STAT 230

May 30 Open Tutorial Solutions

Preview

In this tutorial, you will get some practice calculating probabilities using:

counting techniques, union, intersection, and / or complementary events.

the product rule, the Law of Total Probability, and Bayes' Theorem.

Problem 1

A shipment of 50 circuit boards has just arrived. Suppose that it is known that there are 40 good ones and 10 that are defective. An inspector randomly selects 6 circuit boards without replacement to be tested.

a. In how many ways can the inspector select 6 circuit boards to be tested?
b. How many samples of size 6 would contain exactly 4 good ones?
c. What is the probability that there are exactly 4 good boards in a random sample of 6 taken from this shipment?

Problem 1 Solution

a. There are 50 circuit boards in total, and 6 are to be selected without replacement.

So, there are a total of $\binom{50}{6}$ ways to sample 6 devices from a group of 50.

b. There are $\binom{40}{4}$ ways to select 4 good circuit boards.

Similarly, there are $\binom{10}{2}$ ways to select to defective circuit boards.

So, there are a total of $\binom{40}{4}\binom{10}{2}$ ways to select 4 good circuit boards (and two defective ones).

c. Putting things together, the probability that there are exactly 4 defective circuit boards in a sample of size 6 taken from this shipment is:

$$\frac{\binom{40}{4}\binom{10}{2}}{\binom{50}{6}} = 0.259$$

Problem 2

A music collector is sorting their records on a shelf. The records have been placed in protective cases that are all identical. The cases have been labeled though. There are three different types of records to arrange. There are two rock records (each labeled with an R), six blues records (each labeled with a B), and one heavy metal record (labeled with an M).

a. How many ways can the records be arranged?
b. Let A represent the event that the first and last records are different types. How many outcomes are there in A?
c. What is P(A)?

Problem 2 Solution

a. We have 9 records in total, and there are three types of records (R, B, and M). So, there are a total of 9!/6!2!1! = 252 ways to arrange the records.

b. We can start by determining the number of outcomes in $\bar{A}$. For the first and last records to be the same, they could either be both rock records (R) or both blues records (B). If the arrangement is R…..R, then the middle records can be arranged in 7!/(6!1!) = 7 ways. If it's B…..B, then the middle records can be arranged in 7!/(2!4!1!) = 105 ways. Altogether, there are a total of 7 + 105 = 112 ways.

So, | $\bar{A}$ | = 112. This means that |A| = 252 – 112 = 140.

c. Putting things together, with all arrangements being equally likely:

P(A) = P(first and last records are different types) = 140/252 = 5/9 = 0.556

Problem 3

An audit of a small business by an accounting firm either reveals a problem with the accounts or it doesn't. Also, the audit is either done correctly or incorrectly. The probability that the audit is done correctly is 0.8. The probability that the audit is done incorrectly and it reveals a problem is 0.15. The probability that the audit is done correctly and it does not reveal a problem is 0.3.

a. What is the probability that the audit is done correctly or it reveals a problem?
b. What is the probability that the audit does not reveal a problem?

Problem 3 Solution

Let’s start by defining events.

Let R be the event that an audit reveals a problem.

Let C be the event that an audit is done correctly.

Given:
P(C) = 0.8

$P(R \cap \bar{C})$ = 0.15

$P(\bar{R} \cap C)$ = 0.3

a. We want $P(R \cup C)$.

We can write $P(R \cup C) = P(\bar{R} \cap C)$ + $P(R \cap C)$ + $P(\bar{C} \cap R)$.

We know that P(C) = $P(\bar{R} \cap C)$ + $P(R \cap C)$ (Law of Total Probability).

So, $P(R \cap C)$ = 0.8 – 0.3 = 0.5.

So, $P(R \cup C)$ = 0.3 + 0.5 + 0.15 = 0.95

b. We want $P(\bar{R})$.

From part a., $P(R \cap C)$ = 0.5

Also, P(R) = $P(\bar{C} \cap R)$ + $P(C \cap R)$ = 0.15 + 0.5 = 0.65 (Law of Total Probability)

So, $P(\bar{R}) = 1 - P(R) = 1 - 0.65 = 0.35$.

Problem 4

Suppose it is known that 2% of the population suffers from a particular disease. A blood test is used to help screen patients for this particular disease, but it it’s not perfect. The test has a 95% chance of correctly identifying the disease for diseased individuals, but also has a 10% chance of falsely indicating that a healthy person has the disease (referred to as a false positive).

a. What is the probability that a randomly selected person would test positive for the disease?

b. Of those randomly selected people that test positive for the disease, what proportion would actually have the disease?

Problem 4 Solutions

Let's start by defining events.

Let D be the event that a person has the disease.

Let P be the event that a person tests positive.

Given:

P(D) = 0.02 => $P(\bar{D})$ = 0.98

P(P|D) = 0.95 => $P(\bar{P}|D)$ = 0.05

$P(\bar{P}|\bar{D})$ = 0.9 => $P(P|\bar{D})$ = 0.1

a. We want P(P), and P(P) = $P(\bar{D} \cap P)$ + $P(D \cap P)$ (Law of total probability)

We can use the product rule to write P(P) = $P(P|\bar{D}) * P(\bar{D})$ + P(P|D)*P(D)

Using the given information, P(P) = (0.1)*(0.98) + (0.95)*(0.02) = $\frac{117}{1000}$ = 0.117

b. We want P(D|P). By definition, P(D|P) = $\frac{P(D \cap P)}{P(P)}$.

Note: We can make use of the result from part a. We will rewrite the numerator using the product rule to get:

P(D|P) = $\frac{P(P|Dd)*P(D)}{P(P)} = \frac{(0.95)*(0.02)}{0.117} = \frac{19/1000}{117/1000} = \frac{19}{117}$ = 0.162 (to 3 d.p.)

(This result may seem low, but remember that P(D) = 0.02 before any screening was done!)

Note: This was an application of Bayes' Theorem.

Additional Exercise: Try doing this problem using a tree diagram / probability tree.