Mathematical Probability Models: Chapter 2

Chapter Outcomes:

- Defining sample spaces
- Types of events
- Rules of probability
- Using your sample space to solve for the probability of specific events

- Suppose we roll a 6-sided die.
- Then, $S = \{1, 2, 3, 4, 5, 6\}$.

- Recall that a **sample space, *S***, is a set of **distinct** outcomes for an experiment or process.

- In a single trial, one and only one of these outcomes can occur.

Note: A sample space is NOT necessarily unique. It could be possible to define a sample space more than one way, depending on how you wanted to use it / what you were looking for.

- This sample space, $S = \{1, 2, 3, 4, 5, 6\}$, is said to be **discrete** as it consists of a ***finite*** number of outcomes.

- In general, a discrete sample space is one that *consists of a finite or a countably infinite set of outcomes*.

- We can also have non-discrete sample spaces, like $S = \{x \, \varepsilon \, \mathbb{R} : x > 0\}$. We will discuss these in more detail once we define random variables in Chapter 5.

A Discrete Sample Space with a Countable Infinite Set of Outcomes

- Suppose our experiment consists of tossing a coin until a head appears for the first time. We may have to toss the coin any number of times before a head is obtained. Therefore, the possible outcomes in the sample space are **H, TH, TTH, TTTH, TTTTH,**…. How many outcomes are there? The outcomes are *countable* but *infinite* in number.

In a discrete sample space, we can talk about:

- A simple event (outcome): An event that contains only one point.

- A compound event: An event made up of two or more simple events.

- Now if $S = \{a_1, a_2, a_3, \ldots\}$ is a discrete sample space, the probabilities $P(a_i)$ for $i = 1,2,3, \ldots$ must satisfy the following two conditions:

1. $\mathbf{0 \leq P(a_i) \leq 1}$

2. $\mathbf{\sum_i P(a_i) = 1}$

- Note that *P*(*) is a function whose domain is the sample space *S* and the set of probabilities $\{P(a_i), i = 1,2,3, \ldots\}$ is called the **probability distribution on *S***.
- In addition, the probability *P*(*A*) of an event *A* is defined as

$$\mathbf{P(A) = \sum_{a \epsilon A} P(a)}$$

- **Example:** If we assume we have a fair die, then each number is **equally likely** to occur. Therefore:

$$P(i) = \frac{1}{6} \text{ for } i = 1, 2, 3, 4, 5, 6$$

- We can define the simple event A = "a 2 is obtained on one roll". Therefore, $A = \{2\}$ and

$$P(A) = 1/6$$

- We can define the compound event B = "an even number is obtained on one roll".

Thus, $B = \{2, 4, 6\}$ and $P(B) = 3/6 = 1/2$

Some More Examples:

1. Card example

Experiment: Randomly draw one card from a standard deck of 52 cards.

Find the probability the card is a club.

$S =$

2. Coin example

Experiment: Toss a fair coin twice.

Find the probability of getting <u>exactly</u> one head.

$S =$

You Try:

- A box contains 3 marbles: 1 red, 1 green, and 1 blue. Consider an experiment where a marble is to be selected from the box two times, with replacement.

a) Describe the sample space.

b) If we let A = {exactly one red marble is drawn}, describe event A in terms of all possible outcomes.

c) Calculate $P(A)$.

d) Now describe the sample space if the selections are done without replacement.

You Try Solutions:

- A box contains 3 marbles: 1 red, 1 green, and 1 blue. Consider an experiment where a marble is to be selected from the box two times, <u>with replacement</u>.

a) Describe the sample space.

S = {(R,R),(B,B),(G,G),(R,B),(B,R),(R,G),(G,R),(B,G),(G,B)

b) If we let *A* = {exactly one red marble is drawn}, describe event *A* in terms of all possible outcomes.

A={(R,B),(B,R),(R,G),(G,R)}

c) Calculate *P*(*A*).

P(A) = 4/9 as each outcome is equally likely.

d) Now describe the sample space if the selections are done <u>without replacement</u>.

S={(R,B),(R,G),(B,R),(B,G),(G,R),(G,B)}

Definition:

- We can use the term “odds” to describe probabilities in the following way:
- The **odds in favour** of an event *A* occurring is given by the ratio

$$\boldsymbol{P(A):1-P(A) \ \ or \ \ \frac{P(A)}{1-P(A)}}$$

- Similarly, the **odds against** the event *A* occurring is given by the ratio

$$\boldsymbol{1-\ P(A):P(A) \ \ or \ \ \frac{1-P(A)}{P(A)}}$$

- So in the earlier card example, the **odds in favour** of clubs are:

- And in the earlier die example, the **odds against** rolling a 2 are:

- **Note:** Odds can be used to determine a proper payout to make a game of chance "fair".

- **Note:** It is sometimes useful to list all possible (equally likely) outcomes in *S* and then use the classical definition to solve for the probability of events of interest. (i.e. $P(A) = n(A)/n(S)$)

- When the model is more complex, we will need more efficient techniques to count outcomes without listing them.
- Counting rules can be VERY helpful…..Stay tuned!