Missile Guidance and Control

Proportional Navigation Guidance / 3-Loop Autopilot

MECH 371 - Control System Analysis and Design

March 10, 2021

Missile Guidance Fundamentals

Objective: Make a missile hit it's target - the guidance system ensures that it points in the right direction.

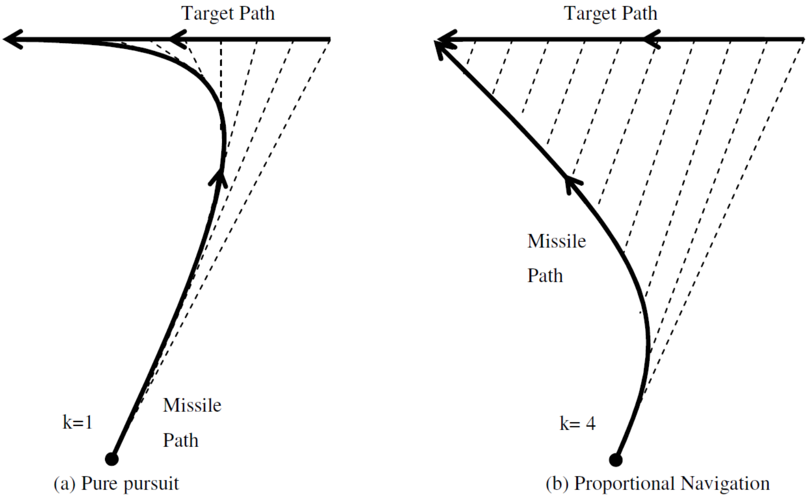
Types:

- Pursuit Guidance (simplest)
- Proportional Navigation Guidance (most popular)
- LQ-optimal Guidance (more recent)
- Q-Kappa Guidance (complex)

For now: Ignore the seeker dynamics (i.e) laser, semi-active homing, passive homing, IR-homing etc.

Pure Pursuit vs. Proportional Navigation

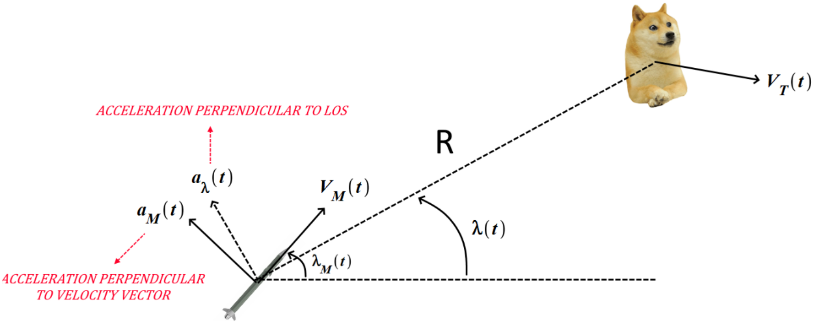
Phases of Flight: Launch, Midcourse, Terminal, Intercept



Planar Guidance - Basics

a = Acceleration, V = Speed, λ = flight path angle, R = range, M = missile, T = target

A missile *seeker* focuses on measuring LOS (line of sight) $= \lambda, \dot{\lambda}, R$



Proportional Navigation Guidance Law

Either: $a_\lambda = NV_c\dot{\lambda}$ **OR**: $a_M = NV_M\dot{\lambda}$

- V_M = Missile Velocity
- V_c = Closing Velocity $\dot{R} = dR/dt \simeq \Delta R/\Delta t$
- For **intercept** to happen: need $\dot{R} < 0$
- $\dot{\lambda}$ = Rate of change of Line of Sight $= d\lambda/dt \simeq \Delta\lambda/\Delta t$
- N = Proportional Navigation Constant $(3-5)$

We will use the formula $a_M = NV_m\dot{\lambda}$, as the control system setpoint is $a_M \triangleq A_{zc}$, i.e: the acceleration command *perpendicular* to the velocity vector.

Implementing PN Guidance - Key Points

The following is very important when implementing PN guidance, to ensure that the missile does not turn steep angles and to avoid damage to the actuators.

- The **commanded** accelerations a_λ or a_M must be **constrained**.
- This is to keep the **actuator** requirement low (avoid steep angles)
- The missile actuator (fin, canard etc.) has **constraints in reality** and cannot achieve **very high** commanded accelerations.
- Based on the change of λ_M i.e $\dot{\lambda}_M$, the vector of a_λ or a_M will be **positive or negative**.

From the **Coriolis** Equation:

$$\dot{\lambda}_M = \frac{a_M}{V_M} \text{ or } \dot{\lambda} = \frac{a_\lambda}{V_\lambda} \text{ (along LOS)}$$

Missile Autopilots - Control System

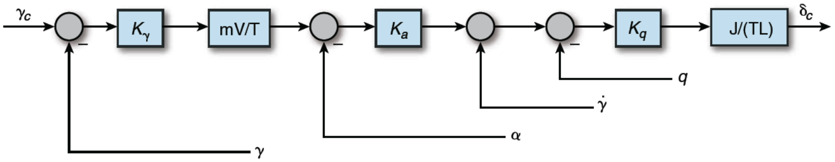


Figure 8. This diagram of a flight-path control system shows the dynamic inversion design approach. The design explicitly uses the fundamental relationships among the missile kinematic and dynamic variables as well as real-time estimates of the missile thrust and mass properties to naturally compensate for the changing missile dynamics as propellant is expended.

Here is an example autopilot. It is a multi-loop structure using linearized transfer function models. Can be designed with root locus (2^{nd}) order systems, or via modern methods (LQR, MPC) etc.

3 Loop Autopilot Design

Inner Loops: Control the pitch rate q or $q = \dot{\theta}$ for 2-D.
Outer Loops: Control the normal acceleration A_{zc} which comes from the PN law.

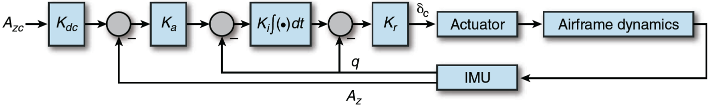


Figure 6. This block diagram illustrates a classical approach to the design of an acceleration control autopilot. The difference between the scaled input acceleration command and the measured acceleration is multiplied by a gain to effectively form a pitch rate command. The difference between the effective pitch rate command and the measured pitch rate is multiplied by a gain and integrated with respect to time. The resulting integral is differenced with the measured pitch rate and multiplied by a third gain to form the control effector command such as desired tail-deflection angle. The gain on the input acceleration command ensures zero steady-state error to constant acceleration command inputs. The final autopilot design would build on this basic structure with the addition of noise filters and other features such as actuator command limits. This basic structure is called the three-loop autopilot.

Figure: Source: JHU APL

Simulation Parameters / Actuator Dynamics

3-Loop autopilot model taken from **JHU APL Paper**: "*Overview of Missile Flight Control Systems*" (see description).

Initial Conditions

- Missile: $V_M(0) = 0$, $\lambda_M(0) = 80^o$, $X_M(0) = 2000m$, $Z_M(0) = 0$
- Target: $V_T(0) = 600m/s$, $\lambda_T(0) = -2.5^0 - 5^0$, $Z_T(0) = 20000m$
- The target is descending at a flight path angle between -2.5 to -5 degrees.
- Launch the missile when $|X_T - X_M| < 200m$, i.e when the target is less than 200m downrange.
- Activate PN Guidance when the missile speed reaches $950m/s$. The missile will cruise at $1021m/s$.
- Actuator is a 2^{nd} order system response (in JHU APL paper)