Stat 230: Probability

Lecture 9

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Review

Example

Roll two fair six sided dice and let the random variable X denote the minimum of the die rolls. Compute $f_X(x)$ and $F_X(x)$. Graph $F_X(x)$.

Review

Last time we talked about:

(1) Random variables: $X : S \to \mathbb{R}$

(2) Probability function: $f_X(x) = P(X = x)$

(3) Cummulative distribution function: $F_X(x) = P(X \leq x)$

For today:

(1) Distributions

(2) Common distributions

- Discrete uniform distribution

Review

- Reminder: No school Monday!

Cumulative Distribution Function

Example

Suppose students A,B and C each independently answer a question on a test. The probability of getting the correct answer is 0.9 for A, 0.7 for B and 0.4 for C. Let X denote the number of people who get the answer correct.

(1) Compute the probability function of X

(2) Compute the CDF of X.

(3) Compute the probability that at least one person gets the answer correct.

Distribution

Definition

The **distribution** *of a random variable X is a probability function (measure) defined on the sample space $\mathbb{R}$, $P_X(\cdot)$, that is defined by $P_X(A) = P(X \in A)$, $A \subset \mathbb{R}$. The distribution of a random variable X is described entirely by its CDF $F_X(x)$.*

Distribution

Definition

Two random variables X and Y are said to **have the same distribution** *if $F_X(x) = F_Y(x)$ for all $x \in \mathbb{R}$. We denote this by*

$$X \sim Y$$

Distribution

Example (Random variables with the same distribution)

Experiment	
Flip a fair coin once	X= # of heads
Roll a fair six sided die	Y= 1 if die roll is even, 0 if die roll is odd

$$X \sim Y$$

Discrete Uniform

Definition

We say that the random variable X has a **discrete uniform distribution** *on a to b (shorthand: $X \sim U(a, b)$) if the range of X is the set of integers z satisfying $a \leq z \leq b$, and the probability that X takes any value in its range is the same.*

Discrete Uniform

Example (Discrete uniform random variables)

Experiment	X	**Distribution**
Roll a 6 sided die	# showing on die	$U(1,6)$
Draw a number between 1 and 50	# Drawn	$U(1,50)$

Discrete Uniform

Example

Suppose $X \sim U(a, b)$. Compute $f_X(x)$, and $F_X(x)$.

Discrete Uniform

Example

Suppose three 6 sided die are rolled. Let X denote the largest number that is rolled. Compute $f_X(x)$.