Stat 230: Probability

Lecture 18

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Review

Example

Compute the expected value and variance for

- Flipping a fair coin
- Rolling a fair die
- Flipping a fair coin 1000 times

Review

Last time:

(1) Variance of Discrete Distributions

For today:

(1) R Applications
(2) Continuous Random Variables

Reading: Chapter 7 and 8

Review

- Monday: TA R tutorial
- Quiz Review

Review

Other Summary Statistics

- **Median**: Middle value (Half observations above/below. If even number of observations then sum of the two divided by 2)
- **Mode**: Most common value (May have multiple)

R Applications

Review

Remark

There exist distributions without expectation: Suppose X is a random variable with probability function

$$f_X(x) = \frac{6}{\pi^2}\frac{1}{x^2}, \quad x = 1, 2, ...$$

Then $E(X) = +\infty$ and $Var(X)$ is not defined.

Continuous Random Variables

Definition

A random variable X is said to be continuous if its range $X(S)$ is an interval $(a, b) \subset \mathbb{R}$.

Continuous Random Variables

Continuous random variables don't exist in "real life", however they may represent useful approximations in many cases:

Examples include:

- Measuring time (time is "continuous")
- Measuring distance or height
- Determining the angle a spinner point comes to rest at.

Continuous Random Variables

Suppose X is a continuous random variable with range $[0, 1]$ (for example). How can we describe probabilities associated with X?

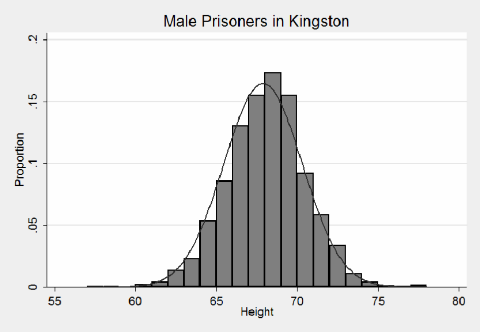
- Try to define the analog of the probability function $f_X(x) = P(X = x)$. In order to satisfy the axioms of probability we would need that

$$\sum_{x \in [0,1]} P(X = x) = 1$$

but this is hard to make sense of. If $P(X = x) > 0$ for all $x \in [0, 1]$, how can this sum converge?

Continuous Random Variables

- Another idea comes from trying to model relative frequency from the histogram.



$$P(a \leq X \leq b) = \int_a^b f(x)dx$$

Probability Density Function

Definition (Probability Density Function)

We say that a continuous random variable X has **probability density function** $f(x)$ *if*

(1) $f(x) \geq 0$

(2) $\int_{-\infty}^{\infty} f(x)dx = 1$

(3) $P(a \leq X \leq b) = \int_a^b f(x)dx$

The probability density function is often abbreviated pdf

Probability Density Function

Definition

The support of a pdf $f(x)$ is defined as

$$supp(f) = \{x \in \mathbb{R} : f(x) \neq 0\}$$

Integrals of a function f over a domain D may be computed on the domain $D \cap supp(f)$.

Probability Density Function

Example

Suppose that X is a continuous random variable with probability density function

$$f(x) = \begin{cases} cx(1-x) & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise} \end{cases}$$

(1) Compute c so that this is a valid pdf

(2) Graph $f(x)$

(3) Compute $P(X \geq 1/2)$

(4) Compute $P(1/4 \leq X \leq 3/4)$

(5) Compute $P(X = 1/2)$