Stat 230: Probability

Lecture 13

Jeremy VanderDoes

University of Waterloo

Friday, June 3rd

Poisson Process

Example

Shiny versions of Pokemon are possible to encounter and catch starting in Generation 2 (Pokemon Gold/Silver). Normal encounters with Pokemon while running in grass occur according to a Poisson process with rate 1 per minute on average. 1 in every 8192 encounters will be a Shiny Pokemon, on average.

(1) If you run around in grass for 15 hours, what is the probability you will encounter at least one Shiny pokemon?

(2) How long would you have to run around in grass so that you have a better than 50 percent chance of encountering at least one Shiny pokemon?

Review

Last time we talked about:

(1) Poisson Process: X_t is the count up to time t, then $X_t \sim Pois(\lambda t)$
  (1) Independence
  (2) Individuality
  (3) Homogeneity

For today:

(1) Examples

(2) Expectation of Discrete Random Variables

Review

- Exam
- Monday: TA tutorial

Examples

Example

Website hits for a given website occur according to a Poisson process with a rate of 100 hits per minute. We say a second is a "break" if there are no hits in that second.

(1) What is the probability of a break in any given second?

(2) Compute the probability of observing exactly 10 breaks in 60 consecutive seconds.

(3) Compute the probability that one must wait for 30 seconds to get 2 breaks.

Examples

Example

(1) Give the distribution of the number of times Jill trips on her 1000 step journey to class if she trips independently on $1/100$ steps.

(2) After eating most of a 10 pound bag of m&ms, Carl looks inside to see 9 brown, 12 yellow, 8 red, 6 orange, 7 green, and 8 blue m&ms remaining. Give the distribution of the next 7 picked if Carl wants yellow or red m&ms (and eating them right after picking of course).

(3) Kyumin is unlucky and gets randomly slapped on 1 day a year. Give the distribution he is slapped on a given day in the upcoming year.

(4) Joyce has been a longtime Blue Jays fan. Give the distribution counting the number of seasons until the Blue jays win the World Series if their probability of winning the World Series is always 0.03.

Examples

Example

(1) If we have a probability function given $f(x) = cx$ for $x \in \{1, 2, 3\}$, what is c?

(2) If X is a discrete random variable, is Y a discrete random variable where $Y \sim 4X + 3$?

R

Chapter 6 discusses using the R programming language for statistical computation. Have a look!

- Upcoming R tutorial

Expected Value

Definition

Suppose X is a discrete random variable with probability function $f_X(x)$. Then $E(X)$ is called the **expected value** *of X, and is defined by*

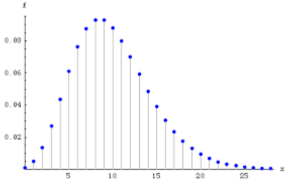
$$E(X) = \sum_{x \in X(S)} x f_X(x)$$

The expected value of X is sometimes referred to as the **mean** of X or the **first moment** of X.

Expected Value

Interpretations of the expected value: $E(X)$

(1) $E(X)$ is the "balancing point" of the probability function $f_X(x)$



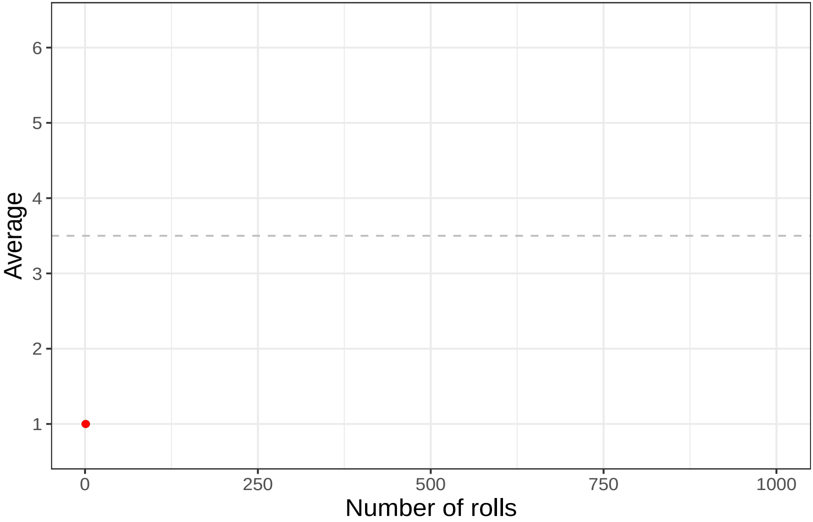
(2) $E(X)$ is what the average of many, many independent realizations of the random variable X would approach (Law of large numbers).

Expected Value

Example

Suppose that X denotes the outcome of one fair six sided die roll. Compute $E(X)$.

Expected Value



Final Estimate: 3.49