Stat 230: Probability

Lecture 6

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Review

Example

Suppose that A and B are independent events with $P(A) = 0.2$ and $P(B) = 0.4$. Compute $P(A \cup B)$.

Review

Last time we talked about

- Inclusion Exclusion
- Independence: A and B are independent if

$$P(A \cap B) = P(A)P(B)$$

Topics for today

- Conditional probability and examples

Reading: Chapter 4

Review

- Reminder: Next tutorial quiz Monday
- Quiz Preparation
  - Review Section 3.6

Conditional Probability

Our ideas/calculation of probabilities may change upon the attainment of some information:

Example

(1) What is the probability it will rain today VS. What is the probability that it will rain today if it is cloudy outside.

(2) What is the chance I will win a LOL game VS. What is the chance I will win a LOL game if my teammate locked Riven top.

(3) What is the probability that the world will end Friday VS. What is the probability the world will end Friday if I eat some cheese.

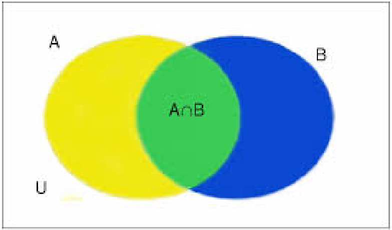
This notion is captured by what is called Conditional Probability.

Conditional Probability

Definition

The conditional probability of A given B is, so long as $P(B) > 0$,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$



Conditional Probability

Example

Consider again rolling two fair six sided dice, and let
$A = \{\text{the sum is } 10\}$, $B = \{\text{the first die is a } 6\}$ $C = \{\text{the sum is } 7\}$.
Determine:

(1) Compute $P(A|B)$

(2) Compute $P(B|A)$

(3) Compute $P(A|C)$

(4) Compute $P(C|B)$

Conditional Probability

Definition (Equivalent definition of independence)

Two events A and B are independent if

$$P(A|B) = P(A).$$

Conditional Probability

Conditional probability behaves the same way as usual probability:

(1) $0 \leq P(A|B) \leq 1$
This follows from the fact that if $A \subset B$ then $P(A) \leq P(B)$

(2) $P(\bar{A}|B) = 1 - P(A|B)$

(3) If A_1 and A_2 are disjoint: $P(A_1 \cup A_2|B) = P(A_1|B) + P(A_2|B)$

(4) $P(S|B) = 1 = P(B|B)$

Conditional Probability

Example

Consider rearranging the letters in the word RACECAR at random to form a word.

(1) What is the probability that the random word ends with an "R" given that the word starts with the three letter sequence "ACE".

(2) Is the event that the word starts with "ACE" independent of the event that it ends with an "R"?

Conditional Probability

Definition

For events A and B,

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$$

This is known as the **product rule**. *It follows directly from the definition of conditional expectation.*

Conditional Probability

Example

Suppose a bag contains 12 red balls and 7 green balls. Suppose that a ball is drawn at random, then, without replacement, a second ball is drawn at random.

(1) What is the probability that both balls are red?

(2) What is the probability that the second ball is red?