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SEMESTER V (B.TECH.)

ACADEMIC YEAR: 2022-23

JAYPEE UNIVERSITY OF ENGINEERING & TECHNOLOGY GUNA

Test-II (Odd Semester 2022)

18B14PH541 INTRODUCTION TO QUANTUM COMPUTING

MAXIMUM DURATION: 1 HOUR 30 MINUTES

MAXIMUM MARKS: 25

IMPORTANT: All the questions are compulsory. The total marks for each question have been indicated next to it.
Note that X, Y, Z are Pauli's spin matrices, H is Hadamard matrix and **I** represents 2×2 unit matrix, symbol $\otimes$ signifies tensor product of two matrices.
In the computational basis $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

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1. Discuss Sequential Stern-Gerlach experiment briefly with necessary schematic diagrams. How outcomes of this experiment form the basis of quantum computation? **[5][CO1]**

2. Let x be a real number and A a matrix such that $A^2 = I$. Show that: **[5][CO2]**

$$\exp(iAx) = \cos(x)I + i\sin(x)A.$$

Further find out matrix representation of the operator defined as $R_y(\alpha) = \exp(-i\alpha Y/2)$, here α is a real number.

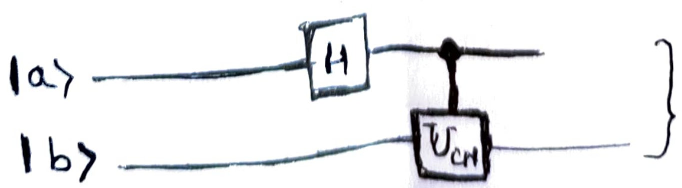
3. In abstract notation define Trace of an operator A, further prove following: **[5][CO2]**
   (a) The trace is cyclic, meaning that $Tr(ABC) = Tr(CAB) = Tr(BCA)$.
   (b) The trace of an outer product is the inner product $Tr(|\psi\rangle\langle\phi|) = \langle\phi|\psi\rangle$.

4. Use the Gram-Schmidt process to construct an orthonormal basis set from: **[5][CO2]**

$$|v_1\rangle = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \quad |v_2\rangle = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \quad \text{and } |v_3\rangle = \begin{pmatrix} 3 \\ -7 \\ 1 \end{pmatrix}.$$

5. Calculate all possible outputs of the following Quantum Circuit: **[5][CO3]**

$a, b \in \{0, 1\}$



$\frac{1}{\sqrt{27}}$

$\frac{1}{\sqrt{3^2 \cdot 3}}$

1+4+1