R Assignment

1) A shipment of paper cups to the DC branch of a large coffee chain contains 5000 cups. 15 of these 5000 cups entitle the buyer to win a car! Suppose you go to there twice a day for a full 5-day work-week.
   i) Compute the probability that...
      a) you win exactly one car during this week?
      b) you win at least one car during this week?
      c) you win no car during this week?
   ii) Repeat the previous part using a Binomial approximation and compare your results.

2) Suppose accidents happen at a big intersection according to a Poisson process with rate 50 per 7 days (=1 Week).
   i) Compute the probability that there are more than 2 accidents on a given day.
   ii) What is the probability that there are between 30 and 70 accidents in a given week?
   iii) What is the probability that you need to wait 5 days until finally there is an accident-free day?

3) Suppose that a box contains 800 electrical switches. Each has a probability of 0.002 of being defective, independent of the others. Let X represent the number of defective switches in a box of 800.
   i) Identify the appropriate model to use and then determine $P(X > 3)$.
   ii) Now, use the Poisson Approximation to the Binomial and approximate $P(X > 3)$.

4) A manufacturing process produces gears for use in automobile transmissions. There is concern that the process is "out of control" and is therefore producing too many defective gears. The quality assurance manager designs a study to examine the process. There are 800 gears available from one day's production, and suppose that there are 50 defective gears in the lot of 800 gears.
   i) Let the random variable X represent the number of defective gears in a random sample of 40 taken from the population of 800. Of course, the gears will be sampled without replacement. Compute $P(X = 4)$, $P(X \geq 10)$ and the probability you find an odd number of defective gears.
   ii) Approximate the probabilities in part i) using a Binomial approximation.
   iii) Approximate the probabilities in part i) using a Poisson approximation to the Binomial.
   iv) Compare your approximations. Which approximation worked best? Does this make sense?
   v) Suppose that we want to sample (again, without replacement) until we have obtained 10 defective gears. Let Y represent the number of gears that we must sample in order to obtain 10 defective gears. Give an expression for $P(Y \geq 150)$ and use R to compute it.