STAT 230: Probability (Sec 02)
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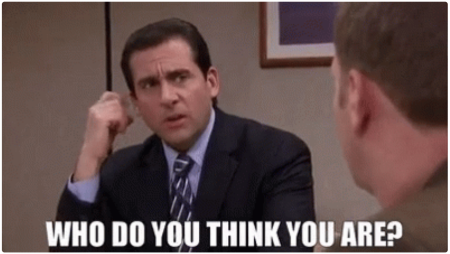
Slide credit for Greg Rice and Nam-Hwui Kim!

Today's Agenda

Today (Lec 1, 05/02):

- "Why do I have to take Stat 230?"
- Go over syllabus
- Chapter 1: What is probability?
- Chapter 2: Probability basics



. . . Erik Hintz!

- Born and raised in Memmingen, Germany
- BSc and MSc in Mathematics and Management, University of Ulm (Germany)
- MMath and fresh PhD in Statistics, University of Waterloo (Go Warriors!)
- Spare time: plants, watching the office, travelling, trying to become funny.

Important Rules

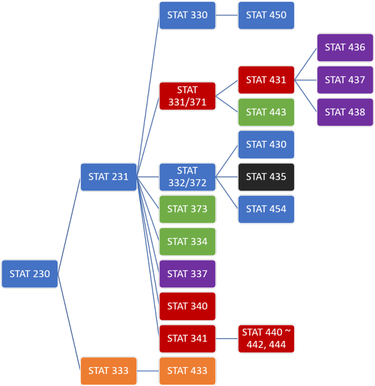
The Questions Rule: A question is stupid only if you have not spent time thinking about it before asking it. This is a class to learn. If you don't know it, first think about it carefully, and if it is still unclear, DO ask it.

The Language Rule: If there is anything you don't understand (due to my accent, choice of words, etc.), do ask for clarification.

The RTFS Rule: Behind every uncomfortable clause in the course outline lies an even more uncomfortable teaching experience!

Why STAT 230?

- There is (almost) no discipline that can get around using probability/statistics!
- One of the most fundamental and important courses in statistics.
- Your next courses (STAT 231, STAT 3xx, STAT 4xx, ACTSC 3xx, ACTSC 4xx . . .) rely heavily on material from this course.



Specific examples of application

- Insurance: Policy pricing
- Logistics: When/Where to deliver goods
- Medical Studies: Clinical trials, Drug testing
- Quality Control: Lowering scrap rates in manufacturing
- Machine Learning: Voice recognition, Facial recognition, "reading" images,...
- Space travels: Predicting trajectory, analyzing soil composition

Course material and communication

- All material is posted on Learn, in particular, the course notes and slides.
- It is imperative that you regularly check Learn announcements!
- Annotated slides (if any) will not be provided.
- The course notes include an abundance of exercises. Do them!
- We will be using Piazza. Please sign up for Piazza and ask and answer questions there.

Syllabus

- Read and read again the course outline on Learn.
- There will be 6 tutorial tests, only the three best ones count.
- There will be 2 term tests.
- There will be a final exam.

For dates, weights, missed assessment policies and all other rules see the course outline.

General advice for doing well in this course:

- Solve the problems in the course note.
- Take all of the assessments, in particular, all quizzes.
- No question is "stupid", so ask away - during the lectures, tutorials, and on Piazza.
- Work every week on the material.

Low-hanging fruits!

- ATTEND LECTURES!
- Skim through slides before the lecture.
- ATTEND LECTURES!
- Practice, practice, practice,. . .

1. INTRODUCTION TO PROBABILITY

What is probability?

Figure: Real Money!

Question: What is the probability that, when Fred flips this coin, it will come up heads?

Randomness, probability versus statistics

An existential question: Does "randomness" really exist?

- Probability: A sub-discipline of mathematics concerned with describing and modelling uncertain experiments.
- Statistics: The study of the collection and analysis of data.
- "Essentially, all models are wrong, but some are useful." - George E. P. Box

$$\text{Statistics} \iff \text{Data} \iff \text{Uncertainty} \iff \text{Probability}$$

Various definitions of probability

The **classical** definition: The probability of some event is

$$\frac{\text{number of ways the event can occur}}{\text{the total number of possible outcomes}},$$

provided all outcomes are equally likely.

For example, the probability of rolling a 2 with a six sided die is $1/6$.

The **relative frequency** definition: The probability of an event is the (limiting) proportion (or fraction) of times the event occurs in a very long series of repetitions of an experiment.

For example, the probability of rolling a 2 is $1/6$ since if you roll the die many times, about $1/6$th of the time the outcome will be a 2.

Various definitions of probability

- Each of the above definitions (incl. subjective definition from the course notes) has pitfalls.
- Solution? Treat probability as a mathematical system defined by a set of axioms.
- Chapter 2 will introduce mathematical foundation of probability.

2. MATHEMATICAL PROBABILITY MODELS

Definition

A **sample space** S is a *set* of distinct outcomes of an experiment with the property that in a single trial of the experiment only one of these outcomes occurs.

Example

For the following experiments, describe the sample space S.

a) Roll a die.

b) Number of coin-flips until heads occurs.

c) Waiting time in minutes until a sunny day.

Definition

A sample space S is said to be **discrete** if it is finite, or "countably infinite" (i.e.,there is a one-to-one correspondence with the natural numbers). Otherwise a sample space is said to be **non-discrete**

Question: Which of the sample spaces S on the previous slide are discrete?

Definition

An **event** is a subset of a sample space S.
Notation: We say A is an event if $A \subseteq S$ (speak: "A is a subset of S", or "A is contained in S").

Example

Write down the event A:

a) The die shows "6".

b) You need 20 or fewer coin tosses until you see heads.

c) It won't be sunny for at least the next hour.

More set notation

Let $A, B, A_1, A_2, \ldots$ be events.

a) Element of: $x \in A$ if the outcome x is in the event A.

b) Union: $A \cup B = \{x \mid x \in A \;\; or \;\; x \in B\}$

c) Intersection: $A \cap B = \{x \mid x \in A \;\; and \;\; x \in B\}$

d) Complement: $A^c = \{x \mid x \in S, \;\; x \notin A\} = A' = \bar{A}$

e) Empty event/set: The empty set is denoted $\emptyset$

f) Disjoint: Two events A and B are said to be disjoint of $A \cap B = \emptyset$.

Example

Suppose two six sided dice are rolled, and the number of dots facing up on each die is recorded.

a) Write down the sample space S.

b) Write down, as a set, the event $A=$"The sum of the dots is 7".

c) Write down, as a set, the event B^c, where $B =$"The sum of the numbers is at least 4".

d) Write down, as a set, $A \cap B^c$, and $A \cup B^c$.