Stat 230: Probability

Lecture 7

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Review

Example

Suppose a bag contains 12 red balls and 7 green balls. Suppose that a ball is drawn at random, then, without replacement, a second ball is drawn at random.

(1) What is the probability that both balls are red?

(2) What is the probability that the second ball is red?

Review

Last time we talked about

- Conditional Probability and Examples

Topics for today

- Total Probability
- Bayes Theorem

Reading: Chapter 4, Chapter 5

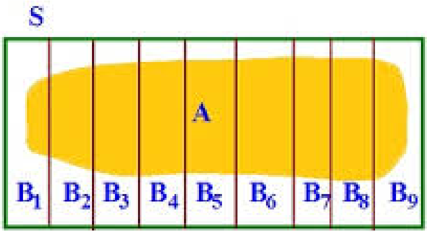
Review

- Office hours

Total Probability

Definition

A sequence of sets $B_1, B_2, ..., B_k$ are said to **partition** *the sample space S if $B_i \cap B_j = \emptyset$ for all $i \neq j$, and $\cup_{j=1}^{k} B_j = S$.*



Total Probability

Theorem (Law of total probability)

Suppose that $B_1, B_2, ..., B_k$ partition S. Then for any event A,

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \cdots + P(A|B_k)P(B_k).$$

A common way in which this is used is that

$$P(A) = P(A|B)P(B) + P(A|\bar{B})P(\bar{B}),$$

since B and $\bar{B}$ partition S.

Total Probability

Example

A rare disease occurs in 0.3% of a population. A test has been developed for the disease. If the test is performed on an individual who has the disease, the probability of a positive test result (i.e. the test says that they have the disease) is 0.96. If the test is performed on an individual who does not have the disease, the probability of a positive test result is 0.05. Suppose the test is performed on a random individual in the population.

(1) What is the probability that the test result will be positive?

(2) Suppose the test comes back positive. What is the probability that the individual actually has the disease?

(3) If the test comes back negative, what is the probability that the individual actually has the disease?

Bayes Theorem

Theorem (Bayes Theorem)

Suppose that $B_1, B_2, ..., B_k$ partition S. Then for any event A,

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_{j=1}^{k} P(A|B_j)P(B_j)}.$$

For the partition B and $\bar{B}$:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\bar{B})P(\bar{B})}.$$

Bayes Theorem

Figure: Reverend Thomas Bayes: 1701-1761

Bayes Theorem

Example

A medical test has been developed for a particular disease that correctly detects the disease in 92% of patients who have the disease, but gives false positives for 6% of patients that do not actually have the disease. Assume that 0.5% of the population has the disease. Suppose the test is applied to a randomly selected individual from the population.

(1) Calculate the probability that the test will be positive.

(2) Given that the test is negative, what is the probability that the individual tested actually has the disease.

(3) Suppose that the doctor who administered the test to the patient believes that the probability that the patient actually has the disease is 50%. Assuming that the doctor's belief is correct, and given that the test is positive, what is the probability that the person actually has the disease?

Bayes Theorem

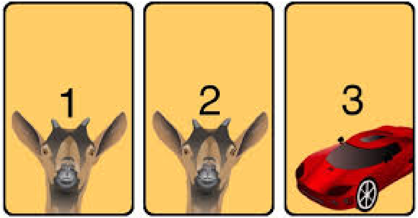
Example

Suppose a room contains 3 chests. One chest contains 2 silver coins, another chest contains a gold coin and a silver coin, and the third chest contains 2 gold coins. Suppose a chest in the room is selected at random, and then a coin is selected at random from the chest.

Given that the first selected coin is gold, what is the probability that the remaining coin in the chest is gold?

Bayes Theorem

This problem is related to the Monty Hall Problem, popularized by the show "Lets Make a Deal".



Moral is you should always switch!