Stat 230: Probability

Lecture 5

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Review

Example (Review Question)

Let A, B, and C be events. Draw the following using a Venn Diagram

$$\overline{(\bar{A} \cup \bar{B} \cup \bar{C})} \cup \left(\overline{(A \cup B)} \cap C\right) \cup ((A \cap B \cap C) \cap \bar{C})$$

Review

Last time we talked about

- Multinomial Coefficient
- Inclusion-Exclusion

Schedule for today:

- Inclusion-Exclusion
- Independence
- Conditional Probability

Reading: End Chapter 3, Start Chapter 4

Review

- Quiz
- Office Hours

Inclusion-Exclusion

Inclusion Exclusion Rules: For arbitrary events A, B, and C

(1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(2) $P(A \cup B \cup C) =$
$P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

(3)

$$P(\cup_{i=1}^{n} A_i) = \sum_i P(A_i) - \sum_{i<j} P(A_iA_j) + \sum_{i<j<k} P(A_iA_jA_k) - \sum_{i<j<k<l} P(A_iA_jA_kA_l) + \cdots$$

Inclusion-Exclusion

Example

Suppose that two fair six sided die are rolled. What is the probability that at least one of the dice shows a 6?

Example

80% of Canadians support Tim Hortons and 63% support Canadian Tire. 51% of Canadians support both Tim Hortons and Canadian Tire. Suppose a Canadian is selected at random, what is the probability:

(1) The individual supports Canadian Tire or Tim Hortons

(2) The individual supports Canadian Tire but not Tim Hortons

Inclusion-Exclusion

Example

Consider rearranging the letters in "FOOD" at random to form a word. Let A denote the event that the two $O's$ appear together, and let B denote the event that the word starts with the letter F. Determine:

(1) P(A)

(2) P(B)

(3) The probability that the resulting word does not start with "F" and that the "O's" do not appear together.

Independence

Definition

Two events A and B are said to be **independent** *if*

$$P(A \cap B) = P(A)P(B)$$

A sequence of events $A_1, A_2, ...$ are said to be independent if

$$P(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}) = P(A_{i_1})P(A_{i_2}) \cdots P(A_{i_k}),$$

for all possible subsets of size k, $i_1, i_2, \ldots, i_k$. Events that are not independent are called **dependent**.

Independence

Example

Consider rolling two fair six sided dice, and let
$A = \{\text{the sum is } 10\}$, $B = \{\text{the first die is a } 6\}$ $C = \{\text{the sum is } 7\}$.
Determine:

(1) Are A and B independent?

(2) Are B and C independent?

(3) Are A and C independent?

Independence

A common misconception is that if A and B are mutually exclusive, then A and B are independent.

Proposition

If A and B are independent and mutually exclusive (disjoint), then either $P(A) = 0$ or $P(B) = 0$.

Independence

Proposition

If A and B are independent, then $\bar{A}$ and $\bar{B}$ are independent, A and $\bar{B}$ are independent, and $\bar{A}$ and B are independent.