Stat 230: Probability

Lecture 2

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Review

Example (Review Question)

Consider the random experiment of rolling two, fair dice. We are interested in the event that the sum of pips is greater than 2. Define:

(1) Sample space

(2) Probability distribution

Also give:

(3) Event

(4) Probability of event

Review

Last time we discussed

- Probability Model:
  - Sample spaces
  - Events
  - Axioms of probability

Schedule for today:

- Review Questions
- Equally likely sample spaces
- Rules of Counting
- (Next Class) Counting

Reading: Chapter 2 – 3

Review

- Discrete vs Non-discrete
- Probability Axioms: Finite Additivity
- Pacing
  - Key definitions (Discussion)
  - Review time
- Examples
- Union ($\cup$) and Intersection ($\cap$)

Probability Definitions

Definition

The **odds** *of an event A is defined*

$$\frac{P(A)}{1 - P(A)}$$

and the **odds against** *event A is defined*

$$\frac{1 - P(A)}{P(A)}$$

Probability Definitions

Example

Consider again rolling a two distinguishable, fair dice

(1) Give the odds of rolling greater than 8

(1) Give the odds against rolling greater than 8

Probability Definitions

Definition

We say a sample space S with a finite number of outcomes is **equally likely** *if the probability of every individual outcome in S is the same.*

Let $|A|$ denote the number of outcomes in an event A. In case of an equally likely sample space,

$$1 = P(S) = \sum_{i=1}^{|S|} P(a_i) = P(a_i)|S|.$$

So

$$P(a_i) = \frac{1}{|S|}.$$

Therefore;

$$P(A) = \sum_{i:\ a_i \in A} P(a_i) = \frac{|A|}{|S|}.$$

Probability Definitions

We can often modify sample spaces to ensure the events are equally likely.

In the context of an equally likely sample space, i.e. when

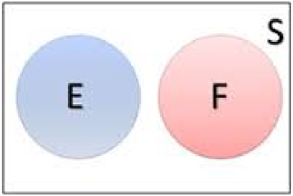
$$P(A) = \frac{|A|}{|S|},$$

computing probabilities is equivalent with "simply" counting outcomes.

Counting Rules

The Addition Rule (OR):
If E and F are disjoint events, then

$$|E \cup F| = |E| + |F|.$$



Example

Suppose two six sided die are rolled, how many outcomes would result in the sum of the die rolls exceeding 8?

The Multiplication Rule (AND):
An ordered k-tuple is an ordered set of k values: $(a_1, a_2, ..., a_k)$. If the outcomes in A can be written as an ordered $k - tuple$ where there are n_1 choices for a_1, n_2 choices for a_2,..., and in general n_i choices for a_i, then

$$|A| = n_1 n_2 \cdots n_k = \prod_{i=1}^{k} n_i.$$

$$\begin{bmatrix} (a_{1,1}, a_{2,1}) & (a_{1,1}, a_{2,2}) & \cdots & (a_{1,1}, a_{2,n_2}) \\ (a_{1,2}, a_{2,1}) & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ (a_{1,n_1}, a_{2,1}) & \cdots & \cdots & (a_{1,n_1}, a_{2,n_2}) \end{bmatrix} \tag{1}$$

Counting Rules

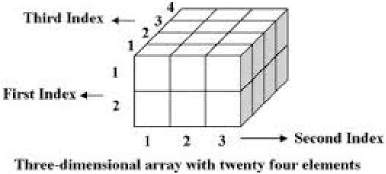


Figure: A three dimensional array.

Figure: A three dimensional array of three dimensional arrays.

Example

A rebel base has been discovered on Hoth and teams of snow troopers dispatched. If each member of a four trooper team can select one of three specializations, how many ways could the team be configured?

Counting Rules

Example

Suppose that three of the numbers $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ are selected at random **without** replacement, and then put together in the order they are drawn to form a three digit number. What is the probability that:

(1) The number is larger than 500?

(2) The number is even?

(3) The number is larger than 700, and is even?