Stat 230: Probability

Lecture 33

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Review

Example

Suppose the the pdf for X is

$$f(x) = \frac{k}{8} \qquad\qquad 1 \leq x \leq 5$$

and 0 otherwise. What is the MGF for X?

Review

Last time we talked about:

(1) Moment generating functions

For today:

(2) Final Review

Review

- Schedule
- No class tomorrow! (Evaluation Day)
- Meetings, emails, Piazza

Final Exam

Final Exam:

- Format:
  - Multiple Choice ($\approx$ 30%, like before)
  - Short Answer Questions (The rest)
  - Bonus Questions
- Coverage: Everything!
  - Slight bias towards material past Midterm 2.
- Calculators: Bring approved calculators!
- Study tips
  - Course schedule
  - Class notes
  - Previous quizzes/midterms
  - Study, and then try the practice final

Review Disclaimer

Attached are many examples. However, this may miss concepts discussed in the course (it took us 12 weeks to cover it after all). All material outlined in the course syllabus is fair game, including section 3.6 that we didn't spend much time on in class.

You know this stuff! If something is asked in a new way, consider drawing pictures, making tables, or listing possible outcomes until you understand it.

Example

Suppose you flip a coin 3 times.

(1) What is a possible sample space? What is a possible event?

(2) Suppose we want to count the number of heads we see. What is the probability we see 2 heads?

Examples

Example

Callie is buying a new phone. She can buy an Android or an iPhone. Among these, there are 4 varieties of Androids, each with 3 colors, and 3 iPhones, each with 2 colors, worth considering.

(1) How many total options does Callie have?

(2) If she decides to narrow her search to 2 Androids and 2 iPhones, how many ways can she do this? What is the probability she would get this if 4 phones are randomly selected?

Examples

Example

Terrence loves alphabet soup. At one point while eating he notices the letters arranged to spell his name. Assume these are the only letters left in the soup.

(1) What are the odds in favor of this happening?

(2) How many ways could the letters be arranged to start with "Te"?

Example

Helga was never one for probability. She finds drawing a picture is really helpful. Suppose she knows $P(A) = 0.3$, $P(B) = 0.6$, and $P(A \cup B) = 0.8$.

(1) Visualize and compute $P(A \cap B)$

(2) Are A and B independent? Mutually exclusive?

Examples

Example

Walter is scared of the disease known as the hokey pokey being played at a party. 23% of the time, it is played if the chicken dance has been played. It is played 68% of the time when the chicken dance has not been played. The chicken dance is played 34% of the time.

(1) What is the probability the hokey pokey will be played?

(2) Suppose his pal Jerry is instead frightened of the chicken dance and is trying to decide if he should leave when Walter leaves due to the hokey pokey. What is the probability the chicken dance will be played given the hokey pokey was played?

Examples

Example

Jesse missed all of class and is frightened for the final. In fear, they reach out and ask about the distributions we named in the course.

(1) Describe the reasons for selecting each of the named distributions.

(2) What are three ways to describe the 'average'?

(3) What are two values we used to describe spread?

Examples

Example

Give the appropriate distributions along with the answers to the following questions

(1) Kathy will work until she makes 4 sells. If she sells to 5% of contacts, and each takes 10 minutes, how long do you expect she will she work?

(2) Oliver has a cookie jar of 12 cookies, 3 chocolate chip, 2 lemon, 5 snicker doodles, and 2 sugar cookies. If he selects 4 cookies, eating them immediately, what is the expected number of lemons eaten?

(3) Paul is tone-deaf. When singing he is equally likely to hit any of 5 possible notes (centered on the correct one). What is the probability he is within 1 note of the correct one? Variance of his singing?

(4) Fran has a bag of candy, with 15 remaining pieces - 7 sweet, 5 sour, and 3 weird ones. If Fran selects 6 pieces and sets them aside, what is the probability Fran selects 3 sweet and 2 weird?

Examples

Example

Dr. Smith has patients coming in according to a Poisson process with 5 patients per 8-hr day.

(1) How many patients are expected per hour?

(2) What is the probability that on 2 of 5 independent days in a week, less than 3 patients come in?

Examples

Example

Suppose we have a random variable X, where $E(X) = 5$ and $Var(X) = 9$. If we also see random variable Y where $Y \sim 2X - 4$

(1) What is $E(Y)$?

(2) What is $E(Y^2)$?

Example

Suppose we have random variable X with pdf $f(x) = \frac{kx^2}{12} + 8$ for $2 \leq x \leq 4$, and 0 otherwise

(1) What is the median of X?

Examples

Example

Solve the following scenarios, along with giving the proper distributions

(1) Jose is terrible at cooking. For example, when recently making a cake, he randomly put in 1-2 cups of flour, with no preference to any amount. What is the probability he puts in more than the necessary 1.25 cups?

(2) Carolyn heard the scores on the last exam were normally distributed with mean 86 and standard deviation of 2. What percentile is her 84 point score?

(3) Gary realizes tears on his previous exam came at a rate of 26 tears a minute. What was the variance of the time between his tears? Given it was 15 seconds since his last tear, what is the probability the next tear will be after 0.1 minutes?

Examples

Example

Suppose we wish to generate an observation x from distribution $f(x) = 1/x^2$ for $x > 1$, 0 for $x \leq 1$.

(1) We generate a u from a $Unif(0, 1)$ with value 0.42. What value of x does this correspond to?

Examples

Example

Suppose we have a X and Y jointly related in the following pf

		x		
	$f_{X,Y}(x,y)$	0	1	3
	0	0.1	0.2	0.2
y	1	0.1	0.1	0.05
	2	0	0.1	0.15

Find:

(1) $P(X > Y)$

(2) The marginal distribution of X

(3) The conditional distribution $f(y|X = 1)$

(4) Determine if X and Y are independent

Examples

Example

Suppose we know $E(X) = 2$, $Var(X) = 4$, $E(Y) = 1$, $Var(Y) = 9$, and $Cov(X, Y) = -1$

(1) What is ρ, the correlation coefficient, between X and Y?

(2) What is $E(3X - 2Y + 4)$ and $Var(3X - 2Y + 4)$?

(3) What is $Cov(X, 3X - 2Y + 4)$?

Examples

Example

Suppose $X \sim G(0,1)$, $Z \sim G(0,1)$ and $Y \sim N(4,9)$, mutually independent.

(1) What is the distribution of W where $W = X + Z + Y$, including all parameters?

(2) Compute $P(W > 5)$.

(3) What is the 41st percentile of W?

Examples

Example

The length of springs produced by a company is assumed to be normally distributed with mean $10cm$ and standard deviation $0.5cm$. Since the ends of the spring do not have the same properties as the middle, a machine cuts off a section at each end of the spring. Suppose the length of the cut-off section is normally distributed with mean $1cm$ and standard deviation $0.2cm$, and is independent of the original length of the spring and the amount cut from the opposite end.

(1) Compute the distribution of the length of the remaining middle section of the spring.

(2) Compute the probability that the remaining middle section of the spring is longer than $7.5cm$.

Examples

Example

In a circuit containing n switches, the ith switch is closed with probability $1/i^2$, $i = 1, \cdots, n$, independently of the rest. Let X be the total number of switches that are closed.

(1) What is $E(X)$?

(2) Suppose this circuit is infinitely large, what is $E(X)$?

Example

A building has 11 floors. Suppose 12 people get into an elevator at floor 1, and each chooses a floor at random to get out (excluding the floor they started on), independently of the others.

(1) At how many floors do you expect the elevator to stop to let out one or more of these 12 people?

Examples

Example

Suppose $X_1, \cdots X_{100}$ are independent and identically distributed continuous random variables such that $X_i \sim F(\mu = 5, \sigma^2 = 9)$–in other words, it is an unknown distribution with $E(X) = 5$ and $Var(X) = \sigma^2$. We define $\bar{X} = \frac{1}{100}\sum_{i=1}^{100} X_i$.

(1) What is the approximate distribution of $\bar{X}$?

(2) Approximate $P(\bar{X} > 5.5)$

Examples

Example

Suppose $Y \sim Pois(30)$.

(1) Give the transformation of this that gets an approximate $N(0, 1)$ random variable

(2) Approximate $P(Y = 30)$ using such a transformation

Examples

Example

In this course we discussed many approximations. For example, how/when/why can we approximate:

(1) A Hypergeometric distribution with a Binomial distribution

(2) A Binomial distribution with a Poisson distribution

(3) A distribution (both discrete and continuous) with a Normal distribution. (Are there special cases?)

What other relations did we see between distributions?

Examples

Example

Suppose we have random variable X with pf

x	-1	0	1	3
f(x)	0.3	0.2	0.1	0.4

(1) Find the MGF of X

Conclusion

Thank you all for a great semester!! Good luck on your exams!