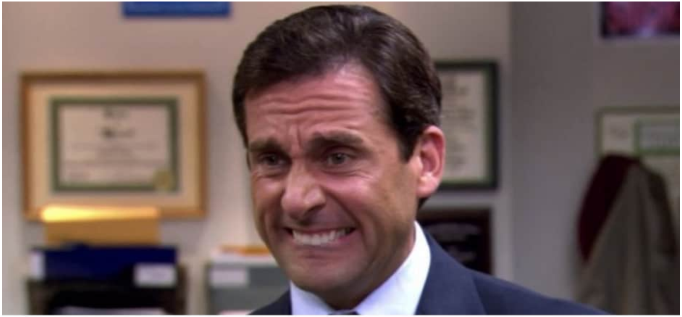
STAT 230 **SECTION 2** **LECTURE 17**



When you realize your instructor needs to quarantine and you will only have fun online lectures instead of fun in person lectures

Today's Agenda

Last time:

- Poisson Distribution
- Poisson Process
- Chapter 5 recap

Today (Lec 17, 06/10):

- Practice, Practice, Practice

Distributions covered so far:

- Discrete Uniform Distribution
- Bernoulli Distribution
- Hypergeometric Distribution
- Binomial Distribution
- Negative Binomial + Geometric Distribution
- Poisson Distribution
  - Limit of the Binomial Distribution ($n \to \infty$, $p \to 0$, $np \to \lambda$)
  - Number of occurrences until t in a Poisson process

Question 1

Suppose traffic accidents on the 401 occur at a rate of 1 per 30minutes.
The number of accidents during an 8-hour day then follows

a) $Poi(8)$

b) $Binom(2/60, 60)$

c) $Poi(16)$ ✓

16×0.5 hours $= 8$ hours
$\Leftrightarrow$ rate of 16 in 8 hours

Question 2a)

3) Homogeneity cars probably arrive uniformly at the same rate and proportional to length of the time interval.

During rush hour the number of cars passing through a particular intersection22 is assumed to follow a Poisson process. On average there are 540 cars per hour.

Discuss briefly whether or not you think the three assumptions for a Poisson process would hold reasonably well in this situation.

1) Independence the number of cars in different non-overlapping intervals is likely independent.

2) Individuality In a very small time interval, the probability of 2 or more cars ≈ 0.

Question 2b)

$1 \text{ hour} = 60 \text{ minutes}$

$= 120 \times 30 \text{ seconds},$

so 540 in 1 hour $\iff \frac{540}{120}$ in 30 seconds

During rush hour the number of cars passing through a particular intersection22 is assumed to follow a Poisson process. On average there are 540 cars per hour. $\lambda = 540/\text{hour}$

Find the probability that 11 cars passed through the intersection in a thirty second interval.

X = # of cars passing throug in 30 sec

$X \sim \text{Poi}\left(540 \cdot \frac{1}{120}\right) = \text{Poi}(4.5)$

$\Rightarrow P(X=11) = e^{-4.5} \frac{(4.5)^{11}}{11!} \approx 0.0043$

Question 2c)

During rush hour the number of cars passing through a particular intersection22 is assumed to follow a Poisson process. On average there are 540 cars per hour.

Find the probability that 11 or more cars passed through the intersection in a thirty second interval.

$$X \sim Poi\left(\frac{540}{120}\right)$$

so $$P(X \geq 11) = 1 - P(X \leq 10)$$

$$= 1 - \underbrace{\sum_{k=0}^{k=10} e^{-4.5} \cdot \frac{4.5^k}{k!}}_{\text{easily computed in R}} \approx 0.0067$$

Question 2d)

During rush hour the number of cars passing through a particular intersection22 is assumed to follow a Poisson process. On average there are 540 cars per hour.

Find the probability that when 20 disjoint thirty second intervals are studied, exactly 2 of them had 11 cars pass through the intersection.

20 disjoint intervals => 20 independent trials with success probability p from b)

$\Rightarrow X = \#$ intervals with 11 cars $\sim \text{Bin}(20, p)$

$$\Rightarrow P(X=2) = \binom{20}{2} p^2 (1-p)^{18} \approx \underline{\underline{0.003}}$$

Question 2e)

During rush hour the number of cars passing through a particular intersection22 is assumed to follow a Poisson process. On average there are 540 cars per hour.

We want to find 12 disjoint thirty second intervals in which 11 cars passed through the intersection. Give an exact expression for the probability that 1000 disjoint 30 second intervals have to be observed to find the 12 having the desired traffic flow.

failures

X = # of 30 sec intervals with ~~not 11 cars until has been found~~ 12th interval with 11 cars $\sim \text{NegBin}(k=12, p)$

$$P(1000 \text{ disj. intervals to find 12 with 11 cars})$$
$$= P(X = 988) = \binom{999}{11} p^{12} (1-p)^{988}$$

Question 3

Let X_1, X_2 and X_3 denote the random variables for the outcome of three independent fair random number generators. Assume that their ranges are $\{1, 2, \ldots, 10\}$. Now let X_{max} denote the maximum value among X_1, X_2 and X_3. Dwight claims that

$$P(X_{max} \leq x) = P(X_1 \leq x)P(X_2 \leq x)P(X_3 \leq x).$$

Is he right?

a) Yea!

b) No, Dwight should really take STAT 230.

$$P(\max\{X_1, X_2, X_3\} \leq x) = P(X_1 \leq x \text{ AND } X_2 \leq x \text{ AND } X_3 \leq x)$$

$$\overset{\text{independence}}{=} P(X_1 \leq x)P(X_2 \leq x)P(X_3 \leq x)$$

Question 4

Website hits for a given website occur according to a Poisson process with a rate of 100 hits per minute. Find

a) $P(1 \text{ hit is observed in a second})$

b) $P(90 \text{ hits are observed in a minute})$

a) X = # hits in one sec, then $X \sim Poi\left(\frac{100}{60}\right)$ so

$= \frac{5}{3}$

$$P(X=1) = e^{-5/3} \frac{(5/3)^1}{1!} \approx 0.315$$

b) X = # hits in one min, then $X \sim Poi(100)$, so

$$P(X=90) = e^{-100} \cdot \frac{100^{90}}{90!} \approx 0.025$$

Question 5

Which of the following is TRUE?

a) The binomial distribution models the number of successes when sampling with replacement.

b) The geometric distribution is a special case of the hypergeometric distribution.

c) A negative binomial distribution can take any value 0,1,2,..., N for known N.

... we don't know how many trials we need at most to get k successes.

Question 6

Every time I think of an example question for STAT 230, there is 50.4% chance of it being about coin flips. What is the probability that I will need more than 5 questions to think of a non-coin-flip example?

$p = 0.496$ success probability. Independent trials until first success.

$X =$ # failures until first success, then $X \sim Geo(p)$.

$P(\text{"need > 5 questions"}) = P(\text{" need > 4 failures"}) = P(X > 4)$

$= 1 - P(X \leq 3) = 1 - \sum_{k=0}^{3} p \cdot (1-p)^k \approx 0.065$

Question 7

Suppose a random variable X has pmf

$$f(0) = 0.1c, \quad f(1) = 0.2c, \quad f(2) = 0.5c, \quad f(3) = c, \quad f(4) = 0.2c,$$

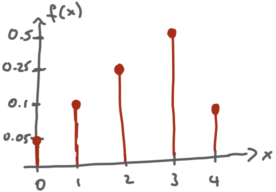
where c is a constant.

a) Find c.
b) Sketch the pmf f
c) Sketch the cdf F
d) Calculate $P(X > 2)$.

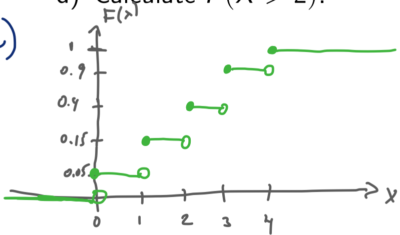
a) $\sum_{x=0}^{4} f(x) = 1 \iff 2c = 1 \iff c = \frac{1}{2}$

d) $P(X > 2) = f(3) + f(4) = 0.6$

b)



c)



0.05; 0.15; 0.4; 0.9; 1

Question 8

In the manufacturing process of commercial carpet, small faults occur at random in the carpet according to a Poisson process at an average rate of 0.95 per 20 m^2. One of the rooms of a new office block has an area of 80 m^2 and has been carpeted using the same commercial carpet described above. What is the probability that the carpet in that room contains at least 4 faults?

X = # faults in $80 m^2$, then $X \sim \text{Poi}(\overbrace{0.95 \times 4}^{3.8})$

$P(X \geq 4) = 1 - P(X \leq 3) = 1 - \sum_{k=0}^{3} e^{-3.8} \cdot \frac{(3.8)^k}{k!} \approx 0.527$