Introduction to Financial Engineering

Report

Group 29

Project 2: Markowitz portfolio optimization

Team Members:

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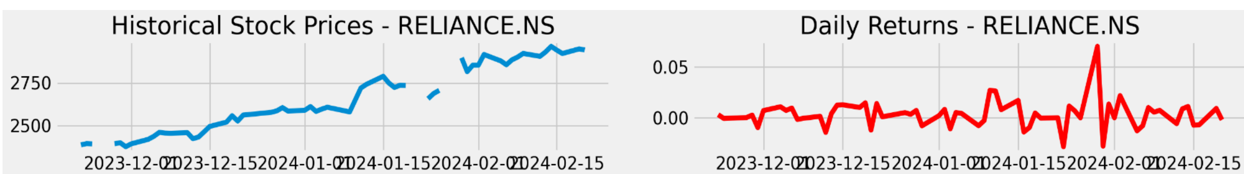
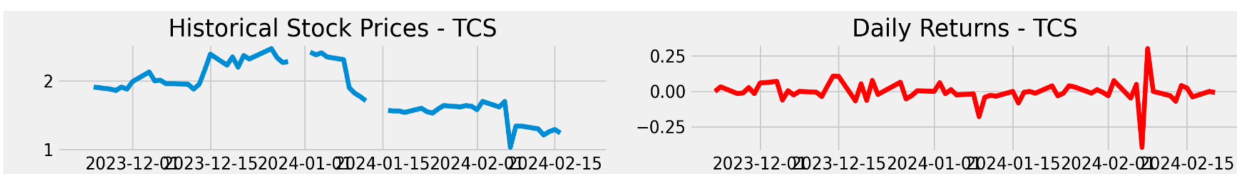
Description of the Chosen Assets

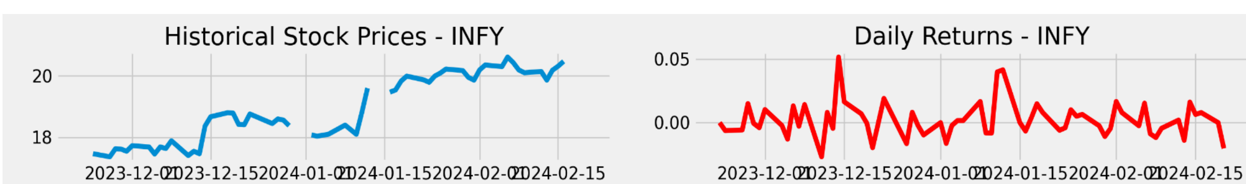
- **RELIANCE.NS -**



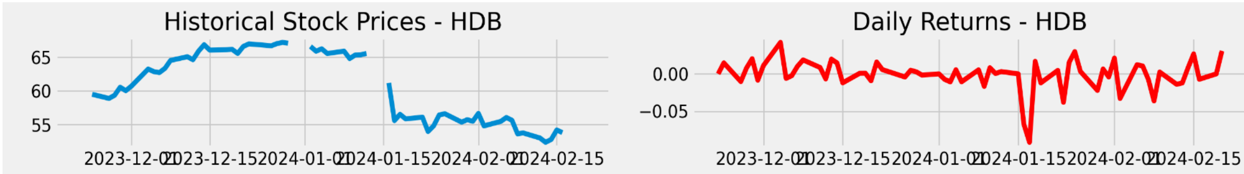
- **TCS**



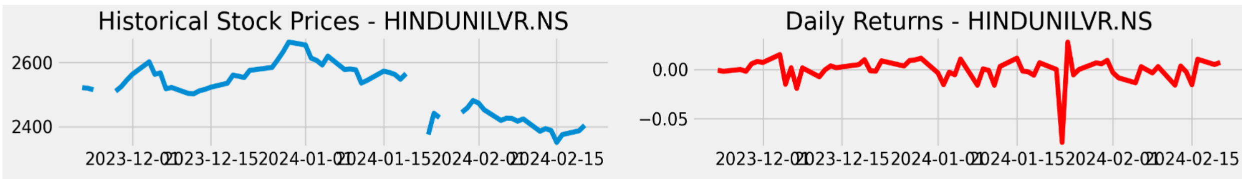
- **INFY**



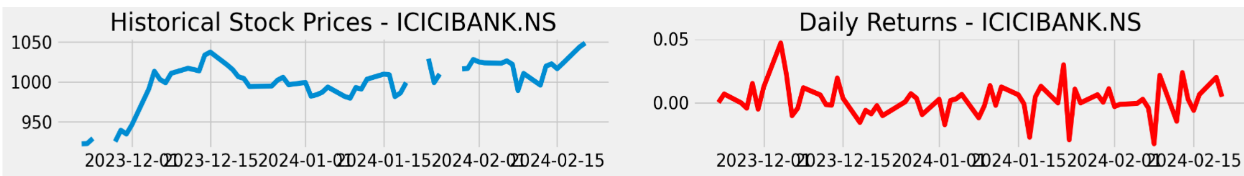
- **HDB**



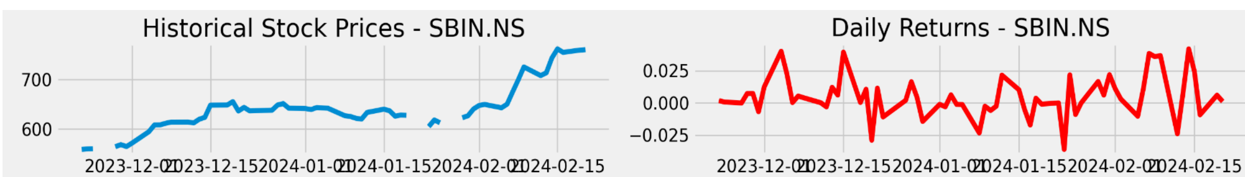
- **HINDUNILVR.NS**



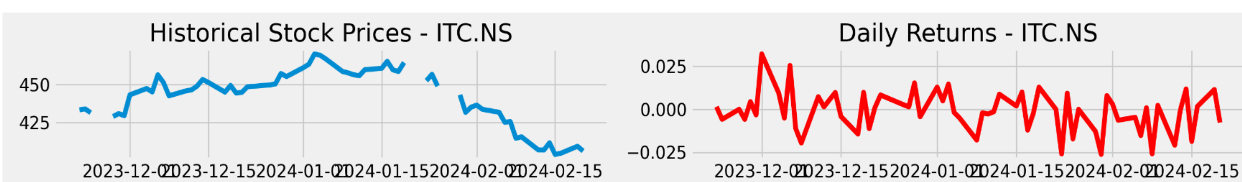
- **ICICIBANK.NS**



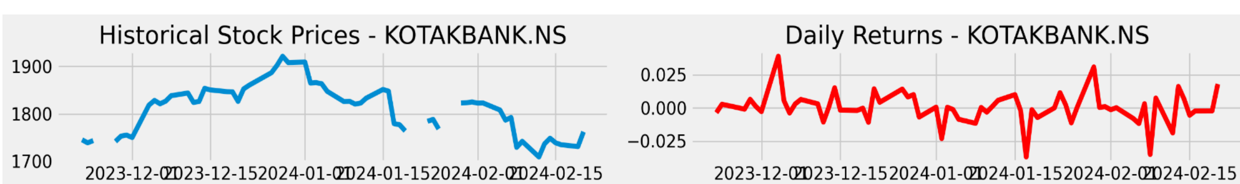
- **SBIN.NS**



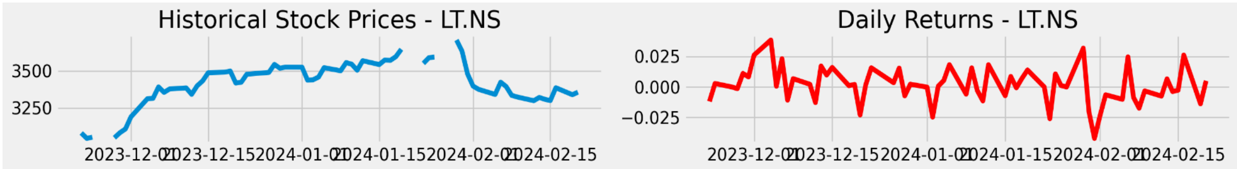
- **ITC.NS**



- **KOTAKBANK.NS**



- **LT.NS**



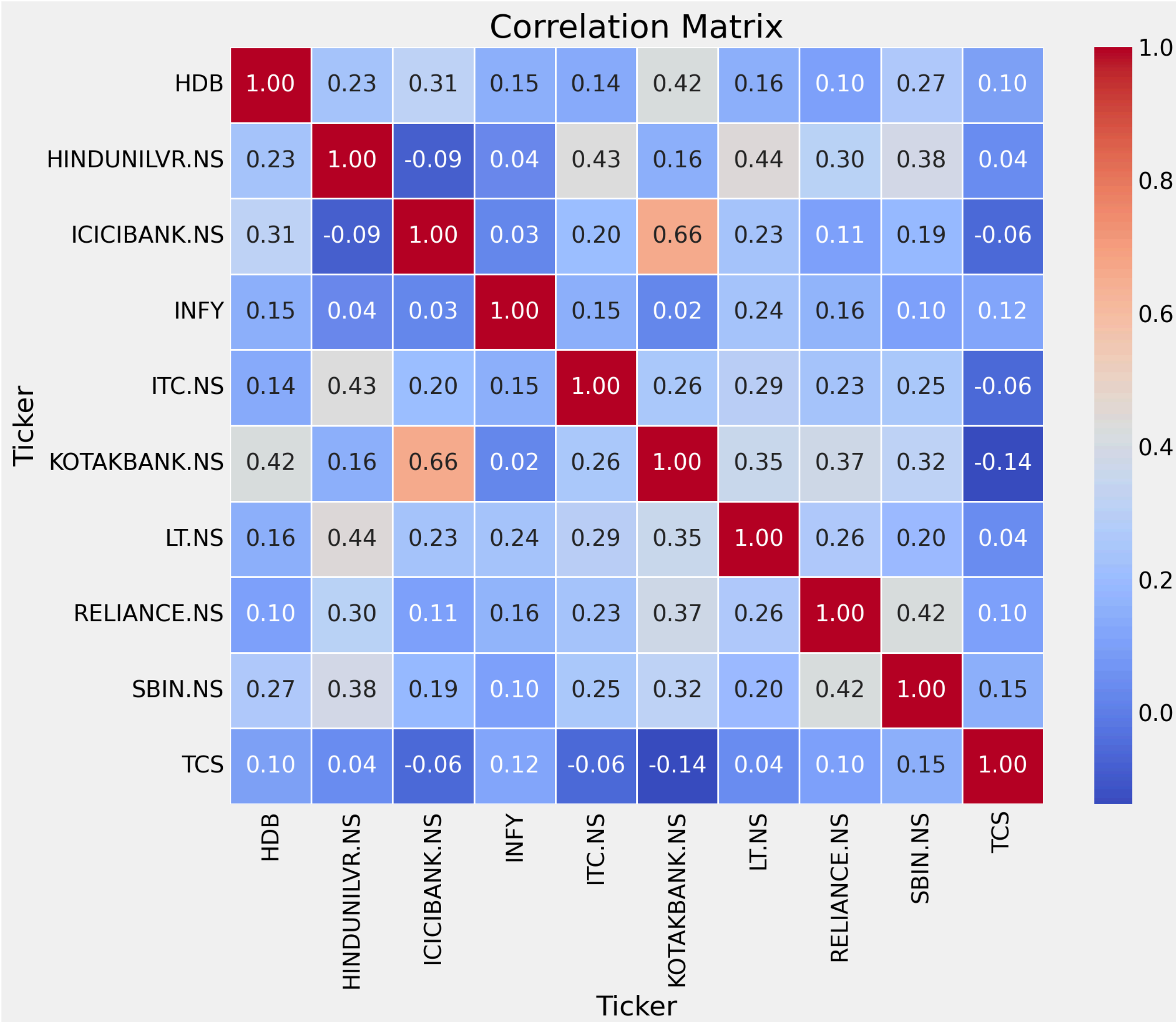
Mean Returns for each Asset over 3 months

```
Ticker
HDB              -0.000669
HINDUNILVR.NS    -0.000676
ICICIBANK.NS      0.002120
INFY              0.002188
ITC.NS           -0.000961
KOTAKBANK.NS      0.000223
LT.NS             0.001464
RELIANCE.NS       0.003407
SBIN.NS           0.005018
TCS              -0.003046
```

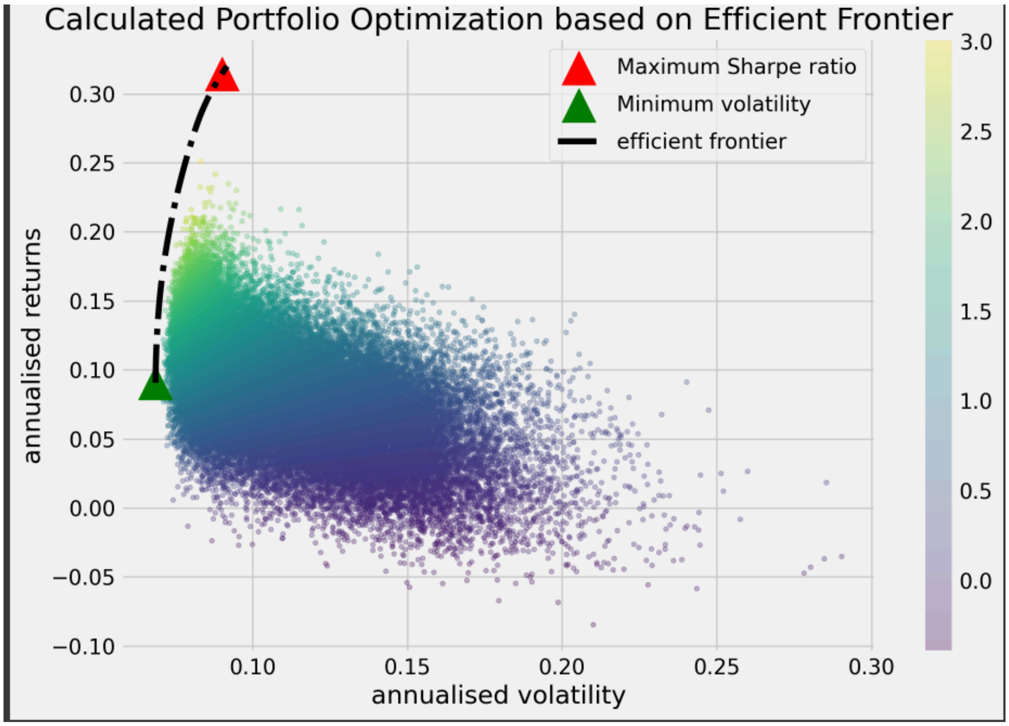
Risk Measure for each Asset over 3 months

```
Ticker
HDB              0.020813
HINDUNILVR.NS    0.012786
ICICIBANK.NS     0.013231
INFY             0.013977
ITC.NS           0.011789
KOTAKBANK.NS     0.011946
LT.NS            0.014925
RELIANCE.NS      0.013689
SBIN.NS          0.016147
TCS              0.078846
dtype: float64
```

Correlation Matrix



Graph Showing the Markowitz Efficient Frontier and the two chosen points



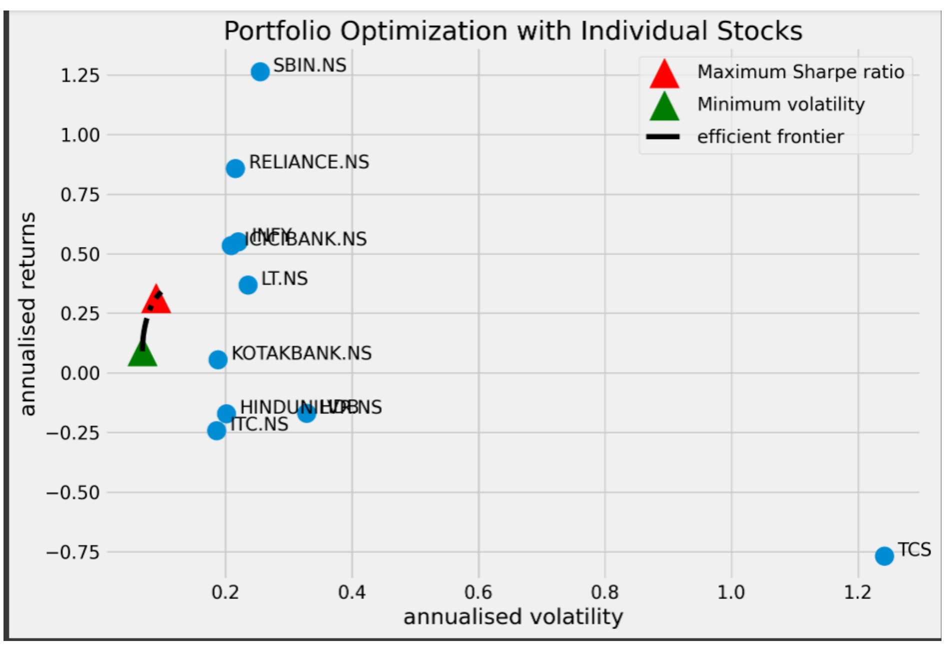
Optimal portfolio weights for each chosen point on the efficient frontier

Maximum Sharpe Ratio Portfolio: This point represents the portfolio with the highest Sharpe ratio, which measures risk-adjusted return. It indicates the portfolio allocation that provides the best trade-off between risk and return.

Minimum Volatility Portfolio: This point represents the portfolio with the lowest volatility or risk. It signifies the portfolio allocation that minimizes risk while aiming to achieve a reasonable level of return. These two points are crucial in portfolio optimization as they represent different objectives: maximizing returns for a given level of risk (Max Sharpe Ratio) and minimizing risk for a given return (Min Volatility). They help investors make informed decisions based on risk tolerance and investment goals.

```
Maximum Sharpe Ratio Portfolio Allocation

Annualised Return: 0.31
Annualised Volatility: 0.09


Ticker      HDB  HINDUNILVR.NS  ICICIBANK.NS   INFY  ITC.NS  KOTAKBANK.NS  \
allocation  0.0            0.0         19.77  20.47     0.0           0.0

Ticker      LT.NS  RELIANCE.NS  SBIN.NS  TCS
allocation    0.0         23.2    36.56  0.0
--------------------------------------------------------------------------------
Minimum Volatility Portfolio Allocation

Annualised Return: 0.09
Annualised Volatility: 0.07


Ticker      HDB  HINDUNILVR.NS  ICICIBANK.NS   INFY  ITC.NS  KOTAKBANK.NS  \
allocation  0.0          23.58         22.93  21.75   12.94          8.58

Ticker      LT.NS  RELIANCE.NS  SBIN.NS  TCS
allocation    0.0         9.63      0.0  0.6
```

Individual Stocks return and volatility



A trade-off between risk and return in your portfolio choices

In the context of the constructed portfolios using Markowitz's mean-variance optimization, the efficient frontier illustrates the optimal combinations of assets that provide the maximum expected return for a given level of risk or the minimum risk for a given level of expected return.

The two chosen points (here used Maximum Sharpe Ratio Portfolio and Minimum Volatility Portfolio) on the efficient frontier represent different risk tolerance levels. As we move along the efficient frontier from left to right, we encounter portfolios with increasing expected returns but also higher levels of risk. The trade-off becomes evident as investors must decide the level of risk they are willing to bear to achieve a desired level of return.

The concept behind Markowitz's Optimization

Basically we have to solve the following constrained optimization problem

$$Obj\ \sigma^2 \quad \text{such that} \quad OW^T = 1, \mu = MW^T$$

By applying Lagrangian to the above problem,

$$L = WCW^T - \lambda_1(OW^T - 1) - \lambda_2(MW^T - \mu)$$

Now solving the lagrangian,

$$1 = \frac{\lambda_1}{2} OC^{-1}O^T + \frac{\lambda_2}{2} OC^{-1}M^T \qquad \text{---------> Equation 1}$$

$$\mu = \frac{\lambda_1}{2} MC^{-1}O^T + \frac{\lambda_2}{2} MC^{-1}M^T \qquad \text{---------> Equation 2}$$

Let us assume,

$$\alpha = OC^{-1}O^T \qquad \gamma = MC^{-1}O^T$$

$$\beta = OC^{-1}M^T \qquad \delta = MC^{-1}M^T$$

Now Equation 1 and 2 will become,

$$1 = \frac{\lambda_1}{2}\alpha + \frac{\lambda_2}{2}\beta$$

$$\mu = \frac{\lambda_1}{2}\gamma + \frac{\lambda_2}{2}\delta$$

Solving the above Equations gives us,

$$\lambda_2 = \frac{2(\gamma - \alpha\mu)}{(\beta\gamma - \alpha\delta)}$$

$$\lambda_1 = 2\,\alpha^{-1}\left(1 - \frac{\lambda_2}{2}\beta\right)$$

$$W_{min}^T = \frac{\lambda_1}{2}\,C^{-1}O^T + \frac{\lambda_2}{2}C^{-1}\mu^T$$

Limitations of Markowitz Optimization

- **Assumption of Normality** - MPT is mean-variance theory. Risk by variance works when returns are normally distributed. Assets that do not follow a normal distribution will not work with the Markowitz Model. Financial returns often exhibit non-normal characteristics such as skewness, kurtosis, and fat tails, leading to inaccuracies in the optimization process.
- **Sensitivity to Inputs** - Markowitz optimization is highly sensitive to the inputs, particularly the expected returns and covariance matrix of assets. Small changes in these inputs can lead to vastly different optimal portfolios, which may not be robust in real-world conditions.
- **Estimation Error** - Estimating expected returns and covariance matrix involves historical data, which may not accurately represent future market

conditions. Estimation errors can lead to suboptimal portfolios or increased volatility.
- **No Consideration of Transaction Costs or Constraints** - Markowitz optimization typically does not account for transaction costs, taxes, or other constraints such as liquidity or regulatory requirements. Ignoring these factors can result in impractical or costly portfolios to implement.
- **Single Period Framework** - Markowitz's optimization is based on a single-period framework and does not consider dynamic changes in investor preferences or market conditions over time. This limitation can lead to suboptimal long-term investment strategies.
- **Finding a practical mix of asset allocations** to achieve efficient frontier optimization may be difficult and doesn't guarantee the portfolio will perform as intended

Real-World Applications of Markowitz Optimization

- **Asset Allocation** - Markowitz optimization is widely used to determine the optimal asset allocation mix within investment portfolios. By considering the expected returns and correlations between different asset classes, such as stocks, bonds, and commodities, investors can construct diversified portfolios that maximize returns for a given level of risk
- **Portfolio Construction** - Investment managers use Markowitz optimization to construct portfolios that meet specific investor objectives and risk preferences. Managers can create efficient portfolios that balance risk and return objectives by optimizing the weights assigned to individual assets within a portfolio.
- **Risk Management** - Markowitz optimization is utilized for risk management purposes, allowing investors to identify and mitigate sources of portfolio risk. By analyzing the covariance structure of asset returns, investors can construct portfolios that minimize exposure to specific risk factors or market downturns
- **Mutual Fund Management** - Mutual fund managers utilize Markowitz optimization to construct diversified portfolios that adhere to specific investment mandates and regulatory requirements. By optimizing portfolio weights within the constraints of fund objectives, managers aim to deliver superior risk-adjusted returns to investors

- **Pension Fund Management** - Pension funds employ Markowitz optimization to efficiently manage large pools of assets and liabilities. By optimizing asset allocation to meet long-term return targets while minimizing funding shortfalls and volatility, pension funds aim to fulfill their obligations to retirees
- **Index Fund Replication** - Index fund managers use Markowitz optimization to replicate the performance of benchmark indices while minimizing tracking error and transaction costs. By optimizing portfolio weights to match the composition of target indices closely, index fund managers aim to provide low-cost, passive investment vehicles to investors