Semester IV (B.Tech)

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Er. No.....
Academic Year: 2021-22

Jaypee University of Engineering & Technology, Guna

T-1 (Even Semester 2022)

18B11CI412 –Algorithms and Problem Solving

Maximum Duration: 1 Hour

Maximum Marks: 15

Notes:

1. This question paper has 5 questions.
2. Write relevant answers only.
3. Do not write anything on question paper (Except your Er. No.).

Q1. Suppose there are N Jobs in a queue to be done, and each job has its own priority. The job with maximum priority will get completed first than others. At each instance we are completing a job with maximum priority and at the same time we are also interested in inserting a new job in the queue with its own priority. So at each instance we have to check for the job with given priority to complete it and also insert if there is a new job. This task can be very easily executed using a heap by considering N jobs as N nodes of the tree. Write an efficient algorithm to delete a particular job from the data set after completion. (Job index given by user). [03]

Q2. The recurrence relation $T(n) = 7T(n/2) + n^2$ describe the running time of an algorithm A. The algorithm A competes with algorithm X which has a running time of [03]

$T(n) = aT(n/4) + n^2$. What is the largest integer value for a *such* that algorithm X is **asymptotically faster** than algorithm A?

Q3. (a) Consider two algorithms A and B that takes time in $O(n^2)$ and $O(n^3)$, respectively. Could there exist an implementation of algorithm B that would be more efficient (in terms of computing time) than an implementation of algorithm A on all instances? Justify your answer. [02]

(b) Explain why the statement, "The running time of algorithm A is at least $O(n^2)$," is meaningless. [01]

Q4. (a) Prove that if $f(n) \in O(n)$ then $[f(n)]^2 \in O(n^2)$. [02]

$[n, \frac{n}{2}, \frac{n}{4}, \ldots\ldots, 4, 2, 1]$

$2^2 \; 2^1 \; 2^0$

(b) Consider an algorithm that takes time in $\Theta(n^{\log 3})$ to solve instances of size n. Is it correct to say that it takes $O(n^{1.59})$ and $\Omega(n^{1.59})$ in $\Theta(n^{1.59})$? Justify your answers. [01] ½

Q5. (a) Solve given recurrence relation using recursion tree method. 1½ [02]

$T(n) = T(n/4) + T(n/2) + c\,n^2$

(b) How much time does insertion sorting take to sort n distinct items in the best case? State your answer in asymptotic notation. [01] 1

Sol.1
```
Heap (A, n, i)
{
  x = A[i]
  A[i] = A[n]
  Maxheapify (A, i, n-1)
  return x
}
```

1, 2, 4, 8, 16, - - -

$T_n = 2 \times n$

$= 2^4 = 16$

2^0 2^1 2^2 2^3 2^4

1, 2, 4, 8, 16

Sol. 2 49

Sol. 3 a) Not Possible

b) At most

Sol. 4 a) Take $f(n) = kn + \lambda$

Square both sides

b)

Sol. 5 a) $T_n = cn^2 + \frac{5}{16}cn^2 + \left(\frac{5}{16}\right)^2 cn^2 + \ldots\ldots$