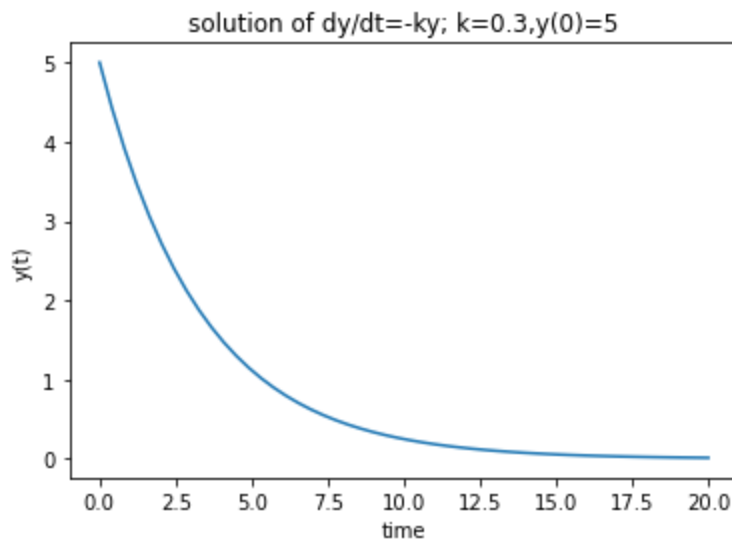


In [15]: *#Program-8: Find the maxima and minima of  $f(x,y)=x^2+y^2+3x-3y+4$*

```
import sympy
from sympy import symbols,solve,Derivative,pprint
x,y=symbols('x,y')
f=x**2+y**2+3*x+3*y+4
d1=Derivative(f,x).doit()
d2=Derivative(f,y).doit()
criticalpoints1=solve(d1)
criticalpoints2=solve(d2)
s1=Derivative(f,x,2).doit()
s2=Derivative(f,y,2).doit()
s3=Derivative(Derivative(f,y),x).doit()
print('function value is')
q1=s1.subs({y:criticalpoints1,x:criticalpoints2}).evalf()
q2=s2.subs({y:criticalpoints1,x:criticalpoints2}).evalf()
q3=s3.subs({y:criticalpoints1,x:criticalpoints2}).evalf()
delta=s1*s2-s3**2
print(delta,q1)
if(delta>0 and s1<0):
    print("f takes maximum")
elif(delta>0 and s1>0):
    print("f takes minimum")
if (delta<0):
    print("The point is a saddle point")
if(delta==0):
    print("Further tests required")
```

```
function value is
4 2.000000000000000
f takes minimum
```

```
In [16]: #Program-9:Solution of first order differential equation  
#Solve dy/dt=-ky with parameter k=0.3 and y(0)=5  
import numpy as np  
from scipy.integrate import odeint  
import matplotlib.pyplot as plt  
#Function returns dy/dt  
def model(y,t):  
    k=0.3  
#dy/dt=-ky  
    return -k*y  
#intial condition  
y0=5  
#values for time  
t=np.linspace(0,20)  
#solve ODE  
y=odeint(model,y0,t)  
  
plt.plot(t,y)  
plt.title('solution of dy/dt=-ky; k=0.3,y(0)=5')  
plt.xlabel('time')  
plt.ylabel('y(t)')  
plt.show()
```



In [ ]: