

Jaypee University of Engineering & Technology, Guna

T-3 (Even Semester 2022)

18B11CI412 –Algorithms and Problem Solving

Maximum Duration: 2 Hours

Maximum Marks: 35

Notes:

1. This question paper has 5 questions.
2. Write relevant answers only.
3. Calculator not allowed.
4. Do not write anything on a question paper (Except your Er. No.).

- Q1. (a) How is Dynamic Programming different from Recursion and Memoization? Illustrate use of Memoization for finding N^{th} Fibonacci number? [03]
- (b) What happens when a top-down approach of dynamic programming is applied to any problem? [02]
- Q2. (a) Design an algorithm to Convert Max-Heap to Min-Heap. Proof the correctness of algorithm using loop invariant method with example. [03]
- (b) Describe the method for analyzing an algorithm and what do you mean by Best case, Average case & Worst-case complexities of an algorithm? [02]
- Q3. (a) The order of a leaf node in a tree B+tree is the maximum number of (value, data record pointer) pairs it can hold. Given that the block size is 1K bytes, the data record pointer is 7 bytes long, the value field is 9 bytes long and a block pointer is 6 bytes long. What is the order of the leaf node? [03]
- (b) Can the master method be applied to solve recurrence $T(N) = 4T(N/2) + N^2 \log N$? Explain why or why not. [02]
- Q4. (a) Four matrices M_1, M_2, M_3 and M_4 of dimensions $p \times q, q \times r, r \times s$ and $s \times t$ respectively can be multiplied in several ways with different numbers of total scalar multiplications. Find the number of scalar multiplications needed, (i) when multiplied as $((M_1 \times M_2) \times (M_3 \times M_4))$, and (ii) when multiplied as $((M_1 \times M_2) \times M_3) \times M_4$. Which combination required the minimum number of scalar multiplication, If $p = 10, q = 100, r = 20, s = 5$ and $t = 80$. [03]

- (b) An algorithm to find the length of the longest monotonically increasing sequence of numbers in an array $A[0:n-1]$ is given below. Let L_i denote the length of the longest monotonically increasing sequence starting at index i in the array. [02]

Initialize $L_{n-1} = 1$

For all i such that $0 \leq i \leq n-2$

$L_i = \begin{cases} 1 + L_{i+1} & \text{if } A[i] < A[i+1] \\ 1 & \text{Otherwise} \end{cases}$

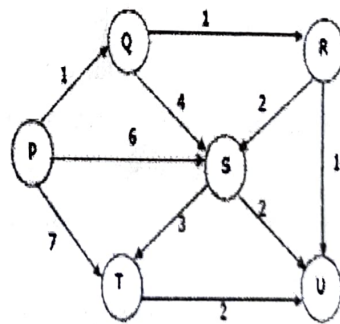
Finally the length of the longest monotonically increasing sequence is $\text{Max}(L_0, L_1, \dots, L_{n-1})$.

Which algorithm design paradigm is used by the above algorithm? Explain

- Q5. (a) Consider the weights (W-in Kgs) and values (V-in Rupees) of four items are given as (10, 60), (7, 28), (4, 20), (2, 24). Note that there is only one unit of each item. The task is to pick a subset of these items such that their total weight is no more than 11 Kgs and their total value is maximized. Moreover, no item may be split. The total value of items picked by an optimal algorithm is denoted by V_{opt} . A greedy algorithm sorts the items by their value-to-weight ratios in descending order and packs them greedily, starting from the first item in the ordered list. The total value of items picked by the greedy algorithm is denoted by V_{greedy} . Find the value of $V_{opt} - V_{greedy}$. [03]

- (b) Explain key factors for preferring B-trees instead of binary search trees for indexing database relations? [02]

- Q6. (a) Suppose we run Dijkstra's single source shortest-path algorithm on the following edge weighted directed graph with vertex P as the source. In what order do the nodes get included into the set of vertices for which the shortest path distances are finalized? [03]



- (b) Given an array where numbers are in range from 1 to n^6 , design an efficient sorting algorithm to sort these numbers in linear time? [02]

Q7. Write short notes on following (with example)

- (a) Greedy choice property Vs Principle of optimality [03]
- (b) Convergence property and Path relaxation property [02]