STAT 230

June 6 Tutorial Questions

Preview

In this tutorial, we will look at the relationship between a pf and cdf for a discrete random variable, and determine how to obtain one from the other.

We will consider three special discrete distributions, the Binomial, Hypergeometric, and Negative Binomial distributions.

Then, we will try to see if we can determine the appropriate distribution to be used for certain probability calculations, playing Name That Distribution!

Problem 1

Suppose you are given the following information regarding the cdf of a discrete random variable, X:

x	0	1	2	3	4	5	6
F(x)	0	0.05	0.2	0.3	0.55	0.8	1

a) Using the information given above, determine $P(2 \le X \le 5)$.

b) Determine the pf of the discrete random variable, X.

c) Verify the calculation in part a) using the pf from part b).

Special Discrete Distribution: Hypergeometric Distribution Set-up

We have a collection of N objects that are classified into two types – “success” types (S) and “failure” types (F). From the N objects, there are r successes and (N-r) failures. Objects are selected **without replacement**.

We let X represent the number of successes obtained in a sample of n objects.

In this case, X has a Hypergeometric distribution with parameters N, r, and n.

The pf of X is given by:

$f(x) = P(X = x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$, for $\max\{0, n - N + r\} \leq x \leq \min\{r, n\}$; 0, otherwise

Problem 2

Fifteen televisions are to be purchased by a local sports bar from a large production run of 500.

Suppose that 450 will last at least 5 years without needing repair.

Find the probability that at least 14 of the 15 televisions will last at least five years without needing to be repaired.

Special Discrete Distribution: Binomial Distribution Set-up

Suppose that we repeat independent Bernoulli trials consisting of two outcomes:

S (success) and F (failure), and P(S) = p in each trial, with $0 < p < 1$.

We are interested in the number of successes in n independent trials, where the probability of success stays constant from trial to trial.

In this case, X has a Binomial distribution. We write X~Binomial(n, p).

The pf of X is given by:

f(x) = P(X = x) = $\binom{n}{x} p^x (1-p)^{n-x}$, for x = 0, 1, 2, ……., n ; 0, otherwise.

Problem 3

A large fast-food chain is having a contest. They have distributed millions of game pieces and will give away thousands of prizes!

Every time a customer purchases a sandwich, side order, and a drink (this is known as a "combo"), they will receive a game piece, for a chance to play and win a prize.

The fast food chain claims that there is a 1 in 5 chance of winning a prize with each game piece.

Note: We will assume that the probability of a win is constant for each game piece, and that the outcomes for each game piece are independent.

a) What is the probability that a customer will win exactly 3 prizes if they purchase 5 combos?

b) What is the probability that a customer will win at least one prize if they purchase 3 combos?

c) How many game pieces are needed so that a customer has at least a 95% chance of winning at least one prize? (Take a guess before this calculation is done!)

Negative Binomial Distribution

Suppose that we repeat independent Bernoulli trials consisting of two outcomes:

S (success) and F (failure), and P(S) = p in each trial, with $0 < p < 1$.

We are interested in the number of failures obtained before the k^{th} success. In this case, X has a Negative Binomial distribution. We can write $X \sim NB(k, p)$.

The pf of X is given by:

$f(x) = P(X = x) = \binom{x+k-1}{x} p^k (1-p)^x$, for x = 0, 1, 2, ……. ; 0, otherwise.

Note: Take a minute and think about the range of the random variable. Remember that X represents the number of failures before the k^{th} success.

Some Comparisons

Binomial and Negative Binomial Distribution Comparisons

These two distributions should be easily distinguishable. They reverse what is specified or known in advance, and what is variable.

Binomial: The number of independent Bernoulli trials is known. However, we don’t know the number of successes until the experiment has been conducted.

Negative Binomial: The number of successes is known in advance. This is k. However, we do not know the number of trials needed to obtain the k successes until the experiment has been conducted.

Problem 4

Suppose that there are two cameras that are identical in appearance. The flash mechanism fails on each camera from time to time though.

In fact, on any given attempt to take a picture, the flash mechanism on camera A fails with probability 0.1, while the mechanism on camera B fails with probability 0.05.

a) Suppose that camera A is selected and the photographer uses it to attempt to take pictures. What is the probability that the flash mechanism works successfully for the second time on the fifth attempt to take a picture?

b) Now suppose that the photographer randomly selects one of the cameras. What is the probability that the photographer needs exactly 10 attempts in order to have exactly 8 of them with the flash working properly? Assume that the 8^{th} came on their 10^{th} attempt.

c) Given that the photographer needed to take exactly 10 attempts in order to have exactly 8 of them with the flash working properly, what is the probability that camera A was used?

Problem 5

Recall from Problem 2: Fifteen televisions are to be purchased by a local sports bar from a large production run of 500.

Suppose that 450 will last at least 5 years without needing repair.

Now, let's use the appropriate distribution to **approximate** the probability that at least 14 of the 15 televisions will last at least five years without needing to be repaired.

Name That Distribution!

Suppose that we are drawing cards from a standard 52 card deck with replacement. That is, the cards are shuffled well after each draw.

We want to calculate the following probabilities. What distribution would be used?

a) If 4 cards are drawn, what is the probability of drawing two kings?

b) What is the probability that the third club will be drawn on the 6^{th} draw?

c) Now, assuming we draw cards <u>without replacement</u>, what is the probability of drawing three hearts in 5 draws?