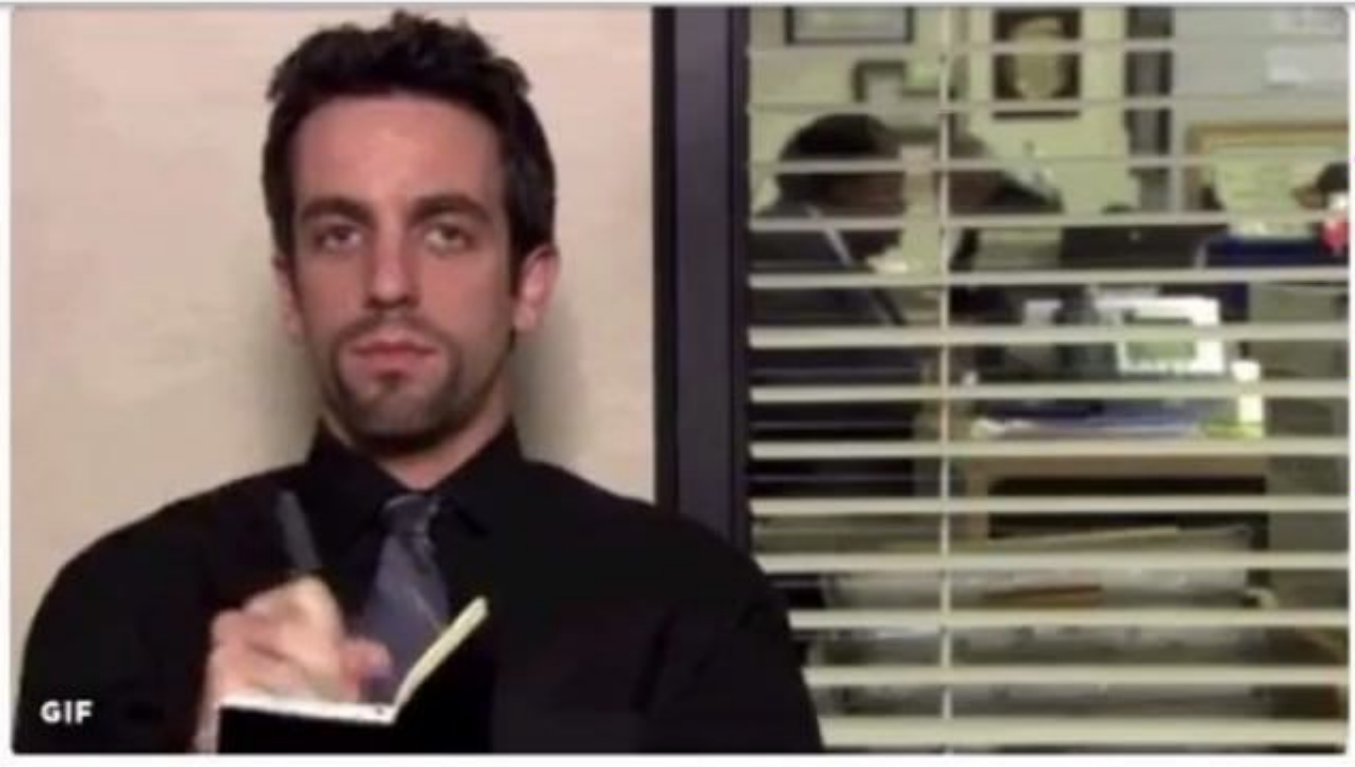
When next Monday is the second Quiz and you're hoping for hints from Hintz



STAT 230

Lec 6

Sec 2

Today's Agenda

Last time:

- Review permutations, binomial coefficient
- Properties of the binomial coefficient, Pascal's triangle
- Birthday problem
- More examples on counting techniques

Today (Lec 6, 05/13):

- Arrangements when objects are of the same type
- More examples and practice

All material from Chapter 3 has been covered after this lecture.

YOU MUST REVIEW SECTION 3.6 YOURSELF! YOU WILL NEED IT!

Example

Consider rearranging the letters at random in the word "HELLOKITTY" to form a single word.

a) How many ways can this be done?

b) What is the probability that all of the letters appear in alphabetic order?

c) What is the probability that the word begins and ends with "T"?

Definition

Consider n objects which consist of k types. Suppose that there are n_1 objects which are of type 1, n_2 which are of type 2, and in general n_i objects of type i. Then there are

$$\frac{n!}{n_1!n_2!\dots n_k!}$$

distinguishable arrangements of the n objects. This quantity is known as a **multinomial coefficient** and denoted by

$$\binom{n}{n_1, n_2, \dots, n_k} = \frac{n!}{n_1!n_2!\dots n_k!}.$$

Summary

a) **Addition rule**: something OR $(+)$ something else

b) **Multiplication rule**: something AND $(\times)$ something else

c) **Factorial**: $n!$ = Number of arrangements of n distinct objects when order matters

d) **Permutation**: $n^{(k)} = \frac{n!}{k!}$ = Number of ways to pick k objects from n distinct objects when order matters

e) **Combination**: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ = Number of ways to choose k objects from n objects when order doesn't matter

f) **Multinomial coefficient**: $\binom{n}{n_1,\ldots,n_k} = \frac{n!}{n_1!\ldots n_k!}$ = Number of ways to arrange n_1 objects of type 1, ..., n_k objects of type k, where $n_1 + n_2 + \cdots + n_k = n$.

Example

Suppose we make a random arrangement of length three of the letters $\{a, b, c, d, e, f, g, h, i, j\}$. What is the probability of the event $B =$ "letters are in alphabetic order" if

a) letters are selected without replacement?

b) letters are selected with replacement?

Example

There are 6 stops on a subway line and 4 passengers on a subway car. Assume the passengers are each equally likely to get off at any stop. Find the probability that

a) the passengers all get off at different stops,

b) 2 passengers get off at stop two and 2 passengers get off at stop five,

c) 2 passengers get off at one stop and the other 2 passengers get off at another same stop.

Example

3 members of the CS department, 2 members of the CO department, and 3 members of the S&AS department sit down at random in a row of 8 seats.

a) What is the probability that each department's members are sitting in consecutive seats?

b) What is the probability that members of the same department are sitting on both ends of the row?