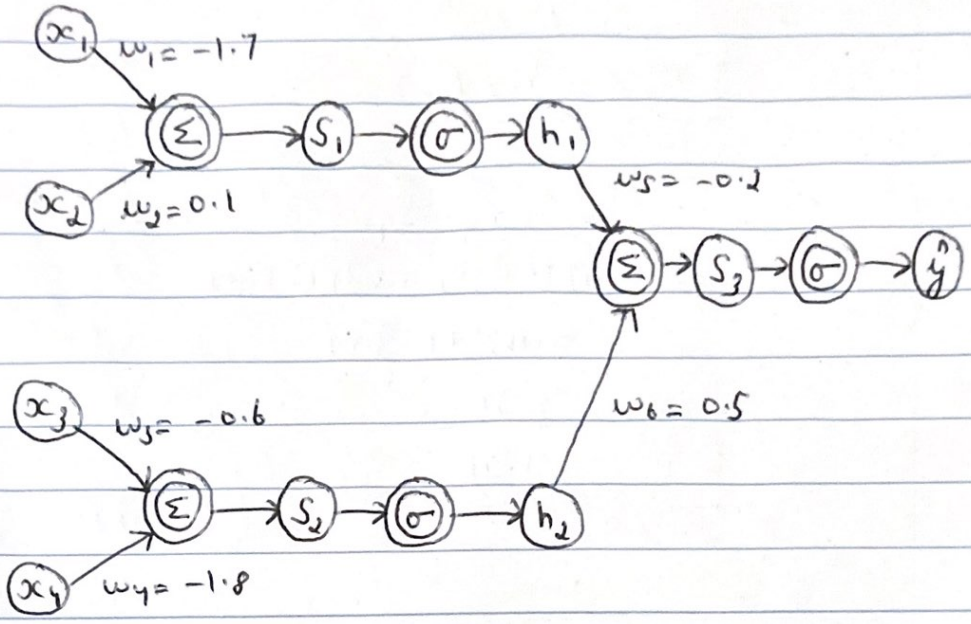
Deep learning

Assignment - 2

Q-1)

Neural Network



$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$h_1 = \frac{1}{1+e^{-w_1 x_1 - w_2 x_2}}$$

$$L2\ Loss\ function\ (y, \hat{y}) = \|\hat{y} - y\|^2$$

$x_1 = 0.7$

$x_2 = 1.2$

$x_3 = 1.1$

$x_4 = 2$

$y = 0.5$

$$S_1 = x_1 w_1 + x_2 w_2$$
$$= (0.7)(-1.7) + (1.2)(0.1)$$
$$= (-1.19) + (0.12)$$
$$= -1.07$$

$$\boxed{S_1 = -1.07}$$

$$S_2 = x_3 w_3 + x_4 w_4$$
$$= (1.1)(-0.6) + (2)(-1.8)$$
$$= (-0.66) + (-3.6)$$
$$= -4.26$$

$$\boxed{S_2 = -4.26}$$

$$h_1 = \frac{1}{1 + e^{-w_1 x_1 - w_2 x_2}}$$

$$= \frac{1}{1 + e^{-(-1.7)(0.7) - (0.1)(1.2)}}$$

$$= \frac{1}{1 + e^{(1.19) - (0.12)}} \qquad = \frac{1}{1 + e^{1.07}}$$

$$= \frac{1}{1 + e^{1.07}} \qquad = \frac{1}{3.915}$$

$$\boxed{h_1 = 0.255}$$

$$h_2 = \frac{1}{1+e^{-x_3 w_5 - x_4 w_4}}$$

$$= \frac{1}{1+e^{-(4.1)(-0.6)-(2)(-1.8)}}$$

$$= \frac{1}{1+e^{0.66+3.6}}$$

$$= \frac{1}{1+e^{4.26}}$$

$$= \frac{1}{1+70.5} = \frac{1}{71.5}$$

$$\boxed{h_2 = 0.0139}$$

$$S_3 = h_1 w_5 + h_2 w_6$$
$$= (0.255)(-0.2) + (0.0139)(0.5)$$
$$= -0.051 + 0.00695$$
$$\boxed{S_3 = -0.0445}$$

Now,

$$\hat{y} = \frac{1}{1+e^{-h_1 w_5 - h_2 w_6}}$$

$$= \frac{1}{1+e^{-(0.255)(0.2)-(0.0139)(0.5)}}$$

$$= \frac{1}{1+e^{0.051-0.00695}} = \frac{1}{1+e^{0.04405}}$$

$$= \frac{1}{1+e^{-0.053}} = \frac{1}{1+1.045} = \frac{1}{2.045}$$

$$\boxed{\hat{y} = 0.4889}$$

gradient of L_2 Loss function

$$\|\hat{y} - y\|^2 \text{ is } 2\|\hat{y} - y\| = \frac{d\epsilon}{dy}$$

Now using Backward propagation

$$\frac{d\epsilon}{dw_1} = \frac{d\epsilon}{d\hat{y}} \times \frac{d\hat{y}}{ds_3} \times \frac{ds^3}{dh_1} \times \frac{dh^1}{ds_1} \times \frac{ds^1}{dw_1}$$

$$\frac{d\epsilon}{dw_1} = 2\|\hat{y} - y\| \times \sigma'(s_3) \times w_5 \times \sigma'(s_1) \times x_1$$

$$w_5 = \frac{ds_3}{dh_1} \qquad x_1 = \frac{ds_1}{dw_1}$$

$$\sigma'(s_3) = \sigma(s_3)(1 - \sigma s_3) \qquad \sigma' s_1 = \sigma(s_1)(1 - \sigma(s_1))$$

$$\frac{d\epsilon}{dw_1} = 2\|0.4889 - 0.5\| \times \sigma s_3 (1 \cdot \sigma s_3)\| \times (-0.2) \times [\sigma(s_1)(1 - \sigma(s_1)] \times (0.7)$$

$$= 2(0.107) \times [(0.2554)(1 - 0.2554)] \times 0.2 \times [(0.4884)(1 - 0.4884)] \times 0.7$$

$$\boxed{\frac{d\epsilon}{dw_1} = -0.00114}$$