# CROWDMARK

**UNIVERSITY OF WATERLOO**

Times: Monday 2022-07-11 at 09:30 to 10:20

Duration: 50 minutes

Exam ID: 4977516

Sections: STAT 230 LEC 001-003

Instructors: Erik Hintz, James Adcock, Jeremy VanderDoes

## Examination
## Quiz 5 (Version A)
## Spring 2022
## STAT 230

## Special Materials

Candidates may bring only the listed aids.

· Calculator - Non-Programmable

## Instructions

1. **All electronic devices are to be turned off and put away. If you haven't already done this, please do so now.**
2. **This is a 50-minute closed-book quiz.**
3. **Answer the questions in the spaces provided.**
4. **Final answers to written questions should be rounded to THREE DECIMAL PLACES or left in fraction form unless otherwise specified.**
5. **Only question pages will be marked.**
6. **You may tear off the last two pages and use them for rough work. All pages will be collected at the end of the quiz.**
7. **Only a non-programmable, non-graphical calculator with a pink-tie or blue-goggle sticker will be allowed. No other aids are allowed.**
8. **A reference sheet has been included with this quiz, and can be found at the end of the quiz. You may CAREFULLY detach the rough work pages and reference sheet if you like.**
9. **DO NOT WRITE ON THIS COVER PAGE.**

## Marking Scheme:

| Questions | Out of |
|-----------|--------|
| 1 – 9 | 9 |
| 10 | 8 |
| 11 | 9 |
| **Total** | 26 |

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**Part 1: Multiple Choice – Each question is worth 1 mark**

**Please CLEARLY CIRCLE your answer selection on this page. Do NOT simply write your choice next to the question. If you do, the question will not be graded. Each question has one correct answer. Choose the best answer.**

1. Suppose $X$ is normally distributed with mean equal to -2 and variance equal to 4. Which of the following is equal to $P(X \leq -1)$?

   A.   $P(X \geq -1)$    B.   $P(X \geq 1)$       C.   $P(X \geq -3)$       D.   More than one of these

2. Suppose we wish to generate a random observation, $x$, from a distribution with pdf:
$$f(x) = \frac{4}{x^5}, \text{ for } x > 1; 0 \text{ for } x \leq 1$$
   We generate an observation, $u$, for a continuous $Uniform(0,1)$ distribution and get 0.6357. What is the value, $x$, this value, $u$, will produce?

   A.     1.445         B.     1.287         C.     1.120         D.     1.095

3. Let $Y \sim N(100, 64)$. What is the 66th percentile of the distribution of $Y$?

   A.  124.659          B.  105.231          C.  103.300          D.  96.918

4. Suppose $X$ is a standard normal random variable with CDF $\Phi(x)$. What is the value of $x$ such that $\Phi(x) = 0.31$?

   A.     0.4399        B.     $-0.4959$          C.     $-0.4677$     D.     $-0.2275$

5. Suppose $X \sim N(3, 4)$, if $Y = X - 2$, what is the $P(-2 \leq Y < 1)$?

   A.     0.433         B.     0.933         C.     0.567         D.     0.84134

6. Suppose X is a random variable that follows a normal distribution with mean equal to 5 and standard deviation equal to 8. What is $P(|X - 5| > 4.4)$?

A. 41.77%       B. 29.12%       C. 58.23%       D. 94.42%

7. Suppose the weight of bouncy balls in a bag is normally distributed with a mean of 70 grams and a variance of 100. Roughly 70 percent of bags will weigh at least how much?

A. 77       B. 60       C. 65       D. 75

8. If $U \sim Uniform(0,1)$, what is the distribution of $1 - U$?

A. Uniform(0,1)       B. Uniform(-1,0)
C. Normal(0,1)        D. Cannot be determined

9. At a large grocery store, the time of employment for a cashier is normally distributed with a mean of 8.3 years and a variance of 7.76. If a cashier is randomly selected, what is the probability the cashier has worked at the store for more than 10 years?

A. 32.51%       B. 45.29%       C. 36.3%       D. 27.09%

**Part 2: Written Answer / Short Answer Questions**

**Final answers can be left in fraction form OR rounded to THREE DECIMAL PLACES.**

**For full credit, you need to show all necessary work. Unjustified answers will not receive full credit.**

10. **(8 marks)** Suppose the height of trees in a local forest is normally distributed with mean 260 cm and standard deviation of 10 cm. Let the height be represented by the random variable $X$.

a. Give the probability density function (pdf) of $X$ with all necessary parameters.
What is $P(X = 260)$? **(2 marks)**

b. What is the probability a randomly selected tree has height greater than 245 cm? **(2 marks)**

c. What is the 80th percentile of the distribution of $X$? **(2 marks)**

d. What is the probability a tree is taller than 245 cm given we know the tree is no taller than 265 cm tall? **(2 marks)**

**11. (9 marks)** Suppose that scores on an exam can be modeled by a random variable $X$ that is normally distributed with a mean of 82 and a variance of 36.

**a.** Determine the z-score for an exam score of 85. Are there any units associated with this z-score? If yes, state the units. If no, state that there are no units. **(2 marks)**

**b.** Suppose students that score in the top 1% for an exam will receive a special award. What is the minimum score necessary (rounded to the nearest integer) to receive the award? **(2 marks)**

**c.** In this class, it was determined 5 students scored 70 or less on an exam. Given this information, what would you estimate the size of the class to be? Please round your answer to the nearest integer. **(2 marks)**

**d.** Suppose quiz scores in this class are also normally distributed with the same mean, but a different variance. If 15% of the students scored at least 88 on a quiz, determine the variance associated with quiz scores. **(3 marks)**

This page may be CAREFULLY removed and used for rough work.

It will not be graded, but MUST be handed in.

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### N(0,1) Cumulative Distribution Function
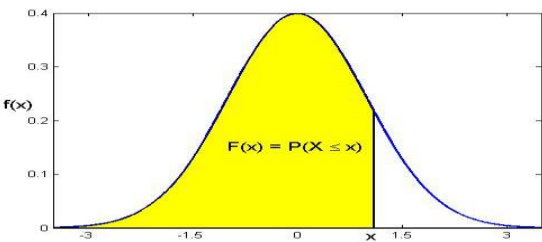


This table gives values of $F(x) = P(X \le x)$ for $X \sim N(0,1)$ and $x \ge 0$

| x | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|------|------|------|------|------|------|------|------|------|------|
| 0.0 | 0.50000 | 0.50399 | 0.50798 | 0.51197 | 0.51595 | 0.51994 | 0.52392 | 0.52790 | 0.53188 | 0.53586 |
| 0.1 | 0.53983 | 0.54380 | 0.54776 | 0.55172 | 0.55567 | 0.55962 | 0.56356 | 0.56749 | 0.57142 | 0.57535 |
| 0.2 | 0.57926 | 0.58317 | 0.58706 | 0.59095 | 0.59483 | 0.59871 | 0.60257 | 0.60642 | 0.61026 | 0.61409 |
| 0.3 | 0.61791 | 0.62172 | 0.62552 | 0.62930 | 0.63307 | 0.63683 | 0.64058 | 0.64431 | 0.64803 | 0.65173 |
| 0.4 | 0.65542 | 0.65910 | 0.66276 | 0.66640 | 0.67003 | 0.67364 | 0.67724 | 0.68082 | 0.68439 | 0.68793 |
| 0.5 | 0.69146 | 0.69497 | 0.69847 | 0.70194 | 0.70540 | 0.70884 | 0.71226 | 0.71566 | 0.71904 | 0.72240 |
| 0.6 | 0.72575 | 0.72907 | 0.73237 | 0.73565 | 0.73891 | 0.74215 | 0.74537 | 0.74857 | 0.75175 | 0.75490 |
| 0.7 | 0.75804 | 0.76115 | 0.76424 | 0.76730 | 0.77035 | 0.77337 | 0.77637 | 0.77935 | 0.78230 | 0.78524 |
| 0.8 | 0.78814 | 0.79103 | 0.79389 | 0.79673 | 0.79955 | 0.80234 | 0.80511 | 0.80785 | 0.81057 | 0.81327 |
| 0.9 | 0.81594 | 0.81859 | 0.82121 | 0.82381 | 0.82639 | 0.82894 | 0.83147 | 0.83398 | 0.83646 | 0.83891 |
| 1.0 | 0.84134 | 0.84375 | 0.84614 | 0.84849 | 0.85083 | 0.85314 | 0.85543 | 0.85769 | 0.85993 | 0.86214 |
| 1.1 | 0.86433 | 0.86650 | 0.86864 | 0.87076 | 0.87286 | 0.87493 | 0.87698 | 0.87900 | 0.88100 | 0.88298 |
| 1.2 | 0.88493 | 0.88686 | 0.88877 | 0.89065 | 0.89251 | 0.89435 | 0.89617 | 0.89796 | 0.89973 | 0.90147 |
| 1.3 | 0.90320 | 0.90490 | 0.90658 | 0.90824 | 0.90988 | 0.91149 | 0.91309 | 0.91466 | 0.91621 | 0.91774 |
| 1.4 | 0.91924 | 0.92073 | 0.92220 | 0.92364 | 0.92507 | 0.92647 | 0.92785 | 0.92922 | 0.93056 | 0.93189 |
| 1.5 | 0.93319 | 0.93448 | 0.93574 | 0.93699 | 0.93822 | 0.93943 | 0.94062 | 0.94179 | 0.94295 | 0.94408 |
| 1.6 | 0.94520 | 0.94630 | 0.94738 | 0.94845 | 0.94950 | 0.95053 | 0.95154 | 0.95254 | 0.95352 | 0.95449 |
| 1.7 | 0.95543 | 0.95637 | 0.95728 | 0.95818 | 0.95907 | 0.95994 | 0.96080 | 0.96164 | 0.96246 | 0.96327 |
| 1.8 | 0.96407 | 0.96485 | 0.96562 | 0.96638 | 0.96712 | 0.96784 | 0.96856 | 0.96926 | 0.96995 | 0.97062 |
| 1.9 | 0.97128 | 0.97193 | 0.97257 | 0.97320 | 0.97381 | 0.97441 | 0.97500 | 0.97558 | 0.97615 | 0.97670 |
| 2.0 | 0.97725 | 0.97778 | 0.97831 | 0.97882 | 0.97932 | 0.97982 | 0.98030 | 0.98077 | 0.98124 | 0.98169 |
| 2.1 | 0.98214 | 0.98257 | 0.98300 | 0.98341 | 0.98382 | 0.98422 | 0.98461 | 0.98500 | 0.98537 | 0.98574 |
| 2.2 | 0.98610 | 0.98645 | 0.98679 | 0.98713 | 0.98745 | 0.98778 | 0.98809 | 0.98840 | 0.98870 | 0.98899 |
| 2.3 | 0.98928 | 0.98956 | 0.98983 | 0.99010 | 0.99036 | 0.99061 | 0.99086 | 0.99111 | 0.99134 | 0.99158 |
| 2.4 | 0.99180 | 0.99202 | 0.99224 | 0.99245 | 0.99266 | 0.99286 | 0.99305 | 0.99324 | 0.99343 | 0.99361 |
| 2.5 | 0.99379 | 0.99396 | 0.99413 | 0.99430 | 0.99446 | 0.99461 | 0.99477 | 0.99492 | 0.99506 | 0.99520 |
| 2.6 | 0.99534 | 0.99547 | 0.99560 | 0.99573 | 0.99585 | 0.99598 | 0.99609 | 0.99621 | 0.99632 | 0.99643 |
| 2.7 | 0.99653 | 0.99664 | 0.99674 | 0.99683 | 0.99693 | 0.99702 | 0.99711 | 0.99720 | 0.99728 | 0.99736 |
| 2.8 | 0.99744 | 0.99752 | 0.99760 | 0.99767 | 0.99774 | 0.99781 | 0.99788 | 0.99795 | 0.99801 | 0.99807 |
| 2.9 | 0.99813 | 0.99819 | 0.99825 | 0.99831 | 0.99836 | 0.99841 | 0.99846 | 0.99851 | 0.99856 | 0.99861 |
| 3.0 | 0.99865 | 0.99869 | 0.99874 | 0.99878 | 0.99882 | 0.99886 | 0.99889 | 0.99893 | 0.99896 | 0.99900 |
| 3.1 | 0.99903 | 0.99906 | 0.99910 | 0.99913 | 0.99916 | 0.99918 | 0.99921 | 0.99924 | 0.99926 | 0.99929 |
| 3.2 | 0.99931 | 0.99934 | 0.99936 | 0.99938 | 0.99940 | 0.99942 | 0.99944 | 0.99946 | 0.99948 | 0.99950 |
| 3.3 | 0.99952 | 0.99953 | 0.99955 | 0.99957 | 0.99958 | 0.99960 | 0.99961 | 0.99962 | 0.99964 | 0.99965 |
| 3.4 | 0.99966 | 0.99968 | 0.99969 | 0.99970 | 0.99971 | 0.99972 | 0.99973 | 0.99974 | 0.99975 | 0.99976 |
| 3.5 | 0.99977 | 0.99978 | 0.99978 | 0.99979 | 0.99979 | 0.99981 | 0.99981 | 0.99982 | 0.99983 | 0.99983 |

### N(0,1) Quantiles:   This table gives values of $F^{-1}(p)$ for $p \ge 0.5$

| p | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.075 | 0.08 | 0.09 | 0.095 |
|---|------|------|------|------|------|------|------|------|-------|------|------|-------|
| 0.5 | 0.0000 | 0.0251 | 0.0502 | 0.0753 | 0.1004 | 0.1257 | 0.1510 | 0.1764 | 0.1891 | 0.2019 | 0.2275 | 0.2404 |
| 0.6 | 0.2533 | 0.2793 | 0.3055 | 0.3319 | 0.3585 | 0.3853 | 0.4125 | 0.4399 | 0.4538 | 0.4677 | 0.4959 | 0.5101 |
| 0.7 | 0.5244 | 0.5534 | 0.5828 | 0.6128 | 0.6433 | 0.6745 | 0.7063 | 0.7388 | 0.7554 | 0.7722 | 0.8064 | 0.8239 |
| 0.8 | 0.8416 | 0.8779 | 0.9154 | 0.9542 | 0.9945 | 1.0364 | 1.0803 | 1.1264 | 1.1503 | 1.1750 | 1.2265 | 1.2536 |
| 0.9 | 1.2816 | 1.3408 | 1.4051 | 1.4758 | 1.5548 | 1.6449 | 1.7507 | 1.8808 | 1.9600 | 2.0537 | 2.3263 | 2.5758 |

# Summary of Discrete Distributions

| Notation and Parameters | Probability Function $f(x)$ | Mean $E(X)$ | Variance $Var(X)$ |
|---|---|---|---|
| Discrete Uniform$(a, b)$ $b \geq a$ $a, b$ integers | $\dfrac{1}{b - a + 1}$ $x = a, a + 1, \dots, b$ | $\dfrac{a+b}{2}$ | $\dfrac{(b - a + 1)^2 - 1}{12}$ |
| Hypergeometric$(N, r, n)$ $N = 1, 2, \dots$ $n = 0, 1, \dots, N$ $r = 0, 1, \dots, N$ | $\dfrac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$ $x = max(0, n - N + r),$ $\dots, min(r, n)$ | $\dfrac{nr}{N}$ | $\dfrac{nr}{N}\left(1 - \dfrac{r}{N}\right)\dfrac{N - n}{N - 1}$ |
| Binomial$(n, p)$ $0 \leq p \leq 1, q = 1 - p$ $n = 1, 2, \dots$ | $\binom{n}{x}p^x q^{n-x}$ $x = 0, 1, \dots, n$ | $np$ | $npq$ |
| Bernoulli$(p)$ $0 \leq p \leq 1, q = 1 - p$ | $p^x q^{1-x}$ $x = 0, 1$ | $p$ | $pq$ |
| Negative Binomial$(k, p)$ $0 < p \leq 1, q = 1 - p$ $k = 1, 2, \dots$ | $\binom{x + k - 1}{x}p^k q^x$ $= \binom{-k}{x}p^k(-q)^x$ $x = 0, 1, \dots$ | $\dfrac{kq}{p}$ | $\dfrac{kq}{p^2}$ |
| Geometric$(p)$ $0 < p \leq 1, q = 1 - p$ | $pq^x$ $x = 0, 1, \dots$ | $\dfrac{q}{p}$ | $\dfrac{q}{p^2}$ |
| Poisson$(\lambda)$ $\lambda \geq 0$ | $\dfrac{e^{-\lambda}\lambda^x}{x!}$ $x = 0, 1, \dots$ | $\lambda$ | $\lambda$ |

# Summary of Continuous Distributions

| Notation and Parameters | Probability Density Function $f(x)$ | Mean $E(X)$ | Variance $Var(X)$ |
|---|---|---|---|
| Uniform$(a, b)$ $b > a$ | $\dfrac{1}{b - a}$ $a \leq x \leq b$ | $\dfrac{a + b}{2}$ | $\dfrac{(b - a)^2}{12}$ |
| Exponential$(\theta)$ $\theta > 0$ | $\dfrac{1}{\theta}e^{-x/\theta}$ $x \geq 0$ | $\theta$ | $\theta^2$ |
| $N(\mu, \sigma^2) = G(\mu, \sigma)$ $\mu \in \mathbb{R}, \sigma^2 > 0$ | $\dfrac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ $x \in \mathbb{R}$ | $\mu$ | $\sigma^2$ |