Stat 230: Probability

Lecture 22

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Review

Example

Suppose for a random variable X defined on $0, 1, 2$ (0 otherwise), the CDF is given

x	0	1	2
F(x)	0.1	0.4	p

Compute $E(X^3)$

Review

Last time we talked about:

(1) Transformations
(2) Uniform Distribution
(3) Exponential Distribution

For today:

(1) Exponential Distribution
(1) Normal Distribution

Reading: Middle of Chapter 8

Review

- Monday: Quiz 4
- Review

Exponential Distribution

Theorem

Suppose that $X \sim exp(\theta)$. *Then* $E(X) = \theta$, *and* $Var(X) = \theta^2$.

Exponential Distribution

Example

Suppose the time in hours a work computer functions before you must deal with IT is exponentially distribution with mean of 100 hours. Find the probability:

(1) A computer will function for fewer than 100 hours?

(2) A computer will function between 50 and 150 hours before a problem?

(3) If a computer survives more than 100 hours, what is the probability it survives (at least) another 50 hours?

Exponential Distribution

Remark (Memoryless Property)

The **memoryless property** *of the exponential distribution means*

$$P(X > c + b | X > b) = P(X > c)$$

Example

Suppose the lifetime of a seat belt motor in a car follows an exponential distribution with mean 14 years. Compute the probability:

(1) The lifetime of the seat belt motor will last more than 20 years.

(2) If a seat belt motor will last another 6 years given it has lasted 14 years.

Quantile

Definition

The $100 \times q$th percentile (or $100 \times q\%$ quantile) of the distribution of X is the value, c_q, such that

$$F_X(c_q) = q$$

The quantile is the "inverse" of the CDF.

Quantile

Example

Suppose $X \sim exp(5)$, calculate the 25th percentile of the distribution of X.