# Project-2 (Markowitz Portfolio Optimization)

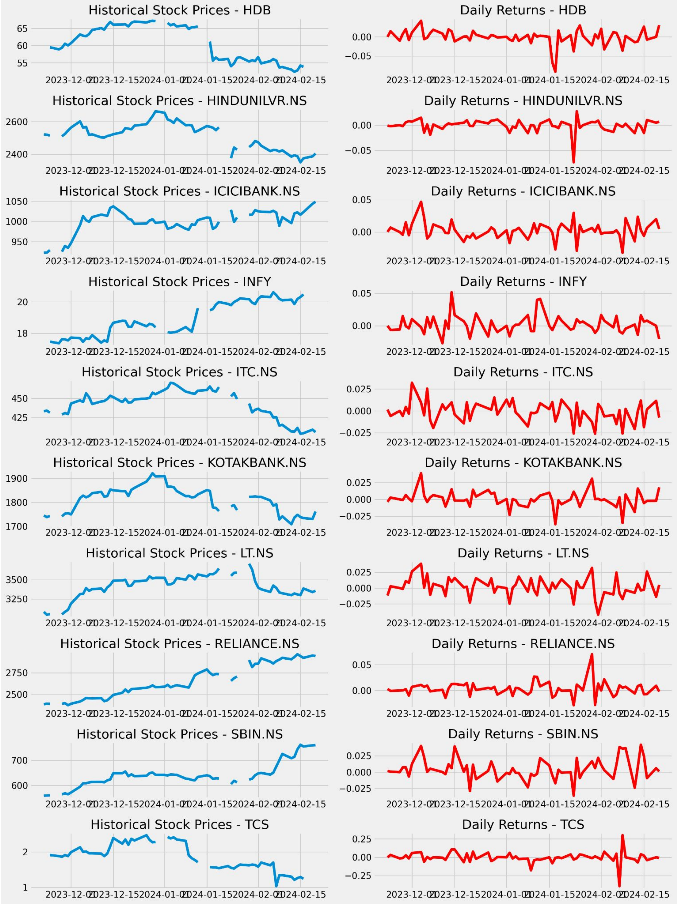
**Group 29**
Devendra Jadhav
Nakkina Vinay
Nitish Bharadwaj
Mayank Jonwal

# Overview

- Choosing the Assets
- Returns and risk measures of each asset
- Exploratory Data analysis
- Graph showing the Markowitz efficient frontier and the two chosen points
- Optimal portfolio weights for each chosen point on the efficient frontier
- Individual Stocks and volatility
- Concept behind Markowitz Optimization
- Trade-off between risk and return
- Limitations of Markowitz optimization
- Real-world Applications

# Chosen assets

- RELIANCE.NS
- TCS
- INFY
- HDB
- HINDUNILVR.NS
- ICICIBANK.NS
- SBIN.NS
- ITC.NS
- KOTAKBANK.NS
- LT.NS



Historical Stock Prices - HDB
Daily Returns - HDB
Historical Stock Prices - HINDUNILVR.NS
Daily Returns - HINDUNILVR.NS
Historical Stock Prices - ICICIBANK.NS
Daily Returns - ICICIBANK.NS
Historical Stock Prices - INFY
Daily Returns - INFY
Historical Stock Prices - ITC.NS
Daily Returns - ITC.NS
Historical Stock Prices - KOTAKBANK.NS
Daily Returns - KOTAKBANK.NS
Historical Stock Prices - LT.NS
Daily Returns - LT.NS
Historical Stock Prices - RELIANCE.NS
Daily Returns - RELIANCE.NS
Historical Stock Prices - SBIN.NS
Daily Returns - SBIN.NS
Historical Stock Prices - TCS
Daily Returns - TCS

# Returns and Risk Measures of Each Asset

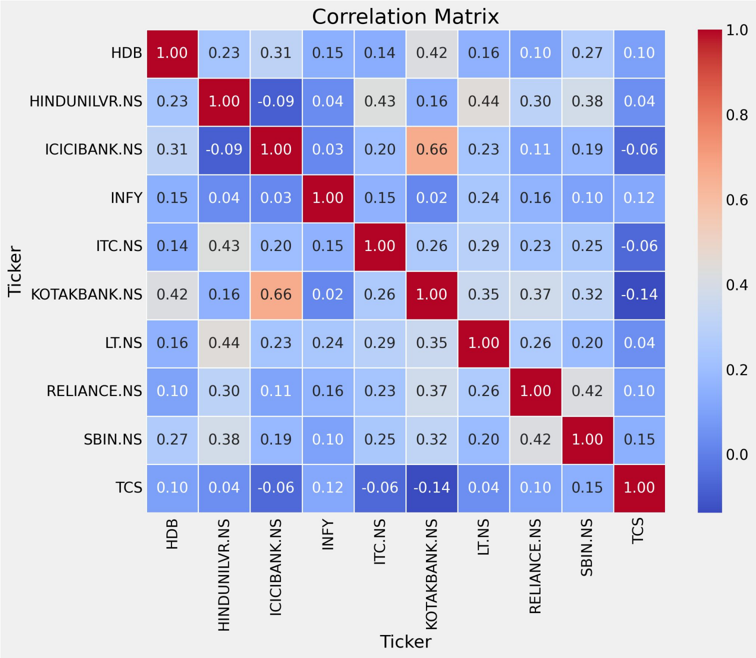
Mean Returns of the assets over 3 months

```
Ticker
HDB             -0.000669
HINDUNILVR.NS   -0.000676
ICICIBANK.NS     0.002120
INFY             0.002188
ITC.NS          -0.000961
KOTAKBANK.NS     0.000223
LT.NS            0.001464
RELIANCE.NS      0.003407
SBIN.NS          0.005018
TCS             -0.003046
```

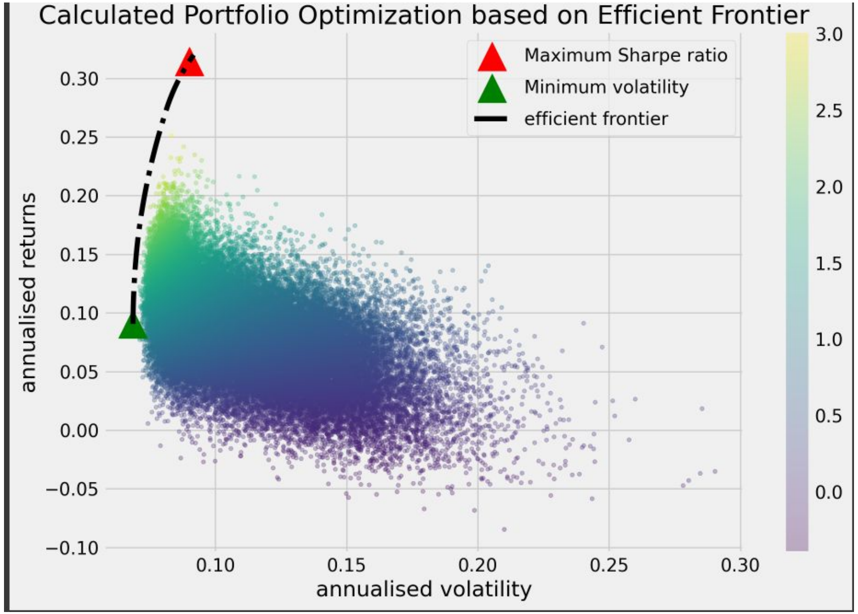
Risk Measure of the assets over 3 months

```
Ticker
HDB              0.020813
HINDUNILVR.NS    0.012786
ICICIBANK.NS     0.013231
INFY             0.013977
ITC.NS           0.011789
KOTAKBANK.NS     0.011946
LT.NS            0.014925
RELIANCE.NS      0.013689
SBIN.NS          0.016147
TCS              0.078846
dtype: float64
```

# Exploratory Data Analysis



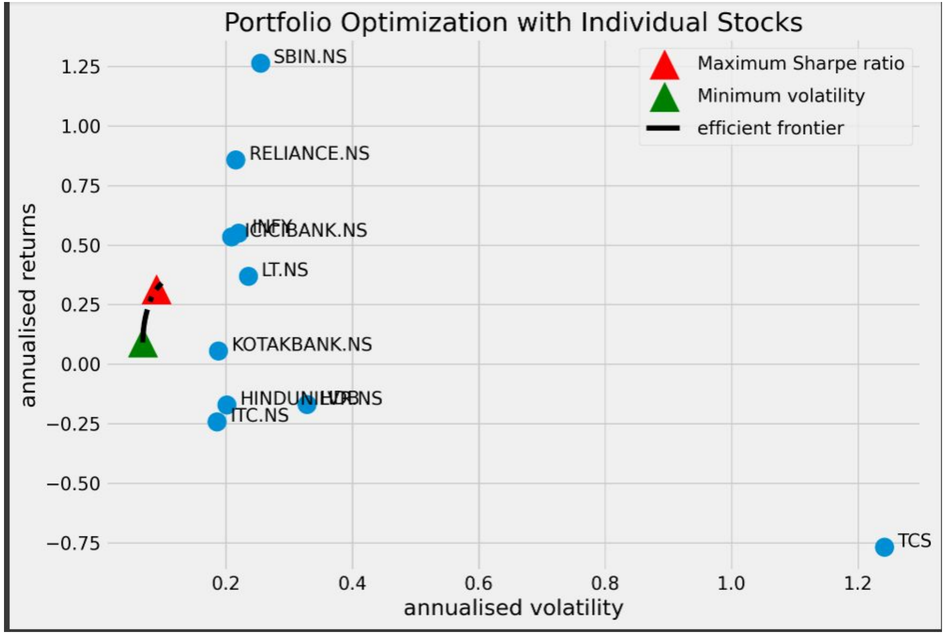
# Efficient Frontier with two chosen points



# Optimal portfolio weights for chosen points

```
Maximum Sharpe Ratio Portfolio Allocation

Annualised Return: 0.31
Annualised Volatility: 0.09


Ticker      HDB  HINDUNILVR.NS  ICICIBANK.NS   INFY  ITC.NS  KOTAKBANK.NS  \
allocation  0.0            0.0         19.77  20.47     0.0           0.0

Ticker      LT.NS  RELIANCE.NS  SBIN.NS  TCS
allocation    0.0         23.2    36.56  0.0
--------------------------------------------------------------------------------
Minimum Volatility Portfolio Allocation

Annualised Return: 0.09
Annualised Volatility: 0.07


Ticker      HDB  HINDUNILVR.NS  ICICIBANK.NS   INFY  ITC.NS  KOTAKBANK.NS  \
allocation  0.0          23.58         22.93  21.75   12.94          8.58

Ticker      LT.NS  RELIANCE.NS  SBIN.NS  TCS
allocation    0.0         9.63      0.0  0.6
```

# Individual Stocks return and volatility



# Concept behind Markowitz Optimization

Basically we have to solve the following constrained optimization problem

$$Obj\ \sigma^2 \quad \text{such that } OW^T = 1, \mu = MW^T$$

By applying Lagrangian to the above problem,

$$L = WCW^T - \lambda_1(OW^T - 1) - \lambda_2(MW^T - \mu)$$

Now solving the lagrangian,

$$1 = \frac{\lambda_1}{2} OC^{-1}O^T + \frac{\lambda_2}{2} OC^{-1}M^T$$ ————-> Equation 1

$$\mu = \frac{\lambda_1}{2} MC^{-1}O^T + \frac{\lambda_2}{2} MC^{-1}M^T$$ ————-> Equation 2

Let us assume,

$$\alpha = OC^{-1}O^T \qquad \gamma = MC^{-1}O^T$$

$$\beta = OC^{-1}M^T \qquad \delta = MC^{-1}M^T$$

Now Equation 1 and 2 will become,

$$1 = \frac{\lambda_1}{2}\alpha + \frac{\lambda_2}{2}\beta$$

$$\mu = \frac{\lambda_1}{2}\gamma + \frac{\lambda_2}{2}\delta$$

Solving the above Equations gives us,

$$\lambda_2 = \frac{2(\gamma - \alpha\mu)}{(\beta\gamma - \alpha\delta)}$$

$$\lambda_1 = 2\,\alpha^{-1}\left(1 - \frac{\lambda_2}{2}\beta\right)$$

$$W_{min}^T = \frac{\lambda_1}{2}\,C^{-1}O^T + \frac{\lambda_2}{2}C^{-1}\mu^T$$

# A brief discussion of the trade-off between risk and return in your portfolio choices.

In the context of the constructed portfolios using Markowitz's mean-variance optimization, the efficient frontier illustrates the optimal combinations of assets that provide the maximum expected return for a given level of risk or the minimum risk for a given level of expected return.

The two chosen points (here used Maximum Sharpe Ratio Portfolio and Minimum Volatility Portfolio) on the efficient frontier represent different risk tolerance levels. As we move along the efficient frontier from left to right, we encounter portfolios with increasing expected returns but also higher levels of risk. The trade-off becomes evident as investors must decide the level of risk they are willing to bear to achieve a desired level of return.

# Limitations of Markowitz optimization

- Assumption of Normality
- Sensitivity to Inputs
- Estimation Error
- No Consideration of Transaction Costs or Constraints
- Single Period Framework
- Finding a practical mix of asset allocations

# Real-world Applications of Markowitz optimization

- Asset Allocation
- Portfolio Construction
- Risk Management
- Mutual Fund Management
- Pension Fund Management
- Index Fund Replication

Markowitz optimization remains a valuable tool for constructing diversified portfolios and managing investment risk

# Thank You