Stat 230: Probability

Lecture 25

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Review

Example

Suppose $X \sim N(6, 7^2)$, how can this be standardized? What is $P(X > 6)$?

Review

Last time we talked about:

(1) Joint Probability Distribution

For today:

(1) Review
(2) Independence
(3) Multinomial distribution

Review

- Today: TA Tutorial
- Tomorrow: Test 2
- Review

Independence

Definition

Suppose that X and Y are discrete random variables with joint probability function $f(x, y)$ and marginal probability functions $f_X(x)$ and $f_Y(y)$. X and Y are said to be **independent random variables** *if*

$$f(x, y) = f_X(x)f_Y(y), \quad \textit{for all } \ x \in X(S), y \in Y(S)$$

This is the same as saying

$$P(X = x, Y = y) = P(X = x)P(Y = y) \text{ for } \textbf{all } x \text{ and } y$$

More generally X and Y are independent if for all subsets $A_x, A_y \subset \mathbb{R}$:

$$P(X \in A_x \cap Y \in A_y) = P(X \in A_x)P(Y \in A_y)$$

Independence

Example

Suppose that X and Y have a joint probability function defined in the table below:

	f(x,y)	x 0	1	2
y	0	.2	.3	.1
	2	.25	.13	.02

Are X and Y independent?

Independence

Independence of more than two random variables: If $X_1, ..., X_n$ have joint probability function $f(x_1, ..., x_n)$, and marginal probability functions $f_{X_1}(x_1), ..., f_{X_n}(x_n)$, then $X_1, ..., X_n$ are said to be **independent** if

$$f(x_1, ..., x_n) = f_{X_1}(x_1) f_{X_2}(x_2) \cdots f_{X_n}(x_n)$$

Conditional Probability

Definition

The **conditional probability function** *of X given $Y = y$ is denoted $f_X(x|y)$, and is defined to be*

$$f_X(x|y) = P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)} = \frac{f(x, y)}{f_Y(y)}$$

$f_Y(y|x)$ is similarly defined.