Structural Crack Analysis Using Bilinear Dynamics

B.Tech Project Report submitted
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by

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under the guidance of

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<u>**Certificate**</u>

*This is to certify that the work contained in this thesis titled "**Structural Crack Analysis Using Bilinear Dynamics**" is a bonafide research work of Yashraj (2101ME84) and Vinay Sobarad (2101ME79), carried out in the Department of Mechanical Engineering, Indian Institute of Technology Patna, under my supervision and that it has not been submitted elsewhere for a degree.*

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This is to certify that Mr. Yashraj and Vinay Sobarad.

1. have sincerely worked on their project,
2. have contacted me regularly to update me on the progress of the assigned project,
3. have received my comments on the preliminary version of the report and presentation and will address those prior to final submission/presentation,
4. be allowed to present/defend the project before the department.

Remarks (if any):

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Abstract

This report presents an investigation into the bilinear dynamics behavior in a spring-mass system, designed to replicate the bilinear dynamics of cantilever beams. The study focuses on the transition from linear to nonlinear behavior, where small vibrations induce linear responses, and increased displacement introduces nonlinear effects due to changes in the effective spring constant (k). The experimental setup will involve precise measurement of the beam's displacement and frequency responses using advanced sensors and analysis tools. Displacement-time and frequency-displacement plots will be generated to capture the system's behavior across different vibrational amplitudes. These experimental results will be systematically compared with the theoretical and numerical models (equivalent spring-mass system) developed to validate predictions about nonlinearities, including resonance shifts and energy conservation patterns. This investigation aims to provide critical insights into the influence of structural imperfections, such as cracks, on the dynamic performance of real-world structures. The outcomes are expected to enhance the understanding of bilinear dynamics, offering valuable contributions to the fields of structural engineering and applied mechanics.

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Nomenclature

ω_{n1} = Semi-major axis of the ellipse, mm
ω_{n2} = Semi-minor axis of the ellipse, mm
ω_{n3} = Radius of the circle, mm
k_1 = Spring Constant of Spring 1
k_2 = Spring Constant of Spring 2
k_3 = Spring Constant of Spring 3
k_4 = Spring Constant of Spring 4
k_{eff1} = Effective Spring Constant of Spring 1 and Spring 3
k_{eff2} = Effective Spring Constant of Spring 1, Spring2 and Spring 3
k_{eff3} = Effective Spring Constant of Spring 1, Spring3 and Spring 4

Greek Symbols

ω = frequency

Chapter 1

Introduction

1.1 Introduction

Investigating cracks in structures using bilinear dynamics is crucial for understanding how these imperfections affect a structure's stability and dynamic behaviour. Cracks introduce nonlinearity into the system, particularly through mechanisms such as crack breathing, where the crack opens and closes in response to cyclic loads. This behaviour results in bilinear dynamics, with the structure exhibiting different stiffness when the crack is open versus when it is closed. These nonlinearities can lead to complex and unbounded oscillatory patterns, and under certain conditions, can give rise to chaotic behaviour. In such cases, even small variations in load or displacement can cause unpredictable and irregular oscillations. This chaotic response is often marked by sensitivity to initial conditions and non-periodic vibrations. Understanding this dynamic behaviour is critical for structural health monitoring, as it helps detect cracks and assess the risk of failure in bridges, buildings, and other critical infrastructure. By studying how cracks influence oscillations and chaotic dynamics, engineers can develop more accurate models for predicting structural failure and ensuring safety.

1.1.1 Literature Review

Sr. No.	Author	Journal Name	Title
1.	S. W. Shaw	Journal of Sound and Vibration. (1985)	Forced Vibrations of a Beam with One-Sided Amplitude Constraint: Theory and Experiment
2.	R. M. Lin and D. J. Ewins	Mechanical Systems and Signal Processing. (1993)	Chaotic Vibration of Mechanical Systems with Backlash

1.1.2 Comments

The research (Forced Vibrations of a Beam with One-Sided Amplitude Constraint: Theory and Experiment) explores the dynamics of an elastic beam constrained on one side and subjected to periodic external excitation, focusing on its non-linear behaviours, including subharmonic resonances, period doublings, and chaotic motion. A theoretical framework based on a single-mode model was established, which was then validated against experimental data obtained from an electromagnetic shaker setup. Strain gauges were strategically placed at the beam's base to measure the resulting motion, allowing for qualitative analysis of the periodic and chaotic behaviours exhibited by the beam.

The results demonstrate that the simple single-mode model can effectively predict the system's response to periodic forcing, capturing the influence of non-linearities in the restoring force. Overall, the research contributes to the understanding of complex vibrational behaviours in constrained systems and highlights the usefulness of straightforward models in analysing such phenomena.

Another research (Chaotic Vibration of Mechanical Systems with Backlash) focuses on the chaotic behaviour of non-linear mechanical systems that exhibit backlash, which is a common phenomenon in mechanical components where there is a gap between parts. It was found that if certain excitation conditions are met, the system can display chaotic vibrations, making it difficult to predict its behaviour. The study employed numerical methods and experimental analyses to explore this chaos, utilizing tools like Poincaré maps and Lyapunov exponents to confirm the chaotic nature of the motion.

The parameters of the system, such as mass, stiffness, and damping, were varied to observe their effects on the chaotic behaviour. The findings indicate that there are extensive regions of forcing and system parameters that can lead to chaotic responses. The experimental validation of these chaotic behaviours in a simple mechanical structure underlines the practical implications of the research, suggesting that engineers should carefully design mechanical systems to avoid unintended chaos and consider using statistical methods for stress analysis to ensure reliability and predictability in their performance.

1.1.3 Motivation

The study of cracks in structures using bilinear dynamics is essential in addressing critical issues such as gear backlash, loose parts, and structural damage, as we learn from the papers discussed above, particularly in systems like cantilever beams with cracks. These cracks introduce nonlinear behaviour, affecting the dynamic response of mechanical systems and leading to irregular oscillations or chaotic behaviour, especially when components are misaligned or loose. In gear systems, backlash can cause vibration anomalies, while loose parts can further amplify dynamic instability, risking failure. In structures like cantilever beams, cracks significantly alter stiffness and dynamic behaviour, making early detection vital for ensuring system integrity. The challenge lies in accurately modelling these nonlinearities and detecting structural damage in real time, all while balancing the need for cost-effective and scalable solutions to monitor and predict failures in diverse mechanical and structural systems.

1.1.4 Objective

The objective of this report is to analyse the bilinear dynamics of cracks in structures by examining a nonlinear spring-mass system. The system involves four springs and one block, where crack behaviour is modelled through the engagement of springs based on displacement thresholds. Springs 1 and 3 are always connected, while Springs 2 and 4 simulate crack-induced nonlinearity by engaging beyond specific displacements. Differential equations were used to model the system's behaviour, and numerical simulations were conducted to study frequency and displacement responses under various initial conditions. The analysis also focuses on energy conservation in scenarios where one spring exhibits high stiffness, mimicking the effects of crack propagation. Frequency vs. displacement plots and resonance analysis were generated to explore how structural dynamics are influenced by crack-induced nonlinearities.

Chapter 2

Methodology

2.1 Bilinear Free Vibration

2.1.1 Setup

The setup involves a nonlinear mechanical system consisting of four springs connected to a single block. Each spring has distinct characteristics, and the engagement of some springs is conditional upon the displacement of the block. The springs are labelled as Spring 1, Spring 2, Spring 3, and Spring 4, with each having its spring constant denoted as k_1 , k_2, k_3, and k_4 respectively.

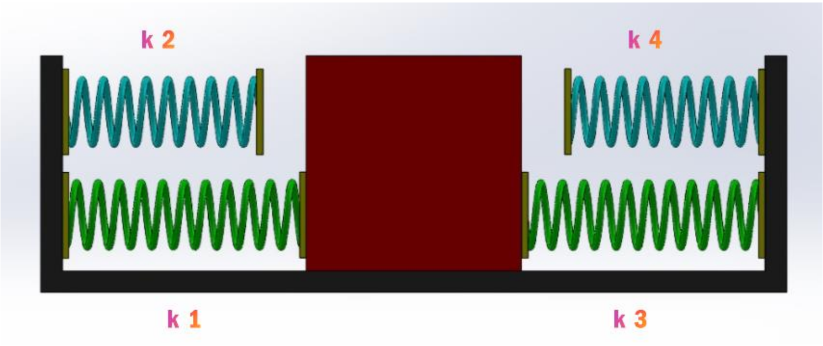


Figure 2.1: Bilinear Free Vibration Setup

- **Spring 1 and Spring 3:** Always connected to the block, contributing to the system's stiffness regardless of the block's displacement.
- **Spring 2 and Spring 4:** Engage only when the block's displacement exceeds a defined threshold, *d* either in the positive or negative direction.
- **Engagement distance *(d)*:** The critical displacement at which additional springs (Spring 2 or Spring 4) engage. For this system, *d* is a specified distance beyond which these springs contribute to the overall restoring force.
- **Nonlinear behaviour:** The system exhibits nonlinear dynamics as the stiffness changes when the block moves beyond the engagement distance, activating Spring 2 or Spring 4.

2.1.2 Analysis

Case 1: (if Id < d)

$$Keff1 = k1 + k3 \qquad \text{.......... (1)}$$

$$\omega 1 = \frac{1}{2\pi} * \sqrt{\frac{Keff1}{m}} \qquad \text{.......... (2)}$$

Equation of motion:

$$Id \sin\left(\frac{1}{2\pi} * t\right) \qquad \text{.......... (3)}$$

Case 2: (if Id >= d, i.e. the loose springs get engaged in the motion)

Let’s divide the whole motion into 3 parts:

Part 1: (when the right-hand side spring is engaged)

$$Keff2 = k1 + k3 + k4 \qquad \text{.......... (4)}$$

$$\omega 2 = \frac{1}{2\pi} * sqrt\sqrt{\frac{Keff2}{m}} \qquad \text{.......... (5)}$$

Equation of motion:

$$Id \sin(\omega 2 * t) \qquad \text{.......... (6)}$$

Time taken to go from Id to d:

$$\Delta t1 = \sin^{-1}\left(\frac{Id}{Id}\right)\frac{1}{\omega 2} - \sin^{-1}\left(\frac{d}{Id}\right)\frac{1}{\omega 2} \qquad \text{.......... (7)}$$

Part 2: (block loses contact with the spring 2 and continue in natural oscillation)

$$Keff1 = k1 + k3 \qquad \text{.......... (8)}$$

$$\omega 1 = \frac{1}{2\pi} * \sqrt{\frac{Keff1}{m}} \qquad \text{.......... (9)}$$

Here the initial velocity is not equal to zero.

Therefore, to account for the energy from the k2 spring we use energy conservation to get the effective amplitude A_2 for this part:

$$(k1 + k3) * Id^2 + k2 * (Id - d)^2 = (k1 + k3) * A2^2 \qquad \text{……….} (10)$$

$$A2 = sqrt(Id^2 + \frac{k2}{k1 + k3} * (Id - d)^2) \qquad \text{……….} (11)$$

Time taken to go from -d to d:

$$\Delta t2 = 2 * \sin^{-1}\left(\frac{d}{A2}\right)\frac{1}{\omega 1} \qquad \text{……….} (12)$$

Part 3: (when the left-hand side spring is engaged)

$$Keff3 = k1 + k2 + k3 \qquad \text{……….} (13)$$

$$\omega 3 = \frac{1}{2\pi} * \sqrt{\frac{Keff3}{m}} \qquad \text{……….} (14)$$

Here the initial velocity is not zero and therefore we need to apply energy conservation to get the effective amplitude A_3 for this part:

$$(k1 + k3) * A2^2 = (k1 + k3)A3^2 + k2(A3 - d)^2 \qquad \text{……….} (15)$$

$$A3 = \frac{k2 * d \pm \sqrt{(k4 * d)^2 - Keff3 * (k2 * d^2 - Keff1 * A2^2)}}{Keff3} \qquad \text{……….} (16)$$

Time taken to go from -d to -A3:

$$\Delta t3 = \sin^{-1}\left(\frac{A3}{A3}\right)\frac{1}{\omega 3} - \sin^{-1}\left(\frac{d}{A3}\right)\frac{1}{\omega 3} \qquad \text{……….} (17)$$

The net time taken for one complete revolution:

$$\Delta tnet = 2 * (\Delta t1 + \Delta t2 + \Delta t3) \qquad \text{……….} (18)$$

Effective Frequency:

$$\frac{1}{2 * \Delta tnet} \qquad \text{……….} (19)$$

The overall equation of motion, in one equation, would be:

Spring k1 exerts a force F1 = −k1.x to the left.

Spring k3 exerts a force F3 = −k3.x to the left.

Spring k2 will only exert a force if x < -d, and the force is F2 = −k2(x – d) to the right.

Spring k4 will only exert a force if x > d, and the force is F4 = −k4(x – d) to the left.

So, the differential equation, in various cases would be:

$$m\left(\frac{dt2}{d2x}\right) + k1x + k3x = 0 \ \ , -d < x < d \qquad \text{……….} (20)$$

$$m\left(\frac{dt2}{d2x}\right) + k1x + k3x + k4(x-d) = 0\ , \ x > d \qquad \text{……….} (21)$$

$$m\left(\frac{dt2}{d2x}\right) + k1x + k3x + k2(|x|-d) = 0\ , \ x < -d \qquad \text{……….} (22)$$

So, the equation of motion for each case would be:

$$x1 = id * \cos\left(\sqrt{\frac{Keff1}{m}} * t\right) + id(initial\ position) \qquad \text{……….} (23)$$

$$x2 = A2 * \cos\left(\sqrt{\frac{Keff2}{m}} * t + \ \Delta\mathrm{t1}\right) \qquad \text{……….} (24)$$

$$x3 = A3 * \cos\left(\sqrt{\frac{Keff3}{m}} * t + \Delta\mathrm{t1} + \Delta\mathrm{t2}\right) \qquad \text{……….} (25)$$

All the *constants* like A1, A2, keff1, keff2, keff3, etc. are already defined above.

2.2 Bilinear Forced Vibration

2.2.1 Setup

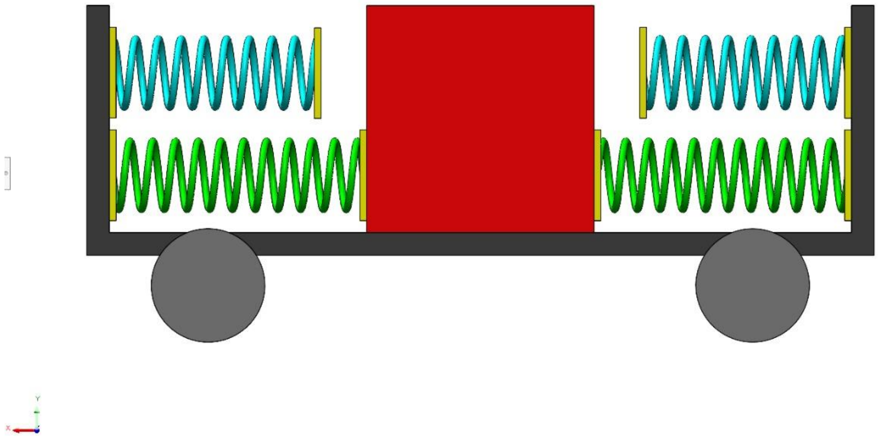


Figure 2.2: Bilinear Forced Vibration Setup

2.2.2 Analysis

Now, consider the whole setup is on a wheel, and there is an external force.

Where force,

$$F= Fe * \sin(\mathrm{wo} * \mathrm{t}) \qquad \text{..........} (26)$$

For now, consider a setup, where only one spring is present so that the solution will be:

$$x(t) = \left(\frac{vo}{wn}\right) * \sin(wn * t) + (xo - \frac{Fe}{m} * \frac{1}{(wn)^2 - (wo)^2)} * \cos(wn * t) + \frac{Fe}{m\left(\frac{1}{(wn)^2 - (w)^2)}\right)} * \cos(wo * t) \qquad \text{..........} (27)$$

Now, consider the whole setup (with all 4 springs)

As we are giving some initial displacement to the spring ‘id’, so there will be a lot of cases, we need to consider, where there will be different engagement and disengagement of the springs (like earlier we had considered)

For $id < -d$(when the left side of the spring is engaged):

$$Keff3 = k1 + k2 + k3 \qquad \text{..........} (13)$$

Here Vo= 0.

$$\omega n3 = \frac{1}{2\pi} * \sqrt{\frac{Keff3}{m}} \qquad \text{..........} (28)$$

Therefore, the solution will be:

$$x(t) = \left(id - \frac{Fe}{m} * \frac{1}{(wn)^2 - (wo)^2)}\right) * \cos(wn3 * t) + \left(\frac{Fe}{m} * \frac{1}{(wn)^2 - (w)^2)}\right) * \cos\,(wo * t)$$

For $|x| < d$:

$$Keff1 \;=\; k1 \;+\; k3 \qquad \text{……….} (1)$$

$$\omega_{n1} = \frac{1}{2\pi} * \sqrt{\frac{Keff1}{m}} \qquad \text{……….} (29)$$

Here V_o is not equal to zero, so:

$$mVo^2 + (k1 + k3) * d^2 + (k1 + k3) * (id)^2 + k4(id - d)^2 \qquad \text{……….} (30)$$

$$Vo = \sqrt{\frac{(k1 + k3)(id^2 - d^2) + k4(id^2 - d^2)}{m}} \qquad \text{……….} (31)$$

Therefore,

$$x(t) = \frac{Vo}{wn2} * (\sin(wn2 * t)) + \left(d - \frac{Fe}{m} * \frac{1}{(wn)^2 - (wo)^2)}\right) * \cos(wn2 * t) + \qquad \text{……….} (32)$$

$$\left(\frac{Fe}{m} * \frac{1}{(wn)^2 - (wo)^2)}\right) * \cos\,(wo * t)$$

For $x > d$:

$$Keff2 = \; k1 \;+\; k3 \;+\; k4 \qquad \text{……….} (33)$$

$$\omega n2 = \frac{1}{2\pi} * \sqrt{\frac{Keff2}{m}} \qquad \text{……….} (34)$$

Assuming from end (|x|= id), that means that Vo= 0

Therefore,

$$x(t) = \left(id - \frac{Fe}{m} * \frac{1}{(wn)^2 - (wo)^2)}\right) * \cos(wn1 * t) + \qquad \text{……….} (35)$$

$$\left(\frac{Fe}{m} * \frac{1}{(wn)^2 - (w)^2)}\right) * \cos\,(wo * t)$$

Chapter 3

Simulation Results and Conclusions

3.1 Numerical Simulation

To investigate the dynamics of the bilinear spring-mass system and understand the influence of nonlinearities caused by cracks in structures, we developed a computational model that simulates the system's motion. The model is based on differential equations governing the displacement and forces in the system, where the engagement of springs reflects the nonlinear, bilinear characteristics of crack behaviour. Specifically, Springs 1 and 3 remain connected to the block, while Springs 2 and 4 engage only beyond specific displacement thresholds, simulating the effect of crack propagation and closure.

The purpose of this code is to generate graphical representations of the system's motion over time, focusing on the displacement and frequency responses under different initial conditions. By varying parameters such as the initial displacement and spring stiffness, we produced a series of plots that illustrate the behaviour of the system as springs engage and disengage. These graphs provide critical insights into how the system responds to varying conditions, highlighting key aspects such as the shifting frequency of oscillations as the displacement changes, and the nonlinearities that arise from crack-induced dynamics.

Through this numerical simulation, the displacement-time graphs and frequency-displacement plots reveal how the system's motion evolves, including the onset of primary resonance and other nonlinear effects. The graphical outputs serve as a crucial tool in understanding the dynamic interaction between the mass and springs, offering a detailed visualization of how cracks influence structural performance. This analysis lays the foundation for further exploration and comparison with experimental data, providing a visual interpretation of the bilinear dynamic behaviour without attempting validation against theoretical models at this stage.

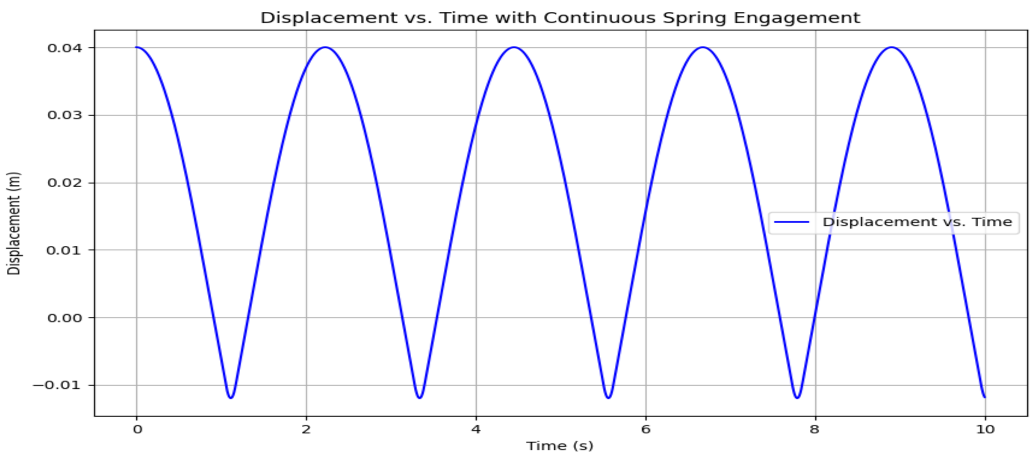
3.1.1 Free Bilinear Vibration

Displacement V/S Time

```
m_val= 1, d_val= 0.01
k1_val= 1, k2_val= 1, k3_val= 1, k4_val= 100
# Define the time span for simulation
tspan = [0, 10], t_eval = np.linspace(0, 10, 1000)# points for smoother plot
# Define the ODE function that dynamically changes based on displacement
def dynamic_ode(t, y):
    x, v = y  # Displacement and velocity
    # Determine which springs are engaged based on the current displacement x
    if abs(x) < d_val:  # Case 1: Not in contact with Spring 2 or Spring 4
        k_eff = k1_val + k3_val
    else:
        if d_val <= x:  # Case 2: Spring 2 engages
            k_eff = k1_val + k2_val + k3_val
        else:  # Case 3: Spring 4 engages
            k_eff = k1_val + k3_val + k4_val

    dxdt = v
    dvdt = -k_eff / m_val * x
    return [dxdt, dvdt]
# Initial conditions: [initial displacement, initial velocity]
initial_conditions_40mm = [0.04, 0]  # 40 mm
# Solve the ODE for the initial condition with increased solver accuracy
sol = solve_ivp(dynamic_ode, tspan, initial_conditions_40mm, t_eval=t_eval, rtol=1e-9, atol=1e-12)
# sol = solve_ivp(dynamic_ode, tspan, initial_conditions_40mm, t_eval=t_eval)
```

Plot Result

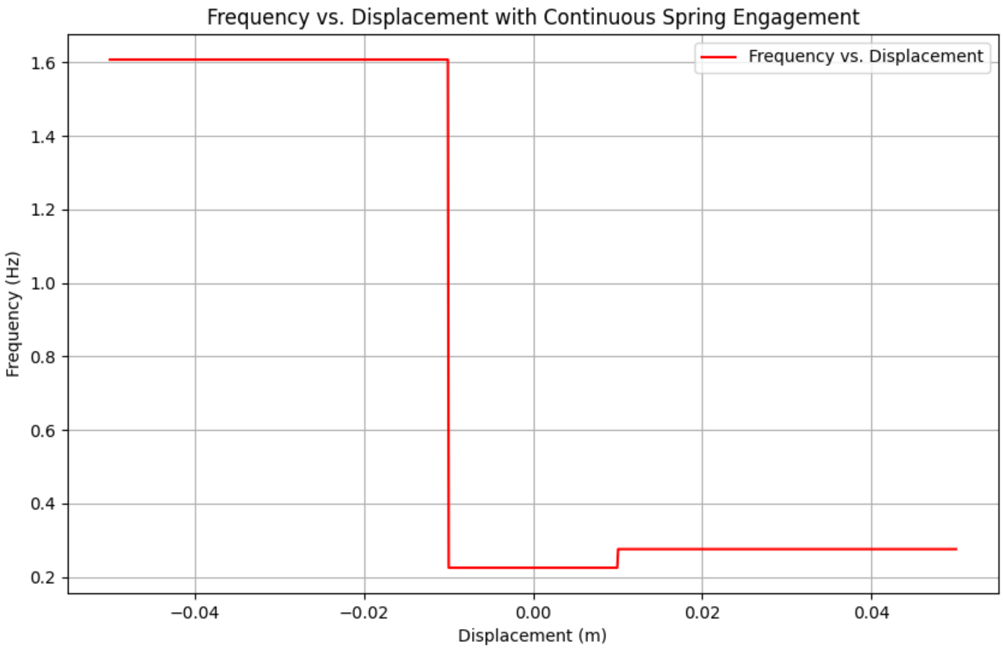


Frequency V/S Displacement

```
m_val= 1, d_val= 0.01
k1_val= 1, k2_val= 1, k3_val= 1, k4_val= 100
# Define a range of displacements to test
displacements = np.linspace(-0.05, 0.05, 1000)  # From -50 mm to 50 mm
# Initialize an array to store frequencies
frequencies = np.zeros_like(displacements)
# Calculate the frequency for each displacement
for i, x in enumerate(displacements):
    # Determine which springs are engaged based on the current displacement x
    if abs(x) < d_val:  # Case 1: Not in contact with Spring 2 or Spring 4
        k_eff = k1_val + k3_val
    else:
        if d_val <= x:  # Case 2: Spring 2 engages
            k_eff = k1_val + k2_val + k3_val
        else:  # Case 3: Spring 4 engages
            k_eff = k1_val + k3_val + k4_val

    # Calculate the natural frequency
    frequency = (1 / (2 * np.pi)) * np.sqrt(k_eff / m_val)
    frequencies[i] = frequency
```

Plot Result

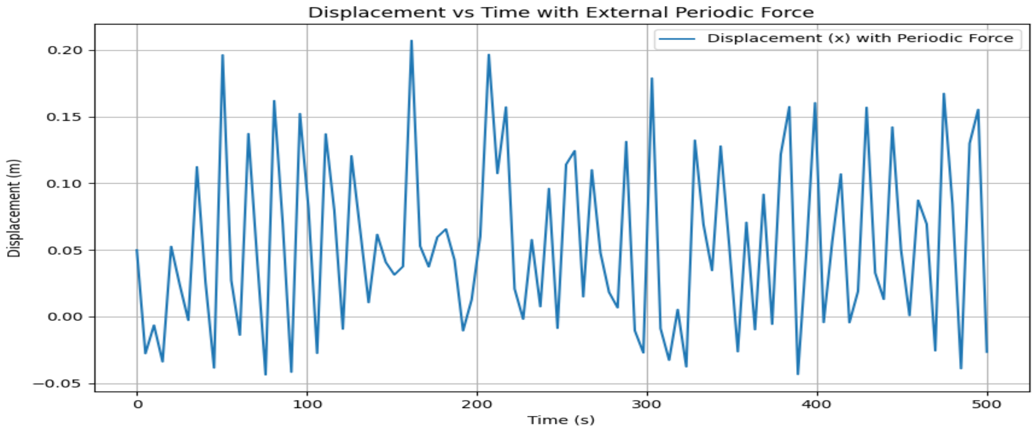


3.1.2 Forced Bilinear Vibrations

Code Snippet

```
# Re-define constants and initial conditions
m = 1.0   # Mass of the block
k1 = 1.0, k2 = 3.0, k3 = 1.0, k4 = 100.0 # Spring constants
x_threshold_k3 = 0.04 # Threshold for k3 to engage
x_threshold_k4 = 0.04 # Threshold for k4 to engage
F0 = 1   # Amplitude of the periodic force
omega = 10 # Angular frequency of the periodic force (adjust as needed)
# Initial conditions
x0 = 0.05 # Initial displacement (m)
v0 = 0.0  # Initial velocity (m/s)
initial_conditions = [x0, v0]
# Time span
t_span = (0, 500)
t_eval = np.linspace(*t_span, 100)
# Modified system of differential equations with external periodic force
def spring_mass_system_with_force(t, y):
    x, v = y
    # Forces from the springs
    F1 = -k1 * x
    F2 = -k2 * x
    F3 = -k3 * (x - x_threshold_k3) if x > x_threshold_k3 else 0
    F4 = -k4 * (x - x_threshold_k4) if x < -x_threshold_k4 else 0
    # External periodic force F(t) = F0 * sin(omega * t)
    F_external = F0 * np.sin(omega * t)
    # Net force including the external periodic force
    F_net = F1 + F2 + F3 + F4 + F_external
    a = F_net / m  # Accelerationw
    return [v, a]
# Solve the modified system with external periodic force
sol_with_force = solve_ivp(spring_mass_system_with_force, t_span, initial_conditions, t_eval=t_eval,
rtol=1e-9, atol=1e-12)
# Extract results
t = sol_with_force.t
x = sol_with_force.y[0]
v = sol_with_force.y[1]
# Calculate energy
kinetic_energy = 0.5 * m * v**2
potential_energy = 0.5 * k1 * x**2 + 0.5 * k2 * x**2 + \
                   0.5 * k3 * np.where(x > x_threshold_k3, (x - x_threshold_k3)**2, 0) + \
                   0.5 * k4 * np.where(x < -x_threshold_k4, (x - (-x_threshold_k4))**2, 0)
total_energy = kinetic_energy + potential_energy
```

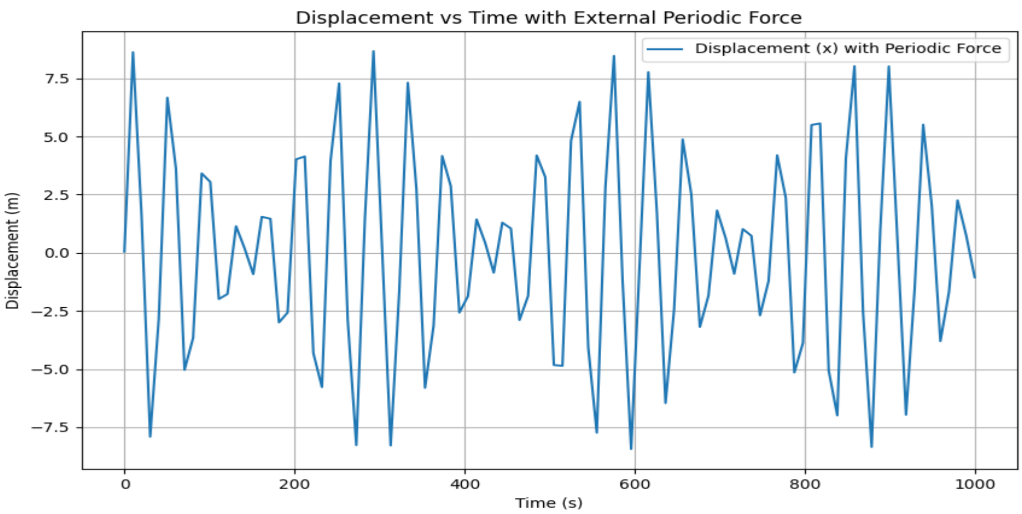
Plot Results (Expecting to be chaotic)



If we change the frequency for the applied force and bring it to closer proximity to the natural frequencies of the system, then we witness a beating frequency.

```
# Re-define constants and initial conditions
m = 1.0   # Mass of the block
k1 = 1.0, k2 = 1.0, k3 = 1.0, k4 = 1.0 # Spring constants
x_threshold_k3 = 0.04 # Threshold for k3 to engage
x_threshold_k4 = 0.04 # Threshold for k4 to engage
F0 = 1   # Amplitude of the periodic force
omega = 1.4 # Angular frequency of the periodic force (adjust as needed)
# Initial conditions
```

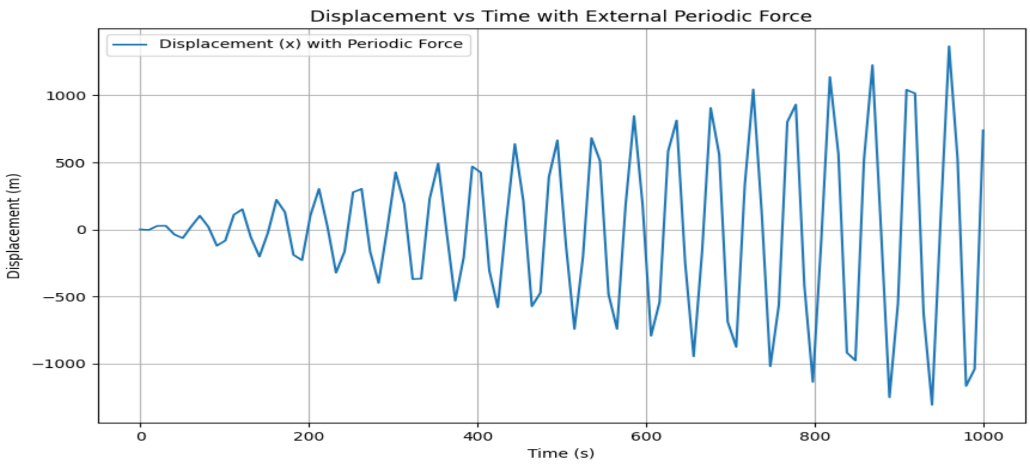
Plot Results (Beat Frequency)



Changing the frequency of the oscillating force to natural frequency results in resonance and uncontrolled oscillation.

```
# Re-define constants and initial conditions
m = 1.0   # Mass of the block
k1 = 1.0, k2 = 1.0, k3 = 1.0, k4 = 1.0
x_threshold_k3 = 0.04 # Threshold for k3 to engage
x_threshold_k4 = 0.04 # Threshold for k4 to engage
F0 = 5   # Amplitude of the periodic force
omega = 1.732050 # Angular frequency of the periodic force (Natural Freq)
```

Plot Results (Uncontrolled)



3.2 Conclusion

The numerical simulations and graphs generated by the code demonstrate that the system's behaviour aligns with the theoretical equations derived for bilinear free and forced vibration. The plots accurately reflect the expected displacement and frequency responses, confirming that the nonlinear engagement of springs due to displacement thresholds produces the anticipated bilinear dynamics. The results provide a strong correlation between the numerical and analytical approaches, showcasing the impact of crack-induced nonlinearities on the system's dynamic response. These findings support the effectiveness of the bilinear model in predicting structural behaviour under free and forced vibration scenarios.

Chapter 4

Future Experimentation

4.1 Introduction

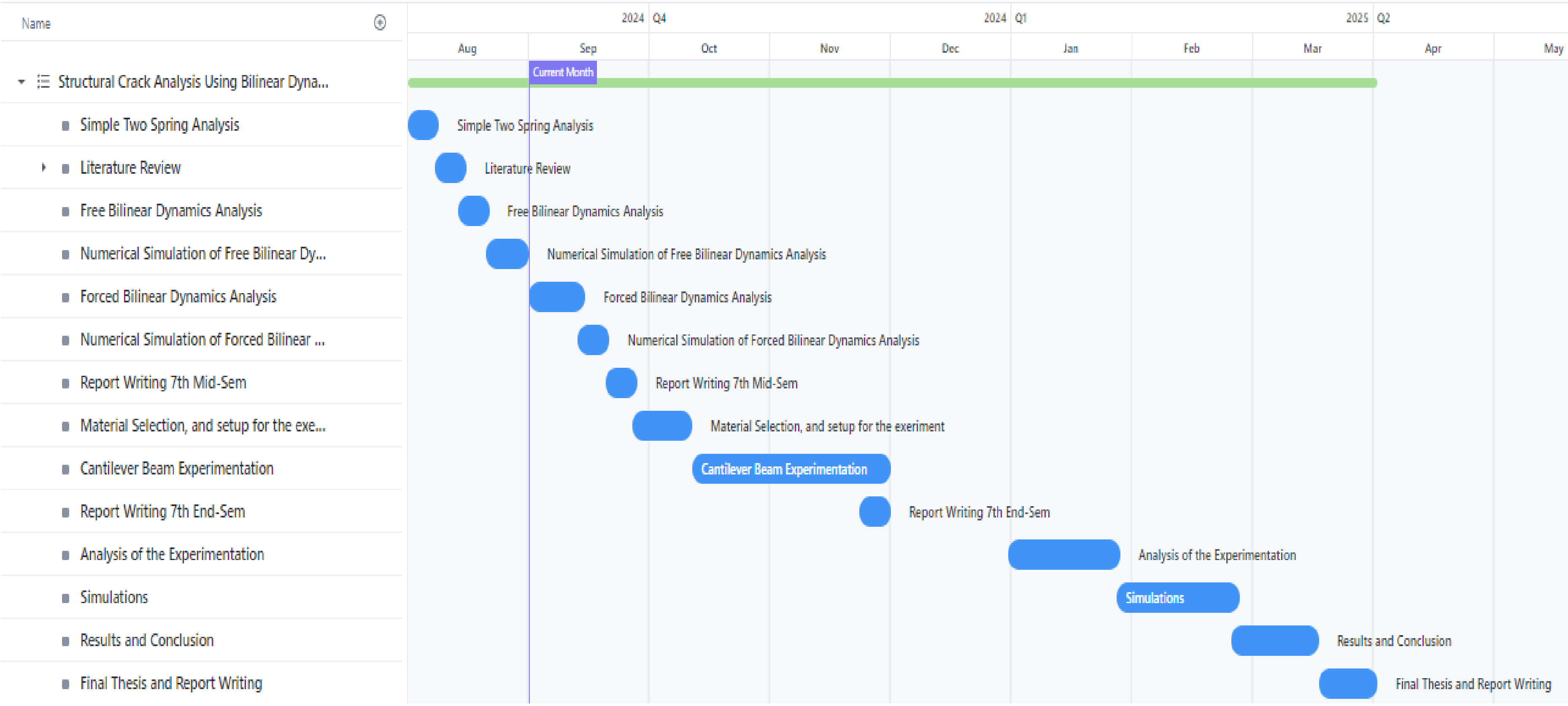
This report outlines the upcoming experimental investigation of nonlinear dynamics in cantilever beams with 250-micrometer grooves. The experiment aims to replicate bilinear dynamics observed in a spring-mass system, where small vibrations result in linear behaviour, and increased displacement causes the grooves to engage, leading to nonlinearity by altering the effective spring constant (k).

Planned Steps:

- **System Setup:**
  Cantilever beams will be prepared, and vibrational input will be applied. The system is expected to transition from linear to nonlinear dynamics as grooves engage under increasing displacement.
- **Data Collection:**
  Displacement sensors and frequency analysis will record system behaviour, focusing on the shift from linear to nonlinear motion. Multiple tests will be conducted to ensure complete data.
- **Graphical Analysis:**
  Displacement-time and frequency-displacement plots will be generated, allowing visualization of system behaviour and resonance shifts.
- **Comparison with Models:**
  Experimental results will be compared with theoretical and numerical models, validating predictions of nonlinear effects.

The experiment is expected to provide valuable insights into how localized imperfections (grooves and cracks) influence structural dynamics

4.2 Gantt Chart



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