**Instructions**

1. **All electronic devices are to be turned off and put away.**
2. **This is a closed-book quiz.**
3. **Answer the questions in the spaces provided.**
4. **Final answers to written questions should be rounded to THREE DECIMAL PLACES or left in fraction form.**
5. **Only question pages will be marked.**
6. **You may tear off the last page and use it for rough work. All pages will be collected at the end of the quiz.**
7. **Only a non-programmable, non-graphical calculator with a pink-tie or blue-goggle sticker will be allowed. No other aids are allowed.**
8. **DO NOT WRITE ON THIS COVER PAGE.**

**Marking Scheme:**

| **Questions** | **Out of** |
|---|---|
| 1 – 9 | 9 |
| 10 | 8 |
| 11 | 9 |
| **Total** | 26 |

**Part 1: Multiple Choice – Each question is worth 1 mark**

**Please CLEARLY CIRCLE your answer selection on this page. Do NOT simply write your choice next to the question. If you do, the question will not be graded. Each question has one correct answer. Choose the best answer.**

**1.** Which of the following quantities is the smallest?

A) 5! B) $\binom{7}{4}$ **C)** $\boldsymbol{\binom{8}{2}}$ D) $7^{(3)}$

- **5! = 120, $\binom{7}{4}$ = 35, $\binom{8}{2}$ = 28, $7^{(3)}$ = 210**

**2.** Five chairs arranged in a straight line are to be painted. Four different colours of paint are available (with an unlimited amount of each colour of paint). How many arrangements of painted chairs are there if no two adjacent chairs are painted the same colour?

A) 108 B) 24 C) 576 D) 243

- **We have five tasks to perform (painting chairs) with the condition / restriction that adjacent chairs cannot be painted the same colour**
- **Thus, the total # of arrangements = 4*3*3*3*3 = 324**
- **Note: The correct answer was not one of the options for this question. Everyone received credit for this question.**

**3.** How many five-digit numbers that are greater than 55,000 can be made using the digits 1, 3, 5, 7, and 9, where each digit may be used more than once?

A) 2,250 **B) 1,625** C) 2,375 D) 3,125 E) None of these

- **We will consider both cases here:**
- **First Case: The first digit is a 5. We then need to choose the other 4 digits, under the condition that the number is greater than 55,000**
- **I will place the 5 in the first spot and then note the number of choices for the other digits: 5 __(3)___ __(5)____ __(5)_____ __(5)____**
- **Second Case: Now, I will consider the case where the first digit is not a 5:**
- **__(2)___ ___(5)___ __(5)___ __(5)___ __(5)___ (The first digit can be either a 7, or 9. Then I'm free to choose any of the 5 digits for the other slots.**
- **So, the total number of ways is $3*5^3 + 2*5^4 = (5^3\,(3 + 10)) = 13*5^3$ = 1,625**

**4.** Consider the following simple game: A fair six-sided die is tossed until the first 4 appears. In general, if a 4 is tossed on the $k^{th}$ toss (k = 1, 2, 3, .....), then you will receive $\$6^k$ dollars. For example, suppose that a die is tossed and the first 4 appears on the $2^{nd}$ toss. You will receive $\$6^2$ = \$36.

Suppose you were asked to pay \$1,000 to play this game. What is the probability that you would make a profit? **Note:** You would make a profit if you receive more than \$1,000.

A) 0.4213 **B) 0.5787** C) 0.9035 D) 0.0965 E) None of these

- **You will make a profit so long as you receive more than \$1,000. We see that this corresponds to tossing a 4 in four or more tosses.**
- **We can determine this probability in terms of the sum of an infinite Geometric series.**
- **P(make a profit) = $\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right)\ldots=\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)\left[1+\left(\frac{5}{6}\right)+\left(\frac{5}{6}\right)^2+\cdots.\right]$**
- $=\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)\left[\frac{1}{1-\frac{5}{6}}\right]=\frac{125}{216}$ **= 0.5787**

**5.** Four women and three men are to be arranged in a straight line for a photo. How many possible arrangements are there if the third person in the line must be a woman?

A) 2,160 B) 186,624 C) 1,440 **D) 2,880**

- **We have seven tasks to perform as there are seven people to be arranged in a straight line.**
- **Since the third person in the line must be a woman, we begin by assigning the third position (i.e. we deal with the restriction first). There are four choices for this first task. For the next task, we assign a person to position one. There are six choices for this task. The remaining five tasks are handled in the same manner.**
- **So, by the multiplication rule, there are: 4*6*5*4*3*2*1 = 4*6! = 2,880**

**6.** In how many ways can 10 different candies be divided between Terry and Alex if all the candies must be given away, and if Alex must receive at least one of them?

**A) 1,023** B) 1,013 C) 19 D) 9

- **There are $2^{10}$ total ways to divide the candies between Terry and Alex. We need to subtract of the number of ways in which Alex receives 0 pieces of candy.**
- **There is one way for Alex to get no candies. So, the number of ways is $2^{10} - 1 = 1{,}023$.**

**7.** A five character password must be made from the following set of seven characters: {%, #, !, @, ^, *, &}. No repetition of the characters is allowed. How many passwords containing the character # are possible?

A) 360 B) 6,480 **C) 1,800** D) 480

- **There are 5 tasks to perform. Each password contains #. There are 5 spots to put #. First position, second position, third position, fourth position, and fifth position. So, let's simplify this and just multiply by 5.**
- **So, we have # chosen. Now, we need to select the other characters (four more tasks).**
- **We have 6 choices for the second task, then 5 choices for the third task, 4 choices for the fourth task, and 3 choices for the fifth task.**
- **This gives us 5*6*5*4*3 = 1,800**

**8.** What is the probability that a student gets an A or B in a particular University Math course if 75% of the class passes and 60% of the class gets at most a C?

**Hint:** You can define the sample space as S = {A or B, C or D, F}, where F is a failing grade.

A) 0.35 B) 0.65 C) 0.6 **D) 0.4**

- **From the given information, $P(\overline{F}) = 0.75 = 1 - P(F) \Rightarrow P(F) = 0.25$.**
- **Also, $0.60 = P(C \text{ or } D) + P(F) \Rightarrow P(C \text{ or } D) = 0.6 - 0.25 = 0.35$.**
- **We know that the probabilities must sum to 1.**
- **So $P(A \text{ or } B) = 1 - P(C \text{ or } D) - P(F) = 1 - 0.35 - 0.25 = 0.4$**

**9.** Two cards are to be selected from a well shuffled deck of playing cards (without replacement). What is the probability that neither card is black?

**A) 0.245** B) 0.25 C) 0.755 D) 0.490

- **There are 26 black cards in the deck and 26 red cards in the deck.**
- **We are choosing two cards without replacement.**
- **So P(neither card is black) = P(both cards are red) = (26/52)*(25/51) = 0.245**

**Part 2: Written Answer / Short Answer Questions**

**Final answers can be left in fraction form OR rounded to THREE DECIMAL PLACES.**

**For full credit, you need to show all necessary work. Unjustified answers will not receive full credit.**

**10.** A fair six-sided die is rolled 8 times. After each roll, the number of dots (pips) showing on the upturned face is recorded, until a final sequence of 8 numbers is obtained.

**a.** How many outcomes are possible? For full credit, please express your final answer as a number. ***(2 marks)***

- **There are 8 tasks to perform (rolls). There are 6 outcomes for each roll.**
- **By the multiplication rule, there are $6^8$ = 1,679,616 possible outcomes.**

**b.** In how many ways can at least one of the rolls be a five? For full credit, please express your final answer as a number. ***(3 marks)***

- **We know that at least one of the rolls need to be a five.**
- **There are $6^8$ outcomes in total.**
- **So, the number of rolls where there is at least one five**
- **= total # of outcomes - # of outcomes where no fives are rolled.**
- **If no fives are rolled, there are 5 possibilities for each roll. By the multiplication rule, we have $5^8$ possibilities.**
- **So, the number of rolls where there is at least one five equals $6^8 - 5^8$ = 1,288,991**

**c.** In how many ways can exactly 2 of the rolls be ones, exactly 3 of the rolls be twos, and at least 2 of the rolls be fives? For full credit, please express your final answer as a number.

***(3 marks)***

- **We know that exactly 2 of the rolls are ones, 3 of the rolls are twos, and at least 2 of the rolls are to be fives.**
- **We will start by considering the case that there are 2 fives.**
- **To start, there are $\binom{8}{2}$ ways to arrange the ones.**
- **Of the remaining 6 rolls, there are $\binom{6}{3}$ ways to arrange the twos.**
- **Of the remaining 3 rolls, there are $\binom{3}{2}$ ways to arrange the 2 fives.**
- **Then, there is a single roll that must be a three, four, or six. There are three ways to roll one of these values, and $\binom{1}{1}$ way to arrange it.**
- **Now, we consider the case where there are 3 fives.**
- **There are $\binom{8}{2}$ ways to arrange the ones.**
- **Of the remaining 6 rolls, there are $\binom{6}{3}$ ways to arrange the twos.**
- **Of the remaining 3 rolls, there are $\binom{3}{3}$ ways to arrange the 3 fives.**
- **When rolling a fair die 8 times, the total number of ways that exactly 2 of the rolls are ones, 3 of the rolls are twos, and at least 2 of the rolls are fives is: $3\binom{8}{2}\binom{6}{3}\binom{3}{2}\binom{1}{1} + \binom{8}{2}\binom{6}{3}\binom{3}{3}$ = 5,600**

**11.** A bridge hand consists of 13 cards randomly dealt from a standard deck of 52 playing cards.

**a.** How many possible bridge hands are there? This represents the total number of outcomes in the sample space. For full credit, please express your final answer as a number. ***(2 marks)***

- **There are a total of $\binom{52}{13}$ possible bridge hands. This represents the total number of outcomes in the sample space.**

**b.** What is the probability that a bridge hand contains exactly 4 clubs? For full credit, please express your final answer to three decimal places. ***(2 marks)***

- **From part a., there are a total of $\binom{52}{13}$ possible bridge hands.**
- **We need 4 clubs from the group of thirteen possible clubs. Then we need 9 cards from the remaining non-clubs.**
- **So, there are a total of $\binom{13}{4}\binom{39}{9}$ possible hands containing exactly 4 clubs.**
- **If we let A represent the event that there a bridge hand contains exactly 4 clubs, then P(A) = $\frac{\binom{13}{4}\binom{39}{9}}{\binom{52}{13}}$ = 0.239 (to 3 d.p.)**

**c.** What is the probability that a bridge hand contains exactly 4 clubs, and no aces? For full credit, please express your final answer to three decimal places. ***(2 marks)***

- **As noted in part a., there are a total of $\binom{52}{13}$ possible bridge hands.**
- **There are a total of $\binom{12}{4}\binom{36}{9}$ possible hands containing exactly 4 aces and no clubs. Note that we cannot have the ace of clubs in our hand, so we need to select 4 clubs from the 12 remaining clubs. Then we select 9 cards from the remaining non-aces and non-clubs (24 red cards + 12 spades = 36 remaining cards).**
- **If we let B represent the event that there a bridge hand contains exactly 4 clubs and no aces, then P(A) = $\frac{\binom{12}{4}\binom{36}{9}}{\binom{52}{13}}$ = 0.073 (to 3 d.p.)**

**d.** What is the probability that a bridge hand contains exactly four times as many diamonds as clubs? For full credit, please express your final answer to three decimal places. ***(3 marks)***

- **As noted in part a., there are a total of $\binom{52}{13}$ possible bridge hands. This represents the total number of outcomes in the sample space.**
- **Now we need to consider all possible cases where there are exactly four times as many diamonds as clubs, keeping in mind that there are 13 cards in total in the bridge hand.**
- **We can have 4 diamonds, 1 club, and 8 "other" cards. We can also have 8 diamonds, 2 clubs, and 3 "other" cards.**
- **We can use the addition rule to obtain the total number of ways for a bridge hand to contain exactly four times as many diamonds as clubs.**
- **There are a total of: $\binom{13}{4}\binom{13}{1}\binom{26}{8}+\binom{13}{8}\binom{13}{2}\binom{26}{3}$ ways.**
- **If we let C represent the event that there are three times as many hearts as spades, then P(C) = $\frac{\binom{13}{4}\binom{13}{1}\binom{26}{8}+\binom{13}{8}\binom{13}{2}\binom{26}{3}}{\binom{52}{13}}$ = 0.023 (to 3 d.p.)**