Stat 230: Probability

Lecture 14

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Review

Example

Website hits for a given website occur according to a Poisson process with a rate of 100 hits per minute. We say a second is a "break" if there are no hits in that second.

(1) What is the probability of a break in any given second?

(2) Compute the probability of observing exactly 10 breaks in 60 consecutive seconds.

(3) Compute the probability that one must wait for 30 seconds to get 2 breaks.

Review

Last time we talked about:

(1) Expectation

For today:

(1) Expectation
- Expectation of functions of a RV
- Expectation of “named” distributions

Review

- TA tutorial today
- Next Monday: Quiz 3
- Monte Carlo Simulations
- R Codes

Review

Example

(1) Give the distribution to count the number of failures until Katy bakes 3 successful cupcakes if the probability of failure is 0.2.

(2) Give distribution to count the number of 100 popcorn kernels Diego can catch if the probability of success on each trial is 0.62.

Expected Value

Example

A lottery is conducted in which 7 numbers are drawn without replacement between the numbers 1 and 50. A player wins the lottery if the numbers selected on their ticket match all 7 of the drawn numbers. A ticket to play the lottery costs \$1, and the jackpot is valued at \$5,000,000. Compute the expected return for this bet.

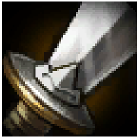
Expected Value

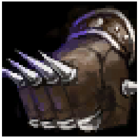
Example

In League of Legends, the damage of an attack from a champion (ignoring armor) is equal to

$$D(1+X), \quad \textit{where} \;\; P(X=1) = p = 1 - P(X=0)$$

p is the probability of a critical strike, and D is the attack damage of the champion. Suppose a champion has initial damage $D = 70$. They may buy Brawler's gloves that cause the probability p of a critical strike to be 0.10 for 400g, or a Long Sword that increases their damage by 10 for 350g (but $p = 0$). Which is better in terms of increasing average damage output?





Expected Value

Remark

If

$$g : \mathbb{R} \to \mathbb{R}$$

then for a random variable X *with p.f.* $f_X(x)$, $g(X)$ *is a random variable taking values* $g(X(S))$. *Therefore,*

$$E[g(X)] = \sum_{x \in X(S)} g(x) f_X(x)$$

Expected Value

Example

If $g(x) = x^2$, and X is the result of a fair six sided die roll, then compute $E[g(X)]$.

Expected Value

Remark

If $g(x)$ is a linear function $g(x) = ax + b$, then for a random variable X

$$E[aX + b] = aE[X] + b$$

It is not true in general that $g(E[X]) = E[g(X)]$, although this is a common mistake.

Distributions Expected Value

Going forward, we would like to compute $E[W]$, $E[X]$, $E[Y]$, and $E[Z]$ if

(1) $W \sim Bin(n, p)$

(2) $Z \sim Pois(\mu)$

(3) $X \sim Hyp(N, r, n)$

(4) $Y \sim NB(k, p)$

Distributions Expected Value

Theorem (Expected Value of Binomial)

If $W \sim Binomial(n, p)$, then $E[W] = np$.

Example

You roll 2 dice 24 times. What is the expected value that the sum of the dice is 7?