Stat 230: Probability

Lecture 4

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Review

Example (Review Question)

Consider drawing 3 numbers at random WITH replacement from the digits $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. What is the probability that there is a repeated number among the three?

Review

Last time we talked about

- Permutations and Combinations
- Birthday Problem

Schedule for today:

- Multinomial Coefficient
- Inclusion/Exclusion Principle

Reading: End of chapter 3

Review

- Piazza
- (Next Time) Quiz

Review

Example

There are 4 passengers on an elevator that services 5 floors. Assume that each of the 4 passengers are equally likely to get off the elevator on any of the 5 floors. Give the probability:

(1) The passengers all get off on different floors
(2) Two passengers get off on floor two, and two get off on floor three
(3) Two passengers get off on one floor, and two passengers get off on another, different floor

Multinomial Coefficient

Example

Consider rearranging the letters at random in the name "ZENYATTA" to form a single 'word'.

(1) How many ways can this be done?

(2) What is the probability that all of the letters appear in alphabetic order?

(3) What is the probability that the word begins and ends with "T"?

Multinomial Coefficient

Definition

Consider n objects which consist of k indistinguishable types. Suppose that there are n_1 objects which are of type 1, n_2 which are of type 2, and in general n_i objects of type i. Then there are

$$\frac{n!}{n_1!n_2!\ldots n_k!}$$

distinguishable arrangements of the n objects. This quantity is known as a **multinomial coefficient**.

Multinomial Coefficient

Example

3 members of the C-S department, 2 members of the math department, and 3 members of the stats department sit down at random in a row of 8 seats.

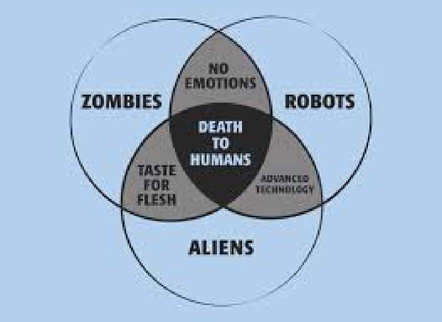
(1) What is the probability that each department's members are sitting in consecutive seats?

(2) What is the probability that members of the same department are sitting on both ends of the row?

Venn Diagram

Probability is a function on sets: So set theory is of central importance in probability theory.

We can visualize sets and set theoretic operations using a Venn Diagram.



Venn Diagram

Example

Visualize the following set theoretic operations using a Venn diagram:

(1) $\bar{A}$

(2) $A \cap B$

(3) $A \cap \bar{B}$

(4) $(A \cap B \cap C) \cup (\overline{(B \cup C)} \cap A)$

DeMorgan's Laws

DeMorgan's Laws:

- $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$
- $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

Inclusion-Exclusion

Inclusion Exclusion Rules: For arbitrary events A, B, and C

(1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(2) $P(A \cup B \cup C) =$
$P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$

(3)

$$P(\cup_{i=1}^{n} A_i) = \sum_i P(A_i) - \sum_{i<j} P(A_iA_j) + \sum_{i<j<k} P(A_iA_jA_k) - \sum_{i<j<k<l} P(A_iA_jA_kA_l) + \cdots$$