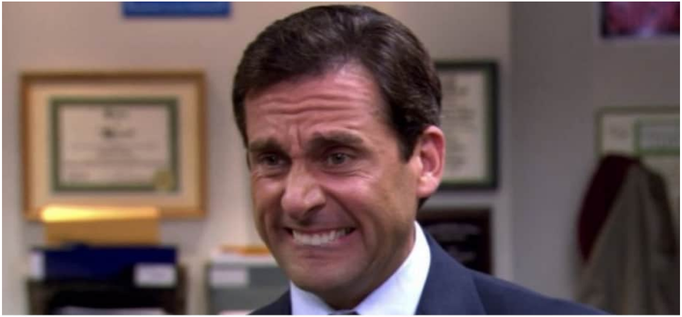
STAT 230 **SECTION 2** **LECTURE 16**



When you realize your instructor needs to quarantine and you will only have fun online lectures instead of fun in person lectures

Today's Agenda

Last time:

- Poisson Distribution
- Practice

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Next Lectures:

- Lec 17 (synchronous online lecture on June-10) consists of practice again.

Poisson Distribution

A random variable X follows a Poisson distribution with intensity $\lambda > 0$, if X has probability function

$$f(x) = e^{-\lambda}\frac{\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$$

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b) Poisson Process

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Consider counting the number of occurrences of an event that happens at random points in time (or space). Examples include

1. Counting emissions of radioactive particles from a radioactive substance
2. Hits on a web site during a given time period.
3. Counting how many pokemon you encounter in an hour.

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c) **Homogeneity or Uniformity:** events occur at a uniform or homogeneous rate λ and proportional to time interval Δ_t, i.e.

$$\frac{P\left(\text{one event in } (t, t+\Delta_t)\right) - \lambda\Delta_t}{\Delta_t} \rightarrow 0.$$

If $X =$occurrences in a time period of length t, then

$$X \sim Poi(\lambda t).$$

Poisson Process

Definition

A process that satisfies the prior conditions on the occurrence of events is often called a **Poisson process**. More precisely, if X_t, for $t \geq 0$, (a random variable for each t) denotes the number of events that have occurred up to time t, then X_t is called a Poisson process.

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Note that sometimes, t may not represent time, but area, volume, ...

Question

A website is giving out prizes to every 300th visitor. Suppose that visitors visit the website at random at a rate of 10 visitors per minute on average, and they visit the site independently and individually from each other. Let X denote the number of prizes given out after 10 minutes. Then

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A) $X \sim Poi(100)$

B) $X \sim Poi(10)$

C) Neither A nor B are true.

Question

Shiny versions of Pokemon are possible to encounter and catch starting in Generation 2 (Pokemon Gold/Silver). Normal encounters with Pokemon while running in grass occur according to a Poisson process with rate 1 per minute on average. 1 in every 8192 encounters will be a Shiny Pokemon, on average.

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a) If Ashton runs around in grass for 15 hours, what is the probability he will encounter at least one Shiny pokemon?

b) How long would Ashton have to run around in grass so that he has better than 50 percent chance of encountering at least one Shiny pokemon?

Partial Solution

- Let X be number of poke encountered after 1 hour, Y be the number of shiny encountered after one hour.
- $X \sim Poi(60)$ and $Y \sim Poi(60/8192)$
- If Z is the number of shiny encountered after 15 hours, then $Z \sim Poi(\frac{60}{8192} \cdot 15)$, and you know how to compute $P(Z \geq 1)$.
- If Z is the number of shiny encountered after t hours, then $Z \sim Poi(\frac{60}{8192} \cdot t)$. You solve $P(Z \geq 1) \geq 0.5$ for t.

Question

Website hits for a given website occur according to a Poisson process with a rate of 100 hits per minute. We say a second is a "break" if there are no hits in that second.

a) What is the probability p of a break in any given second?

b) Compute the probability of observing exactly 10 breaks in 60 consecutive seconds.

c) Compute the probability that one must wait for 30 seconds to get 2 breaks.

- Break means zero hits in one sec. If X = number of hits in one sec, then $X \sim Poi(100/60) = Poi(5/3)$ and

$$p = P(X = 0) = e^{-\frac{5}{3}} \frac{\left(\frac{5}{3}\right)^0}{0!} \approx 0.189$$

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- Take 60 one-sec intervals. Each interval has a probability of p of having a break. The number of one-sec intervals (from 60 one-sec intervals) with a break, say Y, then follows a binomial, $Y \sim Bin(60, p)$, and

$$P(Y = 10) = \binom{60}{10} p^{10} (1 - p)^{50} \approx 0.124$$

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- Let Z be the number of one-sec intervals one needs to wait until observing the second break ("success"). Then, $Z \sim NegBin(k = 2, p)$ and

$$P(Z = 30) = \binom{30+2-1}{2-1} p^2(1-p)^{30} \approx 0.002.$$

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So far, every problem could be solved by using one distribution.

However, many real-life problems may require more than one distribution to be modelled properly. Even within one setting, the type of questions you ask could change the required distribution.

Question

At a super busy coffee chain, customers arrive at a rate of 5 per minute.

a) Find the probability that that there are more than 2 customers in one minute.

b) Suppose you record the number of customers in 5 consecutive one-minute intervals. What is the probability that in at least 3 of them there were more than 2 customers?

c) Suppose you are waiting until finally, there is one minute with more than 2 customers. Denote by X the the number of minutes you need to wait. Find the probability function of X.

d) Find the probability that a minute with more than 2 customers actually had 6 customers.

e) Suppose in 3 minutes, there were n customers. Find the probability that x of these came in the first two minutes.

a) If $X =$ number of customers in one minute, then $X \sim Poi(5 \cdot 1)$. Thus,

$$\begin{aligned} p &= P(X > 2) = 1 - P(X \leq 2) \\ &= 1 - P(X = 0) - P(X = 1) - P(X = 2) \approx 0.875 \end{aligned}$$

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d) Let $X =$ number of customers in one minute. We are looking for

$$P(X = 6 \mid X > 2) = \frac{P(X = 6 \text{ and } X > 2)}{P(X > 2)} = \frac{P(X = 6)}{P(X > 2)} \approx 0.167$$

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- Combining and simplifying gives

$$P(x \text{ in first 2min} \mid n \text{ in 3min}) = \binom{n}{x}\left(\frac{2}{3}\right)^x\left(\frac{1}{3}\right)^{n-x}, \quad x = 0, 1, \ldots, n,$$

which is the pf of $Bin(n, 2/3)$.

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- We have seen some important special cases (uniform, bernoulli, binomial, hypergeometric, negative binomial, geometric, poisson)

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- If X is the element obtained, then$X \sim \mathrm{U}[a, b]$ with probability function

$$f(x) = P(X = x) = \begin{cases} \frac{1}{b-a+1}, & x = a, a+1, \dots, b, \\ 0, & \text{otherwise.} \end{cases}$$

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- Examples: Roll a fair die, pick a number between 1 and 49,...

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- Example: Number of aces in a hand of 7 cards

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- Example: Result when flipping a coin

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- Example: Number of heads in 5 coin tosses.

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- Special Case: For $k = 1$ (one success), we have a Geometric distribution.

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Poisson distribution

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$$f(x) = P(X = x) = e^{-\lambda}\frac{\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots.$$

- If n is large and p is small in a Binomial model, the number of success follows approximately a $Poi(np)$ distribution.
- Generally, a Poisson models the number of occurrences of an event.
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- Example: Number of customers at TH 5 minutes after opening.

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