Instructions

1. **All electronic devices are to be turned off and put away. If you haven't already done this, please do so now.**
2. **This is a 50-minute closed-book quiz.**
3. **Answer the questions in the spaces provided.**
4. **Final answers to written questions should be rounded to THREE DECIMAL PLACES or left in fraction form unless otherwise specified.**
5. **Only question pages will be marked.**
6. **You may tear off the last two pages and use them for rough work. All pages will be collected at the end of the quiz.**
7. **Only a non-programmable, non-graphical calculator with a pink-tie or blue-goggle sticker will be allowed. No other aids are allowed.**
8. **A reference sheet has been included with this quiz, and can be found at the end of the quiz. You may CAREFULLY detach the rough work pages and reference sheet if you like.**
9. **DO NOT WRITE ON THIS COVER PAGE.**

Marking Scheme:

Questions	**Out of**
1 – 9	9
10	8
11	9
Total	26

*** *This page was intentionally left blank.* ***

Part 1: Multiple Choice – Each question is worth 1 mark

Please CLEARLY CIRCLE your answer selection on this page. Do NOT simply write your choice next to the question. If you do, the question will not be graded. Each question has one correct answer. Choose the best answer.

1. Suppose X is normally distributed with mean equal to -3 and variance equal to 9. Which of the following is equal to $P(X \leq -2)$?

A. $P(X \geq -2)$ B. $P(X \geq 2)$ **C.** $\boldsymbol{P(X \geq -4)}$ D. More than one of these

- $P(X \leq -2) = P(X \leq -3 + 1)$
- By symmetry $P(X \leq -3 + 1) = P(X \geq -3 - 1)$
- Thus $P(X \leq -1) = P(X \geq -4)$. Note the others can be checked and are different

2. Suppose we wish to generate a random observation, x, from a distribution with pdf:

$$f(x) = \frac{3}{x^4}, \text{ for } x > 1; 0 \text{ for } x \leq 1$$

We generate an observation, u, for a continuous $Uniform(0,1)$ distribution and get 0.4561. What is the value, x, this value, u, will produce?

A. 1.356 B. 1.164 **C.** $\mathbf{1.225}$ D. 1.300

- To generate the value, x, we need to find $F(x) = \int_1^x \frac{3}{s^4} ds = -(s^{-3})|_1^x = 1 - x^{-3}$, for $x > 1$.
- Next, we let $u = 0.4561 = 1 - x^{-3}$, and then we solve for x.
- This gives us $x = (1 - 0.4561)^{-\frac{1}{3}} = 1.225$ (to 3 d.p.)

3. Let $W \sim N(60, 81)$. What is the 85th percentile of the distribution of W?

A. 78.323 **B.** $\mathbf{69.328}$ C. 50.672 D. 64.541

- We know that $W \sim N(60, 81)$ and we need the value, w^*, such that $P(W \leq w^*) = 0.85$
- Using the bottom of the Normal Table, the corresponding z-score is $z = 1.0364$.
- We transform W into r.v. Z, getting $P(\frac{W-\mu}{\sigma} \leq \frac{w^*-60}{9}) = 0.65$
- Now set $\frac{w^*-60}{9} = 1.0364$ and solve for w^*, $w^* = 60 + 9(1.0364) = 69.3276$
- This seems reasonable as we know that our answer must be greater than 60, since that is the 50th percentile of the distribution

4. Suppose X is a standard normal random variable with CDF $\Phi(x)$. What is the value of x such that $\Phi(x) = 0.22$?

A. -1.1503 B. 0.4677 **C.** $\mathbf{-0.7722}$ D. -0.2533

- We see $F(X) = P(X \leq x) = 0.22$, which means $P(X \leq x) = P(X \geq -x) = 1 - P(X \leq -x) = 0.22$. Thus $P(X \leq -x) = 0.78$. Looking at the N(0,1) quantile table, $-x = 0.7722$, therefore $x = -0.7722$.

5. Suppose $X \sim N(6, 16)$, if $Y = X - 3$, what is the $P(-2 \leq Y < 3)$?

A. 0.89435 B. 0.10565 C. 0.42565 **D.** $\mathbf{0.39435}$

- We know $Y \sim N(3, 4^2)$.
- $P(-2 < Y < 3) = P(Y < 3) - P(Y \leq -2) = P\left(Z < \frac{3-3}{4}\right) - P\left(Z \leq \frac{-2-3}{4}\right) = P(Z < 0) - P\left(Z \leq -\frac{5}{4}\right) = \frac{1}{2} - \left(1 - P(Z \leq 1.25)\right) = 0.5 - (1 - 0.89435) = 0.39435$

6. Suppose X is a random variable that follows a normal distribution with mean equal to 6 and standard deviation equal to 8. What is $P(\,|\,X\ - 6\,|\geq\ 5.2)$?

A. $\mathbf{51.57}\%$ B. 74.22% C. 25.79% D. 63.58%

- We know that $X \sim N(6, 64)$.
- We want $P(|X-6| \geq 5.2) = P(\frac{|X-6|}{8} \geq \frac{5.2}{8}) \ = \ P(|Z| \geq 0.65)$
- $P(|Z| \geq\ 0.65) \ = \ P(Z \leq -0.65) \ + \ P(Z \geq\ 0.65) \ = \ (1 - P(Z \leq\ 0.65)) \ + \ (1 - P(Z \leq 0.65)) \ = \ 2 * (1 - P(Z \leq\ 0.65)) \ = \ 2 * (1 - 0.74215) \ = \ 0.5157$

7. Suppose the weight of candy in a bag is normally distributed with a mean of 90 grams and a variance of 121. Roughly 65 percent of bags will weigh at least how much?

A. **86** B. 82 C. 76 D. 89

- Let Y be the weight of a bag where $Y \sim N(90{,}121). We$ need y^*, such that $P(Y > y^*) = 0.65$
- This is equivalent to $P(Y \geq y^*) = 0.65$. This is equivalent to $P(Y \leq\ y^*) = 0.35$
- We can make use of the table at the bottom of the Normal Table and look up the z-score for the 65th percentile, and then use the negative of this value (due to symmetry)
- From the Table, the $z - score\ = -0.3853$.
- Now, transform Y into the r.v. Z to get $P\left(\frac{Y-\mu}{\sigma} > \frac{y^*-90}{11}\right) = 0.65$.
- Set $\frac{y^*-90}{11} = -0.3853$ and solve for $y^*,\ y^* = 90 - 11 * (0.3853) = 85.7617$
- So approximately 65 percent of bags will have a weight of 86 grams.

8. If $U \sim Uniform(0{,}1)$, what is the distribution of $1 - U$?

A. Uniform(−1,0) **B. Uniform(0,1)**
C. Normal(0,1) D. Cannot be determined

- Imagine any value in (0,1). 1 minus that value will flip to around a 0.5 axis. Thus, the answer is $Uniform(0{,}1)$
- We also can compute $F_X(x) = P(X \leq x) = P(1-U \leq x) = P(U \geq 1-x) = 1 - P(U \leq 1 - x)$. If we plug this value into the CDF, we will achieve a $Uniform(0{,}1)$ random variable

9. At a large clothing store, the time of employment for a manager is normally distributed with a mean of 6.8 years and a variance of 5.33. If a manager is randomly selected, what is the probability the manager has worked at the store for more than 8 years?

A. 32.56% B. 70.35% C. 28.67% **D.** $\mathbf{30.15}\%$

- Let X represent the length of employment for a manager. $X \sim N(6.8, 5.33)$
- We want $P(X\ >\ 8) = 1 - P(X < 8) = 1 - P\left(\frac{X-\mu}{\sigma} < \frac{8-6.8}{\sqrt{5.33}}\right)$
- Thus, we examine $1 - P(Z < 0.5197) \approx 1 - P(Z < 0.52) = \ 1 - 0.69847 = \ 0.30153$

Part 2: Written Answer / Short Answer Questions

Final answers can be left in fraction form OR rounded to THREE DECIMAL PLACES.

For full credit, you need to show all necessary work. Unjustified answers will not receive full credit.

10. (8 marks) Suppose the height of cliffs in a national park is normally distributed with mean 350 m and standard deviation of 10 m. Let the height be represented by the random variable X.

a. Give the probability density function (pdf) of X with all necessary parameters. What is $P(X = 350)$? **(2 marks)**

- $X \sim N(350, 10^2)$
- $f(x) = \frac{1}{10\sqrt{2\pi}} e^{\frac{-(x-350)^2}{2(100)}}$ for $x \in \mathbb{R}$
- $P(X = 350) = 0$ since this is a continuous distribution

b. What is the probability a randomly selected cliff has height greater than 325 m? **(2 marks)**

- By symmetry, $P(X > 325) = P(X > 350 - 25) = P(X < 350 + 25) = P(X < 375)$
- Through standardization, $P(X < 375) = P\left(Z < \frac{375-350}{10}\right) = P(Z < 2.5) = 0.99379$

c. What is the 93rd percentile of the distribution of X? **(2 marks)**

- $P(X < x) = P\left(Z < \frac{x-350}{10}\right) = 0.93$
- Using the table, $\frac{x-350}{10} = 1.4758$
- $X = 350 + 10\,(1.4758) = 364.758$
- Therefore, 93% of the cliffs are no taller than 364.758 m (or 93% are shorter than this)

d. What is the probability a cliff has a height greater than 340 m given we know its height is no more than 355 m? **(2 marks)**

- $P(X > 335\ |X < 355) = \frac{P(X>340 \cap X<355)}{P(X<355)} = \frac{P(340<X<355)}{P(X<355)} = \frac{F_X(355)-F_X(340)}{F_X(355)}$
- After standardization $\frac{F_X(355)-F_X(340)}{F_X(360)} = \frac{F_Z\left(\frac{355-350}{10}\right)-F_Z\left(\frac{340-350}{10}\right)}{F_Z\left(\frac{355-350}{10}\right)} = \frac{F_Z(0.5)-F_Z(-1)}{F_Z(0.5)}$
- Using symmetry to allow use of table $\frac{F_Z(0.5)-F_Z(-1)}{F_Z(0.5)} = \frac{F_Z(0.5)-(1-F_Z(1))}{F_Z(0.5)}$
- Using the table $\frac{F_Z(0.5)-(1-F_Z(1))}{F_Z(0.5)} = \frac{0.69146-(1-0.84134)}{0.69146} \approx 0.771$

11. (9 marks) Suppose that scores on an exam can be modeled by a random variable X that is normally distributed with a mean of 84 and a variance of 36.

a. Determine the z-score for an exam score of 88. Are there any units associated with this z-score? If yes, state the units. If no, state that there are no units. **(2 marks)**

- Let Z be a standard normal random variable, $Z = \frac{X-84}{6}$
- Therefore, the z-score is $\frac{88-84}{6} = 0.667$
- There are no units as the numerator has units that cancel with the denominator

b. Suppose students that score in the top 3% for an exam will receive a special award. What is the minimum score necessary (rounded to the nearest integer) to receive the award? **(2 marks)**

- $F_Z\left(\frac{x-84}{6}\right) = 1 - 0.03$
- Using the table, we get 1.8808
- Solving, $\frac{x-84}{6} = 1.8808$, $x = 84 + 6(1.8808) = 95.2848 \approx 95$
- The top 3% requires a score of at least 95.

c. In this class, it was determined 18 students scored 76.5 or less on an exam. Given this information, what would you estimate the size of the class to be? Please round your answer to the nearest integer. **(2 marks)**

- $P\left(Z < \frac{76.5-84}{6}\right) = P(Z < -1.25)$
- $P(Z < -1.25) = P(Z > 1.25) = 1 - P(Z < 1.25) = 1 - 0.89435 = 0.10565$
- So we know 10.565% of students scored 76.5 or less.
- Since we know there were 18 students, $(0.10565)x = 18$ and thus $x = \frac{18}{0.10565} = 170.3739$
- Therefore, the class has approximately 170 students.

d. Suppose quiz scores in this class are also normally distributed with the same mean, but a different variance. If 19% of the students scored at least 92 on a quiz, determine the variance associated with the distribution of quiz scores. **(3 marks)**

- Let Y be the random variable associated with the previous quiz scores
- $P(Y > 92) = P\left(Z > \frac{92-84}{\sigma}\right) = 0.19$
- $P\left(Z < \frac{8}{\sigma}\right) = 0.81$
- $\frac{8}{\sigma} = 0.8779$
- So $\sigma = 9.113$, and $\sigma^2 \approx 83.040$.

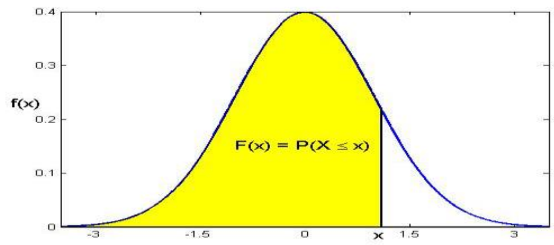
This page may be CAREFULLY removed and used for rough work.

It will not be graded, but MUST be handed in.

This page may be CAREFULLY removed and used for rough work.

It will not be graded, but MUST be handed in.

N(0,1) Cumulative Distribution Function



This table gives values of $F(x) = P(X \leq x)$ for $X \sim N(0,1)$ and $x \geq 0$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99979	0.99981	0.99981	0.99982	0.99983	0.99983

N(0,1) Quantiles: This table gives values of $F^{-1}(p)$ for $p \geq 0.5$

p	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.075	0.08	0.09	0.095
0.5	0.0000	0.0251	0.0502	0.0753	0.1004	0.1257	0.1510	0.1764	0.1891	0.2019	0.2275	0.2404
0.6	0.2533	0.2793	0.3055	0.3319	0.3585	0.3853	0.4125	0.4399	0.4538	0.4677	0.4959	0.5101
0.7	0.5244	0.5534	0.5828	0.6128	0.6433	0.6745	0.7063	0.7388	0.7554	0.7722	0.8064	0.8239
0.8	0.8416	0.8779	0.9154	0.9542	0.9945	1.0364	1.0803	1.1264	1.1503	1.1750	1.2265	1.2536
0.9	1.2816	1.3408	1.4051	1.4758	1.5548	1.6449	1.7507	1.8808	1.9600	2.0537	2.3263	2.5758

Summary of Discrete Distributions

Notation and Parameters	**Probability Function** $f(x)$	**Mean** $E(X)$	**Variance** $Var(X)$
Discrete Uniform(a, b) $b \geq a$ a, b integers	$\frac{1}{b - a + 1}$ $x = a, a + 1, \ldots, b$	$\frac{a+b}{2}$	$\frac{(b - a + 1)^2 - 1}{12}$
Hypergeometric(N, r, n) $N = 1,2, \ldots$ $n = 0,1, \ldots, N$ $r = 0,1, \ldots, N$	$\frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$ $x = max(0, n - N + r),$ $\ldots, min(r, n)$	$\frac{nr}{N}$	$\frac{nr}{N}\left(1 - \frac{r}{N}\right)\frac{N - n}{N - 1}$
Binomial(n, p) $0 \leq p \leq 1, q = 1 - p$ $n = 1,2, \ldots$	$\binom{n}{x} p^x q^{n-x}$ $x = 0,1, \ldots, n$	np	npq
Bernoulli(p) $0 \leq p \leq 1, q = 1 - p$	$p^x q^{1-x}$ $x = 0,1$	p	pq
Negative Binomial(k, p) $0 < p \leq 1, q = 1 - p$ $k = 1,2, \ldots$	$\binom{x + k - 1}{x} p^k q^x$ $= \binom{-k}{x} p^k (-q)^x$ $x = 0,1, \ldots$	$\frac{kq}{p}$	$\frac{kq}{p^2}$
Geometric(p) $0 < p \leq 1, q = 1 - p$	pq^x $x = 0,1, \ldots$	$\frac{q}{p}$	$\frac{q}{p^2}$
Poisson(λ) $\lambda \geq 0$	$\frac{e^{-\lambda}\lambda^x}{x!}$ $x = 0,1, \ldots$	λ	λ

Summary of Continuous Distributions

Notation and Parameters	**Probability Density Function** $f(x)$	**Mean** $E(X)$	**Variance** $Var(X)$
Uniform(a, b) $b > a$	$\frac{1}{b - a}$ $a \leq x \leq b$	$\frac{a + b}{2}$	$\frac{(b - a)^2}{12}$
Exponential(θ) $\theta > 0$	$\frac{1}{\theta} e^{-x/\theta}$ $x \geq 0$	θ	θ^2
$N(\mu, \sigma^2) = G(\mu, \sigma)$ $\mu \in \mathbb{R}, \sigma^2 > 0$	$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ $x \in \mathbb{R}$	μ	σ^2