Stat 230: Probability

Lecture 15

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Wednesday, June 8th

Review

Example

You roll 2 dice, 24 times. What is the expected value the sum of the dice is 7?

Review

Last time we talked about:

(1) Expectations
- Expectation of functions of a RV
- $Z \sim Bin(n, p)$, $E(Z) = np$

For today:

(1) Expectation of named distributions
- $W \sim Pois(\lambda)$, $E(W) = \lambda$
- $Y \sim Hyp(N, r, n)$, $E(Y) = \frac{nr}{N}$
- $X \sim NB(k, p)$, $E(X) = \frac{k(1-p)}{p}$
- $D \sim U(a, b)$, $E(X) = \frac{a+b}{2}$

(2) Variance

Reading: Chapter 7

Review

- Next Monday: Quiz 3
- R Codes Uploaded

Theorem (Expected Value of Poisson)

If $Z \sim Poisson(\mu)$, then $E[Z] = \mu$.

Example

Suppose that calls to the Canadian Tire Financial call center follow a Poisson process with rate 30 calls per minute. Let X denote the number of calls to the center after 1 hour. Compute $E[X]$.

Theorem (Expected Value of Hypergeometric)

If $X \sim Hypergeometric(N, r, n)$, then $E[X] = n\frac{r}{N}$.

Example

Lucy is a baby. While sitting at her high chair, her parents put 10 carrots, 6 peas, and 2 pieces of cauliflower on her plate. What is the expected number of carrots she will eat in the next 3 bites?

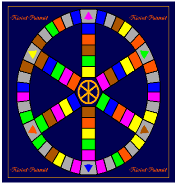
Distributions Expected Value

Theorem (Expected Value of Hypergeometric)

If $Y \sim$ Negative Binomial(k, p), then $E[Y] = \frac{k(1-p)}{p}$.

Example

When playing trivial pursuit, to finish the game one must roll a specified number on a single fair six sided die in order to land on the middle tile. Once in range of the middle tile, the probability of doing this on any one roll is $1/6$. Let X denote the number of rolls required in order to finish the game. Compute $E[X]$.



Distributions Expected Value

Theorem (Expected Value of Discrete Uniform)

If $D \sim$ Discrete Uniform(a, b), then $E[D] = \frac{a+b}{2}$.

Example

Anishka visits a bookstore and finds an interesting 699 page book. After reading the back cover, she completely randomly flips to a page in the book. What is the expected page she flips to?

Variance

Using the expected value is one way of summarizing/predicting the value of a random variable. One may also wonder how much a random variable tends to deviate from its mean. Suppose that $E[X] = \mu$

(1) Deviation

$$E[(X - \mu)] = E[X] - \mu = 0$$

(2) Absolute deviation:

$$E|X - \mu|$$

(3) Squared deviation:

$$E(X - \mu)^2$$

Variance

Definition

The **variance** *of a random variable X is denoted $Var(X)$, and is defined by*

$$Var(X) = E[(X - E[X])^2] = \sigma_X^2$$

A simple calculation gives the "short cut formula" to compute $Var(X)$:

$$Var(X) = E(X^2) - (E(X))^2$$