Stat 230: Probability

Lecture 31

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Review

Example

Suppose $X, W \overset{iid}{\sim} Exp(4)$ and $Y \sim N(2, 9)$ and $Corr(X, Y) = Corr(W, Y) = 0.5$. If $Z \sim W - 2Y$, find

(1) $E(X + Z)$

(2) $Var(X + Z)$

Review

Last time we talked about:

(1) Central Limit Theorem: If $X_1, ..., X_n$ are independent and have a common distribution with $E(X_i) = \mu$, $Var(X_i) = \sigma^2$, then

$$\bar{X} \overset{approx}{\sim} N(\mu, \sigma^2/n)$$

For today:

(1) A couple more examples of CLT

(2) Moment generating function

Review

- Monday: TA Review

Central Limit Theorem

Theorem

If $X_n \sim Binomial(n, p)$, then for large n

$$\frac{X_n - np}{\sqrt{np(1-p)}} \overset{approx}{\sim} N(0,1)$$

Theorem

If $X_\lambda \sim Pois(\lambda)$, then for large λ

$$\frac{X_\lambda - \lambda}{\sqrt{\lambda}} \overset{approx}{\sim} N(0,1)$$

Remark

A continuity correction can be used for more accurate results when modeling a discrete distribution by the normal by modifying the values by a half step, i.e. 0.5 (Particularly for small n).

Central Limit Theorem

Example

Suppose that Billy flips a fair coin 100 times. Approximate the probability that the number of heads is between (inc) 45 and 55.

Central Limit Theorem

Example

Suppose that $X_1, ..., X_{50}$ are independent Geometric random variables with parameter 0.5. Estimate the probability that $\bar{X} > 1.5$.

Central Limit Theorem

Rules of thumb for using the central limit theorem:

- In general if the number of observations exceeds 30, then the central limit theorem often provides a reasonable approximation
- If the distribution of the observations is "close" to being unimodal, not too skewed, and is "close" to being continuous, then the central limit approximation can be reasonable for even smaller values of n (5-15).
- If the distribution is highly skewed, or very discrete, then a larger value of n might be necessary to get reasonably good approximations: ($n > 50$)