Instructions

1. **All electronic devices are to be turned off and put away. If you haven't already done this, please do so now.**
2. **This is a 90-minute closed-book test.**
3. **Answer the questions in the spaces provided.**
4. **Final answers to written questions should be rounded to THREE DECIMAL PLACES or left in fraction form.**
5. **Only question pages will be marked.**
6. **You may tear off the last page and use it for rough work. All pages will be collected at the end of the test.**
7. **Only a non-programmable, non-graphical calculator with a pink-tie or blue-goggle sticker will be allowed. No other aids are allowed.**
8. **The test will be marked out of 43. With the bonus opportunities, it will be possible to score more than 100% on this test.**
9. **DO NOT WRITE ON THIS COVER PAGE.**

Marking Scheme:

Questions	**Out of**
1 – 13	13
14	4
15	7
16	9
17	6
18	4
BONUS 1	2
BONUS 2	2
Total	43

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Part 1: Multiple Choice – Each question is worth 1 mark

Please CLEARLY CIRCLE your answer selection on this page. Do NOT simply write your choice next to the question. If you do, the question will not be graded. Each question has one correct answer. Choose the best answer.

1. I want to avoid catching a cold, so I decide not to go the concert held at the local arena, as I believe that there is an 80% chance that I will be infected there. What is the type of probability that I am using to judge whether to go to the concert?

A) Classical probability
B) Relative frequency probability
C) Subjective probability

- **I am using my personal opinion of the probability that I will be infected so this is an example of subjective probability.**

2. Consider a fair die has faces numbered 1, 2, 3, 4, 5, and 6. The die is rolled five times. After each roll, the number appearing on the upturned face is recorded. Which of the following is / are a possible sample space for the experiment?

I. S = {more than three 5's, fewer than four 5's}
II. S = {more 3's than 6's, fewer 3's than 6's}
III. S = {an even number of 5's, at least one 5}
IV. S = {the sum of the rolls is ≤ 29, five 6's}

A) I. only B) I. and II. C) III. and IV. **D) I. and IV.** E) all of them

- **In order to have a valid sample space, we need a space that consists of all possibilities (complete) and a space that has no overlapping outcomes. That is all outcomes are mutually exclusive.**
- **I. Complete and mutually exclusive, II. Not complete, III. Not mutually exclusive,**
- **IV. Complete and mutually exclusive.**

3. Three players each have a fair coin which they all flip at the same time. If all three coins come up heads, they all toss the coins again. If all three coins come up tails, they will also all toss the coins again. If two heads and one tail are tossed, the person who tossed the tail wins, and the game ends. If two tails and one head are tossed, the person who tossed the head wins, and the game ends. The sample space for this experiment is

A) finite **B) countably infinite** C) continuous D) undefined

- **Let A be the event that all three players toss either all heads or all tails.**
- **Let F be the event that the first player tosses the odd (different) result.**
- **Let S be the event that the second player tosses the odd (different) result.**
- **Let T be the event that the third player tosses the odd (different) result.**
- **We know that the player that tosses the odd (different) result wins the game, and the game ends.**
- **The sample space represents the set of all possible outcomes.**
- **In this case, it would include:**
  - **F, AF, AAF, AAAF, AAAAF, ...**
  - **S, AS, AAS, AAAS, AAAAS, ...**
  - **T, AT, AAT, AAAT, AAAAT, ...**
- **There are an infinite number of points in the sample space, but they can be put in one-to-one correspondence with the positive integers (i.e. order: F, S, T, AF, AS, AT, AAF, AAS, AAT, ...).**
- **As such, this sample space has a countably infinite number of points.**

4. Consider the set S = {1, 2, 3, 4, 5, 6}. How many subsets of S have more than one element?

A) 25 B) 5 **C) 57** D) 63

- **There are a total of $2^6 = 64$ subsets of S.**
- **6 subsets will have exactly 1 element, and 1 subset will have no elements (empty set).**
- **So, there are a total of $2^6 - 6 - 1 = 57$ subsets with more than 1 element.**

5. Consider the set S = {a, b, c, d, e, f, g, h}. How many subsets with exactly 5 elements do not contain BOTH c and d?

A) 36 B) 250 C) 41 D) 80

- **We can determine the number of 5 element subsets that DO NOT contain BOTH c and d by determining the total number of 5 element subsets and then we subtract the number of 5 element subsets that DO contain BOTH c and d.**
- **So, the number of 5 element subsets that DO NOT contain BOTH c and d is equal to $\binom{8}{5} - \binom{2}{2}\binom{6}{3} = 36$**

6. Alicia, Billy, Cally, and Declan are the only four people running for class president. Surveys have indicated that Declan has a 1/8 probability of winning. The probability that Alicia wins is twice that of Billy winning, but only one-half that of Cally winning. What is the probability that either Alicia or Cally wins the election?

A) 1/8 **B) 3/4** C) 1/2 D) 1/4

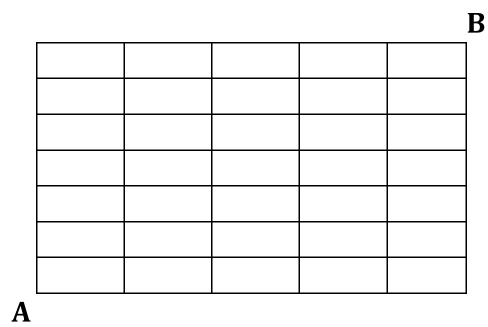
- **Let's define events: A = Alicia wins, B = Billy wins, C = Cally wins, D = Declan wins.**
- **Let's write out the probabilities of winning for each.**
- **P(D) = 1/8, P(A) = 2*P(B), P(C) = 2*P(A).**
- **We will end up expressing the other probabilities in terms of P(B).**
- **So, P(A) = 2*P(B) and P(C) = 2*P(A) = 4*P(B).**
- **We also know that P(A) + P(B) + P(C) + P(D) = 1.**
- **So, 2*P(B) + P(B) + 4*P(B) + 1/8 = 1.**
- **7*P(B) = 7/8. Thus, P(B) = 1/8.**
- **We can then fill in the other probabilities:**
  P(A) = 1/4, P(B) = 1/8, P(C) = 1/2, P(D) = 1/8
- **We want P(A or C wins) = P(A) + P(C) = 1/4 + 1/2 = 3/4**

7. In how many ways can 8 people: {A, B, C, D, E, F, G, H} be arranged in a row if A and B must sit next to each other, and C, D, and E must sit next to each other?

A) 40,320 B) 720 C) 120 **D) 1,440**

- **We know that there are a total of 8! ways to arrange the 8 people without restrictions, but we have restrictions.**
- **A and B must sit together. Also, C, D, and E must sit together.**
- **We will treat A, B like one "block". We will treat C, D, E like another "block".**
- **This leaves F, G, and H to be arranged without restrictions.**
- **There are a total of 5 blocks to arrange the 5 "blocks". These can be arranged 5! ways.**
- **Inside the AB block, we can arrange A and B 2! ways.**
- **Inside the CDE bloc, we can arrange C, D, and E 3! ways.**
- **So, there are a total of 2!3!5! = 1,440 ways to arrange the 8 people with the given restrictions.**

8. Simon leaves corner A in the grid diagram below to join Garfunkel who is waiting at corner B. Each rectangle represents a city block, and Simon only travels east (to the right) or north (up) from each intersection. How many possible routes are available to Simon?



A) $\binom{7}{5}$ B) $7^{(5)}$ C) $12^{(5)}$ **D)** $\boldsymbol{\binom{12}{5}}$

- **In order to get from A to B, moving only to the right or up, Simon must travel through a total of 12 intersections with any combination of 5 east (right) moves and 7 north (up) moves. Of course, $\binom{12}{7}$ would also be correct, but the value is the same due to symmetry.**
- **Try this with a smaller grid, say 2 x 2, and you can trace all $\binom{4}{2}$ = 6 possible ways.**

9. A small group of 6 students: Alex, Bobby, Carl, Diana, Evan, Francis, is divided into 3 groups of size 2. Each group is assigned a topic to research. In how many ways can this be done if each group is assigned the <u>same topic</u>?

A) 90 **B)** **15** C) 30 D) 6

- **Since the topics are the same, the problem reduces to dividing 6 people into 3 groups of size 2. However, we note that each of these groups is to perform the <u>same task</u>. That is, they are to research the same topic.**
- **We see that the first group of size 2 can be selected in $\binom{6}{2}$ ways.**
- **Another group of size 2 can be selected in $\binom{4}{2}$ ways.**
- **The final group of size 2 can be selected in $\binom{2}{2}$ ways.**
- **The order that the groups are selected doesn't matter here.**
- **Our expression: $\binom{6}{2}\binom{4}{2}\binom{2}{2}$ counts the 3! orders in which 3 fixed groups can be picked as being different divisions of 6 people into 3 groups of size 2, when in fact, all such orders give the same partnerships. We need to correct for this overcounting, so we will need to divide the above expression by 3!**
- **So, the number of ways = $\binom{6}{2}\binom{4}{2}\binom{2}{2}\left(\frac{1}{3!}\right)$ = 15**
- **Note: If this seems a little confusing, try the simpler problem of dividing 3 people into 3 groups of size 1 (all performing the same task). Of course, in this case, the answer would be: $\binom{3}{1}\binom{2}{1}\binom{1}{1}\left(\frac{1}{3!}\right)$ = 1.**

10. Given: P(A) = 0.3, P(B) = 0.2, and P(A|B) + P(B|A) = 0.75.

What is P($\bar{A} \cap$ B)?

A) 0.14 B) 0.21 C) 0.29 **D) 0.11**

- **Given: P(A) = 0.3, P(B) = 0.2, P(A|B) + P(B|A) = 0.75.**
- **So,** $\frac{P(A \cap B)}{0.2} + \frac{P(A \cap B)}{0.3}$ **= 0.75**
- **Now,** $\frac{0.5 * P(A \cap B)}{0.06}$ **= 0.75 => P(A $\cap$ B) = 0.09.**
- **We want P($\bar{A} \cap$ B).**
- **We know that P(B) = P($\bar{A} \cap$ B) + P(A $\cap$ B). So, 0.2 = P($\bar{A} \cap$ B) + 0.09.**
- **Thus, P($\bar{A} \cap$ B) = 0.2 – 0.09 = 0.11.**

11. You glance at a poker hand consisting of 5 cards randomly dealt from a standard deck of 52 cards and see that it has exactly 4 red cards and 1 black card. What is the probability that this hand has exactly 3 diamonds?

Note: Diamonds and hearts are red cards, and clubs and spades are black cards.

A) $\frac{\binom{13}{3}\binom{13}{1}}{\binom{26}{4}}$ B) $\frac{\binom{13}{3}\binom{13}{1}\binom{26}{1}}{\binom{52}{5}}$ C) $\frac{\binom{26}{4}\binom{26}{1}}{\binom{52}{5}}$ D) $\frac{\binom{13}{4}\binom{13}{1}\binom{26}{1}}{\binom{26}{4}}$

- **Let's start by defining events.**
- **Let A = exactly 3 diamonds in a 5-card hand.**
- **Let B = exactly 4 red cards and 1 black card in a 5-card hand.**
- **We want P(A|B).**
- **By definition, P(A|B) =** $\frac{\text{P(A}\cap\text{B)}}{\text{P(B)}} = \frac{\text{P(3 diamonds,1 heart,1 black card)}}{\text{P(4 red cards,1 black card)}}$.
- **So, P(A|B) =** $\frac{\binom{13}{3}\binom{13}{1}\binom{26}{1}/\binom{52}{5}}{\binom{26}{4}\binom{26}{1}/\binom{52}{5}} = \frac{\binom{13}{3}\binom{13}{1}}{\binom{26}{4}}$

12. Three machines – A, B, and C – are running independently of each other. If their probabilities of failure on a given day are P(A) = 0.01, P(B) = 0.04, and P(C) = 0.08, what is the probability that at least one of them will fail today?

A) 0.8744 **B) 0.1256** C) 0.13 D) 0.001

- **Let's start by defining events.**
- **Let A = machine A fails.**
- **Let B = machine B fails.**
- **Let C = machine C fails.**
- **Given: P(A) = 0.01, P(B) = 0.04, P(C) = 0.08.**
- **We want P(at least one of the machines fails) = P(A $\cup$ B $\cup$ C).**
- **However, P(at least one fails) = 1 – P(no machines fail)**
- **P(at least one machine fails) = 1 -** $P(\overline{A \cup B \cup C}) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$
- **= 1 -** $[P(\bar{A}) * P(\bar{B}) * P(\bar{C})]$ **(due to independence).**
- **So, P(at least one machine fails) = 1 – [(1-P(A))*(1-P(B))*(1-P(C))]**
- **P(at least one machine fails) = 1 – [(1-0.01)(1-0.02)(1-0.03)] = 1 – 0.874368 = 0.1256**

13. A local delivery service sends letters to Toronto either by plane, train, or by truck. The probability that a letter is not delivered to its destination in Toronto within 24 hours of receipt is 0.05 if it is sent by plane, 0.1 if it is sent by train, and 0.2 if it is sent by truck. The service sends 70% of its letters by truck, 20% by train, and 10% by plane. If the letter fails to reach its destination within 24 hours, what is the probability that it was sent by plane?

A) 1/20 B) 4/33 **C) 1/33** D) 33/200

- **Let's start by defining events.**
- **Let P = letter is sent by plane.**
- **Let T = letter is sent by truck.**
- **Let R = letter is sent by train.**
- **Given: P(P) = 0.1, P(T) = 0.7, P(R) = 0.2.**
- **P(L|P) = 0.05, P(L|T) = 0.2, P(L|R) = 0.1**
- **We want P(P|L).**
- **By definition, P(P|L) =** $\frac{P(P \cap L)}{P(L)}$
- **P(P|L)** $= \frac{P(L|P)*P(P)}{P(L|P)*P(P)+P(L|T)*P(T)+P(L|R)*P(R)} = \frac{(0.05)(0.1)}{(0.05)(0.1)+(0.2)(0.7)+(0.1)(0.2)} = \frac{5}{165} = \frac{1}{33}$

Part 2: Written Answer / Short Answer Questions

Final answers can be left in fraction form OR rounded to THREE DECIMAL PLACES.

For full credit, you need to show all necessary work / justify your answer. Unjustified answers will not receive full credit.

14. ***(4 marks)*** Harold is getting ready for a date tonight. However, living in Waterloo, he doesn't know if the weather will be warm or cold. If Harold dresses for warm weather, he can choose from 6 outfits and 4 hairstyles. If he dresses for cold weather, he has 2 outfits and 3 hairstyles.

a. If he selects a 1 outfit and 1 hair style combination, determine the total number of combinations Harold has to choose from. **Hint**: Consider 2 cases. ***(2 marks)***

Harold can dress for warm or cold weather (OR rule). If he dresses for warm weather, he has 6 outfits and 4 hairstyles (AND rule). If he dresses for cold weather, he has 2 outfits and 3 hairstyles (AND rule). Thus, we see there are a total of $(6)(4) + (2)(3) = 30$

So, Harold has 30 combinations to choose from.

b. If Harold randomly selects 5 outfit-hair combinations, what is the probability there are 4 warm weather combinations and 1 cold weather combination? ***(2 marks)***

Harold has 24 warm weather combinations (6*4) and 6 cold weather combinations (3*2). Thus, he can select 5 random combinations in $\binom{30}{5}$ ways, 4 warm weather combinations in $\binom{24}{4}$ ways, and 1 cold weather combination in $\binom{6}{1}$ ways.

Therefore, the probability Harold selects 4 warm and 1 cold weather combinations (we will call this event 4W1C) is

$$\mathrm{P(4W1C)} = \frac{\binom{24}{4}\binom{6}{1}}{\binom{30}{5}} = \frac{506}{1131} = 0.447 \text{ (to 3 d.p.)}$$

15. ***(7 marks)*** Matilda makes cookies using 3 different cookie recipes. After baking them, she tastes a cookie from each recipe. She then decides if the recipe is yummy (Y) or disgusting (D).

a. Assuming Matilda is equally likely to call a recipe yummy or disgusting, list a sample space, including all 8 equally likely outcomes for Matilda's opinion of the cookie recipes. ***(2 marks)***

Listing the options, we see the 8 equally likely outcomes in the sample space are

$$\mathbf{S = \{YYY, YYD, YDY, YDD, DYY, DYD, DDY, DDD\}}$$

Note, we can verify our sample space is the right size using the AND rule. The size of the sample space, |S|, should be 8 since there are two taste categories, and three different cookie recipes will be sampled. This gives a total of $\mathbf{2 * 2 * 2 = 8}$ (equally likely) outcomes.

b. Determine the odds in favour of Matilda calling exactly 2 of the recipes yummy. ***(2 marks)***

If event E is the event representing exactly two recipes are yummy,

$$\mathbf{E = \{YYD, YDY, DYY\}}$$

All outcomes are equally likely, so P(E) = 3/8.

So, the odds in favour of exactly two yummy recipes is given by

$$\textbf{Odds in favour of 2Y's} = \frac{P(E)}{P(E^C)} = \frac{3/8}{5/8} = \frac{3}{5} = 0.6$$

c. In this "experiment", the sample space is not necessarily unique. Give another possible sample space, where every event has probability greater than 0, based on the number of recipes Matilda calls yummy. ***(1 mark)***

Matilda can call 0, 1, 2, or 3 recipes yummy.

Therefore, the sample space is given by $S = \{0, 1, 2, 3\}$

Note: There are other possibilities here.

For example, you could also state: S = {(0 recipes are yummy), (at least 1 recipe is yummy)}

You could also state: S = {(an even number of recipes are yummy), (an odd number of recipes are yummy)}.

d. Are the outcomes in the sample space in part c. equally likely? Explain / justify your answer. ***(2 marks)***

We note that the first sample space in part c. is valid, but the outcomes are NOT equally likely.

With the first sample space in part c.: P(0) = 1/8, P(1) = 3/8, P(2) = 3/8, P(3) = 1/8.

For the second sample space: P(no recipes are yummy) = 1/8, P(at least one recipe is yummy) = 7/8. The sample space is valid. The outcomes are NOT equally likely.

For the third sample space: P(an even number of recipes are yummy) = P(no recipes are yummy) + P(two recipes are yummy) = 1/2.

Likewise, P(an odd number are yummy) = 1/2. The sample space is valid, and the outcomes ARE equally likely.

6. ***(9 marks)*** The Dean of the faculty owns a total of 7 ties, owning two types of ties (neck ties and bow ties). He has 3 neck ties (1 red, 1 blue, 1 pink) and 4 bow ties (1 red, 1 pink, 1 green, 1 black). All 7 ties are stored in a large box. On Sunday evening, the Dean randomly selects a tie for each of the upcoming 5 days at work in order, that is, for Monday, Tuesday, ..., Friday. He randomly selects one tie after another from the box, <u>without replacement</u>.

a. How many elements are in the sample space for this experiment? ***(1 mark)***

There are 7 ties (objects) and 5 are being selected without replacement. Thus, the number of elements / sample points in the sample space is given by $7^{(5)} = 2,520$

b. What is the probability that the Dean wears a pink neck tie on Monday? ***(2 marks)***

If the Dean picks a pink neck tie to be worn on Monday, there are 6 objects left to choose from for the remaining 4 days.

Since the outcomes in the sample space are equally likely, we find:

P(Dean wears a pink neck tie on Monday) = $\frac{6^{(4)}}{7^{(5)}} = \frac{360}{2520} = \frac{1}{7} \approx 0.143$ (to 3 d.p.)

c. What is the probability that the Dean does not wear his black bow tie? ***(2 marks)***

The Dean does not wear the black tie if he selects one of the 6 "non-black tie" objects.
So, P(Dean does not wear black tie) $= \frac{6^{(5)}}{7^{(5)}} = \frac{6!}{7!/2!} = \frac{2}{7} \approx 0.286$ (to 3 d.p.)

d. What is the probability that the Dean wears a pink tie on Monday and also on Friday? ***(2 marks)***

We can have "Pink neck tie Monday + Pink bow tie Friday" or "Pink bow tie Monday + Pink neck tie Friday". For each of these two possibilities, we can fill the remaining three days in $5^{(3)}$ ways.

Thus, P(Dean wears a pink tie on Monday and also on Friday) = $\frac{2\times 5^{(3)}}{7^{(5)}} = \frac{1}{21} = 0.048$ (to 3 d.p.)

e. What is the probability that the Dean wears a different colour tie each day? ***(2 marks)***

In order for this to happen, the Dean needs to pick the blue neck tie, the green bow tie, the black bow tie and (a red neck tie and a pink bow tie OR a pink neck tie and a red bow tie) in any order. Let's identify the ties with letters, and 1 and 2 subscripts for the ties that have more than 1 type (i.e. R_1, R_2 and P_1, P_2).

P(Dean wears 5 different colors of ties) = (P(R_1,BL,P_1,G,BK) + P(R_1,BL,P_2,G,BK)

+ P(R_2,BL,P_1,G,BK) + P(R_2,BL,P_2,G,BK)) / (# arrangements with no restrictions)

These 4 cases to consider and 5! ways to arrange.

So, the required probability = (4×5!)/(7*6*5*4*3) = 4/21 (= 0.190 to 3 d.p.)

17. ***(6 marks)*** Consider the word "FLUFFY" whose letters are being randomly rearranged.

a. What is the probability that the letters are randomly arranged to form the word "FLUFFY"? ***(2 marks)***

There are 6 letters, 3 of which are the same, so that $|S| = \frac{6!}{3!1!1!1!} = \frac{6!}{3!} = 120$.

So $P(\text{FLUFFY occurs}) = 1/|S| = 1/120 \approx 0.008$ (to 3 d.p.)

b. Given that "FF" occurs together, what is the probability that the word "FLUFFY" is formed? ***(2 marks)***

This one's a little tricky! Let's use counting first principles to reason through this question.

We know there are a total of 6!/(3!1!1!1!) = 120 distinguishable arrangements (as there are 3 F's).

Now the question is, of the 120 equally likely arrangements, how many have FF in them?

Important Note: If we have FFF, then we have FF.

Let's look at FFF together.

FFF...: We can place FFF in the first "slot", then there are 3 choices for the fourth letter, 2 choices for the fifth letter, and 1 choice for the sixth letter.

Then, we can multiply this by 4, as FFF can "slide down the line", so to speak.

So, there are 4*3! = 4! = 24 ways to achieve that.

If we don't have FFF, then the FF could appear in any of 5 places.

Each has a different number of possibilities:

FF....: 3 choices for the third letter, then 3! for the rest: 18.

.FF...: 3 choices for the first, 2 choices for the fourth, then 2!: 12.

..FF..: 3 choices for the second, 2 choices for the fifth, then 2!: 12.

...FF.: 12 again (same as the second and third cases)

....FF: 18 again (same as first case).

So there are 24 + 18 + 12 + 12 + 12 + 18 = 96 arrangements containing FF, all

equally likely.

So, the required probability is 1/96.

c. Given that "UFFY" occurs together, in that order, what is the probability that the word "FLUFFY" is formed? ***(2 marks)***

Similarly, we regard "UFFY" as a block, so under the condition that "UFFY" occurs together, we are rearranging the 3 distinct objects "F", "L", "UFFY".

Thus $P(\text{FLUFFY occurs}|\text{ UFFY together}) = \frac{1}{3!} = \frac{1}{6} \approx 0.167$ (to 3 d.p.)

18. ***(4 marks)*** A family-owned breakfast restaurant in Timbuktu is using 3 different toasters for their bagels. We will refer to them as Toaster 1, 2 and 3. The three toasters were purchased at different times, so they have different ages. As such, they have different capacities. In particular, the staff uses Toaster 1 only twenty-five percent of the time, while they use Toaster 2 thirty-five percent of the time and Toaster 3 forty percent of the time. The toasting quality of the three toasters also differs: Toaster 1 burns 5% of the bagels, Toaster 2 burns 10% of the bagels and Toaster 3 burns 1% of the bagels.

If a randomly selected bagel is burnt, what is the probability that it was toasted using Toaster 2? Please round your final result to 3 decimal places.

Let A_i denote the event "Toaster i was used to toast the bagel" for $i = 1, 2, 3$ and let B denote the event "Bagel is burnt".

We are given

$$P(B|A_1) = 0.05, P(B|A_2) = 0.1, P(B|A_3) = 0.01$$

and

$$P(A_1) = 0.25, P(A_2) = 0.35, P(A_3) = 0.4$$

We are looking for $P(A_2|B)$. We can use Bayes' formula to determine this probability.

We will make use of the product rule and law of total probability to determine the required probability.

By definition, $P(A_2|B) = \frac{P(A_2 \cap B)}{P(B)} = \frac{P(B|A_2) * P(A_2)}{P(B)}$

By the law of total probability:

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3) = 0.0515$$

So that we get $P(A_2|B) = \frac{0.035}{0.0515} = \frac{70}{103} \approx 0.680$ (to 3 d.p.)

Final answers can be left in fraction form OR rounded to THREE DECIMAL PLACES.

For full credit, you need to show all necessary work / justify your answer. Unjustified answers will not receive full credit.

BONUS QUESTION 1:

Alex recently hosted a party and asked everyone to wear a hat. After taking pictures, and having some laughs, he collected everyone's hat shortly after they arrived to the party. At the end of the night, there was a power outage. As a result, everyone decided to leave all at once, so Alex ended up blindly handing hats back to people as they were quickly leaving in the darkness. Because of this, some people may or may not have been handed their own hat. If there were 50 people at the party, what is the chance that EXACTLY 49 out of 50 people got the correct (their own) hat back?
(2 marks)

This one may seem trickier than it actually is. If exactly 49 people get the correct (their own) hat, then the 50th person will automatically receive the correct hat.

So, P(EXACTLY 49 people get the correct hat) = 0.

BONUS QUESTION 2:

Two roommates, Tom and Jerry are going to roll a fair die to see who has to clean the bathrooms in their house every week until the end of the year! They will take turns rolling the die until an odd number is rolled. The person who rolls the first odd number wins and won't have to clean the bathrooms for the rest of the year.

Jerry has decided to let Tom go first. What is the probability Tom will have to clean the bathrooms for the rest of the year? ***(2 marks)***

P(Tom loses) = P(Jerry rolls an odd number on the 2nd roll) + P(Jerry rolls an odd number on the 4th roll) + P(Jerry rolls an odd number on the 6th roll) + ……..

P(Tom loses) = $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)+\cdots$

This probability can be calculated by evaluating the above infinite sum.

P(Tom loses) = $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left[1+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+\cdots\right]=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left[\frac{1}{1-\left(\frac{1}{2}\right)^2}\right]=\frac{1}{3}$

So, the probability that Tom will have to clean the bathrooms for the rest of the year is equal to the probability that Tom loses. This is 1/3 (or 0.333 to 3 d.p.).

Note: P(Tom loses) = 1/3 => P(Jerry loses) = 2/3.

Jerry should not have let Tom go first! There is an advantage to going first!