ROHINI COLLEGE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF CIVIL ENGINEERING

CE 8602 STRUCTURAL ANALYSIS II

(VI SEMESTER)

COURSE MATERIAL

COMPILED BY

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UNIT-I

INFLUENCE LINES FOR DETERMINATE BEAMS

2/12/19

Definition:

An influence is a graph showing, for any given beam, frame or truss, the variation of any force or displacement quantity (such as shear force, bending moment,) for all position of a moving unit load as it crosses the structure from one end to the other.

Problems 1:

A single rolling load of "100kN" moves on a girder of span 20m. a) Construct the influence lines for shear force and bending moment for a section "5m" from the left support. b) Construct the influence lines for points at which the absolute max shear and absolute max bending moment develop. Determine these absolute max values.

Soln:

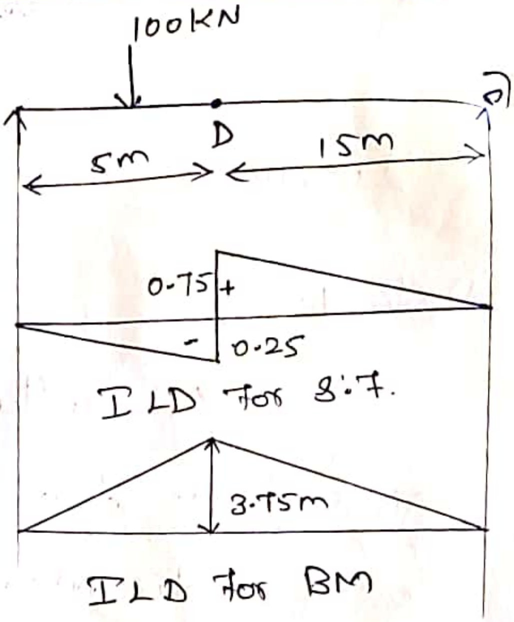
a) To find max shear force and Bending Moment at 5m from the left support

Influence lines Diagram for Shear Force

IL ordinate to the right of D

$$= \frac{l-x}{l}$$

$= \frac{l-x}{l} = \frac{20-5}{20} = 0.75$



IL ordinate to the left of D

$= \frac{x}{l} = \frac{5}{20} = 0.25$

IL for BM

IL ordinate at D

$= \frac{x(l-x)}{l} = \frac{5(20-5)}{20}$

$= 3.75m$

Max positive Shear force = load x ordinate

$= 100 \times 0.75$

$= 75 kN$ (+)

Max Negative Shear force = load x Ordinate

$= 100 \times 0.25$

$= 25 KN$ (-)

Max Bending Moment = load x ordinate

$= 100 \times 3.75$

$= 375 kNm$.

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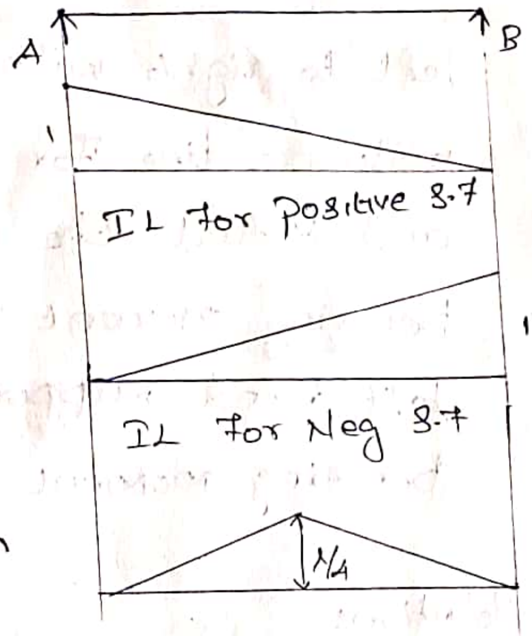
b) Absolute Max Shear Force and Bending Moment

For Shear Force.

IL ordinate at A

$= \frac{20}{20} = 1$



IL ordinate at B.

$= \frac{20}{20} = 1$

IL ordinate at BM. at Midspan

$= \frac{l}{4} = \frac{20}{4} = 5$

Positive Shear Force.

= load x ordinate

= 100 x 1

= 100 kN (+)

Negative Shear Force

= load x ordinate

= 100 x 1

= 100 kN (-)

Absolute maximum BM

= load x ordinate

= 100 x 5

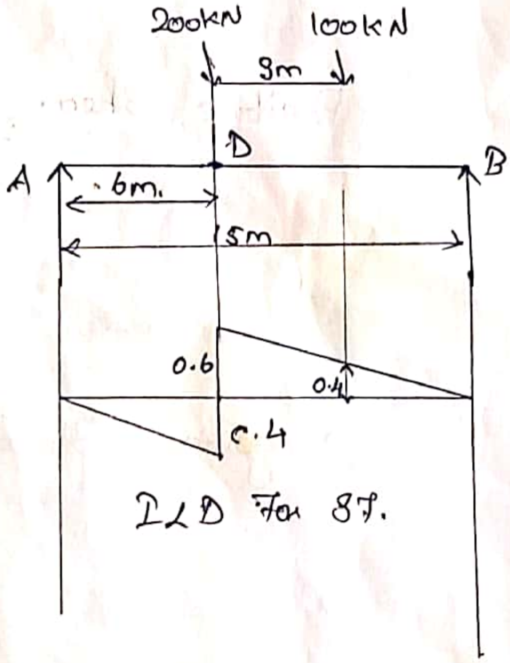
= 500 kNm.

Problem 2:

Two point loads of 100KN and 200KN spa[illegible] 3m apart are cross a girder of span 15m from the left to right with the 100KN load leading. Draw the influence line for shear force and bending moment and find the value of max shear force and bending moment at a section D, 6m from the left hand support. Also, find the absolute max bending moment due to the given load system.

Solution:

a) Find Max Shear Force.



ILD for SF.

Shear increament.

$$S_i = \frac{W_e}{l} - W_1$$

$$= \frac{300}{15} - 200 = -180$$

(i) Positive Shear Force.

$$\frac{l-x}{l} = \frac{15-6}{15} = 0.6$$

$$\frac{x}{l} = \frac{6}{15} = 0.4$$

Ordinate under 200KN $= 0.6$

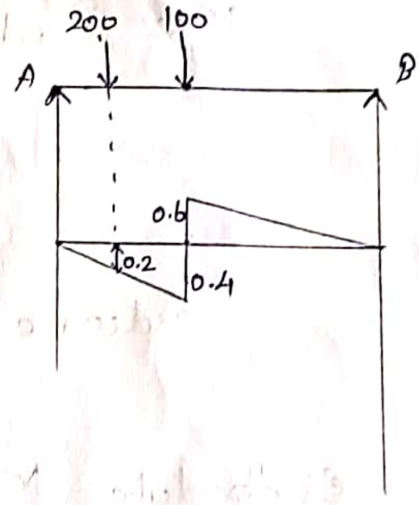
Ordinate Under 100KN

$$= \frac{0.6}{9} \times 6 = 0.4$$

Max positive Shear Force $= (200 \times 0.6) + (100 \times 0.4)$

$= 160 kN.$

(ii) Negative Shear force.



Shear increament.

$$S_i = \frac{W_e}{l} - W_1$$

$$= \frac{300 \times 3}{15} - 100$$

$$= -40.$$

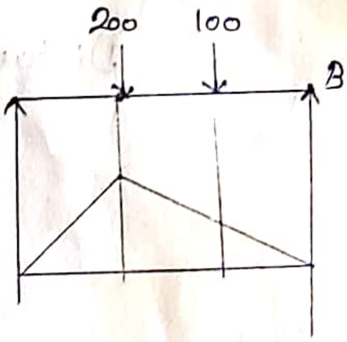
Ordinate under 200kN $= \frac{0.4}{6} \times 3$

$= 0.2$

Max Negative Shear force $= (200 \times 0.2) + (100 \times 0.4)$

$= 80 kN$ (Neg).

b) Max Bending Moment.



Find critical load

loading rate $L_r = \frac{W_{left}}{x} - \frac{W_{right}}{l-x}$

$$= \frac{200}{6} - \frac{100}{9} = 22 (+ve)$$

loading rate $L_r = \frac{0}{6} - \frac{300}{9} = -33.33 (-ve)$

Ordinate under 100 kN $= \frac{3.6}{9} \times 6 = 2.4m$

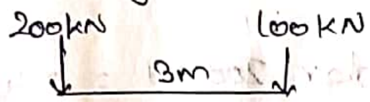
Max Bending Moment = Load × ordinate

$= (200 \times 3.6) + (100 \times 2.4)$

$= 960 kNm$

Ordinate of ILD $= \frac{x(l-x)}{l} = \frac{9 \times 6}{15} = 3.6m$

c) Absolute Max Bending Moments.



Taking Moment about 200 kN

$100 \times 3 = R.\bar{x}$

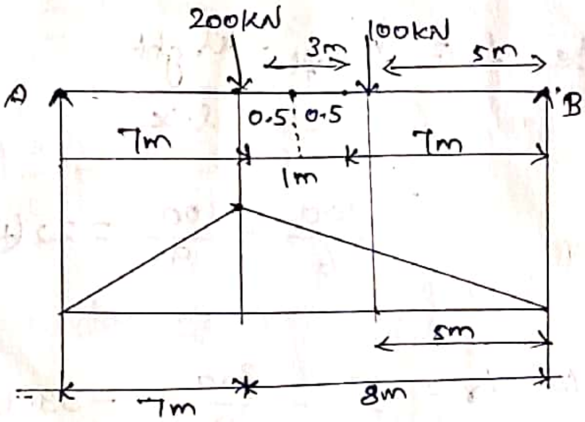
$300 = R\bar{x}$

$300 = 300\,\bar{x}$

$\bar{x} = 1m.$

Distance of this 200kN from c.

$= \frac{\bar{x}}{2} = \frac{1}{2} = 0.5$



Max ordinate under 200kN $= \frac{(l-x)x}{l} = \frac{8 \times 7}{15} = 3.73m$

ordinate under 100kN $= \frac{3.73}{8} \times 5 = 2.33m$

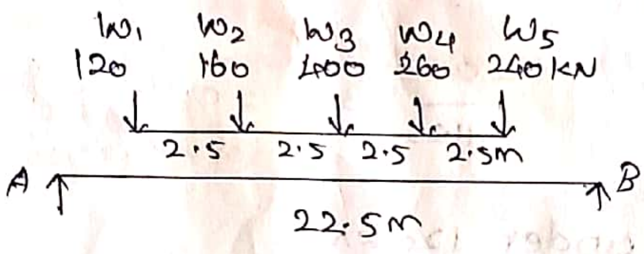
Absolute Max Bending Moment $= (200 \times 3.73) + (100 \times 2.33)$

$= 979.3$ kNm

—x—

Problem No: 3

A train of 5 wheel loads crosses a ss beam of span 22.5m. Using influence lines, calculate the Max positive and negative shear forces at mid span and absolute max bending moment anywhere in the span.



Solution.

a) Max Shear force.

Find Shear increament.

W = 1180kN, c = 2.5

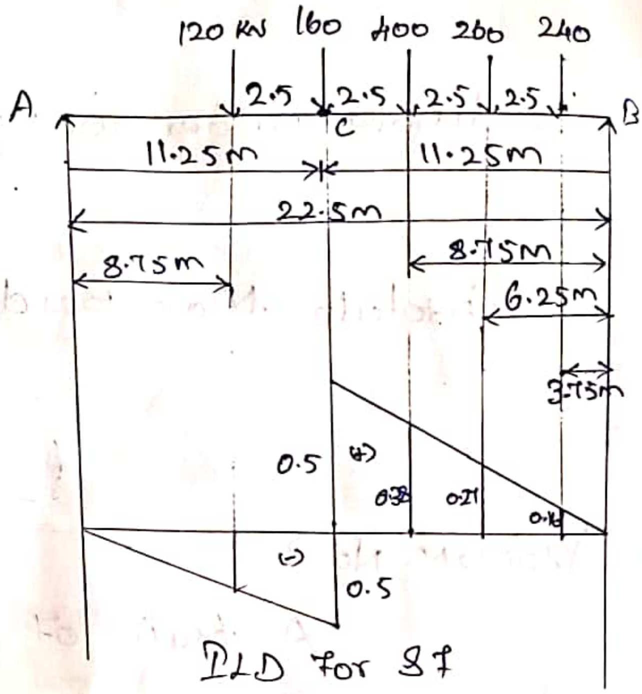
$S_i = \frac{Wc}{l} - W_1 = \frac{1180 \times 2.5}{22.5} - 120 = 11.11$ (+ve)

$S_i = \frac{1180 \times 2.5}{22.5} - 160 = -28.8$ (−ve)

Ordinate under 'C' =

Right Side $= \frac{l-x}{l} = \frac{11.25}{22.5} = 0.5$

Left Side $= \frac{x}{l} = \frac{11.25}{22.5} = 0.5$



ILD for SF

Ordinate under 400KN.

$= \frac{0.5}{11.25} \times 8.75 = 0.38$

Ordinate under 260 KN

$= \frac{0.5}{11.25} \times 6.25 = 0.27$

Ordinate under 240 KN

$= \frac{0.5}{11.25} \times 3.75 = 0.16$

Ordinate under 120 KN

$= -\frac{0.5}{11.25} \times 8.75 = -0.38$ (-ve)

Max positive Shear Force

$= (-120 \times 0.38) + (400 \times 0.38) + (260 \times 0.27) + (240 \times 0.16) + (160 \times 0.5)$

$=$ ~~266.10KN~~ 295 KN

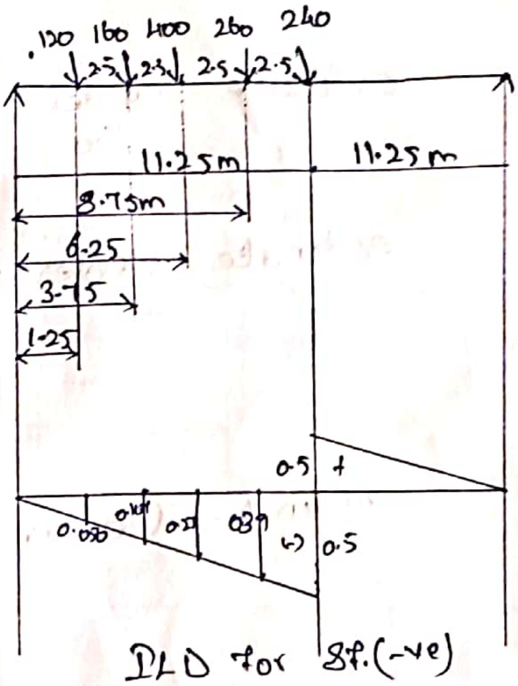
(ii) Negative Shear Force.

Find Shear increament

$S_i = \frac{Wc}{l} - W_1 = \frac{1180 \times 2.5}{22.5} - 240 = -108.89$ (-ve)

Ordinate under 260 kN

$$= \frac{0.5}{11.25} \times 8.75 = 0.39$$



Ordinate under 400 kN

$$= \frac{0.5}{11.25} \times 6.25 = 0.27$$

Ordinate under 160 kN

$$= \frac{0.5}{11.25} \times 3.75 = 0.167$$

Ordinate under 120kN

$$= \frac{0.5}{11.25} \times 1.25 = 0.056$$

Max Negative Shear force =

$$= (240 \times -0.5) + (260 \times -0.39) + (400 \times -0.27) + (160 \times 0.167) + (120 \times 0.056)$$

$$= -366.04 \text{ kN } (-ve)$$

b) Absolute Max Bending Moment

120 160 400 260 240

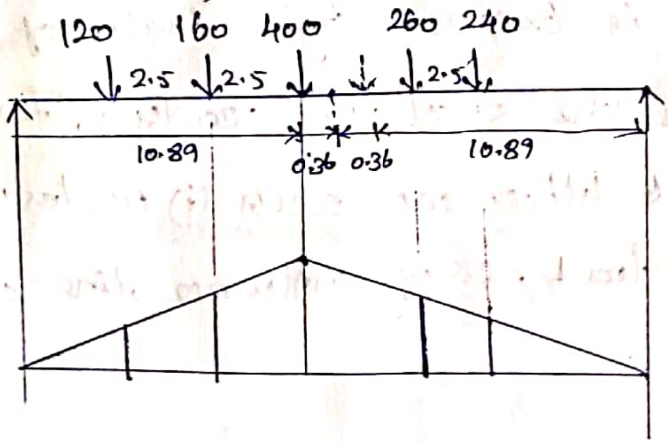
2.5, 2.5, 2.5, 2.5

Taking Moment about 120kN

$$(160 \times 2.5) + (400 \times 5) + (260 \times 7.5) + (240 \times 10) = R \cdot \bar{x}$$

$$6750 = 1180\,\bar{x}$$

$$\bar{x} = 5.72\text{m}$$



5

Max ordinate of ILD $= \dfrac{x(l-x)}{l} = \dfrac{10.89\,(22.5-10.39)}{22.5} = 5.62$

Ordinate under 160 $= \dfrac{5.62}{10.89} \times 8.39 = 4.33$

Ordinate under 120 kN

$$= \frac{5.62}{10.89} \times 5.89 = 3.04$$

Ordinate Under 260 kN

$$= \frac{5.62}{10.89} \times 9.11 = 4.41$$

Ordinate under 240kN

$$= \frac{5.62}{10.89} \times 6.61 = 3.20$$

Absolute Maximum Bending Moment

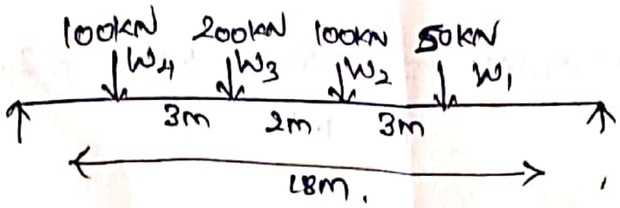
$= 120(3.04) + 160(4.33) + 400(5.62) + 260(4.41) + (240 \times 3.2)$

$= 5220.2$ kNm

———x———

PROBLEM : 4

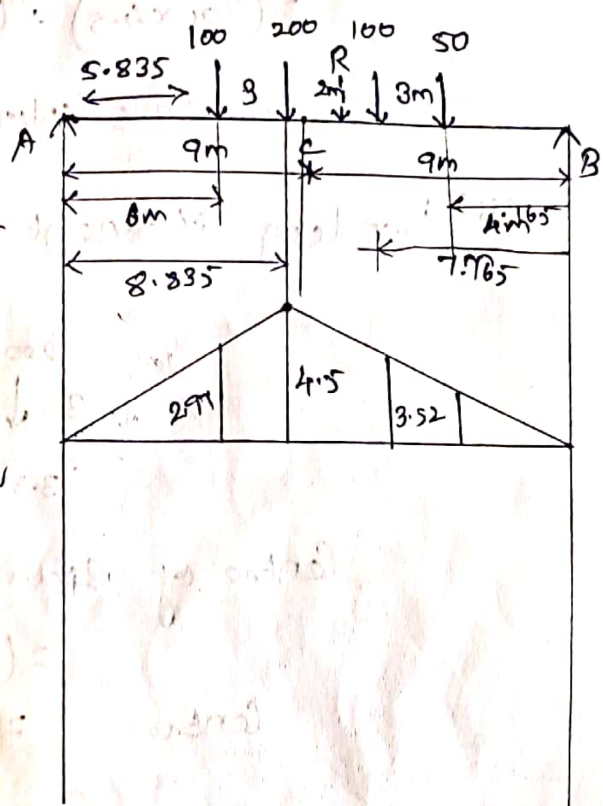
A girder having a span of 18m is ss at the ends. It is traversed by a train of loads as shown in figure. the 50kN load leading. Find the maximum Bending Moment which can occur (i) under the 200kN load (ii) Under 50kN load, using influence line diagrams.



Solution.

Max Bending Moment

(i) Under 200kN load



Resultant loads = 100+200+100+50 = 450kN

Taking Moment about W_4.

100kN 200 100 50kN

↓ 3m ↓ 2m ↓ 3m ↓

$(200\times3)+(100\times5)+(50\times8) = R\bar{x}$

$1500 = 450\,\bar{x}$

$\bar{x} = 3.33\text{m}$

Ordinate Max $= \dfrac{x(l-x)}{l} = \dfrac{9(9)}{18}$

$= 4.5$

Distance between C and 200kN = Dist B/n C and R

$\dfrac{0.33}{2} = 0.165$

Ordinate under 100kN $= \dfrac{4.5}{8.835}\times 5.835 = 2.97$

Ordinate under 100kN $= \dfrac{4.5}{9.165}\times 7.165 = 3.52$

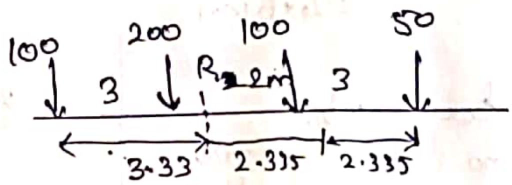
Ordenate under 50KN $= \frac{4.5}{9.165} \times 4.165 = 2.05$

BM und 200kN load

$= (200 \times 4.5) + (100 \times 2.97) + (100 \times 3.51) + 50(2.05)$

$\therefore = 1650.5$ kNm.

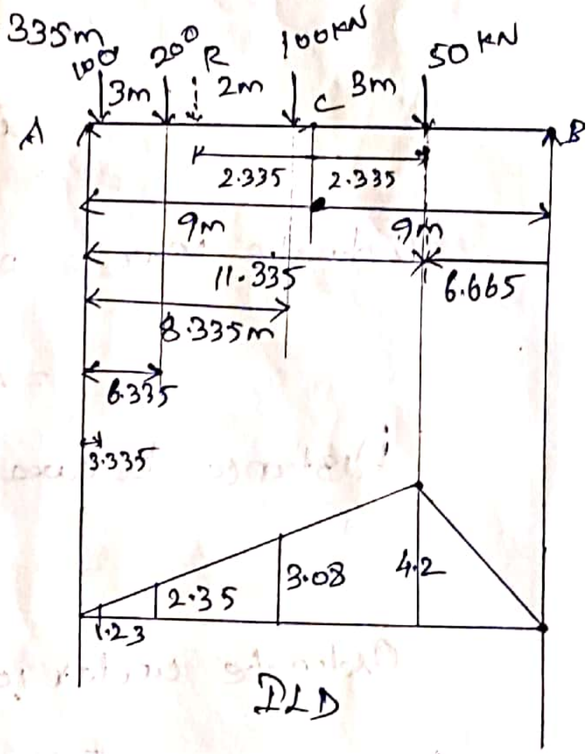
(ii) Bending Moment under 50kN load



Centre of span to BM. Equal Distance

$= (5 - 0.33)$

Centre $= 4.67/2 = 2.335$m



Ordenate under 50KN

$= \frac{x(l-x)}{l} = \frac{11.335 \times 6.665}{18} = 4.2$

Ordenate under 100KN

$= \frac{4.2}{11.335} \times 8.335$

$= 3.08$

Ordenate Under 200KN

$= \frac{4.2}{11.335} \times 6.335 = 2.35$

Ordenate Under 100kN

$= \frac{4.2}{11.335} \times 3.335 = 1.23.$

Max Bending Moment $= (50\times4.2) + (100\times3.09) + (200\times2.35) + (100\times1.24)$

$= 1113 kNm.$

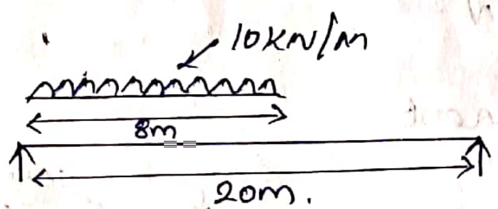
PROBLEM : 5

Using the relevant influence line diagrams find (i) The max bending moment, (ii) the max positive and negative shear at 4m from the left support of a SS girder of span 10m. When a train of 4 wheel loads of 10kN, 15kN, 30kN, 30kN, spaced at 2m, 3m, 3m respectively. Cross the span with the 10kN load leading.

Problem : 6

Draw the influence line Diagram for Shear Force and Bending moment for a section at 5m from the left hand support of a simply supported beam, 20m long. Hence calculate the max Bending Moment and shear force at the section, due to an uniformly distributed rolling load of length 8m and intensity 10kN/m run.

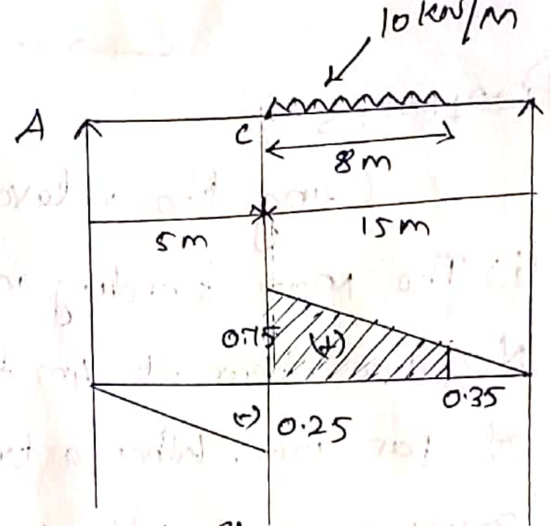
Solution:



a) Maximum Shear Force.

(i) Positive Shear Force.

$$= \frac{l - x}{l} = \frac{15}{20} = 0.75$$



Ordinate under C_1

$$= \frac{0.75}{15} \times 7 = 0.35$$

Maximum Positive Shear Force

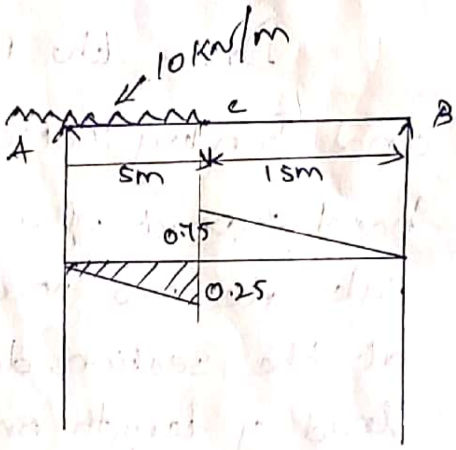
$$= 10 \times \left[\frac{h}{2}\left[a+b\right]\right]$$

Trapezoidal Shape $\frac{(a+b)h}{2}$

$$= 10 \times \frac{(0.75 + 0.35)\,8}{2}$$

$$= 44\,kN$$

(ii) Negative Shear Force.



$$= \frac{x}{l} = \frac{5}{20} = 0.25$$

Max Negative Shear Force

$$= 10 \times \left(\tfrac{1}{2} \times 5 \times 0.25\right)$$

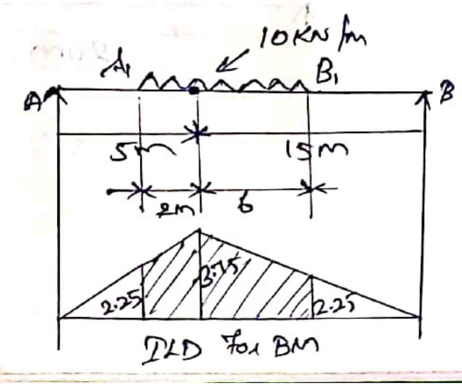
$$= 6.25\,kN.$$

b) Max Bending Moment.

Equal Ratio.

$$\frac{20}{4} = 5$$

$$\frac{8}{4} = 2$$



Max ordinate $= \frac{x(l-x)}{l} = \frac{5(15)}{20} = 3.75m$

Ordinate under A_1 $= \frac{3.75}{5} \times 3 = 2.25m$

Ordinate under B_1 $= \frac{3.75}{15} \times 9 = 2.25m$

Max Bending Moment

$= 10 \times \left[\frac{(2.25+3.75)2}{2} + \frac{(2.25+3.75)6}{2} \right]$

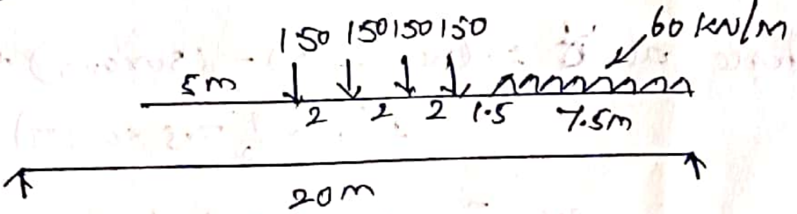
$= 10 \times \left[6 + 18 \right]$

$= 240$ KNM.

---x---

PROBLEM : 7

Four Equal loads of 150KN each equally spaced at 2M apart followed by a U.D.L of 60 KN/M at a distance of 1.5m from the last 150KN load across a girder of 20m span from Right to left. Using influence lines. Calculate the shear force and bending moments at a distance 8m from the left hand support, when the leading 150KN load is at 5m from the left hand support.



Solution:

a) Shear Force at this section.

Positive Shear Force

$= \frac{l-x}{l} = \frac{20-8}{20} = 0.6$

Ordinate Under 'C' left

$= \frac{x}{l} = \frac{8}{20} = 0.4$

Ordinate under 'F'

$= \frac{0.6}{12} \times 11 = 0.55$

Ordinate under 'G'

$= \frac{0.6}{12} \times 9 = 0.45$

Ordinate under 'H'

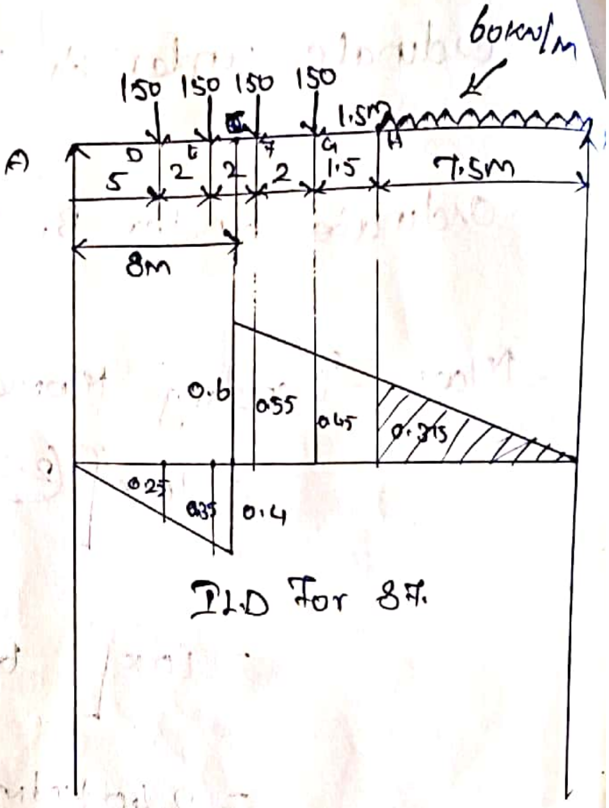
$= \frac{0.6}{12} \times 7.5 = 0.375$

Ordinate under 'E'

$= \frac{-0.4}{8} \times 7 = -0.35$

Ordinate under 'D'

$= \frac{-0.4}{8} \times 5 = -0.25$

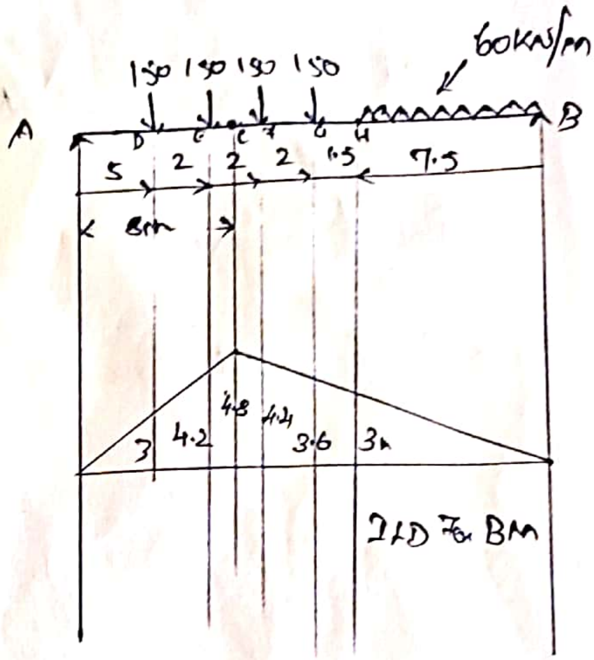


ILD for SF.

Shear Force at C $= -(150 \times 0.25) - (150 \times 0.35) + (150 \times 0.55) + (150 \times 0.45) + (60 \times \frac{1}{2} 7.5 \times 0.375)$

$SF_c = 144.375$ kN

b) Bending Moment for Given load position



Max Ordinate at 'C'

$$= \frac{x(l-x)}{l} = \frac{8(12)}{20} = 4.8$$

Ordinate under 'E'

$$= \frac{4.8}{8} \times 7 = 4.2m$$

Ordinate under 'D'

$$= \frac{4.8}{8} \times 5 = 3m$$

Ordinate under 'F'

$$= \frac{4.8}{12} \times 11 = 4.4m$$

Ordinate under 'G'

$$= \frac{4.8}{12} \times 9 = 3.6m$$

Ordinate under 'H'

$$= \frac{4.8}{12} \times 7.5 = 3m$$

Bending Moment at C

$$= (150 \times 3) + (150 \times 4.2) + (150 \times 4.4) + (150 \times 3.6) + (60 \times 1/2 \times 7.5 \times 3)$$

$$= 2955\, kNm$$

1.

UNIT-II

INFLUENCE LINES FOR INDETERMINATE STRUCTURES

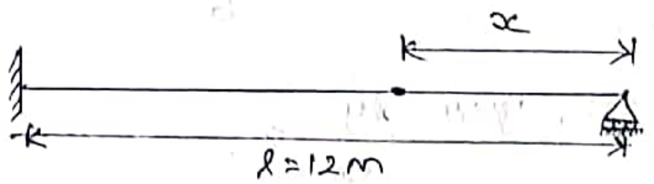
Muller Breslau Principle:

It states that, if we want to sketch the influence line for any force quantity (like shear, reaction, Bending Moment) in a structure.

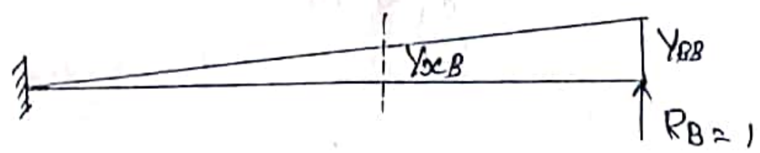
* We remove from the structure the restraint to that force quantity
* We apply on the remaining structure a unit displacement corresponding to that force quantity

PROBLEM 1:

Draw the influence line for reaction at B and for the support moment M_A at A for the propped cantilever as shown in fig. Computes the IL ordinates at 1.5m intervals.



Solution:



When $R_B = 1$, Y_{xB} is displacement at x section.

due to unit load applied at B

$$M_x = -EI\frac{d^2y}{dx^2}$$

$$R_B x = -EI\frac{d^2y}{dx^2}$$

$$1x = -EI\frac{d^2y}{dx^2}$$

$$EI\frac{d^2y}{dx^2} = -x$$

Integrating on both sides

$$EI\frac{dy}{dx} = -\frac{x^2}{2} + C_1 \quad -(1)$$

Again Integrate on Both sides

$$EI\,y = -\frac{x^3}{6} + C_1 x + C_2 \quad -(2)$$

Sub at $x = 12$, $\cancel{y=0}$ $\frac{dy}{dx} = 0$,

$$0 = -\frac{12^2}{2} + C_1$$

$$C_1 = 72.$$

Sub $x=12$, $y=0$ in (2)

$$0 = -\frac{12^3}{6} + 72\times 12 + C_2$$

$$0 = 576 + C_2$$

$$\therefore C_2 = -576.$$

Apply C_1 & C_2 in (2)

$$Y_{xB} = \frac{1}{EI}\left[-\frac{x^3}{6} + 72x - 576\right].$$

At $x=0$

$$Y_{BB} = \frac{-576}{EI}$$

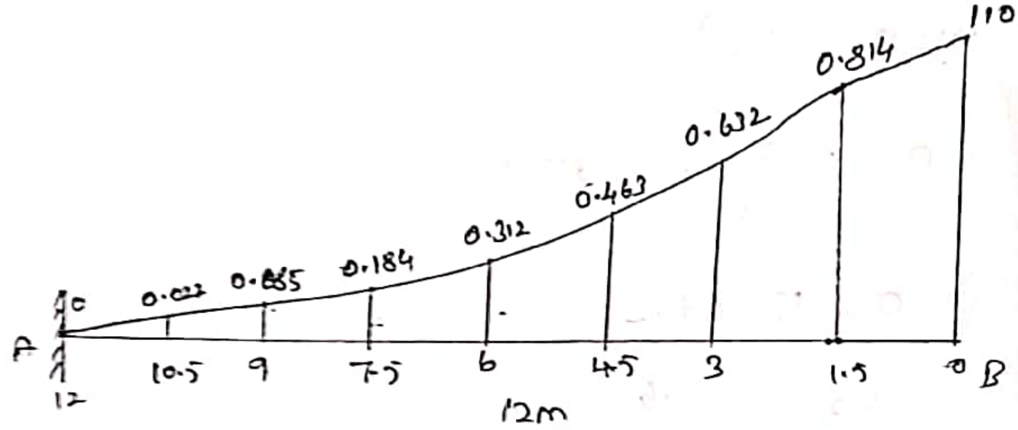
ILD for R_B at x

$$x = \frac{Y_{xB}}{Y_{BB}} = \frac{\frac{1}{EI}\left[-\frac{x^3}{6} + 72x - 576\right]}{\frac{-576}{EI}}$$

$$= \frac{-\frac{x^3}{6} + 72x - 576}{-576}$$

Ordinates of ILD for R_B at 1.5m intervals.

x (m)	0	1.5	3	4.5	6	7.5	9	10.5	12
R_B	1	0.814	0.632	0.463	0.312	0.184	0.085	0.022	0.0



We have to apply a unit rotation at A.

$$M_A = 1$$

$$R_B = -R_A = \frac{1}{12}$$

$$M_x = -EI\frac{d^2y}{dx^2}$$

$$\frac{x}{12} = -EI\frac{d^2y}{dx^2}$$

$$EI\frac{d^2y}{dx^2} = -\frac{x}{12}$$

Integrate on Both sides

$$EI\frac{dy}{dx} = -\frac{x^2}{24} + C_1 \quad \text{—} (1)$$

Again Integrate:

$$EIy = -\frac{x^3}{72} + c_1 x + c_2$$

At $x=0$, $y=0$

$x=12$, $y=0$

Hence $c_2 = 0$, $c_1 = 2$.

$$Y_{xA} = \frac{1}{EI}\left[-\frac{x^3}{72} + 2x\right]$$

$$\theta_{\substack{\cancel{AA} \\ xA}} = \frac{dy}{dx} = \frac{1}{EI}\left[-\frac{x^2}{24} + 2\right]$$

$\theta^{AA}_{\substack{\cancel{AA} \\ \cancel{x}A}}$ at $x = 12$.

$$\theta_{AA} = \frac{1}{EI}\left[-\frac{12^2}{24} + 2\right]$$

$$= \frac{-4}{EI}$$

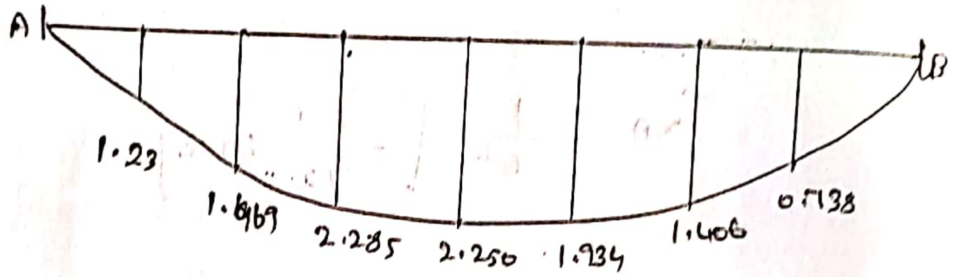
When we divide Y_{xA} by θ_{AA} We get the ILO at x

$$\text{ILO for } M_A = \frac{-\frac{x^3}{72} + 2x}{-4}$$

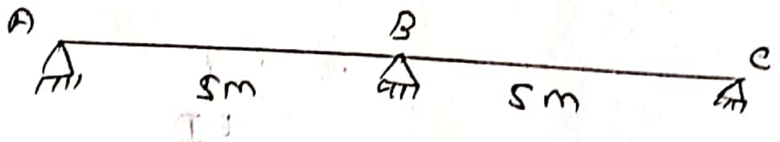
$$= \left[+\frac{x^3}{288} - \frac{x}{2}\right]$$

Ordinates of the ILD for M_A at 1.5m.

x (m)	0	1.5	3	4.5	6	7.5	9	10.[illegible]
ILD	0	-0.738	-1.406	-1.934	-2.250	-2.285	-1.669	-1.[illegible]



PROBLEM 2: Determine the influence line for R_A for continuous beam shown in fig: Compute the IL ordinates at 1m intervals.



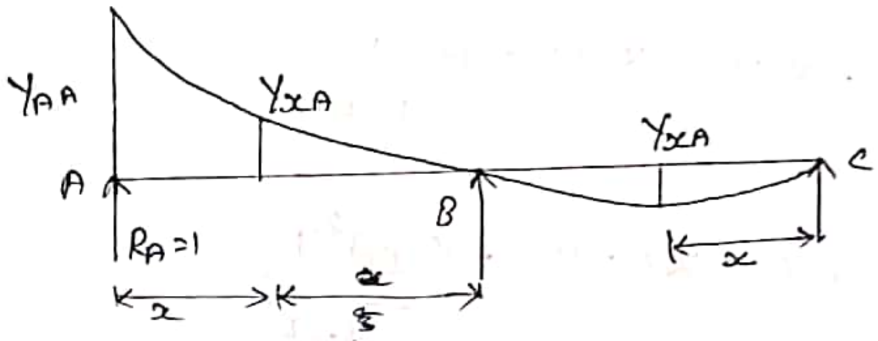
Solution.

(i) Remove support A.

(ii) Apply a unit force at A and compute the deflection at any "x". on CB and BA.

(iii) Divide these deflections by the displacement at A.

Elastic curve due to $R_A = 1$.



Taking Moment about C.

$$R_A \times 10 + R_B \times 5 = 0$$

$$10 + R_B 5 = 0$$

$$R_B = -10/5$$

$$R_B = -2$$

$$R_A + R_B + R_C = 0$$

$$1 - 2 + R_C = 0$$

$$-1 + R_C = 0$$

$$R_C = 1$$

$$M_x = -\frac{d^2y}{dx^2} EI$$

$$M_x = R_C x + R_B (x - 5)$$

$$1x - 2(x-5) = -\frac{d^2y}{dx^2} EI$$

$$-x + 2(x-5) = EI\frac{d^2y}{dx^2}.$$

$$\therefore -x + 2x - 10 = EI\frac{d^2y}{dx^2}.$$

Integrate on Both side

$$EI\frac{dy}{dx} = \frac{-x^2}{2} + \frac{2x^2}{2} - 10x + C_1$$

$$EI\frac{dy}{dx} = \frac{-x^2}{2} + x^2 - 10x + C_1 \quad -(1)$$

Again integrate on Both sides

$$EIy = \frac{-x^3}{6} + \frac{x^3}{3} - \frac{10x^2}{2} + C_1x +$$

$$EIy = \frac{-x^3}{6} + \frac{x^3}{3} - 5x^2 + C_1x + \quad -(2)$$

Apply conditions

$x = 0, y = 0$

$0 = C_2$

$x=5,\ y=0$

$$(2) \Rightarrow \quad 0 = \frac{-5^3}{6} + \frac{5^3}{3} - 5(5)^2 + C_1 \times 5 + 0$$

$$0 = -104.16 \times C_1 \times 5$$

$$C_1 = 20.83$$

Apply C_1 & C_2.

$$Y_{xA} = \frac{1}{EI}\left[-\frac{x^3}{6} + \frac{x^3}{3} - 5x^2 + 20.83x + 0\right]$$

At $x = 10$

$$Y_{AA} = \frac{1}{EI}\left[\frac{-10^3}{6} + \frac{10^3}{3} - 5(10)^2 + 20.83(10) + 0\right]$$

$$= -125.033$$

$$M_x = -\frac{d^2y}{dx^2} \cdot EI$$

$$1x + 2(x \cdot 5) = \frac{d^2y}{dx^2}\,EI$$

$$EI\frac{d^2y}{dx^2} = -x + 2(x-5)$$

$$\cancel{EI\frac{d^2y}{dx}}$$

Integrate on Both side:

$$EI\frac{dy}{dx} = -\frac{x^2}{2} + c_1 + 2\frac{(x-5)^2}{2}$$

$$EI\frac{dy}{dx} = -\frac{x^2}{2} + (x-5)^2 + c_1 \quad -(1)$$

Again Integrate Both side

$$EIy = -\frac{x^3}{6} + \frac{(x-5)^3}{3} + c_1x + c_2 \quad -(2)$$

Apply condition.

(i) $x=0,\ y=0$

$(2) \Rightarrow \quad 0 = c_2.$

(ii) $x=5,\ y=0.$

$(2) \Rightarrow \quad 0 = \frac{-5^3}{6} + \frac{(5-5)^3}{3} + c_1 5 + c_2$

$0 = -20.83 + 0 + c_1 5 + 0$

$c_1 = \frac{20.83}{5}$

$c_1 = 4.167.$

Apply C_1 & C_2.

$$y_{xA} = \frac{1}{EI}\left[-\frac{x^3}{6} + 4.167x + \frac{(x-5)^3}{3}\right] \quad -③$$

At $x = 10$ in ③

$$Y_{AA} = \frac{1}{EI}\left[-\frac{10^3}{6} + 4.167(10) + \frac{(10-5)^3}{3}\right]$$

$$= \frac{1}{EI}\left[-83.33\right] \quad -④$$

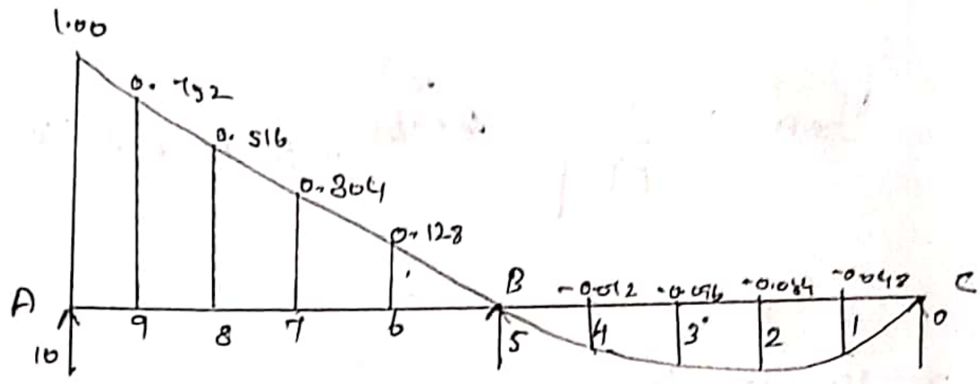
$$\text{ILO at } x = \frac{Y_{xA}}{Y_{AA}}$$

$$= \frac{\frac{1}{EI}\left[-\frac{x^3}{6} + 4.167x + \frac{(x-5)^3}{3}\right]}{-83.33}$$

$$= \frac{1}{EI}\left[-\frac{x^3}{6} + 4.167x + \frac{(x-5)^3}{3}\right]\frac{1}{83.33}$$

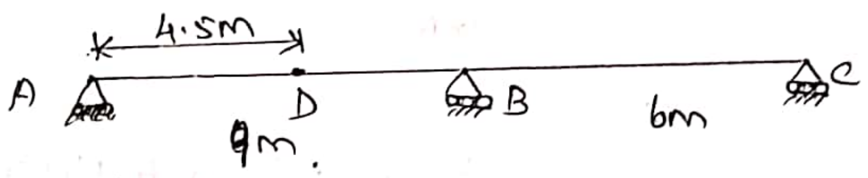
Ordinate at ILD for R_A

x(m)	Support C 0	1	2	3	4	Support B 5	6	7	8	9	Support at A 10
ILO (R_A)	0	-0.048	-0.084	0.096	-0.072	0	0.128	0.128	0.304	0.516	1



PROBLEM:

Using Muller Breslau Principle, draw the influence line for bending moment at the mid-point of span AB of the continuous beam ABC shown in figure. Determine the influence line ordinates at suitable intervals and plot them.



Solution.

To get the influence line for M_D

(i) Introduce a hinge at D.

(ii) Apply a unit bending moment at D.

(iii) Determine the ~~influence~~ deflection y_{xD} and the slope θ_{DD} at D.

(iv) $\dfrac{y_{xD}}{\theta_{DD}}$ is the influence line ordinate at any s

Bending Moment at any x is

$$M_x = -EI\frac{d^2y}{dx^2} \cancel{+0.333x} \; \cancel{+0.0}$$

$$0.333x - 0.555(x-6) = -EI\frac{d^2y}{dx^2}$$

$$EI\frac{d^2y}{dx^2} = -0.333x + 0.555(x-6)$$

Integrate on both sides

$$EI\frac{dy}{dx} = -\frac{0.333x^2}{2} + \frac{0.555(x-6)^2}{2}$$

$$EI\frac{dy}{dx} = -0.1665x^2 + 0.2775(x-6)^2 + C_1 \quad — (1)$$

Again integrate on both sides.

$$EIy = \cancel{0.33} - \frac{0.1665x^3}{3} + \frac{0.2775(x-6)^3}{3} + C_1x + C_2$$

$$EIy = -0.0555x^3 + 0.0925 + C_1x + C_2 \quad — (2)$$

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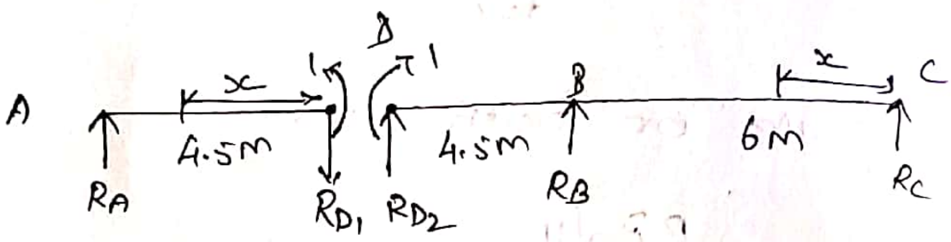
Find $R_A, R_B, R_C, R_{D_1}, R_{D_2}$

$M = 1$ at D.

$R_A \times 4.5 = 1$

$R_A = \frac{1}{4.5} = 0.222$ KN



$R_{D_1} = 0.222 \downarrow$

$R_{D_2} = 0.222 \uparrow$

Taking Moment about C.

$0.222 \times 10.5 + 1 + R_B \times 6 = 0$

$R_B = -0.555$ KN

$R_A + R_B + R_C = 0$

$0.222 - 0.555 + R_C = 0$

$R_C = 0.333$ KN

Two regions AD and DBC will be considered seperately (Because of discontinuity at D).

Boundary conditions

(i) $x = 0, y = 0,$

(3) $\Rightarrow$ $0 = c_2$

(ii) $x = 6, y = 0$

(3) $\Rightarrow$ $0 = -0.0555(6)^3 + 0.0925 \cdot \frac{(6-6)^3}{3} + c_1 6 + 0$

$0 = -11.988 + c_1 6$

$c_1 = 2$

Apply c_1 & c_2 in Slope of Deflection Value

(1) $\Rightarrow$ $EI \frac{dy}{dx} = -0.1665x^2 + 0.2775 \cdot (x-6)^2 + 2$

$x = 10.5$

$\theta_{DC} = \frac{dy}{dx} = \frac{1}{EI}\left[0.1665(10.5^2) + 0.2775(10.5-6)^2 + 2\right]$

$= \frac{1}{EI}\left[-10.78\right]$

Apply (3) $\Rightarrow$

$y = \frac{1}{EI}\left[-0.0555(10.5^3) + 0.0925(10.5-6)^3 + 2(10.5) + 0\right]$

$= \frac{1}{EI}\left[-34.8\right]$

For the zone AD

$$M_x = 1 - 0.222x$$

$$EI \frac{d^2y}{dx^2} = 0.222x - 1$$

Integrate on Both sides

$$EI \frac{dy}{dx} = \frac{0.222x^2}{2} - x + C_3$$

$$EI \frac{dy}{dx} = 0.111x^2 - x + C_3 \quad \text{--- (3)}$$

Again Integrate on Both sides.

$$EIy = \frac{0.111x^3}{3} - \frac{x^2}{2} + C_3x + C_4$$

$$EIy = 0.037x^3 - \frac{x^2}{2} + C_3x + C_4 \quad \text{--- (4)}$$

Boundary conditions.

(i) $x = 0, \; y = \frac{-34.82}{EI}$

(4) $\Rightarrow$

$$\frac{-34.82\, EI}{EI} = 0.037(0) - 0 + 0 + C_4$$

$$C_4 = -34.82.$$

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(ii) $x = 4.5$, $y = 0$

④ ⇒

$$0 = 0.037 \times 4.5^3 - \frac{4.5^2}{2} + c_3 \times 4.5 - 34.82$$

$$0 = -41.57 + 4.5c_3$$

$$c_3 = 9.24.$$

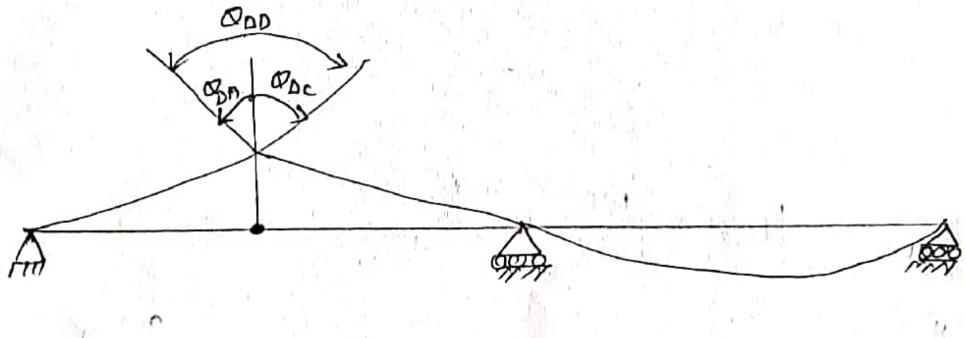
Apply c_3 & c_4 in ③

③ ⇒ $$EI\frac{dy}{dx} = 0.222\left(\frac{x^2}{2}\right) - x + 9.24.$$

$$\theta_{DA} = \frac{dy}{dx} = \frac{9.24}{EI}. \quad \text{at } x = 0$$

④ ⇒ $$EIy = 0.037x^3 - \frac{x^2}{2} + 9.24x - 34.82$$

$$\theta_{DD} = \theta_{DA} - \theta_{DC}$$



$$= \frac{9.24}{EI} + \frac{10.738}{EI}$$

$$= \frac{19.978}{EI}$$

For the region CD,

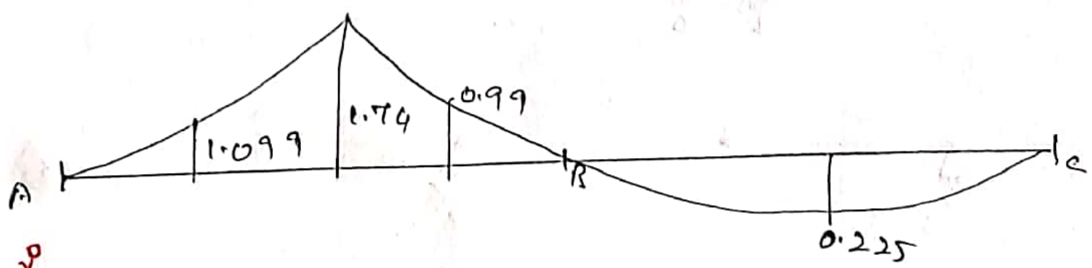
$$ILO \text{ for } M_D = \frac{Y_{xD}}{\delta_{DD}} \cdot \left[\frac{0.333\frac{x^3}{6} + 2x + 0.555\frac{(x-6)^3}{6}}{19.978} \right]$$

For the region DA.

$$ILO \text{ for } M_D = \left[\frac{0.222\frac{x^3}{6} - \frac{x^2}{2} + 9.24x - 34.832}{19.978} \right]$$

Influence line ordinate:

x (m)	0	3	6	9	10.5	12	15
ILO	0	0.225	0.0	-0.999	-1.743	-1.099	0



ILD for MD.

2/1/2020

UNIT-III

ARCHES.

Arches:

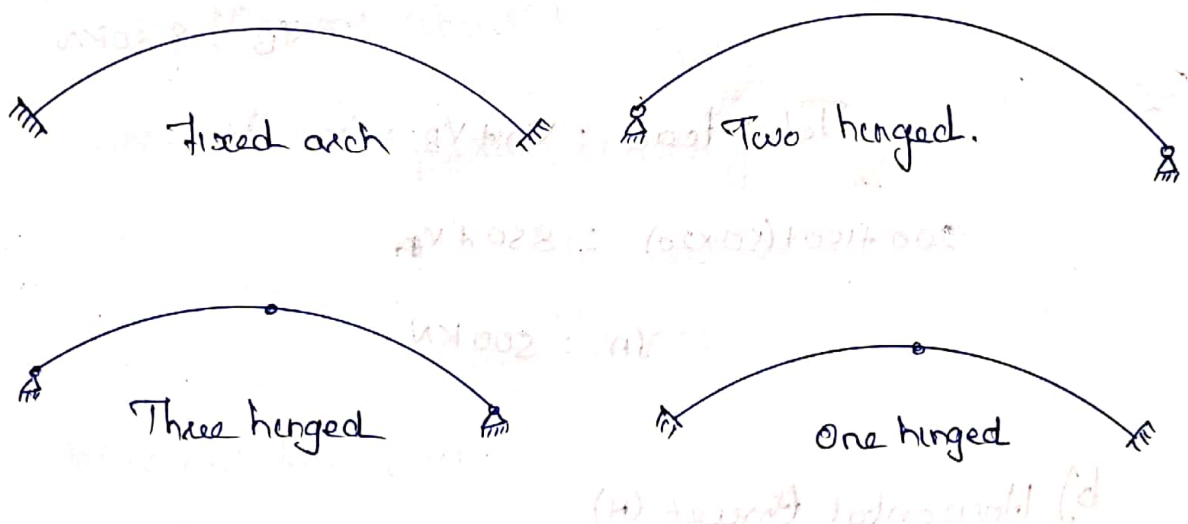
Arches are shaped to take the load above them and develop only compression. Arches to develop bending moment and shear too.

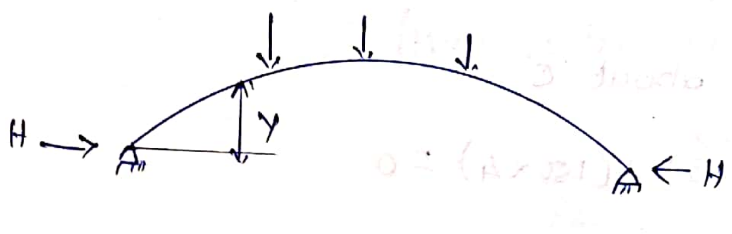
Arches can be:

* Circular
* Parabolic
* Polygonal
* Elliptical
* Any other curved shape.

Arches can be build of masonry, RCC or steel.

Hinges in Arches:

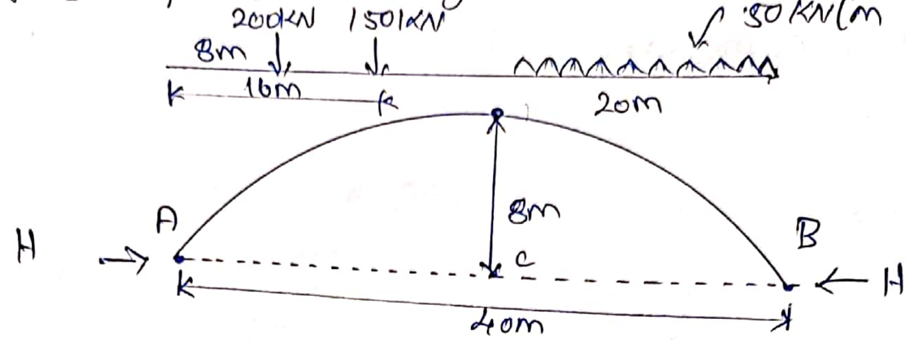




H provides arch action

PROBLEM 1:

A 3 hinged arch of span 40m and rise 8m carries concentrated load of 200KN and 150KN at distance of 8m and 16m from the left end and an udl of 50KN/m on the right half of the span.



Solution:

a) Vertical Reactions V_A and V_B

Taking Moment about A,

$$(200\times8)+(150\times16)+[50\times20\times(20+20/2)]-V_B\,40=0$$

$$1600+2400+30000-V_B\,40=0$$

$$V_B=850KN$$

$$\text{Total load}=V_A+V_B$$

$$200+150+(50\times20)=850+V_A$$

$$V_A=500KN$$

b) Horizontal thrust (H)

Taking Moment about 'C'

$$H\times8-V_A(20)+(200\times12)+(150\times4)=0$$

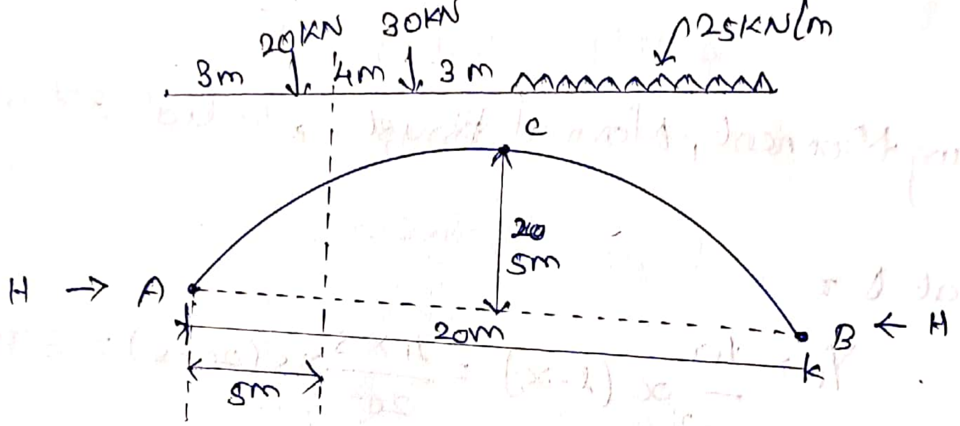
ACB4

$$8H - 4000 + 2400 + 1600 = 0$$

$$H = 325 KN$$

PROBLEM 2:

A parabolic 3 hinged arch carries loads as shown in fig: Determine the resultant reactions at supports. Find Bending moment, normal thrust, and radial shear at D, 5m from A. What is the max bending moment.



Solution:

Vertical Reactions V_A and V_B

Taking Moment about A.

$$(20\times3) + (30\times7) + [25\times10\times(10+10/2)] - V_B\times20 = 0$$

$$V_B = 201 KN$$

$$V_A = 99 KN$$

Horizontal thrust (H)

$$(H\times5) + (20\times7) + (30\times3) - V_A\times10 = 0$$

$$5H + 140 + 90 - 990 = 0$$

$$H = 152 KN$$

Resultant reactions (R_A and R_B)

$$R_A = \sqrt{H^2 + V_A^2} = \sqrt{152^2 + 99^2} = 181.39\,KN$$

$$R_B = \sqrt{H^2 + V_B^2} = \sqrt{152^2 + 201^2} = 252\,KN$$

$$\theta_A = \tan^{-1}\frac{V_A}{H} = \tan^{-1}\frac{99}{152} = 33^\circ 4' 36''.6$$

$$\theta_B = \tan^{-1}\frac{V_B}{H} = \tan^{-1}\frac{201}{152} = 52^\circ 54' 9''.86.$$

Bending Moment, Normal thrust, radial SF at D.

BM at D :

$$Y_D = \frac{4r}{l^2} x(l-x) = \frac{4 \times 5}{20^2} \times 5(20-5) = 3.75\,m$$

$$BM_D = +V_A \times 5 \mp H y_D \mp 20 \times 2$$

$$= +495 \mp 570 \mp 40$$

$$= -115\,KN\,m$$

Slope of the arch at D.

$$\theta = \tan^{-1}\left[\frac{4r}{l^2}(l-2x)\right].$$

$$\theta = \tan^{-1}\left[\frac{4 \times 5}{20^2}(20 - 2 \times 5)\right]$$

$$\theta = 26^\circ 33' 55''.18.$$

Normal thrust:

$$P = V_x \sin\theta + H\cos\theta$$

V_x = Net beam shear force.

$$V_x = V_A - 20 = 99 - 20 = 79\,KN$$

$$P = 79 \sin 26^{\circ}33'55''.18 + 152 \cos 26^{\circ}33'55''.18 = 171.28\,KN$$

Radial Shear force.

$$F = V_x \cos\theta - H\sin\theta$$

$$= 79\cos\theta - 152\sin\theta$$

$$= 2.683\,KN$$

Max BM in CB.

$$BM_x = V_B x - \frac{Wx^2}{2} - H y_x$$

$$y_x = \frac{4r}{l^2} x(l-x)$$

$$= \frac{4\times 5}{20^2} \times x(20-x)$$

$$= 0.05x(20-x)$$

$$M_x = 201x - \frac{25\times x^2}{2} - \overset{152}{\cancel{H}}[0.05x(20-x)]$$

$$= 201x - 12.5x^2 - 7.6x(20-x)$$

$$= 201x - 12.5x^2 - 152x + 7.6x^2$$

$$M_x = 49x - 4.9x^2$$

Diff. w.r.to. x.

$$\frac{dm}{dx} = 49 - 9.8x$$

∴ BM to be Max

$$9.8x - 49 = 0$$

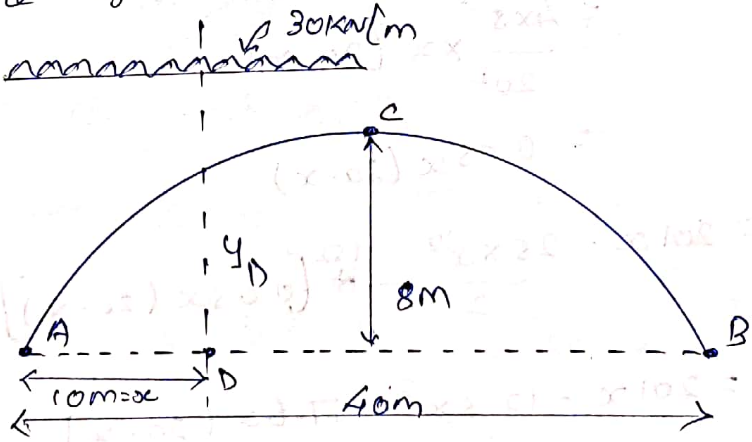
$$x = 5m$$

$$M_x = 49(\times 5) - 4.9(5^2)$$

$$= 122.5 kN.$$

PROBLEM 8:

A Symmetrical three hinged parabolic arch of span 40m and rise 8m carries an U.D.L of 30kN over the left half of the span. The hinges are pro at the supports and at the centre of arch. Calcu the reactions at the supports. Also calculate the bending moment, radial shear and normal thrust a distance of 10m from the left support.



Solution:

Vertical Components V_A and V_B

Taking Moment about A.

$$V_B \times 40 - 30 \times \frac{20^2}{2} = 0$$

$$V_B = 150\,kN$$

$$V_A = \text{Total load} - V_B$$

$$= 30 \times 20 - 150$$

$$= 450\,kN.$$

Horizontal Components.

$$V_A \times 20 - H \times 8 - 30 \times 20 \times 20/2 = 0$$

$$H = 375\,KN.$$

Resultant Reactions R_A & R_B.

$$R_A = \sqrt{H^2 + V_A^2} = \sqrt{450^2 + 375^2} = 585.77\,kN$$

$$R_B = \sqrt{V_B^2 + H_B^2} = \sqrt{150^2 + 375^2} = 403.89\,kN$$

Bending Moment at 10m from A.

$$y = \frac{4r}{l^2} x (l - x)$$

$$= \frac{4 \times 8}{40^2} \times 10 (40 - 10)$$

$$= 6m.$$

Bending Moment. at 10m.

$= V_A(10) - H_A(y) - 30 \times 10 \times 10/2$

$= 450 \cdot (10) - (375y) - 30(50)$

$= 3000 - 375y$

$= 3000 - 375(6)$

$= 750 \, KNM$

Radial Shear force at $x = 10m$

$R = V_x \cos\theta - H \sin\theta$

$V_x = V_A - 30 \times 10$

$= 450 - 300 = 150 \, KN$

Slope at D.

$\theta = \tan^{-1}\left[\frac{4r}{l^2}(l - 2x)\right]$

$= \tan^{-1}\left[\frac{4 \times 8}{40^2}(40 - 2(10))\right]$

$= 21°48'$

R.

$R = 150 \cos 21°48' - 375 \sin 21°48'$

$= 0$

Normal thrust.

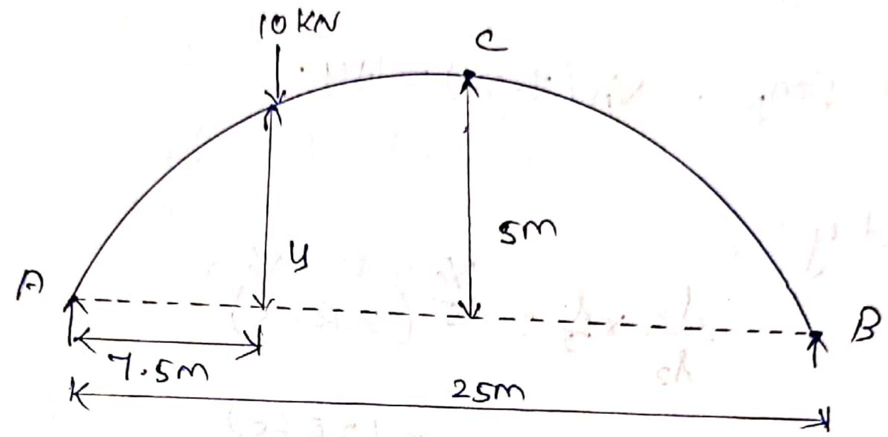
$P = V_x \sin\theta + H\cos\theta$

$= 150 \sin 21°48' + 375 \cos 21°48'$

$= 403.89 \, KN.$

oblem 4:

A 3 hinge arch is circular, 25m in span with a central rise of 5m. It is loaded with a concentrated load of 10KN at 7.5m from the left hand hinge. Find the horizontal thrust, Reactions at each end hinge, Bending Moment under the load.



Solution

Vertical Reactions. V_A & V_B

Taking Moment about A.

$$V_B \times 25 - 10 \times 7.5 = 0$$

$$V_B = 3\,KN$$

$$V_A = 7\,KN$$

Horizontal thrust. H

$$V_B \times 12.5 - H \times 5 = 0$$

$$3 \times 12.5 - H \cdot 5 = 0$$

$$H = 7.5\,KN$$

Reactions R_A and R_B.

$$R_A = \sqrt{V_A^2 + H^2} = \sqrt{7^2 + 7.5^2} = 10.26\,kN$$

$$R_B = \sqrt{V_B^2 + H^2} = \sqrt{3^2 + 7.5^2} = 8.08\,kN$$

Bending Moment Under the load.

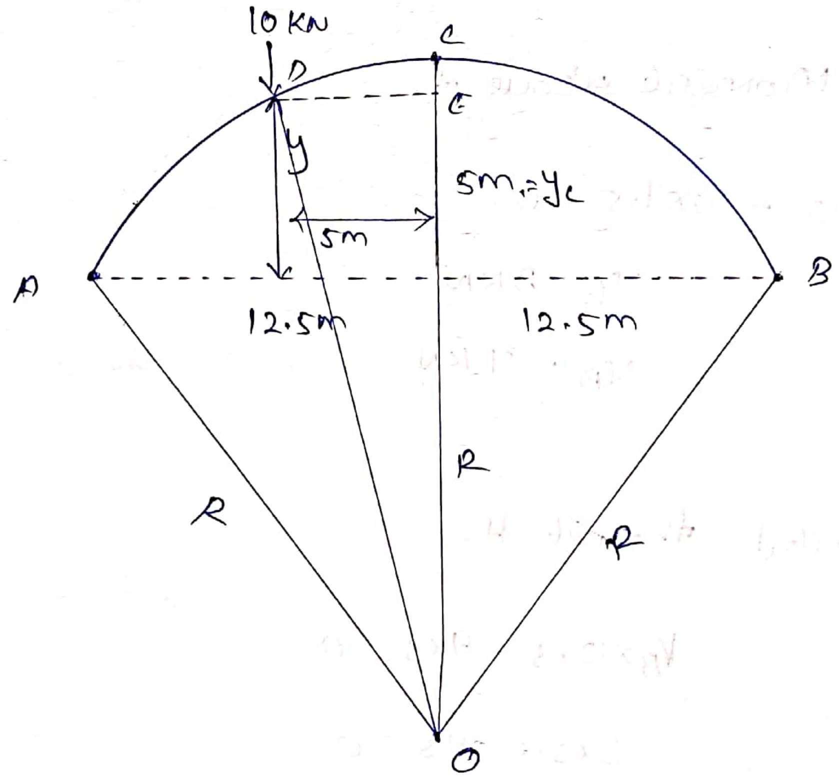
$$BM_D = V_A(7.50) - Hy.$$

Find y:

$$\frac{l}{2} \times \frac{l}{2} = y_c(2R - y_c)$$

$$12.5 \times 12.5 = 5(2R - 5)$$

$$R = 18.125\,m.$$



$$R^2 = (R - Y_c + y)^2 + x^2$$

$$18.125^2 = (18.125 - 5 + y) + 5^2$$

$$303.515 = (13.125 + y)^2$$

$$17.421 = 13.125 + y$$

$$y = 4.3m.$$

$$BM_D = 7(7.5) - 7.5(4.3)$$

$$= 20.25 KNm.$$

PROBLEM 5:

A parabolic arch hinged at ends has a span of 60m and a rise of 12m. A concentrated load of 8KN act at 15m from the left hinge. The second moment of area varies as the seccant of the inclination of arch axis. Calculate the horizontal thrust and the reactions at the hinge. Also calculate the net bending moment of the section.

Solution.

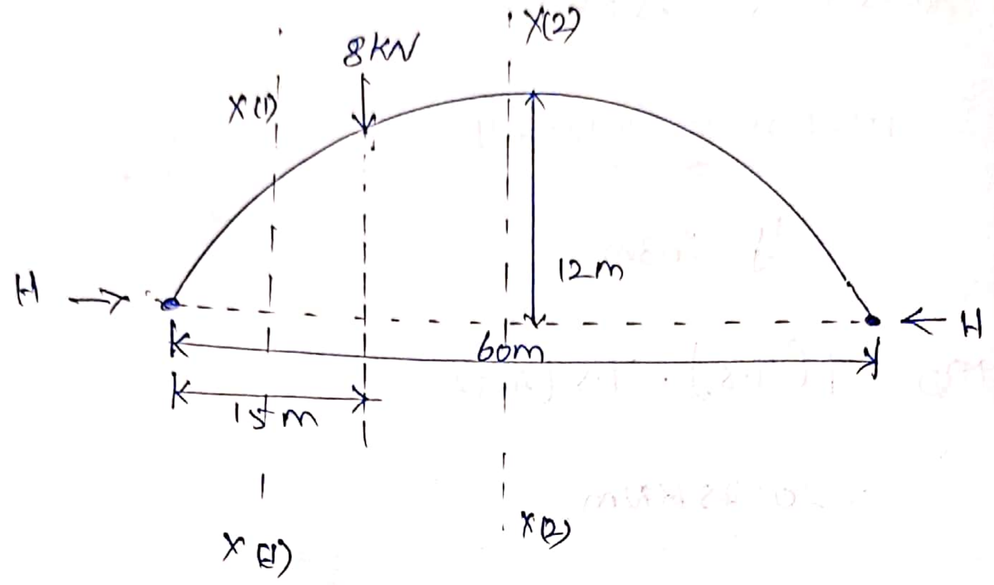
Vertical Reaction: V_A & V_B.

Taking Moment about A.

$$-8(15) + V_B \times 60 = 0$$

$$V_B = 2\,kN$$

$$V_A = 6\,kN.$$



Horizontal thrust (H)

$$H = \frac{\int_0^l M y\,dx}{\int_0^l y^2\,dx}.$$

$$\int_0^l M y\,dx = \int_0^{15} M_1 y\,dx + \int_{15}^{60} M_2 y\,dx.$$

$$y = \frac{4r}{l^2} x (l - x)$$

$$\int_0^{60} y^2\,dx = \int_0^{60} \left[\frac{4r}{l^2} x (l - x)\right]^2 dx.$$

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$$= \int_0^{60} \left[\frac{4\times12}{60^2} x(60-x)\right]^2 dx$$

$$= \int_0^{60} \quad y = (0.8x - 0.0133x^2)^2$$

$$= 0.64x^2 - 0.0213x^3 + (1.76\times10^{-4})x^4$$

$$= \int_0^{60} (0.64x^2 - 0.0213x^3 + 1.76\times10^{-4}x^4)\, dx$$

$$= \left[\frac{0.64x^3}{3} - \frac{0.0213x^4}{4} + \frac{1.76\times10^{-4}x^5}{5}\right]_0^{60}$$

$$= \cancel{12.18} + 138.24\times10^3 - 276.0\times10^3 + 27136.87\times10^3$$

$$= 46.08\times10^3 - 69.012\times10^3 + 27.37\times10^3$$

$$= 4439.52.$$

$$\int_0^{15} M_1 y\, dx$$

$$M_1 = V_A x_1 = 6x.$$

$$= \int_0^{15} 6x(0.8x - 0.0133x^2)\, dx$$

$$= \int_0^{15} (4.8x^2 - 0.079x^3)\, dx$$

$$= \left[\frac{4.8x^3}{3} - \frac{0.079x^4}{4}\right]_0^{15}$$

$$= 5400 - 999.84$$

$$= 4400.$$

$$\int_{15}^{60} M_2 y \, dx$$

$$M_2 = Y_A x_2 - 8(x_2 - 15)$$

$$= 6x - 8x + 120$$

$$= -2x + 120$$

$$= \int_{15}^{60} (120 - 2x)(0.8x - 0.0133x^2) \, dx$$

$$= \int_{15}^{60} (96x - 1.596x^2 - 1.6x^2 + 0.0266x^3) \, dx$$

$$= \int_{15}^{60} (0.0266x^3 - 3.196x^2 + 96x) \, dx$$

$$= \left[\frac{0.0266x^4}{4} - \frac{3.196x^3}{3} + \frac{96x^2}{2} \right]_{15}^{60}$$

$$= [(86184 - 230112 + 172800) - (336.6 - 359$$

$$+1080$$

$$= 21330.9$$

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$$H = \frac{4400 + 21330.9}{4439.52}$$

$$= 5.79 \text{ KN}$$

Reactions.

$$R_A = \sqrt{V_A^2 + H^2} = \sqrt{6^2 + 5.79^2} = 8.18 \text{ KN}$$

$$R_B = \sqrt{V_B^2 + H^2} = \sqrt{2^2 + 5.57^2} = 5.91 \text{ KN}$$

Max Bending Moment

$$M_x = V_A(15) - Hy$$

$$y = \frac{4 \times 12}{60^2} \times 15 \times (60 - 15) = 9 \text{m}$$

$$M_x = 6(15) - 5.79(9)$$

$$= 39.87 \text{ KNm.}$$

Fixed Arches:

Fixed arches are more common than hinged arches.

Analysis of fixed arches.

1. Castigliano's theorem

2. Elastic centre method

3. Column analogy method.

PROBLEM.6:

A parabolic arch fixed at both ends has a span of 42m and a central rise of 8.5m. It is subjected to concentrated loads of 75kN and 100kN at 8m and 16m respectively from the left end. The moment of inertia of the arch rib varies as the secant of the inclination of rib axis. Analyse the arch and find the Bending Moment at either support and at the crown.

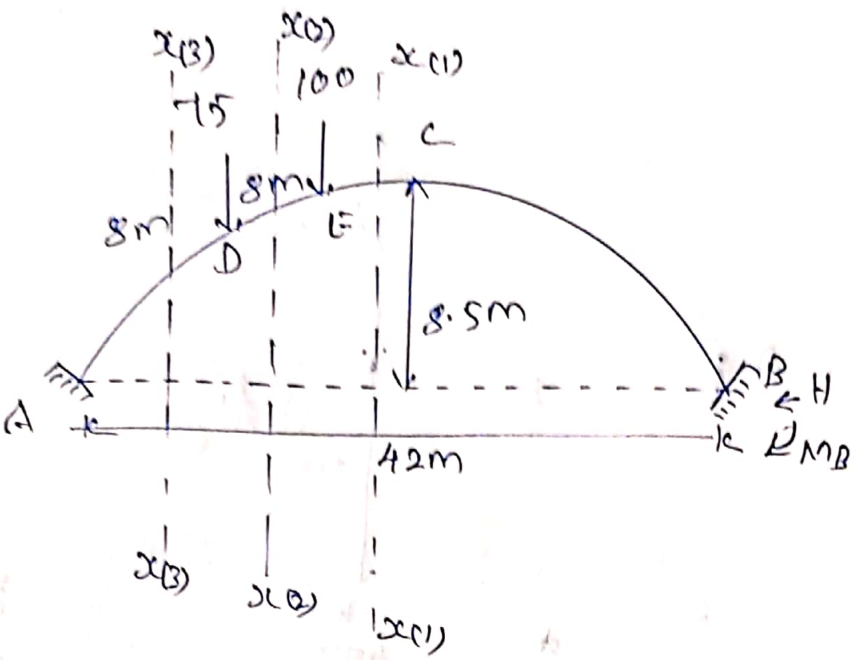
Solution

Find y:

$$y = \frac{4r}{l^2} x (l-x)$$

$$= \frac{4 \times 8.5}{42^2} \times x (42-x)$$

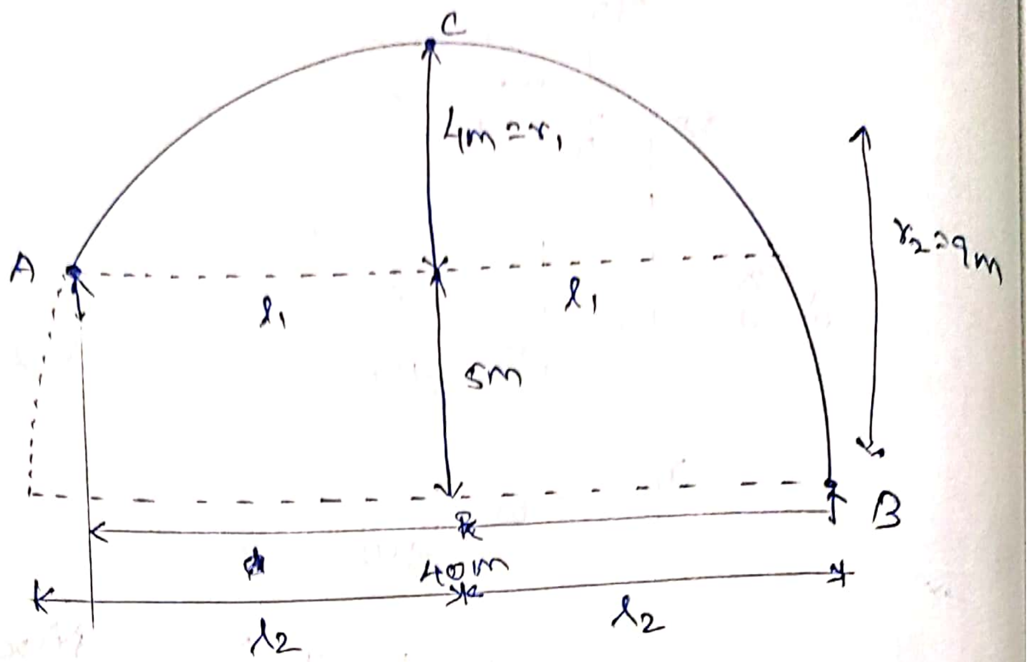
$$= 0.81x - 0.0192x^2$$



Portion	Origin	Limits (m)	M_x
BE	B	0-26	$V_B x - H_B y - M_B$
ED	B	26-34	$V_B x - H_B y - M_B - 100(x-26)$
DA	B	34-42	$V_B x - H_B y - M_B - \{100(x-26) - 75(x-34)\}$

PROBLEM: 7

A three hinged parabolic arch of 40m span has abutments at unequal levels. The highest point of the arch is 4m above the left support and and 9m above right support abutments. The arch is subjected to an udl of 15kN/m over its entire horizontal span. Find the horizontal thrust and bending moment at a point 8m from the left support.



Solution.

Reactions: A, B and H.

Find l_1, l_2,

$$\frac{l_1}{l_2} = \sqrt{\frac{r_1}{r_2}}$$

$$\frac{l_1}{40 - l_1} = \sqrt{\frac{4}{9}}$$

$$l_1 = (40 - l_1) \times 2/3$$

$$l_1 = 16m$$

$$l_2 = 40 - l_1$$
$$= 40 - 16$$
$$= 24m$$

Considering left side of C.

$$V_A(16) - H(4) - 1.5 \times 16 \times 16/2 = 0$$

$$16V_A - 4H - 1920 = 0$$

$$4V_A - H - 480 = 0 \quad \text{--- (1)}$$

Considering right side of C.

$$-V_B(24) + H(9) + 15 \times 24 \times 24/2 = 0$$

$$-24V_B + 9H + 4320 = 0$$

$$-8V_B + 3H + 1440 = 0 \quad \text{--- (2)}$$

$$V_A + V_B = 600$$

$$V_B = 600 - V_A \quad \text{--- (3)}$$

sub (3) in (2)

$$-(600 - V_A) + 3H + 1440 = 0$$

$$8V_A - \cancel{600}^{4800} + 3H + 1440 = 0$$

$$8V_A + 3H - 3360 = 0 \quad \text{--- (4)}$$

(1) & (4) ⇒ Solve

$$4V_A - H - 480 = 0$$

$$V_A + 3H - \cancel{480}^{3360}_{\cancel{1440}} = 0$$

$$\Rightarrow \times 4 \quad -4V_A + 12H + \cancel{1920}_{13440} = 0$$

$$11H + \cancel{1440}_{12960} = 0$$

$$H =$$

$$8V_A - 2H - 960$$

$$8V_A + 3H - 3360$$

$$(-) \quad (-) \quad (+)$$

$$-5H = 2400$$

$$H = 480\,kN$$

$4V_A - H - 480 = 0$

$4V_A - 480 - 480 = 0$

$V_A = 240\,KN$

$V_B = 360\,KN.$

Bending Moment $x = 8.$

$BM_x = V_A(8) - 15 \times 8 \times 8/2 - Hy$

$y = \frac{4r}{l^2} x(l - x)$

$= \frac{4 \times 4}{(2 \times 16)^2} \times 8\,(2 \times 16 - 8)$

$y = 3m.$

$BM = 240 \times 8 - 15 \times 32 - 480 \times 3$

$= 0.$

Radial shear.

$R = V_x \cos\theta - H \sin\theta$

$V_x = V_A - 15 \times 8$

$= 240 - 15 \times 8$

$= 120\,KN.$

~~[illegible]~~ $\theta = \tan^{-1}\left[\frac{4r}{l^2}(l - 2x)\right]$

$$\theta = \tan^{-1}\left[\frac{4\times4}{(2\times16)^2}(32-2\times8)\right]$$

$$= 14^\circ 2'$$

$$F = 120\cos 14^\circ 2' - 480\sin 14^\circ 2'$$

$$F = 0$$

Normal thrust N at x = 8m

$$P_N = V_x \sin\theta + H\cos\theta$$

$$= 120\times\sin 14^\circ 2' + 480\cos 14^\circ 2'$$

$$= 494.77\,KN$$

Formulas:

1.) Resultant Reactions

$$R_A = \sqrt{H^2 + V_A^2}$$

$$R_B = \sqrt{H^2 + V_B^2}$$

Diff Level. Length

$$\frac{l_1}{l_2} = \sqrt{\frac{r_1}{r_2}}$$

2.) Slope.

$$\theta = \tan^{-1}\left[\frac{V_A}{H}\right].$$

$$\theta = \tan^{-1}\left[\frac{4r}{l^2}(l-2x)\right].$$

3.) Find y

$$y = \frac{4r}{l^2}x(l-x)$$

4.) Normal thrust

$$P_N = V_x \sin\theta + H\cos\theta$$

V_x = Shear force.

5.) Radial Shear force.

$$R_t = V_x \cos\theta - H\sin\theta$$

6.) Three hinged circular arch.

Find Radius 'R'

$$\frac{l}{2} \times \frac{l}{2} = Y_c [2R - Y_c]$$

Find y

$$R^2 = (R - y_c + y)^2 + x^2$$

7.) Two hinged arch

Horizontal thrust $H = \dfrac{\int_0^l M y\,dx}{\int_0^l y^2\,dx}$

8.) Fixed arches:

$$\frac{\partial U}{\partial V_B} = \frac{1}{EI}\int_0^l M_x \times \frac{\partial M_x}{\partial V_B} \cdot dx$$

$$\frac{\partial U}{\partial H_B} = \frac{1}{EI}\int_0^l M_x \times \frac{\partial M_x}{\partial H_B} \cdot dx.$$

$$\frac{\partial U}{\partial M_B} = \frac{1}{EI}\int_0^l M_x \times \frac{\partial M_x}{\partial M_B} \cdot dx.$$

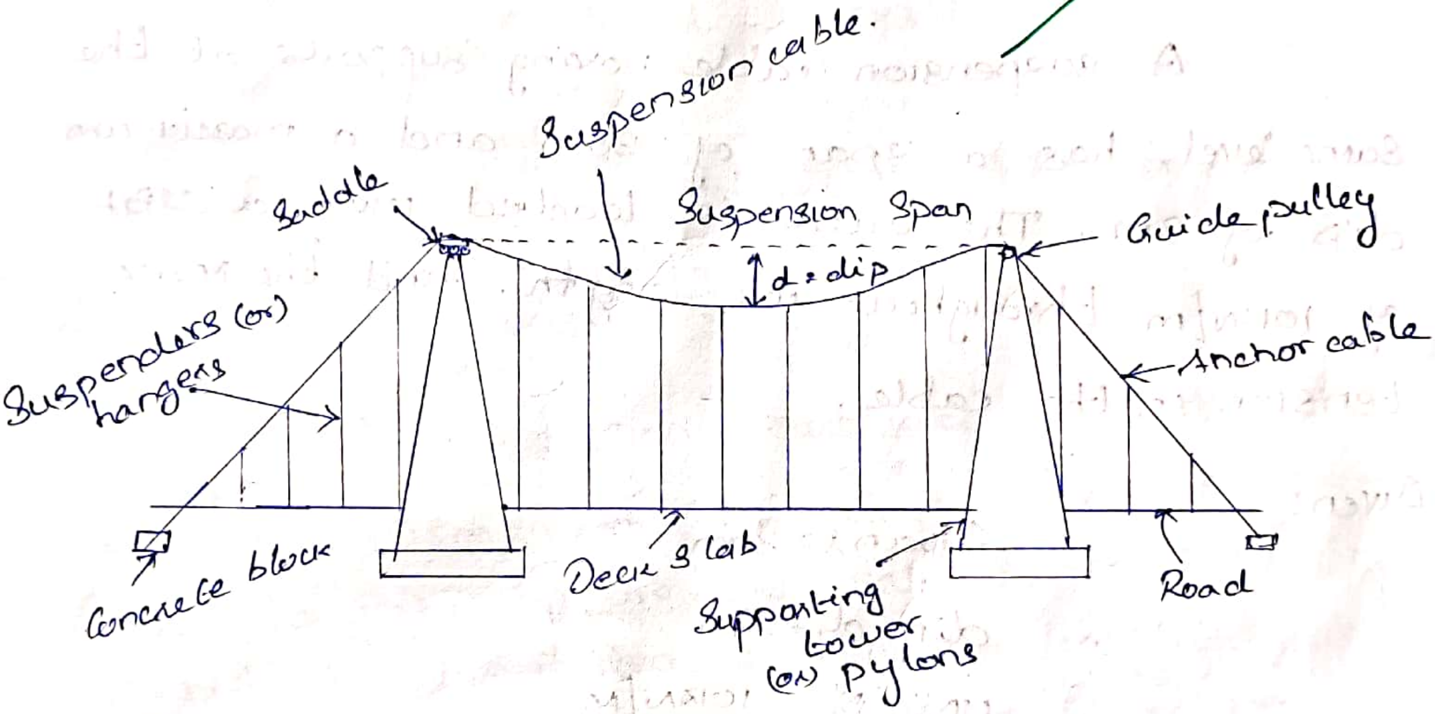
2/1/2020

UNIT-IV

CABLES AND SUSPENSION BRIDGES

Suspension cable:

Components and their functions



Suspension Cable : Main load Bearing Member flexible, take direct tension only.

Suspenders : Girder with deck slab is suspended from the the suspension cable, (or) hangers, transfer load from deck to suspenders.

Anchor cables: After passing over pylons, cables anchored to the bed rock.

Passing the cable over pylons

(i) Guide pulley support

(ii) Roller (or) Saddle Support

Anchoraging into a huge mass of concrete.

Problems 1:

A suspension cable having supports at the same level, has a span of 30m and a maximum dip of 3m. The cable is loaded with a UDL of 10kN/m throughout its length. Find the max tension in the cable.

Given:

Span 'l' = 30m

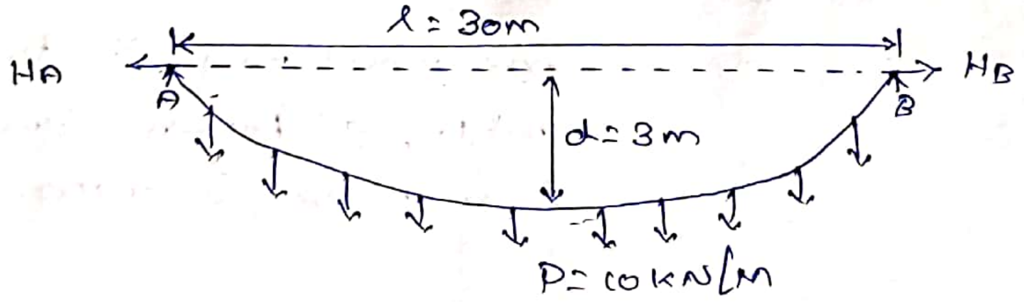
dip d = 3m

UDL 'p' = 10kN/m

To find

Max tension in the cable

Solution:



Find Vertical Reactions.

$$V_A = V_B = \frac{pl}{2} = \frac{10 \times 30}{2} = 150 kN$$

Max Tension in cable

$$T_{max} = \sqrt{V_A^2 + H^2}$$

Horizontal pull in the cable

$$H = \frac{pl^2}{8d} = \frac{10 \times 30^2}{8 \times 3} = 375 kN$$

$$T_{max} = \sqrt{150^2 + 375^2} = 403.88 kN$$

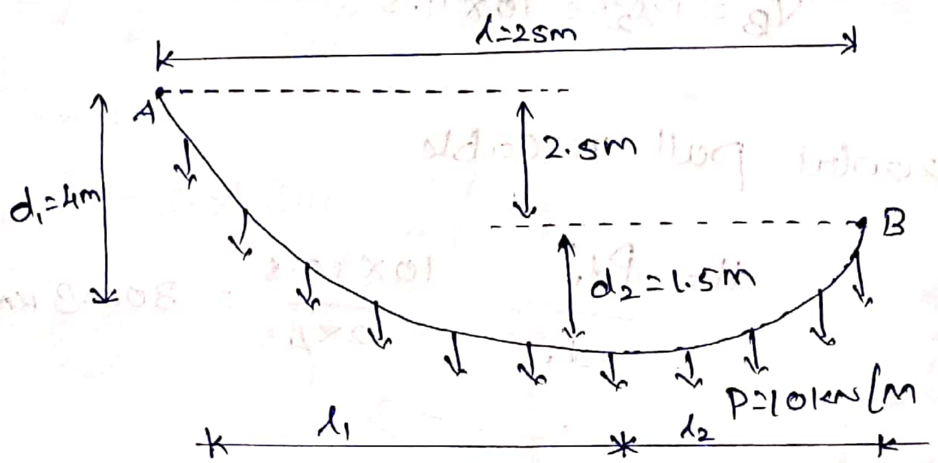
Problems 2:

A suspension cable is supported at two points 25m apart. The left support is 2.5m above the right support. The cable is loaded with a uniformly distributed load of 10kN/m througout the span. The max dip in the cable from the left support is 4m. Find max and min tension in cable.

Given:

UDL = 10 kN/m

$d_1 = 4m$



To find

Max and Min tension in cable.

Solution

Find length l_1 and l_2

$$\frac{l_1}{l_2} = \sqrt{\frac{d_1}{d_2}} \quad \text{+++1 (V)}$$

$$l_1 = \sqrt{\frac{4}{1.5}} \times l_2$$

$$l_1 = 1.63\, l_2 \quad \text{— (1)}$$

$$l = l_1 + l_2$$

$$25 = 1.63 l_2 + l_2$$

$$25 = 2.63 l_2$$

$$l_2 = 9.5\,m$$

$$l_1 = l - l_2$$
$$= 25 - 9.5$$
$$= 15.5\,m$$

Vertical Reactions

$$V_A = Pl_1 = 10 \times 15.5 = 155\ KN.$$

$$V_B = Pl_2 = 10 \times 9.5 = 95\ KN.$$

Horizontal pull in cable

$$H = \frac{Pl_1^2}{2d_1} = \frac{10 \times 15.5^2}{2 \times 4} = 800.3\ KN$$

$$H = \frac{Pl_2^2}{2d_2} = \frac{10 \times 9.495^2}{2 \times 1.5} = 300.3 \text{ KN}$$

Tension in cable.

$$T_A = \sqrt{V_A^2 + H^2} = \sqrt{155^2 + 300.3^2} = 337.9 \text{ KN}$$

$$T_B = \sqrt{V_B^2 + H^2} = \sqrt{300.3^2 + 95^2} = 314.96 \text{ KN}$$

Max Tension $= 337.9$ KN

Min Tension $= 300.3$ KN

Problem 3:

A suspension cable of 130m horizontal span is supported at the same level. It is subjected to a uniformly distributed load of 28.5kN / horizontal metre. If the max tension in the cables is limited to 5000kN. Calculated central dip needed.

Given data:

Span $l = 130$m

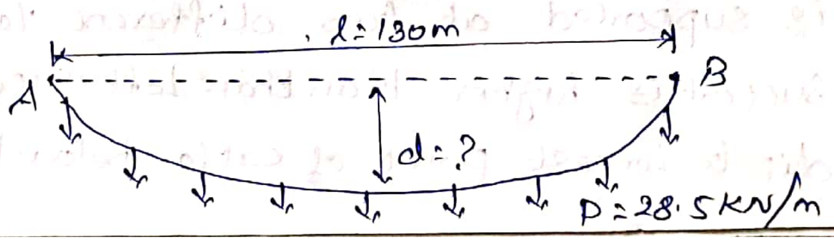
UDL $P = 28.5$ kN/m

$T_{max} = 5000$ kN

To find

Central Dip.

Solution:



Vertical Reactions:

$$V_A = V_B = \frac{Pl}{2} = \frac{28.5 \times 130}{2} = 1852.5\,KN$$

Horizontal pull (tensio).

$$H = \frac{Pl^2}{8d} = \frac{28.5 \times 130^2}{8d} = \frac{60206.25}{d}\,KN$$

Max Tension

$$T_{max} = \sqrt{V^2 + H^2}$$

$$5000 = \sqrt{1852.5^2 + \left(\frac{60206.25}{d}\right)^2}$$

$$5000^2 = 1852.5^2 + \frac{60206.25^2}{d^2}$$

$$\frac{5000^2 - 1852.5^2}{60206.25^2} = \frac{1}{d^2}$$

$$5.95 \times 10^{-3} = \frac{1}{d^2}$$

$$d^2 = \frac{1}{5.95 \times 10^{-3}}$$

$$d = 168. = 12.96m.$$

PROBLEM 4:

A suspension cable of horizontal span 95m is supported at two different levels. The right support is higher than than Left support by 4m. The dip to Lowest point of cable below the left support is 5m. The cross sectional area of the cable is 3500 mm^2. Find the uniformly distributed load that can be carried by the cable if the max stress is limited to 600 N/mm^2.

Given data:

Span $l = 95m$

$d_1 = 5m$

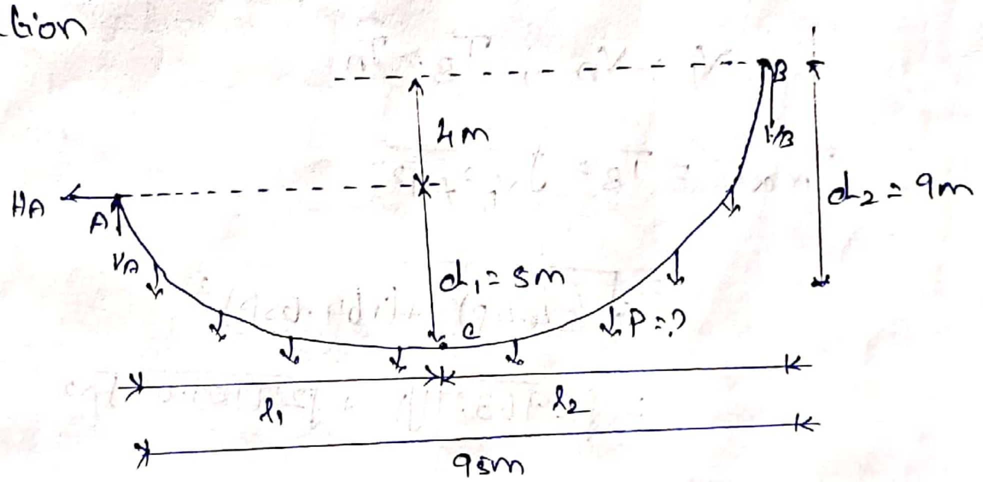
$A = 3500mm^2$

Stress $\sigma = 600 N/mm^2$.

To find

Uniformly Distributed load.

Solution



Find Length

$$\frac{l_1}{l_2} = \sqrt{\frac{d_1}{d_2}}$$

$$\frac{l_1}{l_2} = \sqrt{\frac{5}{9}}$$

$l_1 = 0.745 l_2$

$$l = l_1 + l_2$$

$$95 = 0.745\, l_2 + l_2$$

$$95 = > 1.745\, l_2$$

$$l_2 = 54.4\,m$$

$$l_1 = 40.56\,m$$

Vertical Reactions

$$V_A = Pl_1 = 40.56P.$$

$$V_B = Pl_2 = 54.4P$$

Horizontal pull.

$$H = \frac{Pl^2}{2d} = \frac{P(54.4)^2}{2\times 9} = 164.4\,P.$$

Max tension will occur at right support

$$\because V_B > V_A \;,\; T_B > T_A$$

$$T_{max} = T_B = \sqrt{V_B^2 + H^2}$$

$$= \sqrt{(54.4P)^2 + (164.65P)^2}$$

$$= \sqrt{2963.71}\sqrt{P^2} + \sqrt{27109.62}\sqrt{P^2}$$

$$T_{max} = 173.4\,P\;N$$

$$\sigma = \frac{T_{max}\;kN}{A}$$

$$600\,N/mm^2 = \frac{173.4P}{3500\,mm^2}$$

$N/mm^2 \times mm^2$

$$P = 12110\;N/m$$

$$= 12.11\;kN/m$$

PROBLEMS: 5

A cable of horizontal span 21m is to be used to support six equal loads of 40kN each at 3m spacing. The central dip of the cable is limited to 2m. Find the length of the cable required and also its sectional area if the safe tensile stress is 750 N/mm^2.

Given data:

Span $l = 21m$

dip 'd' = 2m

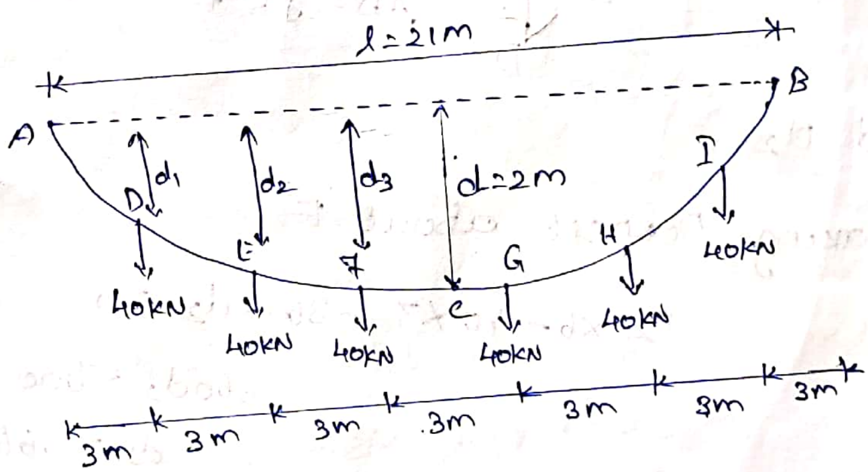
$\sigma = 750 N/mm^2$

To find

Length of cable

Sectional Area

Solution.



Vertical reaction

$$V_A = V_B = \frac{\text{Total load}}{2} = \frac{6 \times 40}{2} = 120kN$$

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Horizontal pull.

Taking Moment about 'C'

$V_A \times 10.5 - 40\times 7.5 - 40\times 4.5 - 40\times 1.5 - H\times 2 = 0$

$120\times 10.5 - 540 - 2H = 0$

$720 = 2H$

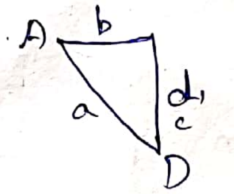
$H = 360\,kN.$

Find D_1

Taking Moment about D.

$120\times 3 - 360\times d_1 = 0$

$d_1 = 1m.$



$AD = \sqrt{b^2+c^2} = \sqrt{3^2+1^2} = 3.16m$

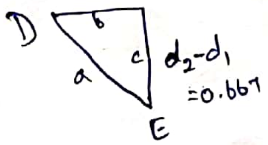
Find d_2

Taking Moment about 'E'

$120\times 6 - 40\times 3 - 360\times d_2 = 0$

$360 d_2 = 600$

$d_2 = 1.667\,m$



$DE = \sqrt{b^2+c^2} = \sqrt{3^2+0.667^2} = 3.073\,m$

Find d_3

Taking Moment about F

$120\times 9 - 40\times 6 - 40\times 3 - 360\times d_3 = 0$

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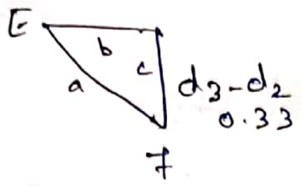
$720 - 360\, d_3 = 0$

$d_3 = 2\text{m}$



$E7 = \sqrt{b^2 + c^2} = \sqrt{3^2 + 0.33^2} = 3.018\text{m}$

Length of cable $= 2\,(AD + DE + E7 + 7C)$

$= 2\,(3.162 + 3.073 + 3.018 + 1.5)$

$= 21.506\,\text{m}$

Max Tension in cable

$T_{max} = \sqrt{V^2 + H^2} = \sqrt{120^2 + 360^2} = 379.47\,\text{kN}$

Area

$\sigma = \dfrac{T_{max}}{A}$

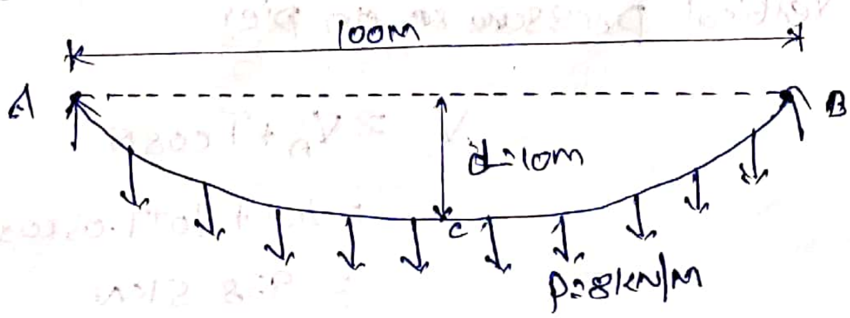
$750 = \dfrac{379.47 \times 10^3}{A}$

$A = 0.505\,\text{m}^2$

$= 505\,\text{mm}^2$

PROBLEM: 6.

A suspension cable of span 100m and dip 10m carries a uniformly distributed load of 8kN/m of horizontal span over the full span. Find the vertical and horizontal forces transmitted to the supporting pylons.



a) If the cable is passed over a smooth pulley

b) If the cable is clamped to a saddle with rollers on the top of the piers. The anchor cable makes 30° to the horizontal at the pylons.

Given data:

~~Data~~

Span $l = 100m$

dip $d = 10m$

$P = 8 kN/m.$

$\theta = 30^0$

Solution:

Vertical Reactions:

$$V_A = V_B = \frac{Pl}{2} = \frac{8 \times 100}{2} = 400 kN$$

Horizontal pull

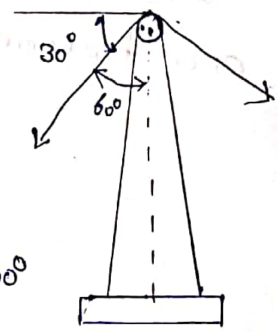
$$H = \frac{Pl^2}{8d} = \frac{8 \times 100^2}{8 \times 10} = 1000 kN$$

Tension in cable.

$$T = \sqrt{V^2 + H^2} = \sqrt{400^2 + 1000^2} = 1077.03 kN$$

a) · Anchor cable passing over pulley.

Vertical pressure ~~on~~ on pier



$$V = V_A + T\cos\theta$$

$$= 400 + 1077.03 \cos 60^0$$

$$= 938.5 kN$$

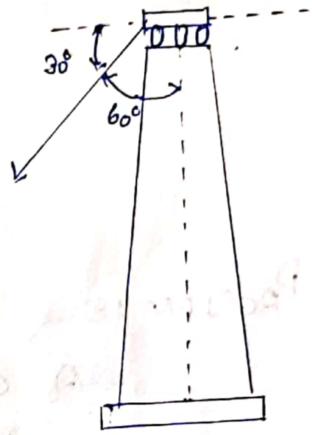
Horizontal force at top of pylon.

$= H - T \sin\theta$

$= 1000 - 1077 \sin 60^\circ$

$= 67.29$ kN.

b) Cable passing over saddle support



W.K.T $T_1 = H/\sin 60^\circ$

$= \frac{1000}{\sin 60^\circ}$

$= 1154.7$ kN

Vertical pressure

$= V + T_1 \cos\theta$

$= 400 + 1154.7 \cos 60^\circ$

$= 977.35$ kN

PROBLEM: 7

A suspension cable of horizontal span 210m is supported ~~at~~ at the same level and has a central dip of 20m. Find the increase in dip of the cable if the cable is subjected to a rise in temperature of 28°c. Take $\alpha = 12 \times 10^{-6}$ per °c.

Given data:

Span $l = 210$m

$d = 20$m

$t = 28$°c

$\alpha = 12 \times 10^{-6}$ per °c

Solution.

Change in dip.

$$\delta d = \frac{3l^2}{16d} \alpha t$$

$$= \frac{3 \times 210^2}{16 \times 20} \times 12 \times 10^{-6} \times 28$$

$$= 0.138 m.$$

$$= 138 mm.$$

PROBLEM: 8

A cable supported at the same level on either end is of 140m horizontal span with a central dip of 14m. It carries a load of 15kN/m on the horizontal span. Calculate the change in the horizontal tension when the temperature rises through 28°c. Co-efficient of linear expansion of the cable materials. $\alpha = 4 \times 10^{-6} /°c$.

Given data:

Span $l = 140m$

$d = 14m$.

$P = 15 kN/m$

$t = 28°c$

$\alpha = 4 \times 10^{-6} /°c$

Solution

$$H = \frac{Pl^2}{8d} = \frac{15 \times 140^2}{8 \times 14} = 2625 kN$$

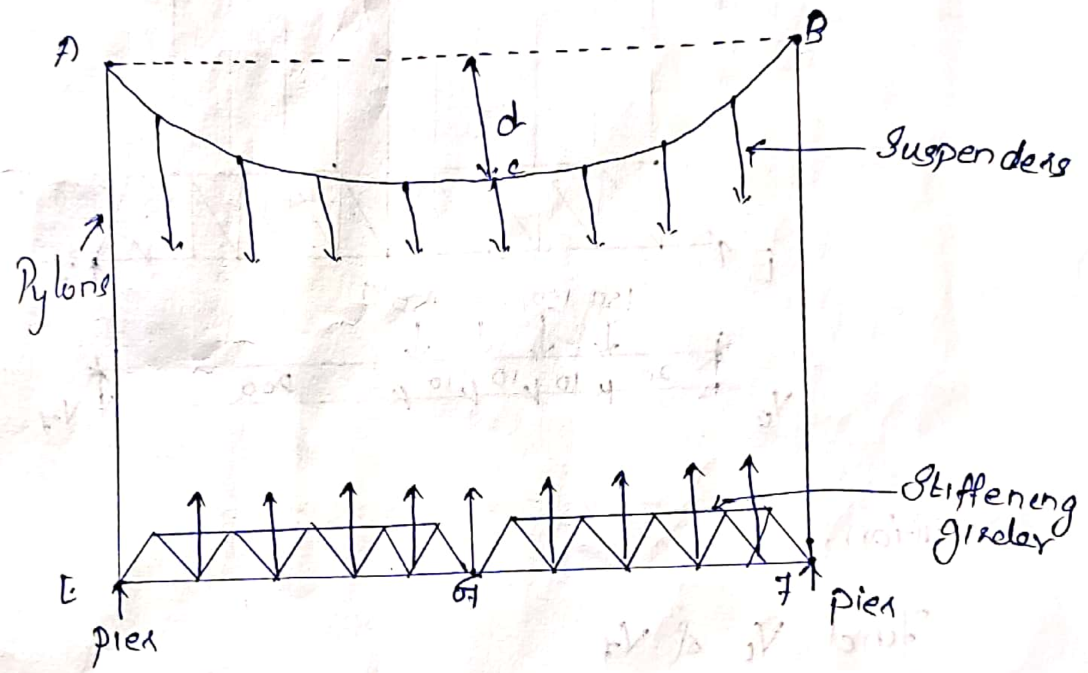
Change in horizontal tension.

$$\Delta H = -\frac{3l^2}{16d^2} \alpha t H$$

$$= \frac{3 \times 140^2}{16 \times 14^2} \times 4 \times 10^{-6} \times 28 \times 2625$$

$= -5.513$ KN

SUSPENSION BRIDGES WITH THREE HINGED STIFFENING GIRDERS.



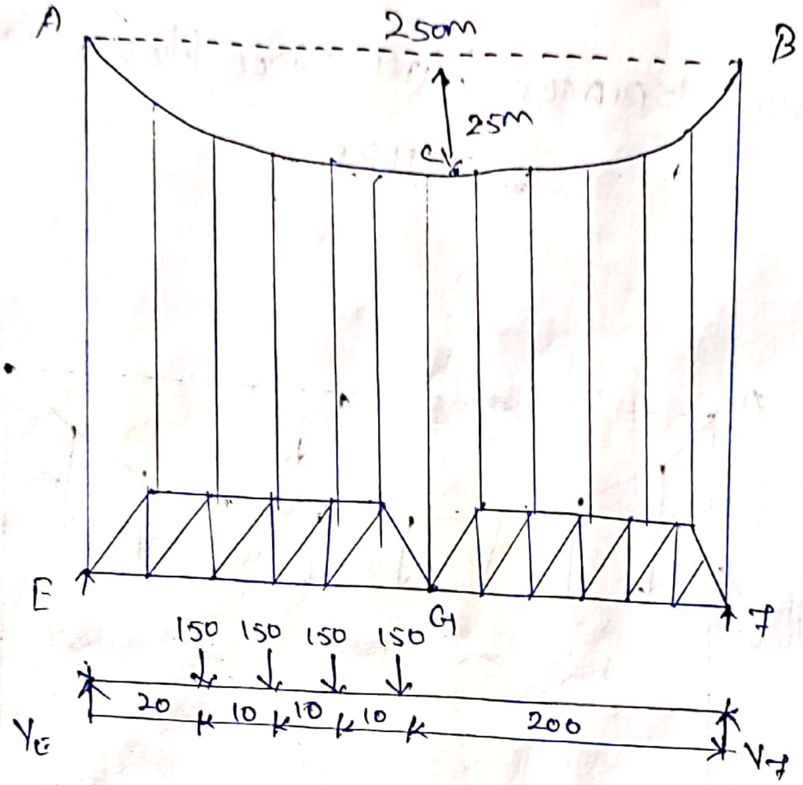
PROBLEM: 8.

A. Suspension bridge of 250m span has two nos. of three hinged stiffening girders supported by cables with a central dip of 25m. If 4 point loads of 300KN each are placed at the centre line of the roadway at 20m, 30m, 40m and 50m from the left hand hinge. Find the shear force and Bending moment in each girder at 62.5m from each end calculate also the max tension in the cable.

Given data:

Span $l = 250m$

dip $'d' = 25m$



Solution:

Find V_E & V_F

Taking Moment about 'F'

$V_E \times 250 - 150 \times 200 - 150 \times 210 - 150 \times 220 - 150 \times 230 = 0$

$V_E = 516\ kN$

Total load $= V_E + V_F$

$600 = 516 + V_F$

$V_F = 84\ kN$

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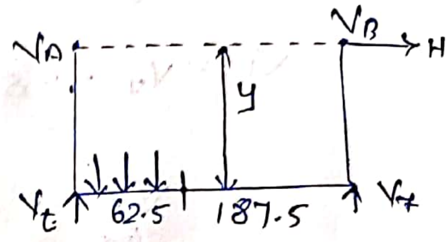
Horizontal pull

$$H = \frac{M_c}{d} = \frac{V_7 \times 125}{25} = \frac{84 \times 125}{25} = 420KN$$

a) Bending Moment

BM @ 62.5m From left hand hinge

$= V_7 \times 187.5 - H \times y$



$$y = \frac{4d}{l^2} x(l-x)$$

$$= \frac{4 \times 25}{250^2} \times 62.5 \times 187.5$$

$= 18.75m$

BM @ 62.5m

$= 84 \times 187.5 - 18.75 \times 420$

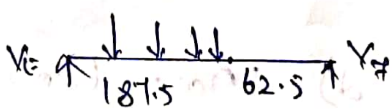
$= 787.5 KNm$

BM @ 62.5 From Right hand hinge

$= V_7 \times 62.5 - H \times y$

$= 84 \times 62.5 - 420 \times 18.75$

$= -2625 KNm.$



b) Shear Force:

SF @ 62.5 From left hand hinge

$V = V_b - \tan\alpha H$

$$\tan\theta = \frac{4d}{l^2}(l-2x)$$

$$= \frac{4\times25}{250^2}(250-2\times62.5) = 0.2$$

$$\tan\theta = 0.2$$

$V_D = V_E - 4\times150 = 516 - 600 = -84\,KN$

$V_D = V_7 = -84\,KN$

$V_{62.5} = -84 - 420\times0.2 = -168\,KN$

SF @ 62.5 from Right side

SF $\times$ $V_{187.5} = -V + H\tan\theta$

$= -84 + 420\times0.2 = 0$

c) Vertical pull on the cable.

$$H = \frac{Pl^2}{8d}$$

$$420 = \frac{P\times250^2}{8\times25}$$

$P = 1.34\,KN/M$

d) Max tension in cable

$$T = \sqrt{V_A^2 + H^2}$$

$$V_A = V_B = \frac{Pl}{2} = \frac{1.34\times250}{2} = 168\,KN$$

$$T = \sqrt{168^2+420^2} = 452.35\,KN.$$

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FORMULAS:

1. Support at Same Level.

$$V_A = V_B = \frac{Pl}{2}$$

$$H = \frac{Pl^2}{8d}$$

$$ds = \sqrt{dx^2 + dy^2}$$

2. Support at different level

$$l = l_1 + l_2$$

$$H = \frac{Pl_1^2}{2d_1}, \quad H = \frac{Pl_2^2}{2d_2}$$

$$\frac{l_1}{l_2} = \sqrt{\frac{d_1}{d_2}}$$

$$V_A = Pl_1$$

$$V_B = Pl_2$$

3. Max Tension

$$T_{max} = \sqrt{V_A^2 + H^2}$$

4. Stress $= \frac{\text{Load}}{\text{Area}}$

$$\sigma = \frac{T_{max}}{A}.$$

5. Cable passing over pulley

Vertical pressure $= V_A + T\cos\theta$

Horizontal force = $H - T\sin\theta$

6. Cable passing over saddle.

T_1

Vertical pressure $= V + T_1 \cos\theta$

$$\therefore T_1 = \frac{H}{\sin 60^\circ}$$

7. Change in dip. (or) increase in dip

$$\delta d = \frac{3l^2}{16d} \alpha t.$$

8. Change in horizontal Tension

$$\Delta H = \frac{-3l^2}{16d^2} \alpha t H$$

9. Three hinged problems.

$$H = \frac{\mu_c}{d}$$

$$y = \frac{4d}{l^2} x(l-x)$$

SF @ left $= V_b - H\tan\theta$

SF @ Right $= H\tan\theta - V_b$

UNIT-V

PLASTIC ANALYSIS

Definition:

Plastic Hinge:

Fully plastic moment is considered to have develop at any section of a structure subjected to a system of loads, when the section is completely yielded or plastified.

Plastic hinge is defined as an yielded zone due to bending in a structural member, at which large rotations can take place at a section at a constant plastic moment. Mp.

Types of Mechanism:

1. Beam Mechanism
2. Panel mechanism (or) Sway mechanism
3. Gable mechanism.

Static method or Virtual Work Method

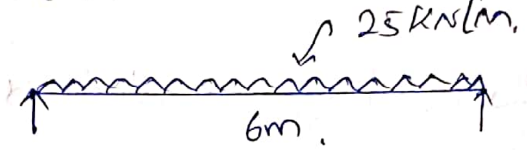
This method is based on the static or lower bound theorem which states that A load computed on the basis of an assumed equilibrium BM diagram in which the moments are not greater than Mp.

Load Factor:

$$\text{Factor of safety} = \frac{\text{Yield Stress}}{\text{Working Stress}}$$

PROBLEM 1:

A beam of span 6m is to be designed fo[r] an ultimate U.D.L of 25 KN/m. The beam is ss at ends. Design a suitable I section using plastic theo[ry] assuming $\sigma_y = 250\ N/mm^2$

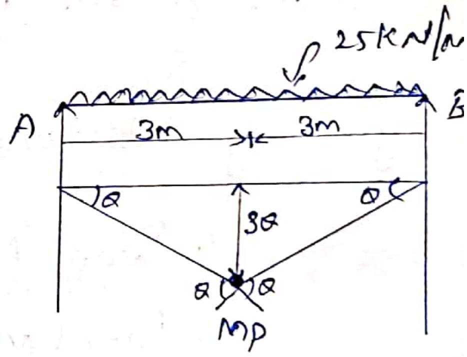


Solution:

Internal Work Done

$= 0 + M_p \times 2\theta + 0$

$= 2M_p\theta.$



External Work Done

= Load intensity × Area of under triangle.

$= 25 \times (\frac{1}{2} \times 6 \times 3\theta)$

$= 225\theta$

Equating I.W.D = E.W.D

$2M_p\theta = 225\theta$

$M_p = \frac{225}{2} = 112.5\ KNm$

$$\frac{M}{I} = \frac{\sigma}{y} \quad , \quad Z = \frac{I}{y}$$

$$M = \sigma \times I/y$$

$$M_p = \sigma \times Z_p$$

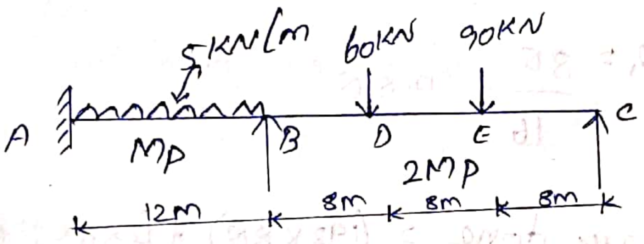
$$Z_p = \frac{M_p}{\sigma} = \frac{1125 \times 10^6 \text{ Nmm}}{250} = 4.5 \times 10^5 \text{ mm}^3$$

$$S = \frac{Z_p}{Z}$$

$$Z = \frac{Z_p}{S} = \frac{4.5 \times 10^5}{1.15} = 391.304 \times 10^3 \text{ mm}^3$$

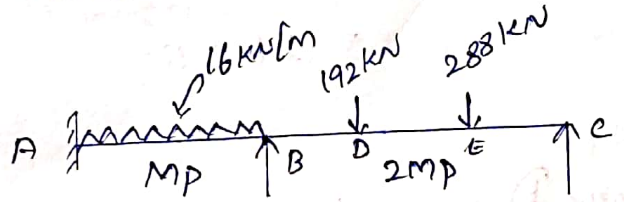
PROBLEM 2:

A continuous Beam ABC is loaded as shown in figure. Determine the required Mp if the load factor is 3.2.

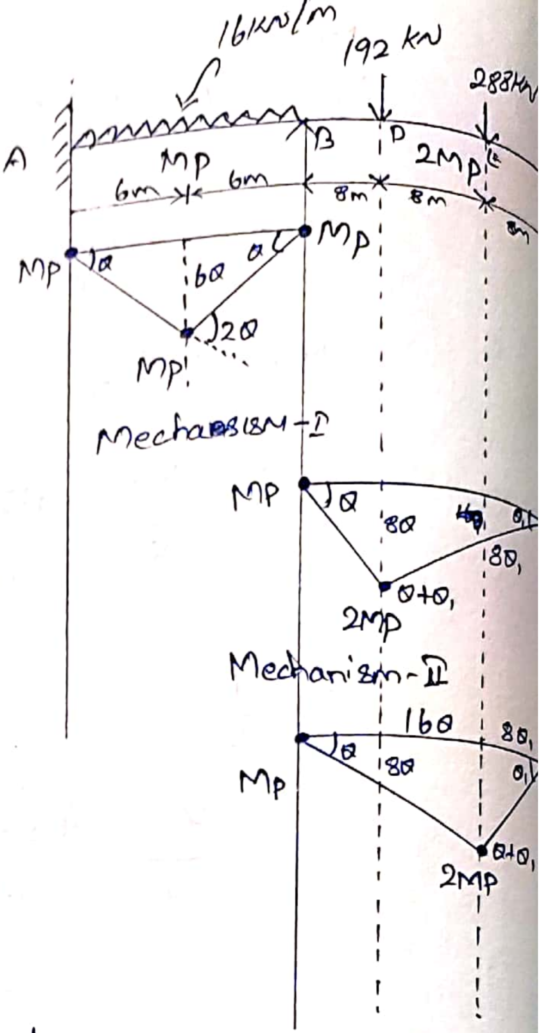


Solution.

Load Factor = 3.2



Mechanism - I



External Work Done

$= 16 \times \left[\frac{1}{2} \times 12 \times 6\theta\right]$

$= 576\theta \quad -(1)$

Internal Work Done

$= \theta M_p + (M_p \times 2\theta) + M_p\theta$

$= 4M_p\theta \quad -(2)$

Equating (1) & (2)

$576\theta = 4M_p\theta$

$M_p = \frac{576}{4} = 144\, kNm$

Mechanism - II

$\theta_1 = \frac{8\theta}{16} = 0.5\theta$

External Work done $= (192 \times 8\theta) + (288 \times 8\theta_1)$

$= 1536\theta + 2304 \times 0.5\theta$

$= 1536\theta + 1152\theta$

$= 2688\theta$

Internal Work Done $= M_p\theta + 2M_p(\theta + \theta_1) + 0$

$= M_p\theta + 2M_p(\theta + 0.5\theta) + 0$

$= M_p\theta + 2M_p(1.5\theta) = 4M_p\theta$

External = Internal

$2688\,\theta = 4M_p\theta$

$M_p = \frac{2688}{4} = 672\,kN/m.$

Mechanism III

External Work Done.

$= (192\times 8\theta) + (288\times 16\theta)$

$= 6144\,\theta$

Internal Work Done:

$= M_p\theta + 2M_p(\theta+\theta_1)$

$\therefore\ \theta_1 = \frac{16\theta}{8} = 2\theta.$

$= M_p\theta + 2M_p 3\theta$

$= M_p\theta + 6M_p\theta$

$= 7M_p\theta$

$6144\,\theta = 7M_p\theta$

$M_p = 877.7\,kN/m.$

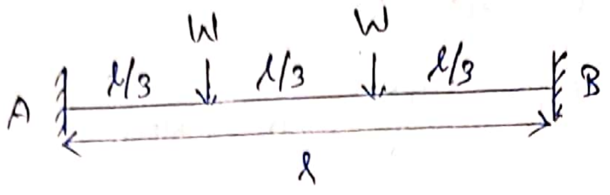
Plastic Moment:

$M_p = 877.71\,kN/m.$

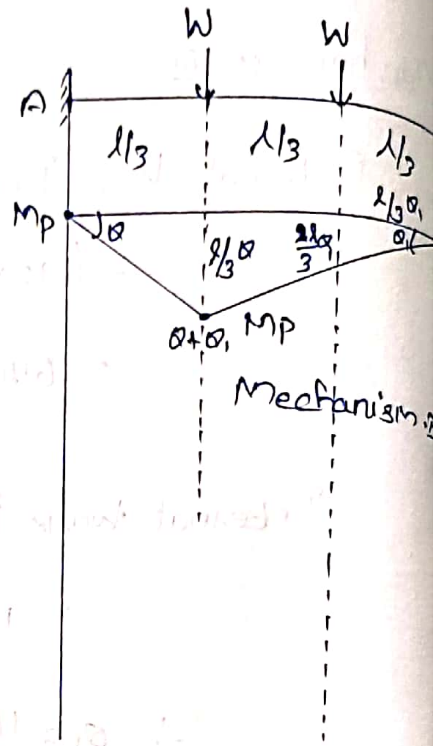
PROBLEM: 3

A beam fixed at both ends is subjected to two concentrated loads, each at $\frac{1}{3}$rd point of the span.

Determine the collapse load for the beam in te
of its Mp.



Solution.



Find θ_1

$l \quad \theta_1 = \frac{\quad}{0.33\theta_1}$

$\frac{l}{3}\theta = \frac{2l}{3}\theta_1$

$0.33l\theta = 0.66l\theta_1$

$\theta_1 = 0.5\theta$

Mechanism I

External Work Done

$= W(0.33l\theta) + W(0.33l\theta_1)$

$= W(0.33l\theta) + W(0.33 \times 0.5 l\theta)$

$= \cancel{l\theta}\ 0.495 l\theta W$

Internal Work Done.

$= M_p\theta + M_p(\theta+\theta_1) + M_p\theta_1$

$= M_p\theta + M_p(\theta + 0.5\theta) + M_p\, 0.5\theta$

$= M_p\theta + 1.5M_p\theta + 0.5M_p\theta$

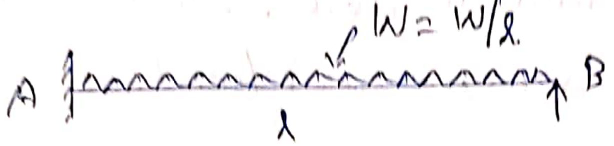
$= 3M_p\theta$

$l\, 0.495 \theta W = 3 M_p \theta$

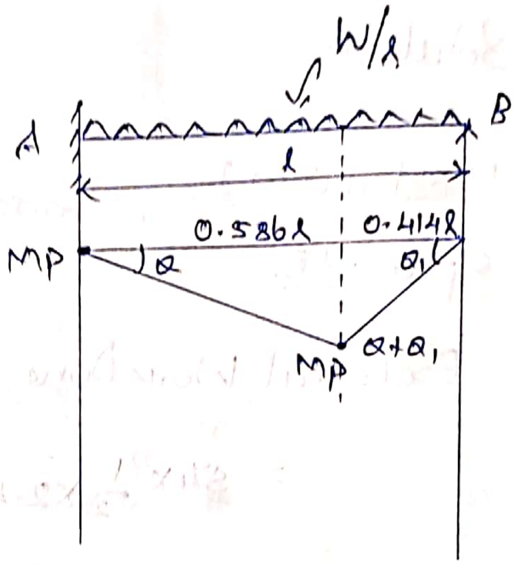
$W = \dfrac{6.06 M_p}{l}$

PROBLEM : 4

Analyse the propped cantilever loaded as shown and determine the collapse load



Solution.



$0.586 l\, \theta = 0.414 l\, \theta_1$

$\theta_1 = 1.415 \theta$

External Work Done

$= \dfrac{W}{l} \times \dfrac{1}{2} \times l \times 0.586 l \theta$

$= 0.293 W l \theta$

Internal Work Done

$= M_p \theta + M_p (\theta + \theta_1)$

$= M_p \theta + M_p (\theta + 1.415 \theta)$

$= 3.415 M_p \theta$

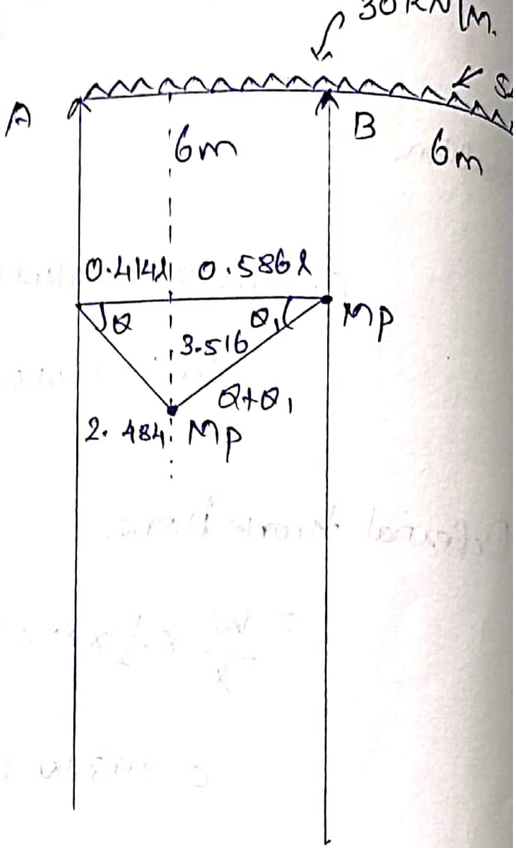
$0.293 W l \theta = 3.415 M_p \theta$

$W = \dfrac{11.65 M_p}{l}$

PROBLEM 5:

A two span continuous beam ABC has span length AB = 6m, BC = 6m, and carries a UDL 30kN/m completely covering the span AB and BC. A and C are SS. If the load factor is 1.80 and shape factor is 1.15 for the I-Section. Find the section modulus needed. Assume yield stress for the material as 250 N/mm^2

Solution.



Mechanism I

Span AB.

External Work Done

$= 54 \times \frac{1}{2} \times 2.484\theta \times 6$

$= 402.408\theta$

$2.484\theta = 3.516\theta_1$

$\theta_1 = 0.706\theta$

Internal Work Done

$= M_p(\theta + \theta_1) + M_p\theta_1$

$= M_p(\theta + 0.706\theta) + M_p 0.706\theta$

$= 2.412 M_p\theta$

$402.408\,a = 2.412\,Mpa$

$$M_p = 166.8\ KNm.$$

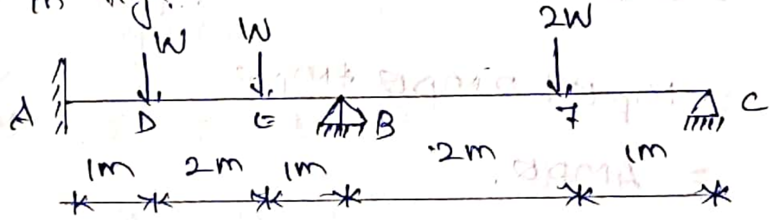
$$Z_p = \frac{M_p}{\sigma_y} = \frac{166.8\times10^6}{250} = 667.2\times10^3\ mm^3$$

$$S = \frac{Z_p}{Z}$$

$$Z = \frac{Z_p}{S} = \frac{667.2\times10^3}{1.15} = 580.18\times10^3\,mm^3.$$

ROBLEM :6

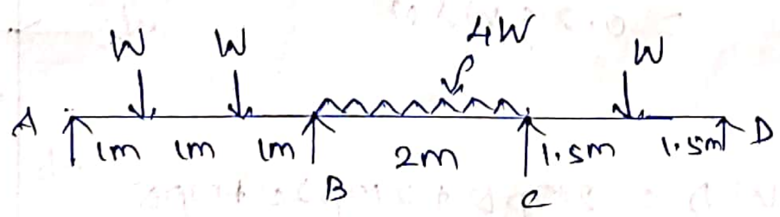
Determine the collapse load of the beam load as shown in fig:



2Mp · 2Mp Mp

PROBLEM :7

Find the collapse load Wc for the continuous beam shown in fig: The beam has uniform plastic moment Mp.

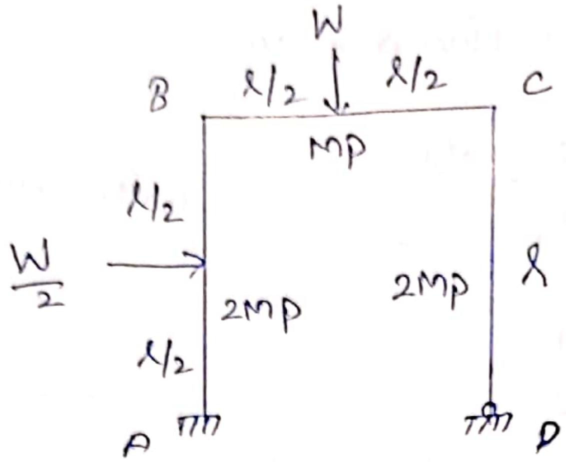


$\frac{4Mp}{3}$ 2Mp $\frac{4}{3}Mp = 1.33Mp$

6

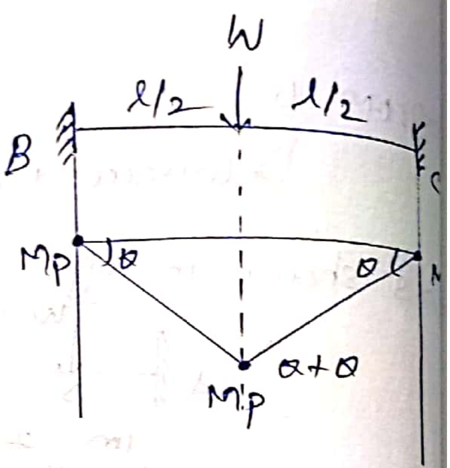
PROBLEM: 8

Find the collapse load for the frame.



Solution.

(i) Beam Mechanism.



$E.W.D = W(0.5\theta)$

$= 0.5lW\theta$

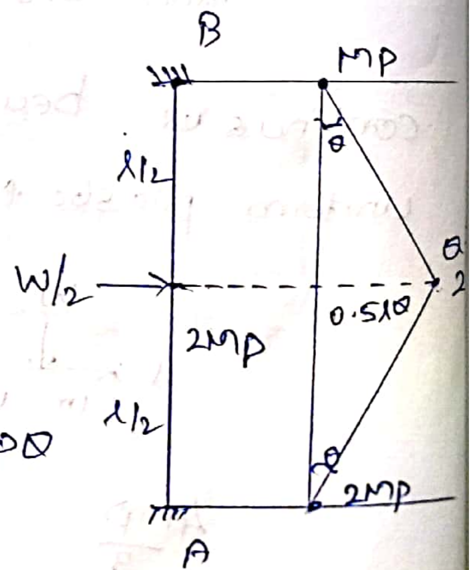
$I.W.D = Mp\theta + 2Mp\theta + Mp\theta$

$= 4Mp\theta.$

$0.5lW\theta = 4Mp\theta$

$W = \frac{8Mp}{l}$

(ii) Colum Mechanism.



$E.W.D = \frac{W}{2}.(0.5l\theta)$

$= 0.25Wl\theta$

$I.W.D = 2Mp\theta + 2Mp2\theta + Mp\theta$

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$= 7M_p\theta$

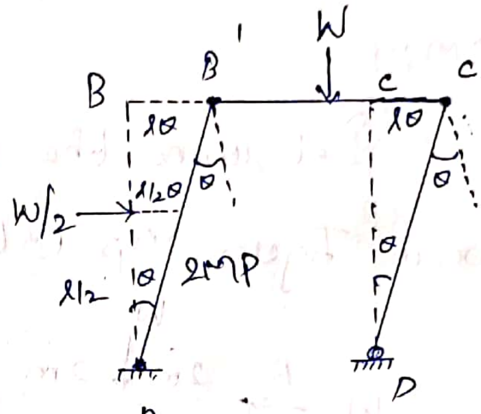
$0.25 W l \theta = 7M_p\theta$

$W = \frac{28 M_p}{l}$

(iii) Panel Mechanism. (Sway Mechanism)

$E.W.D = \frac{W}{2} \times \frac{l}{2}\theta$

$= 0.25 W l \theta$



$I.W.D = 2M_p\theta + M_p\theta + M_p\theta$

$= 4M_p\theta$

$4M_p\theta = 0.25 W l \theta$

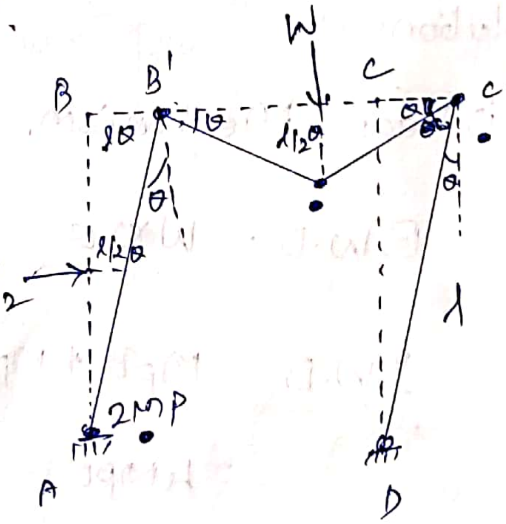
$W = \frac{16 M_p}{l}$

(iv) Combined Mechanism.

$EWD = \frac{W}{2} l/2\theta + W \times \frac{l}{2}\theta$

$= 0.25 W l \theta + 0.5 W l \theta$

$= 0.75 W l \theta.$



$$I.W.D = 2M_p\theta + M_p(\theta+\theta) + M_p(\theta+\theta)$$

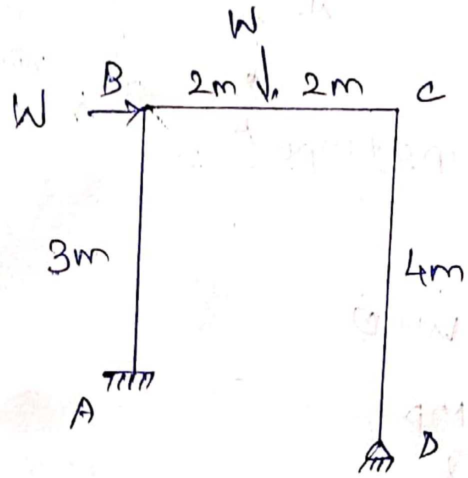
$$= 6M_p\theta$$

$$0.75\,Wl\theta = 6M_p\theta$$

$$W = \frac{8M_p}{l}$$

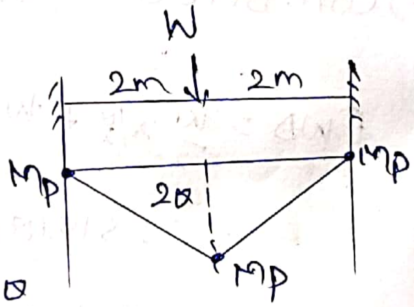
PROBLEM: 9

Determine the collapse load for the frame shown in figure. M_p is the same for all members.



Solution:

(i) Beam Mechanism.



$$E.W.D = W \times 2\theta$$

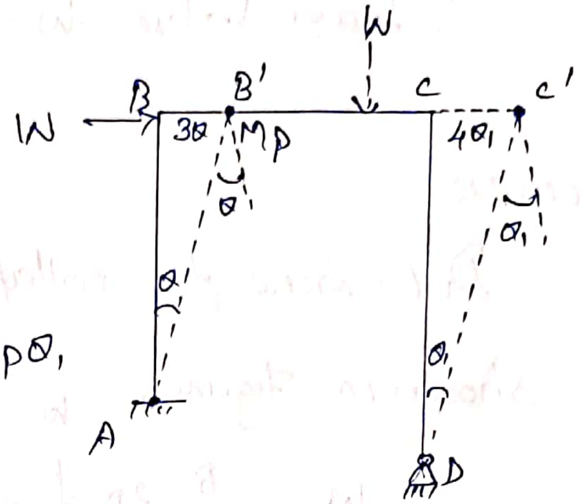
$$I.W.D = M_p\theta + M_p 2\theta + M_p\theta$$

$$= 4M_p\theta$$

$$2W\theta = 4M_p\theta$$

$$W = 2M_p.$$

(ii) Sway Mechanism



$$E.W.D = W3\theta$$

$$I.W.D = M_p\theta + M_p\theta + M_p\theta_1$$

$$3\theta = 4\theta_1$$

$$\theta_1 = 0.75\theta$$

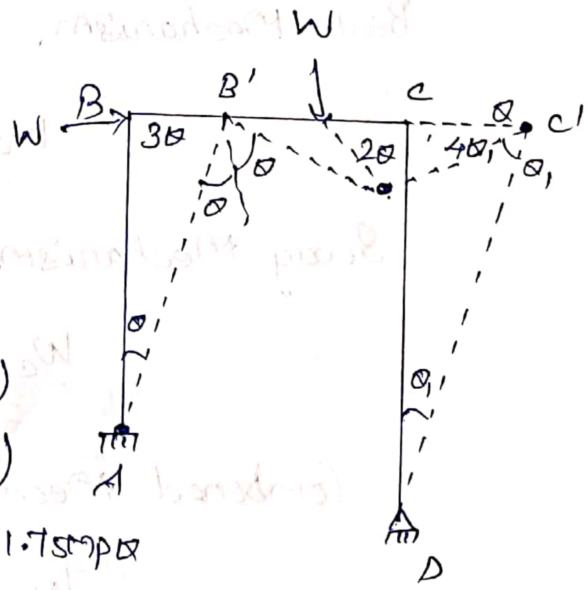
$$= M_p\theta + M_p\theta + 0.750 M_p$$

$$= 2.75 M_p\theta$$

$$W3\theta = 2.75 M_p\theta$$

$$W = 0.916 M_p$$

ii) Combined Mechanism.



$$E.W.D = 3\theta W + W2\theta$$

$$= 5W\theta$$

$$I.W.D = M_p\theta + M_p(\theta+\theta) + M_p(\theta+\theta_1)$$

$$= M_p\theta + 2M_p\theta + 1.75M_p\theta$$

$$= 4.75 M_p\theta$$

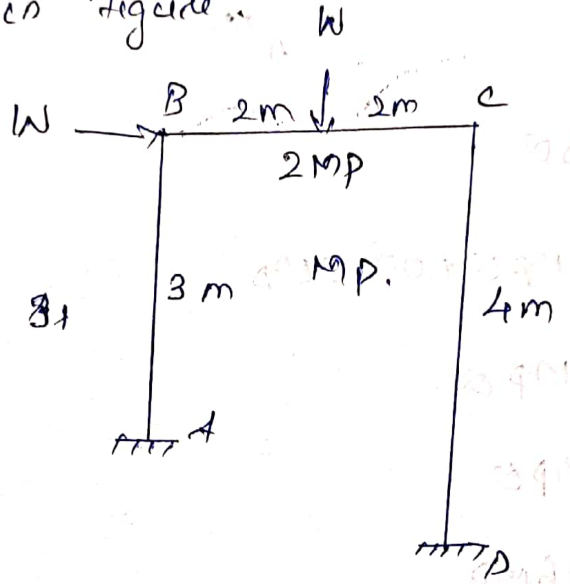
$5W\theta = 04.75M_p\theta$

$W = 0.95 M_p.$

$\therefore$ Least Value $W = 0.916 M_p.$

PROBLEM : 10:

Determine the collapse load. For the frame as shown in figure.



Beam Mechanism.

$W_c = 3M_p$

Sway Mechanism.

$W_c = 7/6 M_p$

Combined Mechanism.

$W_c = 1.5 M_p$

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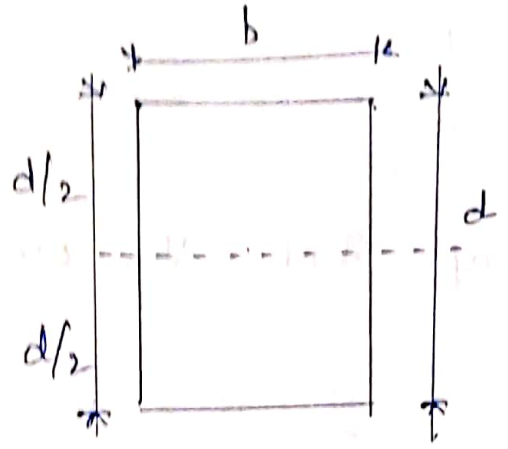
Problem 11:

Find Shape factor for Rectangular section



Solution.

Shape factor $= S = \frac{Z_p}{Z} = \frac{\text{Plastic Modulus of Section}}{\text{Elastic Section Modulus.}}$

Z_p = Plastic Modulus of Section

$= \frac{A}{2} (\bar{y}_1 + \bar{y}_2)$

$= \frac{bd}{2} (d/4 + d/4)$

$= \frac{bd}{2} (\frac{1}{2} d)$

$= \frac{bd^2}{4}$

$Z = I/y \quad \therefore I = \frac{bd^3}{12}$

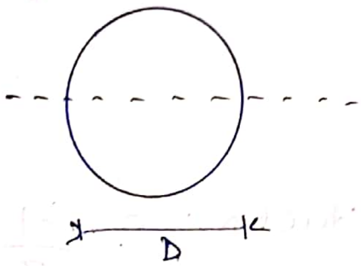
$y = d/2$

$Z = \frac{bd^3}{12} \times \frac{2}{d} = \frac{bd^2}{6}$

$$S = \frac{\frac{bd^2}{4}}{\frac{bd^2}{6}} = \frac{bd^2}{4} \times \frac{6}{bd^2} = 1.5.$$

PROBLEM 12:

Find shape factor for circular section.



Solution:

$$S = \frac{Z_P}{Z}.$$

$$Z_P = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$$

$$= \frac{\frac{\pi D^2}{4}}{2}\left[\frac{2D}{3\pi} + \frac{2D}{3\pi}\right]$$

$$= \frac{\pi D^2}{8}\left[\frac{2D}{3\pi} + \frac{2D}{3\pi}\right]$$

$$= \frac{\cancel{\pi} D^2}{8}\left[\frac{4D}{3\cancel{\pi}}\right]$$

$$= \frac{D^3}{6}$$

$$Z = \frac{I}{y} = \frac{\frac{\pi D^4}{64}}{D/2} = \frac{\pi D^4}{64} \times \frac{2}{D} = \frac{\pi D^3}{32}$$

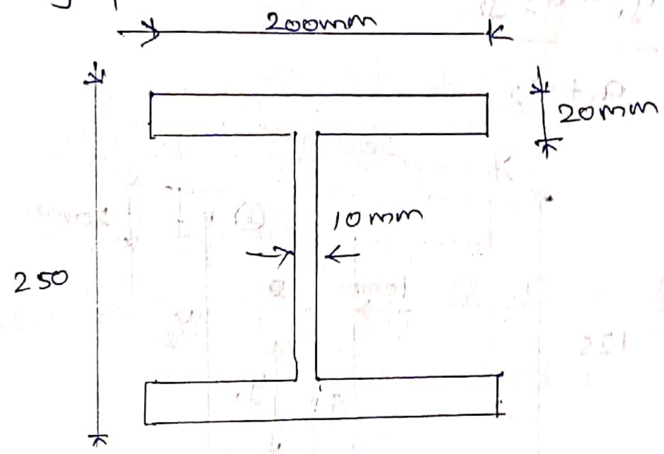
Shape factor $S = \frac{Z_P}{Z} = \frac{\frac{D^3}{6}}{\frac{\pi D^3}{32}}$

$= \frac{D^3}{6} \times \frac{32}{\pi D^3}$

$S = 1.69.$

PROBLEM 18:

A mild steel I-section 200mm wide and 250mm deep has a mean flange thickness of 20mm and a web thickness of 10mm. Calculate the shape factor. Find the fully plastic moment if $\sigma_y = 252\ N/mm^2$.



Solution:

Shape factor $S = \frac{Z_P}{Z}$

Elastic Section Modulus $Z = \frac{I}{Y}$

$I = \frac{200 \times 250^3}{12} - \frac{190 \times 210^3}{12} = 113.78 \times 10^6\ mm^4$

$Y = D/2 = \frac{250}{2} = 125mm$

$$Z = \frac{11378.42 \times 10^4}{125} = 910.27 \times 10^3 \, mm^3$$

Plastic Section modulus.

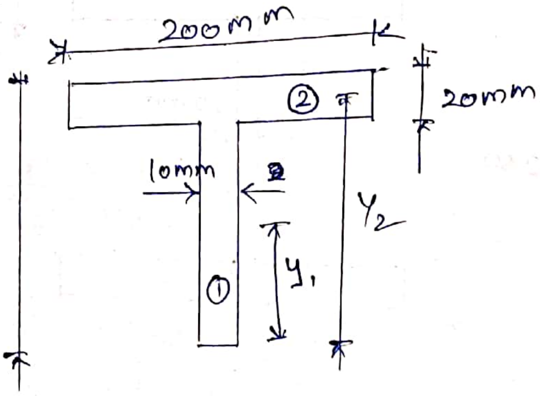
$$Z_p = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$$

$$A = a_1 + a_2 + a_3$$

$$= (200 \times 20) + (200 \times 20) + (210 \times 10)$$

$$= 10100 \, mm^2$$

$$\bar{y}_1 = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$



$$y_1 = \frac{105}{2} = 52.5 mm$$

$$y_2 = 105 + \frac{20}{2} = 115 mm$$

$$\bar{y}_1 = \bar{y}_2 = \frac{(1050 \times 52.5) + (4000 \times 115)}{1050 + 4000} = 102 \, mm$$

$$Z_p = \frac{10100}{2}[102 + 102] = 1.03 \times 10^6 \, mm^3$$

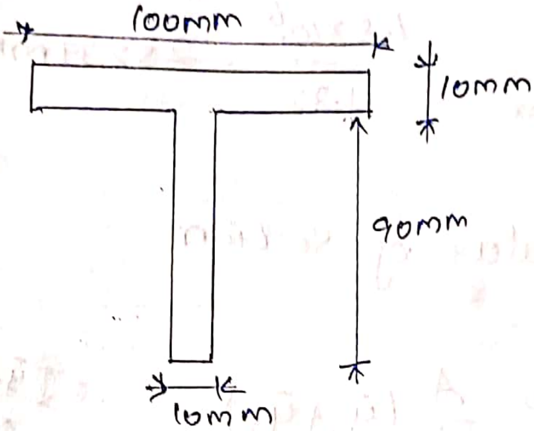
$S = \frac{Z_p}{I} = \frac{1.03\times10^6}{910.27\times10^3} = 1.13.$

Fully plastic Moment.

$$M_p = \sigma_y \times Z_p = 252 \times 1.03\times10^6 = 259.6\times10^6 \text{ Nmm}$$

Problem 14:

Find the shape factor for the Tee Section, as shown in Figure.



Solution

$$\text{Shape factor} = \frac{Z_p}{Z} = \frac{\text{Plastic modulus of Section}}{\text{Elastic modulus}}$$

Elastic Modulus, Z.

$$Z = I/y$$

Location of centroid.

$$\left[\overset{a_1}{(100\times10)} + \overset{a_2}{(90\times10)}\right] y_t = \overset{a_1 y_1}{\left(100\times10\times\frac{10}{2}\right)} + \overset{a_2 y_2}{\left(90\times10\times 10+\frac{90}{2}\right)}$$

$$y_t = 100\times10\times5 + \frac{54500}{1900} = 28.68\text{mm}$$

$Y_b = 100 - 28.68 = 71.32\,mm$

$I_{xx} = \frac{bd^3}{12} + (Ah^2) \quad \therefore h = y - \bar{y}$

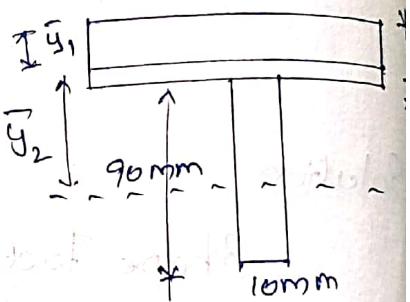
$= \frac{100\times10^3}{12} + 100\times10 \times 23.68^2 + \left[\frac{10\times90^3}{12} + 10\times90\times2\ldots\right.$

$= 1.8\times10^6\,mm^4$

$Z = \frac{I}{y_{max}} = \frac{1.8\times10^6}{71.32} = 25239\,mm^3.$

Plastic modulus of section:

$Z_p = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$



Equal Area axis

$\frac{\text{Total area}}{2} = \text{Width of flange} \times h.$

$\frac{1900}{2} = 100h$

$h = 9.5m.$

$\bar{y}_1 = \frac{9.5}{2} = 4.75\,mm$

$\bar{y}_2 = \frac{\overset{a_1}{100\times0.5} \times \overset{y_1}{\frac{0.5}{2}} + \overset{a_2}{90\times10} \times \overset{y_2}{0.5+45}}{\underset{a_1}{100\times0.5} + \underset{a_2}{90\times10}}$

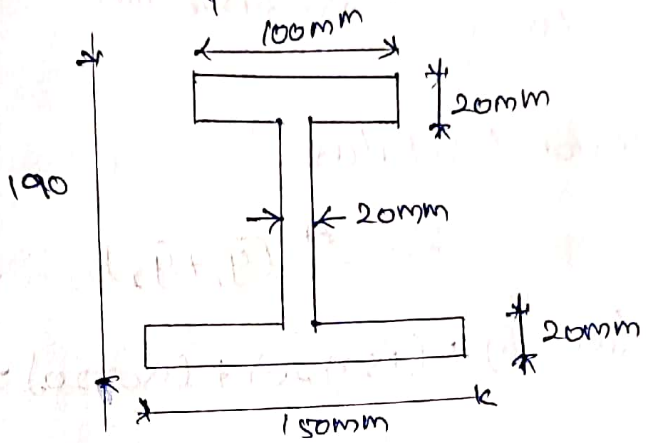
$= 43.12\,mm$

$$Z_p = \frac{1900}{2}(4.75 + 43.12) = 45476.5\,mm^3$$

Shape factor $S = \frac{Z_p}{Z} = \frac{45476.5}{25239} = 1.80$

PROBLEM 15:

Find the shape factor of the I-Section.



Solution:

Shape factor

$$S = \frac{Z_p}{Z}$$

Location of centroid

Moment of areas about top.

$$y_t = \frac{(100\times20\times10) + [150\times20\times(20+150/2)] + [150\times20\times(20+150+20/2)]}{(100\times20) + (150\times20) + (150\times20)}$$

$= 105.6\,mm$

$y_b = 190 - 105.6 = 84.4\,mm$

Moment of Inertia.

$$I = \frac{100\times20^3}{12} + \left[100\times20\times(105.6-10)^2\right] + \frac{20\times150^3}{12} + (2\ldots$$

$$\times(105.6-95)^2 + \frac{150\times20^3}{12} + (20\times150)\times(84.4\ldots$$

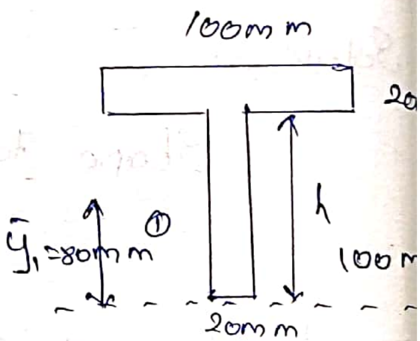
$$= 4101.35\times10^4\ mm^4$$

$$Z = \frac{4101.35\times10^4}{105.6} = 388.39\times10^3 mm^3$$

Plastic Section Modulus

$$Z_p = \frac{A}{2}(\bar{y}_1+\bar{y}_2)$$

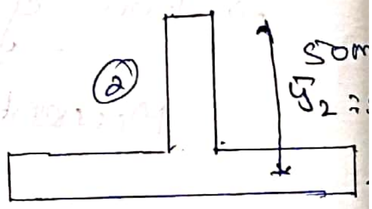
$$A = (100\times20) + (150\times20) + (150\times20) = 8000mm^2$$



$$\frac{8000}{2} = (100\times20) + 20\times h$$

$$20h = 2000$$

$$h = 100mm.$$



$$\bar{y}_1 = \frac{(\overset{a_1}{100\times20}\times\overset{y_1}{50} + 100\times20\times100+10)}{(100\times20)+(100\times20)}$$

$$= 80mm.$$

$$\bar{y}_2 = \frac{(20\times50\times25) + 150\times20\times(50+10)}{(50\times20)+(150\times20)}$$

$$= 51.25mm$$

$$Z_p = \frac{8000}{2}\left[80 + 51.25\right] = 525 \times 10^3\,mm^3$$

$$S = \frac{Z_p}{Z} = \frac{525 \times 10^3}{388.39 \times 10^3} = 1.352.$$

Formulas:

I.W.D = E.W.D

I.W.D = Based on Support.

ss at Ends is ~~No~~ Mp is absent

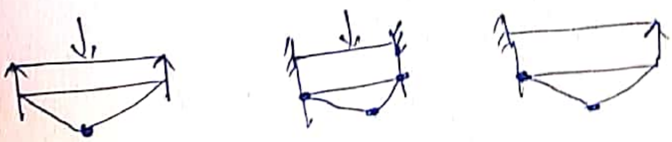
Fixed Support Mp is present

E.W.D for UDL = Load x Area of triangle.

E.W.D for point = Load x term.

2.) $\frac{M}{I} = \frac{\sigma}{y}$, $Z = I/y$

3.) Shape factor $S = \frac{Z_p}{Z}$ = $\frac{\text{Plastic modulus of section}}{\text{Elastic section modulus}}$

4.) 

Prop = $0.414\,l$

Fixed = $0.586\,l$.

5.) $Z_p = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$

Rectangular.

$A = bd$, $\bar{y} = d/4$, $I = \frac{bd^3}{12}$, $y = d/2$

6. Circular.

$$A = \frac{\pi D^2}{4}, \quad \bar{y} = \frac{2D}{3\pi}, \quad I = \frac{\pi D^4}{64}, \quad y = D/2$$

7. I. Section, ~~T.S.~~

$$I = \frac{bd^3}{12} + Ah^2 \; ; \; h = y - \bar{y}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$ [Location of centroid]

8. Equal Area axis.

$$\frac{\text{Total area}}{2} = \text{Width of Flange} \times h$$

9. I. Section Unsymmetric

Equal Area axis

$$\frac{\text{Total Area}}{2} = \text{Flange Area} \times \text{Web thick} \times h.$$

Sign 2/1/2020