STAT 230: Probability (Sec 02)
Spring 2022

Erik Hintz
Department of Statistics and Actuarial Science
erik.hintz@uwaterloo.ca

Lecture 3

Today's Agenda

Last time:

- Simple/compound events
- Probability distribution
- Odds

Today (Lec 3, 05/06):

- Quick review
- Counting techniques

- Sample space S: Set of outcomes of a random experiment.
- Event: Subset $A \subset S$ ("simple" if it consists only of one point).
- If S is discrete, say $S = \{a_1, a_2, \dots\}$ we can assign probabilities $P(a_i)$ such that $0 \leq P(a_i) \leq 1$ for $i = 1, 2, \dots$ and $\sum_{\text{all } i} P(a_i) = 1$. The $\{P(a_i) : i = 1, 2, \dots\}$ is called probability distribution.
- If S is finite and equally likely, then we have for an event A that

$$P(A) = \frac{|A|}{|S|}.$$

READ THE COURSE NOTES!

Example

Suppose that four students with different heights are lining up for office hours and are being called in in random order.

a) Describe a sample space S.

b) What is the probability that they are being helped in increasing height order, i.e., shortest first, then second-shortest,. . . .

c) What is the probability that the tallest one is served first?

d) What is the probability that the tallest one is served first and the shortest one is served last?

e) What is the probability that they are not served in increasing order?

3. COUNTING TECHNIQUES

Example

Suppose I have 3 pairs of pants and 4 pairs of shorts. If I need to pick either a pair of pants or a pair of shorts, how many possible picks are there?

Addition Rule

- If A and B are disjoint events (i.e., $A \cap B = \emptyset$), then

$$|A \cup B| = |A| + |B|$$

- From the addition rule, we can see that "or" means "$+$".

Example

Suppose two six sided die are rolled, how many outcomes would result in the sum of the die rolls being larger than 8?

Example

Suppose I have 4 different pairs of pants and 6 different shirts, and I need to pick one pair of pants and one shirt. How many options do I have?

Multiplication rule

- An ordered k-tuple is an ordered set of k values: $(a_1, a_2, ..., a_k)$. If the outcomes in A can be written as an ordered k-tuple where there are n_1 choices for a_1, n_2 choices for a_2,..., and in general n_i choices for a_i, then
$$|A| = n_1 n_2 \cdots n_k = \prod_{i=1}^{k} n_i.$$
- The multiplication rule indicates that "and" means "$\times$".

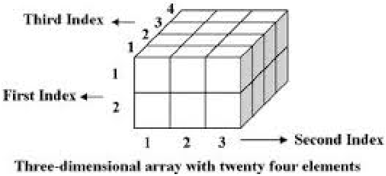

Figure: A three dimensional array.

Figure: A three dimensional array of three dimensional arrays.

With versus without replacement

Suppose you have an urn with n distinct balls, and you select $k \leq n$ balls in order. You can do that

- "**with** replacement": Every time an object is selected, it is put back into the pool of possible objects.
- "**without** replacement": Once an object is selected, it stays out of the pool of possible objects.

Sampling without replacement affects the probabilities thereafter!

Example

Suppose that three of the numbers $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ are selected at random **with** replacement, and then put together in the order they are drawn to form a three digit number. What is the probability that:

a) The number is larger than 500?

b) The number is even?

Example

Suppose that three of the numbers $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ are selected at random **without** replacement, and then put together in the order they are drawn to form a three digit number. What is the probability that:

a) The number is larger than 500?

b) The number is even?

Factorials!!!!!!

Definition

Given n distinct objects, n **factorial** (denoted $n!$) is the number of ordered arrangements of length n that can be made. Mathematically,

$$n! = n \times (n-1) \times \ldots 2 \times 1.$$

By definition, $0! = 1$.

A useful property of $n!$ is $n! = n \times (n-1)!$.

Example

10 people are standing next to each other for a group picture. How many arrangements are there?

Example

Suppose that the 5 members of of the "STATS IS COOL" club must select a president and a secretary. How many ways can they do this?

Definition

Given n distinct objects, a **permutation** of size k is an *ordered* subset of k of the individuals. The number of permutations of size k taken from n objects is denoted $n^{(k)}$ and

$$n^{(k)} = n(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!}.$$