Stat 230: Probability

Lecture 17

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Review

Example

Suppose X is a random variable with probability function

$$f_X(x) = \frac{x}{10} \qquad x = 2, 3, 5$$

If $Z \sim 6X + 4$, what is $E(Z)$?

Review

Last time we talked about:

(1) Variance
(2) Existence of moments

For today:

(1) Variance
(2) Variance of Discrete Distributions

Reading: Chapter 7

Review

- Today: Quiz
- Next Monday: R tutorial
- R Code Modules

Variance Properties

Theorem

Variance of a linear combination:

$$Var(aX + b) = a^2 Var(X)$$

Example

Suppose that X has variance $Var(X) = 2$. Compute the standard deviation of Y, where $Y = -2X + 3$.

Variance Properties

Remark (Properties of the Variance)

(1) *For all random variables X,*

$$Var(X) \geq 0$$

(2) *Larger values of $Var(X)$ indicate that the distribution is more "spread out" around the mean.*

(3) $Var(X) = 0$ *if and only if* $P(X = E(X)) = 1$.

Variance of Discrete Distributions

Theorem (Variance of Binomial)

Suppose that $X \sim Binomial(n, p)$, then

$$Var(X) = np(1-p)$$

Example

Suppose a fair coin is flipped 10,000 times, and let X denote the number of heads observed. Calculate the mean, variance, and standard deviation of X.

Variance of Discrete Distributions

Theorem (Variance of Poisson)

Suppose that $X \sim Poisson(\mu)$, then

$$Var(X) = \mu$$

Example

Suppose you make chocolate cookies with the chocolate chips distributed among the cookies according to a Poisson distribution with 7 chips per cookie on average. What is the expected value, variance, and standard deviation of the number of chips?

Variance of Discrete Distributions

Theorem (Variance of Hypergeometric)

If $X \sim hyp(N, r, n)$, then

$$Var(X) = n\frac{r}{N}\left(1 - \frac{r}{N}\right)\left(\frac{N-n}{N-1}\right)$$

Example

When cleaning your attic, you find a box of Halloween costumes, 5 monsters, 7 superheroes and 3 foods. Suppose you select 3 from the group, without replacement, what is the expected value, variance, and standard deviation of the number of superhero costumes you pick?

Variance of Discrete Distributions

Theorem (Variance of Negative Binomial)

If $Z \sim NB(k, p)$, then

$$Var(Z) = \frac{k(1-p)}{p^2}$$

Example

Suppose the probability you enjoy a random movie is 0.3. What is the expected value, variance, and standard deviation of the number of movies you watch before you find 3 you enjoy?

Considerations

Why do we care about the variance?

- The expectation and the variance give a simple summary of the distribution
- Skewness:

$$E\left[\left(\frac{X - E(X)}{\sqrt{Var(X)}}\right)^3\right] = \frac{E[(X - E(X))^3]}{(E[(X - E(X))^2])^{\frac{3}{2}}}$$

- Kurtosis:

$$E\left[\left(\frac{X - E(X)}{\sqrt{Var(X)}}\right)^4\right] = \frac{E[(X - E(X))^4]}{(E[(X - E(X))^2])^2}$$

$\vdots$

Considerations

Remark

There exist distributions without expectation: Suppose X is a random variable with probability function

$$f_X(x) = \frac{6}{\pi^2}\frac{1}{x^2}, \quad x = 1, 2, \ldots$$

Then $E(X) = +\infty$ and $Var(X)$ is not defined.

Considerations

Other Summary Statistics

- Median(X): Middle value (Half observations above/below. If even number of observations then sum of the two divided by 2)
- Mode(X): Most common value (May have multiple)