STAT 230: ODDS STRIKE BACK

LECTURE 2

MAY THE 4TH BE WITH YOU

Today's Agenda

Last time:

- Course syllabus
- What is probability?
- Sample spaces and events

Today (Lec 2, 05/04):

- Quick review
- Simple events, compound events
- Probability distribution
- Odds
- Examples

2. MATHEMATICAL PROBABILITY MODELS

Definition

A **sample space** S is a *set* of distinct outcomes of an experiment with the property that in a single trial of the experiment only one of these outcomes occurs.

Definition

A sample space S is said to be **discrete** if it is finite, or "countably infinite" (i.e.,there is a one-to-one correspondence with the natural numbers). Otherwise a sample space is said to be **non-discrete**

Definition

An **event** is a subset of a sample space S.
Notation: We say A is an event if $A \subset S$ (speak: "A is a subset of S", or "A is contained in S").

More set notation

Let $A, B, A_1, A_2, \ldots$ be events.

a) Element of: $x \in A$ if the outcome x is in the event A.

b) Union: $A \cup B = \{x \mid x \in A \ \ or \ \ x \in B\}$

c) Intersection: $A \cap B = \{x \mid x \in A \ \ and \ \ x \in B\}$

d) Complement: $A^c = \{x \mid x \in S, \ \ x \notin A\} = A' = \bar{A}$

e) Empty event/set: The empty set is denoted $\emptyset$

f) Disjoint: Two events A and B are said to be disjoint of $A \cap B = \emptyset$.

Corrigendum: PLEASE forget about $\cap_{i=1}^{\infty} A_i$ and $\cup_{i=1}^{\infty} A_i$ (for now)!

Example

Suppose two six sided dice are rolled, and the number of dots facing up on each die is recorded.

a) Write down the sample space S.

b) Write down, as a set, the event $A =$ "The sum of the dots is 7".

c) Write down, as a set, the event B^c, where $B =$ "The sum of the numbers is at least 4".

d) Write down, as a set, $A \cap B^c$, and $A \cup B^c$.

Definition

Let S be discrete and $A \subset S$ an event. If A is indivisible so it contains only one point, we call it a **simple event**, otherwise **compound event**.

Definition

Let $S = \{a_1, a_2, \dots\}$ be discrete. Assign numbers $P(\{a_i\})$ (or short: $P(a_i)$), $i = 1, 2, \dots$, so that

1) $0 \leq P(a_i) \leq 1, \quad i = 1, 2, \dots$
2) $\sum_{\text{all } i} P(a_i) = 1$.

We then call the set of probabilities $\{P(a_i) : i = 1, 2, \dots\}$ a **probability distribution**.

Definition

Let $S = \{a_1, a_2, \dots\}$ discrete and $A \subset S$ an event. Then

$$P(A) = \sum_{a_i \in A} P(a_i).$$

Example

Suppose we roll a fair die. Then we can set $S = \{1, 2, \ldots, 5, 6\}$ and $P(i) = 1/6$ for $i = 1, 2, \ldots, 6$.

Consider the event A="Number odd". What is $P(A)$?

Definition

We say a sample space S with a finite number of outcomes is **equally likely** if the probability of every individual outcome in S is the same.

For a set A, denote by $|A|$ its cardinality. If S is equally likely, then

$$1 = P(S) = \sum_{i=1}^{|S|} P(a_i) = P(a_i)|S|,$$

so that

$$P(a_i) = \frac{1}{|S|}.$$

We thus find

$$P(A) = \sum_{i:\, a_i \in A} P(a_i) = \frac{|A|}{|S|}.$$

Observation: In the context of an equally likely sample space, i.e. when

$$P(A) = \frac{|A|}{|S|},$$

computing probabilities is equivalent to counting outcomes in sets. $\Rightarrow$ Chapter 3.

Example

Suppose that two fair six sided die are rolled.

a) What is the probability that the dots on each die match?
b) What is the probability that the dots sum to 7?
c) What is the probability that the dots do not sum to 7?
d) What is the probability that the dots match and sum to 7?

Odds

"Odds" can be used to describe probabilities.

- Odds **in favour** of an event A occurring is

$$P(A) : (1 - P(A)) \quad \text{or} \quad \frac{P(A)}{1 - P(A)},$$

- Odds **against** an event A is

$$1 - P(A) : P(A) \quad \text{or} \quad \frac{1 - P(A)}{P(A)}.$$

Example

A fair six-sided die is rolled once. What are...

- the odds in favour of rolling a 6?
- the odds against rolling a 6?
- the odds in favour of rolling an even number?

Example

Suppose that four students with different heights are lining up for office hours and are being called in in random order.

a) Describe a sample space S.

b) What is the probability that they are being helped in increasing height order, i.e., shortest first, then second-shortest,. . . .

c) What is the probability that the tallest one is served first?

d) What is the probability that the tallest one is served first and the shortest one is served last?

e) What is the probability that they are not served in increasing order?