Stat 230: Probability

Lecture 29

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Review

Example

Suppose X, Y, and Z are independent random variables where $X \sim N(1,4)$, $Y \sim U(1,3)$, and $Z \sim Exp(3)$.

Give the expected value and variance of $X + 2Y + 3Z + 4$?

Review

Last time we talked about:

(1) Correlation

(1) Mean and variance of linear combinations

$$E\left(\sum_{i=1}^{n} a_i X_i\right) = \sum_{i=1}^{n} a_i E\left(X_i\right)$$

$$\mathrm{Var}\left(\sum_{i=1}^{n} a_i X_i\right) = \sum_{i=1}^{n} a_i^2 \mathrm{Var}\left(X_i\right) + 2 \sum_{1 \leq i < j \leq n} a_i a_j \mathrm{Cov}(X_i, X_j)$$

For today:

(1) Indicator random variables

(2) Sums of normal random variables

Review

- Monday: Quiz 6

Indicator

Example

Show that $E(\mathbb{1}_A) = P(A)$, and $Var(\mathbb{1}_A) = P(A)(1 - P(A))$.

Indicator

Remark

If A and B are events, then $Cov(\mathbb{1}_A, \mathbb{1}_B) = P(A \cap B) - P(A)P(B)$. Moreover, one can show that $\mathbb{1}_A$ and $\mathbb{1}_B$ are independent if A and B are independent.

Therefore, A and B are independent if and only if $\mathbb{1}_A$ and $\mathbb{1}_B$ are uncorrelated.

Example

Suppose $X \sim Binomial(n, p)$. Show $E(X) = np$, and $Var(X) = np(1 - p)$ using indicator random variables.

Indicator

Example

Suppose $X \sim hyp(N, r, n)$. Show $E(X) = n(r/N)$, and $Var(X) = n(r/N)(1 - r/N)(N - n)/(N - 1)$ using indicator random variables.

Sum of Random Variables

Theorem

Suppose that $X_1, ..., X_n$ are independent random variables, and that $X_i \sim N(\mu_i, \sigma_i^2)$. Then

$$\sum_{i=1}^{n} a_i X_i \sim N\left(\sum_{i=1}^{n} a_i \mu_i, \sum_{i=1}^{n} a_i^2 \sigma_i^2\right)$$

We don't quite have enough probability machinery to prove this (Moment generating functions).