Stat 230: Probability Lecture 16

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Example

Let X denote the outcome of a fair six sided die roll. Compute Var(X).

Last time we talked about:

- (1) Expected Value
- (2) Variance

For today:

- (1) Review
- (2) Variance
- (3) Existence of moments

Reading: Chapter 7

- Monday: Quiz 3
- R Code Modules
- Review

Example

Suppose we are give the following probability function:

$$\begin{array}{c|cc}
x & f(x) \\
\hline
1 & \frac{1}{2}p \\
2 & p \\
3 & \frac{3}{2}p \\
4 & 2p \\
5 & \frac{5}{2}p
\end{array}$$

What is P(X > 3)?

Example

Suppose goose droppings are spread across UW according to a Poission process with 6 droppings per 10 steps.

- (1) Define X to be the number of droppings stepped in t steps. Give the probability function and the range of X.
- (2) Give the probability of stepping in exactly 63 droppings in the 100 step journey to class.

Suppose we take a 1000 step journey, segmented into 100 step legs.

- (3) What is the probability in exactly 1 of these 10 legs, you encounter 63 droppings?
- (4) What is the probability you take at least 5 legs before stepping in 63 droppings in 2 legs?

Variance, Moments, and SD

Definition

The k^{th} moment of a random variable X is defined by

 $E(X^k)$

Variance, Moments, and SD

Definition

The **standard deviation** of a random variable X is denoted SD(X), and defined by

$$SD(X) = \sqrt{Var(X)} = \sigma$$

Example

Let X denote the outcome of a fair six sided die roll. Compute SD(X) and compare to Var(X).

Variance Properties

Theorem

Variance of a linear combination:

$$Var(aX + b) = a^2 Var(X)$$

Example

Suppose that X has variance Var(X) = 2. Compute the standard deviation of Y, where Y = -2X + 3.