Stat 230: Probability

Lecture 20

Jeremy VanderDoes

University of Waterloo

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Review

Example

Suppose X has pdf

$$f(x) = \begin{cases} 6x(1-x) & \text{if } 0 \le x \le 1, \\ 0 & \text{otherwise} \end{cases}$$

Compute the CDF.

Review

Last we talked about

(1) CDF for continuous variables

For today:

(1) Summary statistics

(2) Transformations

Reading: Middle of chapter 8

Review

- Today: TA R tutorial
- Next Monday: Quiz 4

Summary Statistics

Definition

If X is a continuous random variable with pdf $f(x)$, and $g : \mathbb{R} \to \mathbb{R}$, then

$$E(g(X)) = \int_{-\infty}^{\infty} g(x)f(x)dx$$

It follows that

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx$$

$$Var(X) = E([X - E(X)]^2) = \int_{-\infty}^{\infty} (x - E(X))^2 f(x)dx$$

Summary Statistics

- The expectation is linear: $E[ag(X) + b] = aE[g(X)] + b$
- The shortcut formula for computing the variance still holds in the continuous case:

$$Var(X) = E(X^2) - (E(X))^2$$

Example

Suppose X has pdf

$$f(x) = \begin{cases} 6x(1-x) & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise} \end{cases}$$

Compute $E(X)$.

Summary Statistics

Example

Suppose X has CDF

$$F(x) = \begin{cases} 0 & x < 0, \\ \frac{x^2}{2} & 0 \leq x < 1/2 \\ \frac{7}{4}x - \frac{3}{4} & 1/2 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

Compute $E(X)$ and $Var(X)$.

Remark

Consider the transformation $Y = h(X)$ *of a continuous random variable* X*, where* $h : \mathbb{R} \to \mathbb{R}$*. It is often of interest to compute the distribution of such transformations. In particular, if* X *is continuous with pdf* $f_X(x)$*, it may be useful to compute the pdf of* Y*,* $f_Y(y)$*.*

Transformations

Strategy for computing the probability density function and/or cumulative distribution function of $Y = h(X)$ in terms of the analogous functions for X:

(1) Determine the range of X, and from this deduce the range of $Y = h(X)$.

(2) Write the CDF of Y in terms of the CDF of X.

(3) If desired, differentiate the CDF of Y to obtain the PDF as a function of $f_X(x)$.

Transformations

Example

Suppose X is a random variable with PDF

$$f_X(x) = \begin{cases} 1 & x \in (0,1) \\ 0 & \text{Otherwise} \end{cases}$$

and we are given $Y = \frac{2}{X} - 1$. What is the range of Y?

Uniform

Definition

We say that X has a uniform distribution on (a, b) if X has pdf

$$f(x) = \begin{cases} \frac{1}{b-a} & x \in (a, b), \\ 0 & \textit{otherwise} \end{cases}$$

This is abbreviated $X \sim U(a, b)$.