Stat 230: Probability
Lecture 10

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Review

Example

After many basic programs, a game based on dice is a logical next step. You decide to code a DnD game. During the adventure, the program checks if a player avoids danger. You consider taking the minimum of two D20s (dice with 20 sides), but want to verify the chance of success is reasonable.

(1) Give the CDF of a D20

(2) Give the PF of the minimum of two D20s

(3) Give the probability the player rolls at least 5

Review

Last time we talked about:

1. Uniform Random Variables

$$X \sim U(a, b), \; X(S) = \{a, ..., b\} \; f(x) = \frac{1}{b - a + 1}$$

For today:

1. Common distributions
   - Hypergeometric Distribution
   - Binomial Distribution
   - Negative Binomial Distribution

Review

- Reminder: Exam on Tuesday
- TA review tutorial on Monday

Hypergeometric

Definition

*Consider a population that consists of N objects, of which r are considered "successes" and the remaining $N - r$ are considered "failures", and suppose that a subset of size n is drawn from the population **without replacement**. We say that the random variable X has a **hypergeometric distribution** if X denotes the number of successes in the subset (shorthand: $X \sim hyp(N, r, n)$).*

Hypergeometric

Example (Hypergeometric random variables)

Experiment	X	Distribution
Drawing 5 cards from a deck of cards	# of Ace's	$hyp(52, 4, 5)$
Lotto where 7 numbers are drawn from 50	# Matches	$hyp(50, 7, 7)$

Hypergeometric

Example

Suppose $X \sim hyp(N, r, n)$. Show that

$$f_X(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}} \quad \max\{0, n - (N - r)\} \leq x \leq \min\{r, n\}.$$

The p.f. of a hypergeometric sums to 1 due to the "Hypergeometric Identity".

The fact that the probability function of the hypergeometric distribution sums to 1 follows from the hypergeometric identity: If $n < r$, $n < N - r$,

$$\binom{N}{n} = \sum_{j=0}^{n} \binom{r}{j} \binom{N-r}{n-j}$$

Hypergeometric

Example

Consider drawing a 5 card hand at random from a standard 52 card deck of playing cards (13 Kinds: A,2,3,4,...,10,J,Q,K, in 4 suits: $\clubsuit, \diamondsuit, \heartsuit, \spadesuit$).

(1) What is the probability that the hand contains at least 3 J's?

(2) What is the probability that the hand contains 1 or fewer A's?

Binomial

Definition

A **Bernoulli trial** *with probability of success p is an experiment that results in either a success or failure, and the probability of success is p.*

Definition

Consider an experiment in which n Bernoulli trials are independently performed each with probability of success p. Then if X denotes the number of successes observed from the n trials, we say that X is **Binomial** *with parameters n and p.*

$$X \sim Binomial(n, p) \sim Bin(n, p)$$

Binomial

Example (Binomial Random Variables)

(1) Flip a fair coin independently 20 times, and let X denote the number of heads observed. Then

$$X \sim Binomial(20, 0.5)$$

(2) Consider drawing numbers from 1-9 **with replacement** to form a 5 number sequence, and let X denote the number odd numbers in the sequence. Then

$$X \sim Binomial(5, 5/9)$$

Example

Compute the probability function of a Binomial random variable with parameters n and p.