Stat 230: Probability

Lecture 19

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Review

Example

What do we know about the random variable given one of the following (only (1), or only (2), or ...):

(1) The distribution (without parameter values)
(2) The expected value
(3) The variance
(4) The median
(5) The mode

Review

Last we talked about

(1) R Applications

(2) Continuous Random Variables - pdf

For today:

(1) Properties of the CDF

Reading: Middle of chapter 8

Review

- Monday: TA R tutorial
- Definitions
- R Code - Poisson Example

Cumulative Distribution Function

Definition

The CDF of a random variable X is

$$F_X(x) = P(X \leq x)$$

If X is continuous with pdf $f(x)$, then

$$F_X(x) = \int_{-\infty}^{x} f(y)dy$$

Moreover, by the fundamental theorem of calculus,

$$\frac{d}{dx}F_X(x) = f(x).$$

where $f(x)$ is continuous. $F(x)$ is a continuous function.

Cumulative Distribution Function

The CDF for a continuous random variable is much more useful, and less difficult to work with, in terms of computing probabilities related to the random variable:

$$
\begin{aligned}
P(a \leq X \leq b) = F_X(b) - F_X(a) &= \int_{-\infty}^{b} f(y)dy - \int_{-\infty}^{a} f(y)dy \\
&= \int_{a}^{b} f(y)dy.
\end{aligned}
$$

Cumulative Distribution Function

Properties of the CDF of a continuous random variable:

(1) $F(x)$ is defined for all real x.

(2) $F(x)$ is non-decreasing. Plus it's continuous. And it's differentiable except at a countable number of points (which many be 0).

(3) $\lim_{x\to-\infty} F(x) = 0$, $\lim_{x\to\infty} F(x) = 1$.

(4) $P(a \leq X \leq b) = F(b) - F(a)$.

Cumulative Distribution Function

Example

Suppose that X is a continuous random variable with probability density function

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq \sqrt{2}, \\ 0 & \text{otherwise} \end{cases}$$

(1) Compute $F_X(x)$

(2) Graph $F_X(x)$

(3) Compute $P(1/2 \leq X < 1)$.