Stat 230: Probability

Lecture 27

Jeremy VanderDoes

University of Waterloo

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Review

Example

Suppose that X and Y have a joint probability function defined in the table below:

	f(x,y)	x = 0	x = 2	x = 3
y	0	.1	.2	.05
	2	.3	.1	.25

Find:

(1) $f_Y(2|X=2)$

(2) $f_U(4)$ where $U \sim 2X - Y$

Review

Last time we talked about:

(1) Conditional probability
(2) Multinomial distribution
(3) Functions of joint variables

For today:

(1) Functions of joint variables
(2) Expectation Functions of joint variables
(3) Covariance
(3) Correlation

Review

- Today: Quiz 5
- Next Monday: Quiz 6

Functions

Example

Suppose X and Y are independent, and that $X \sim Pois(\mu)$ and $Y \sim Pois(\lambda)$. Show $X + Y \sim Pois(\mu + \lambda)$.

Functions

Theorem

If $X \sim Binomial(n, p)$ *and* $Y \sim Binomial(m, p)$*, and* X *and* Y *are independent, then* $X + Y \sim Binomial(n + m, p)$*.*

Expectation

Definition

Suppose X and Y are jointly distributed random variables with joint probability function $f(x, y)$. Then for a function $g : \mathbb{R}^2 \to \mathbb{R}$,

$$E(g(X, Y)) = \sum_{(x,y)} g(x, y) f(x, y)$$

More generally, if $g : \mathbb{R}^n \to \mathbb{R}$, and $X_1, ..., X_n$ have joint probability function $f(x_1, ..., x_n)$, then

$$E(g(X_1, ..., X_n)) = \sum_{(x_1,...,x_n)} g(x_1, ..., x_n) f(x_1, ..., x_n)$$

Expectation

Example

Suppose X and Y have joint probability function given by the following table:

	f(x,y)	x = 0	x = 1	x = 2
y	0	.2	.3	.1
	2	.25	.13	.02

Compute $E(XY)$.

Expectation

Remark

Properties of Expected Values of Jointly Distributed Random Variables:

(1) $E[a \cdot g_1(X,Y) + b \cdot g_2(X,Y)] = a \cdot E(g_1(X,Y)) + b \cdot E(g_2(X,Y))$.

(2) $E(X+Y) = \sum_x x f_X(x) + \sum_y y f_Y(y) = E(X) + E(Y)$

Covariance

Definition

If X and Y are jointly distributed, then $Cov(X, Y)$ denotes the **covariance** *between X and Y. It is defined by*

$$Cov(X, Y) = E[(X - E(X))(Y - E(Y))]$$

Remark (Shortcut formula)

As before it can be shown:

$$Cov(X, Y) = E(XY) - E(X)E(Y)$$

Covariance

Example

Suppose X and Y have joint probability function given by the following table:

	f(x,y)	x = 0	x = 1	x = 2
y	0	0.2	0.3	0.1
	2	0.25	0.13	0.02

Compute $Cov(X, Y)$.

Covariance

Theorem

If X and Y are independent, then $Cov(X, Y) = 0$.

The converse statement is FALSE, namely if $Cov(X, Y) = 0$ then X and Y are not necessarily independent. Counter example: Let X satisfy $P(X = -1) = P(X = 0) = P(X = 1) = 1/3$, and let $Y = X^2$.

Correlation

Definition

The **correlation** *of* X *and* Y *is denoted* $corr(X, Y)$*, and is defined by*

$$corr(X, Y) = \rho = \frac{Cov(X, Y)}{SD(X)SD(Y)}$$

It follows from the Cauchy-Schwarz inequality that $-1 \leq corr(X, Y) \leq 1$*, and if* $|corr(X, Y)| = 1$*,* $X = aY + b$*.*