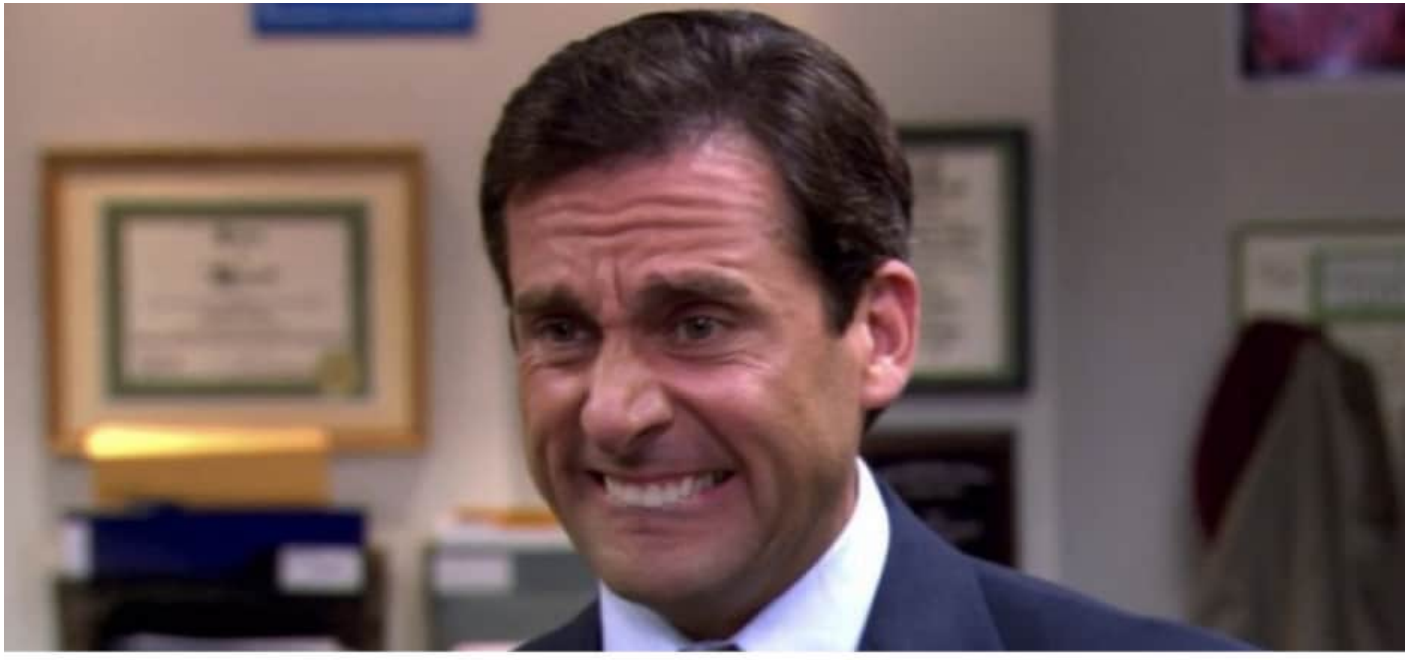
STAT 230 SECTION 2 LECTURE 15



When you realize your instructor needs to quarantine and you will only have fun online lectures instead of fun in person lectures

Today's Agenda

Last time:

- Negative Binomial + Geometric Distribution
- Memoryless property of the geometric distribution
- Poisson distribution

We have covered everything up to and including Section 5.7!

Today (Lec 15, 06/08):

- Practice, Practice, Practice

Next Lectures:

- Lec 16 (recorded) covers the remaining material in Chapter 5 and will be posted by June-09
- Lec 17 (synchronous online lecture on June-10) consists of practice and review of Chapter 5

Distributions covered so far:

- Discrete Uniform Distribution
- Bernoulli Distribution
- Hypergeometric Distribution
- Binomial Distribution
- Negative Binomial + Geometric Distribution
- Poisson Distribution

Question 1

What is the relationship between geometric and negative binomial distribution?

A) They are both discrete ✓

B) They both model the number of failures given a fixed number of successes ✓

C) Geometric is a special case of negative binomial ✓

Question 2

Dwight flips a fair coin until he got k heads. As a result, he got r tails as well. In how many ways could this have happened?

A) $\binom{k}{r}$

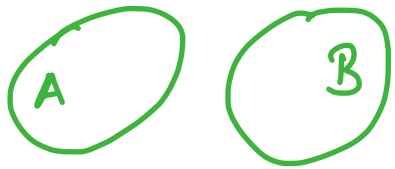
B) $\binom{k+r}{k}$

C) $\binom{k+r}{r}$

D) $\binom{r+k-1}{k-1}$ ✓

last one : head

Question 3



$P(A \cap B) = P(\emptyset) = 0$

$\neq P(A) \cdot P(B) = \frac{1}{16}$

$\Rightarrow A, B$ not independ.

Suppose A and B are mutually exclusive, and that $P(A) = P(B) = 1/4$. Are A and B independent?

A) Yes

B) No ✓

C) This information is not sufficient to determine whether or not A and B are independent.

Question 4

$p = \frac{1}{2}$

Suppose a fair coin is flipped 17 times. Let X denote the the number of heads observed, and let Y denote the number of tails observed. Which of the following is FALSE:

A) $X \sim Binomial(17, .5)$ ✓

B) $Y \sim Binomial(17, .5)$ ✓

C) $X \sim Y$ ✓

D) $X + Y = 17$ ✓

E) $X = Y$ ✗

Question 5

An urn contains 7 red marbles, 13 blue marbles, and 3 green marbles. Suppose that 5 marbles are drawn from the urn with replacement. Let X denote the number of blue marbles observed in the 5 draws. What is the distribution of X?

A) $X \sim U(1, 5)$

B) $X \sim Binomial(5, 7/23)$

C) $X \sim hyp(23, 13, 5)$

D) $X \sim Binomial(5, 13/23)$ ✓

F) $X \sim Binomial(23, 5/13)$

5 independent trials with constant success probability $\frac{13}{23}$

Question 6

The range of a random variable is $A = \{1, 2, 3, \dots\}$. For $x \in \{0, 1, 2, \dots\}$ the cumulative distribution function of X is given by

$$F(x) = P(X \leq x) = 1 - 2^{-x}.$$

A) Find $P(X = 5)$ and $P(X \geq 5)$.

B) Find the probability function $f(x) = P(X = x)$.

A) $P(X=5) = P(X \leq 5) - P(X \leq 4) = F(5) - F(4) = 2^{-4} - 2^{-5}$

$P(X \geq 5) = 1 - P(X \leq 4) = 1 - F(4) = 2^{-4}$

B) $f(x) = P(X=x) = P(X \leq x) - P(X \leq x-1) = F(x) - F(x-1)$

$= 1 - 2^{-x} - (1 - 2^{-(x-1)}) = 2^{-x+1} - 2^{-x} = 2^{-x}$, $x \in \{1, 2, 3, \dots\}$

Question 7

A manufacturer of auto parts just shipped 25 auto parts to a dealer. Later on, it was discovered that 5 of those parts were defective. By the time the company manager contacted the dealer, 4 auto parts from that shipment had been sold. Denote by X the number of good parts sold. What is the distribution of X?

$N = 25$ parts in total

$r = 20$ good parts ("success")

$n = 4$ parts are sampled/sold without replacement.

X = # of good parts

$X \sim \text{HypGeo}(N = 25, r = 20, n = 4)$

Question 8

At Skip The Dishes, home delivery providing high-quality service to customers is the top priority of the management. The company guarantees a refund of all charges if your meal is not delivered within 40 minutes of placing your order. Through past history, it is known that 2% of the deliveries do not arrive within the 40 minutes. Suppose Skip The Dishes receives 10 orders during a specific hour.

A) Find the probability that exactly 1 of these orders is not delivered within 40 minutes.

B) Find the probability that at most 1 of these orders is not delivered within 40 minutes.

X = # of orders not delivered on time

$X \sim Bin(10, 0.02)$

A) $P(X=1) = \binom{10}{1} \cdot 0.02^1 \cdot 0.98^9$

B) $P(X \leq 1) = P(X=0) + P(X=1) = \binom{10}{0} 0.02^0 \cdot 0.98^{10} + \binom{10}{1} 0.02 \cdot 0.98^9$

Question 9

A start-up company is looking for 5 investors. Each investor will independently agree to invest in the company with probability 20%. The founder asks investors one at a time until 5 "yes" responses are obtained. Let X represent the total number of investors asked.

Write down the probability function of X.

Let Y = # disagreeing investors until 5^{th} agreeing one

Then $Y \sim NB(k=5, p=0.2)$, so

$$f_Y(y) = P(Y=y) = \binom{5+y-1}{5-1} 0.2^5 \cdot 0.8^y$$

Since X = total number of investors, $X = Y+5$, so for $x \in \{5,6,\dots\}$,

$$f_X(x) = P(X=x) = P(Y+5=x) = f_Y(x-5) = \binom{x-1}{4} 0.2^5 0.8^{x-5}$$

Question 10

Suppose that the probability is 0.75 that an applicant for a driver's license will pass the road test on any given attempt. What is the probability that an applicant will finally pass the test on the fourth attempt?

X = # failures until success

$X \sim \text{Geo}(p)$

$P(\text{"pass on } 4^{th} \text{ attempt"}) = P(X=3)$

$= 0.25^3 \cdot 0.75$

Question 11

Suppose X is a discrete random variable and $a, b \in \mathbb{R}$ with $a < b$. Then

$$P(X > a) \geq P(X > b).$$

TRUE or FALSE?

$P(X>a) \geq P(X>b)$

$\Leftrightarrow \quad 1 - F(a) \geq 1 - F(b)$

$\Leftrightarrow \quad F(a) \leq F(b)$ which is true, since $a<b$ and cdf is non-decreasing.