Instructions

1. **All electronic devices are to be turned off and put away.**
2. **This is a closed-book quiz.**
3. **Answer the questions in the spaces provided.**
4. **Final answers to written questions should be rounded to THREE DECIMAL PLACES or left in fraction form.**
5. **Only question pages will be marked.**
6. **You may tear off the last page and use it for rough work. All pages will be collected at the end of the quiz.**
7. **Only a non-programmable, non-graphical calculator with a pink-tie or blue-goggle sticker will be allowed. No other aids are allowed.**
8. **DO NOT WRITE ON THIS COVER PAGE.**

Marking Scheme:

Questions	**Out of**
1 – 9	9
10	8
11	9
Total	26

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Part 1: Multiple Choice – Each question is worth 1 mark

Please CLEARLY CIRCLE your answer selection on this page. Do NOT simply write your choice next to the question. If you do, the question will not be graded. Each question has one correct answer. Choose the best answer.

1. Which of the following quantities is the largest?

A) 4! B) $\binom{7}{2}$ **C)** $\binom{8}{3}$ D) $7^{(2)}$

- **4! = 24,** $\binom{7}{2}$ **= 21,** $\binom{8}{3}$ **= 56,** $7^{(2)}$ **= 42**

2. A flagpole is to have 4 flags arranged from top to bottom. Three different colours of flags are available (with unlimited quantities of each colour). In how many ways can the flags be arranged if no two adjacent flags have the same colour?

A) 36 **B) 24** C) 16 D) 9

- **We have three tasks to perform (placing flags) with the condition / restriction that adjacent flags cannot be the same colour**
- **The total # of arrangements = 3*2*2*2 = 24**

3. How many five-digit numbers that are greater than 35,000 can be made using the digits 1, 3, 5, 7, and 9, where each digit may be used more than once?

A) 2,250 B) 875 C) 3,000 D) 105 E) None of these.

- **We will consider both cases here:**
- **First Case: The first digit is a 3. We then need to choose the other 4 digits, under the condition that the number is greater than 35,000**
- **I will place the 3 in the first spot and then note the number of choices for the other digits: 3 _(3)__ __(5)___ __(5)____ _(5)___**
- **Second Case: Now, I will consider the case where the first digit is not a 3:**
- **_(3)__ __(5)__ _(5)__ _(5)__ _(5)__ (The first digit can be either a 5, 7, or 9. Then I'm free to choose any of the 5 digits for the other slots.**
- **So, the total number of ways is $3*5^3 + 3*5^4 = 3*(5^3 + 5^4) = 2{,}250$**

4. Consider the following simple game: A fair coin is flipped until tails appears. In general, if tails is flipped on the k^{th} toss (k = 1, 2, 3, ...), then you will receive $\$2^k$. For example, suppose that a fair coin is flipped and the first tail appears on the 3^{rd} flip. You will then receive $\$2^3 = \$8$.

Suppose you were asked to pay $20 to play this game. What is the probability that you would make a profit? **Note:** You would make a profit if you receive more than $20.

A) 1/32 B) 1/64 **C) 1/16** D) 1/4 E) None of these

- **You will make a profit so long as you receive more than $20. We see that this corresponds to seeing a tail in 5 or more flips.**
- **We can determine this probability in terms of the sum of an infinite Geometric series. The required probability is** $\left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^7 + \left(\frac{1}{2}\right)^8 + \cdots$
- **Taking out the common factor in the infinite sum, this gives:**
- $\left(\frac{1}{2}\right)^5 \left[1 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots\right] = \left(\frac{1}{2}\right)^5 \left(\frac{1}{1-\frac{1}{2}}\right) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

5. Three girls and seven boys are to be arranged in a straight line to have a group photo taken. How many different ways can they be arranged if there must be a girl at one end and a boy at the other end?

A) **1,693,440** B) 60,480 C) 80,640 D) 846,720

- **There are two cases here. We can have a boy at one end and a girl at the other. Then switch them. So, place the girl at one end, place the boy at the other, then place the others, and switch. This gives us:**
- **__(3)__ _(8)__ _(7)__ _(6)__ __(5)__ __(4)__ __(3)_ __(2)__ __(1_ __(7)_**
- **__(7)__ _(8)__ _(7)__ _(6)__ __(5)__ __(4)__ __(3)_ __(2)__ __(1)_ __(3)_**

- **We end up with 2*3*7*8! = 1,693,440**

6. In how many ways can 10 different candies be divided between Aidan and Julia if all the candies must be given away, and if Aidan must receive at least two of them?

A) 1,024 **B)** **1,013** C) 20 D) 9

- **There are 2^{10} total ways to divide the candies between Aidan and Julia. We need to subtract of the number of ways in which Aidan receives 0 or 1 piece of candy.**
- **There is one way for him to get no candies. There are 10 ways for him to get 1 candy. So, the number of ways is 2^{10} – 11 = 1013.**

7. A four-letter password must be made from the following set of seven lower-case letters: {a, b, c, k, s, t, x}. No repetition of the letters is allowed. How many passwords containing the letter x are possible?

A) 30 B) 60 C) 120 **D)** **480**

- **Each password contains the letter x. There are 4 spots to put the x. First position, second position, third position, and fourth position. So, let's simplify this and just multiply by 4.**
- **So, we have the letter x chosen. Now, we need to select the other letters (three more tasks).**
- **We have 6 choices for the second task, then 5 choices for the third task, and 4 choices for the fourth task.**
- **This gives us 4*6*5*4 = 480**

8. What is the probability that a student gets a C or D in a particular University Math course if 75% of the class passes and 60% of the class gets at most a C?

Hint: You can define the sample space as S = {A or B, C or D, F}, where F is a failing grade.

A) **0.35** B) 0.65 C) 0.6 D) 0.4

- **From the given information, P($\overline{F}$) = 0.75 = 1 - P(F) => P(F) = 0.25.**
- **Also, 0.60 = P(C or D) + P(F) => P(C or D) = 0.6 – 0.25 = 0.35**

9. Two cards are to be randomly selected from a well shuffled deck of playing cards (without replacement). What is the probability that neither of the cards are hearts?

A) 0.029 **B)** **0.559** C) 0.563 D) 0.176

- **There are 13 hearts in the deck, so there are 39 non-hearts in the deck.**
- **We are choosing two cards without replacement.**
- **So P(neither card is a heart) = (39/52)*(38/51) = 0.559**

Part 2: Written Answer / Short Answer Questions

Final answers can be left in fraction form OR rounded to THREE DECIMAL PLACES.

For full credit, you need to show all necessary work. Unjustified answers will not receive full credit.

10. A fair six-sided die is rolled 10 times. After each roll, the number of dots (pips) showing on the upturned face is recorded, until a final sequence of 10 numbers is obtained.

a. How many outcomes are possible? For full credit, please express your final answer as a number. ***(2 marks)***

- **There are 10 tasks to perform (rolls). There are 6 outcomes for each roll.**
- **By the multiplication rule, there are $6^{10} = 60{,}466{,}176$ possible outcomes.**

b. In how many ways can exactly 4 of the rolls be sixes? For full credit, please express your final answer as a number. ***(3 marks)***

- **We know that exactly 4 of the rolls need to be sixes.**
- **There are $\binom{10}{4}$ ways to arrange the sixes.**
- **Then, for the other 6 rolls, we know that there are 5 possibilities for each.**
- **So, the number of ways that exactly 4 of the rolls are sixes is:**
- $\binom{10}{4} 5^6 = 3{,}281{,}250$

c. In how many ways can exactly 4 of the rolls be threes, exactly 3 of the rolls be fives, and at least 2 of the rolls be sixes? For full credit, please express your final answer as a number.

(3 marks)

- **We know that exactly 4 of the rolls are threes, 3 of the rolls are fives, and at least 2 of the rolls are to be sixes.**
- **We will start by considering the case that there are 2 sixes.**
- **There are $\binom{10}{4}$ ways to arrange the threes.**
- **Of the remaining 6 rolls, there are $\binom{6}{3}$ ways to arrange the fives.**
- **Of the remaining 3 rolls, there are $\binom{3}{2}$ ways to arrange the 2 sixes.**
- **Then, there is a single roll that must be a one, two, or four. There are three ways to roll one of these values, and $\binom{1}{1}$ way to arrange it.**
- **Now, we consider the case where there are 3 sixes.**
- **There are $\binom{10}{4}$ ways to arrange the threes.**
- **Of the remaining 6 rolls, there are $\binom{6}{3}$ ways to arrange the fives.**
- **Of the remaining 3 rolls, there are $\binom{3}{3}$ ways to arrange the 3 sixes. Then, there is a single roll that must be a one, two, or four.**
- **When rolling a fair die 10 times, the total number of ways that exactly 4 of the rolls are threes, 3 of the rolls are fives, and at least 2 of the rolls are sixes is: $3\binom{10}{4}\binom{6}{3}\binom{3}{2}\binom{1}{1} + \binom{10}{4}\binom{6}{3}\binom{3}{3} = 42{,}000$**

11. A bridge hand consists of 13 cards randomly dealt from a standard deck of 52 playing cards.

a. How many possible bridge hands are there? This represents the total number of outcomes in the sample space. For full credit, please express your final answer as a number. ***(2 marks)***

- **There are a total of $\binom{52}{13}$ possible bridge hands. This represents the total number of outcomes in the sample space.**

b. What is the probability that a bridge hand contains exactly 2 aces? For full credit, please express your final answer to three decimal places. ***(2 marks)***

- **From part a., there are a total of $\binom{52}{13}$ = 635,013,559,600 possible bridge hands.**
- **We need 2 aces from the group of four possible aces. Then we need 11 cards from the remaining non-aces.**
- **So, there are a total of $\binom{4}{2}\binom{48}{11}$ possible hands containing exactly 2 aces.**
- **If we let A represent the event that there a bridge hand contains exactly 2 aces, then P(A) = $\frac{\binom{4}{2}\binom{48}{11}}{\binom{52}{13}}$ = 0.213 (to 3 d.p.)**

c. What is the probability that a bridge hand contains exactly 2 aces and no clubs? For full credit, please express your final answer to three decimal places. ***(2 marks)***

- **As noted in part a., there are a total of $\binom{52}{13}$ possible bridge hands.**
- **There are a total of $\binom{3}{2}\binom{36}{11}$ possible hands containing exactly 2 aces and no clubs. Note that we cannot have the ace of clubs in our hand, so we need to select 2 aces from the 3 remaining aces. Then we select 11 cards from the remaining non-aces and non-clubs (24 red cards + 12 spades = 36 remaining cards).**
- **If we let B represent the event that there a bridge hand contains exactly 2 aces and no spades, then P(A) = $\frac{\binom{3}{2}\binom{36}{11}}{\binom{52}{13}}$ = 0.003 (to 3 d.p.)**

d. What is the probability that a bridge hand contains exactly three times as many clubs as spades? For full credit, please express your final answer to three decimal places. ***(3 marks)***

- **As noted in part a., there are a total of $\binom{52}{13}$ possible bridge hands. This represents the total number of outcomes in the sample space.**
- **Now we need to consider all possible cases where there are exactly three times as many clubs as spades, keeping in mind that there are 13 cards in total in the bridge hand.**
- **We can have 3 clubs, 1 spade, and 9 "other" cards. We can also have 6 clubs, 2 spades, and 5 "other" cards. We can also have 9 clubs, 3 spades, and 1 "other" card.**
- **We can use the addition rule to obtain the total number of ways for a bridge hand to contain exactly three times as many clubs as spades.**
- **There are a total of: $\binom{13}{3}\binom{13}{1}\binom{26}{9} + \binom{13}{6}\binom{13}{2}\binom{26}{5} + \binom{13}{9}\binom{13}{3}\binom{26}{1}$ ways.**
- **If we let C represent the event that there are three times as many clubs as spades, then P(C) = $\frac{\binom{13}{3}\binom{13}{1}\binom{26}{9}+\binom{13}{6}\binom{13}{2}\binom{26}{5}+\binom{13}{9}\binom{13}{3}\binom{26}{1}}{\binom{52}{13}}$ = 0.032 (to 3 d.p.)**