IT-2K19-72

Singoriya

PAGE NO. :
DATE : / /

NAME : VINAYAK SINGORIYA

BRANCH : M.tech(IT) 4th SEM.

ROLL NO : IT-2K19-72

SUBJECT : NUMERICAL ANALYSIS AND DESIGN.

IT-2K19-72

Singoriya

PAGE NO.:
DATE: / /

Q.1 Find the roots of $f(x) = x^2 - x - 1 = 0$

Ans-

(i) Bisection Method :-

$f(x) = x^2 - x - 1 = 0$, $e = 0.0001$

Let $a = 1,\ b = 2$, $f(1) = -1$, (-ve)

$f(2) = 1$, (+ve)

$f(1) \times f(2) < 0$ => Root will lie b/w 1 and 2

Iteration	a	b	m=(a+b)/2	f(m)	f(a)·f(m)
1	1.000	2.000	1.50	-0.25	-ve
2	1.50	2.000	1.75	0.3125	+ve
3	1.50	1.75	1.625	0.0156	+ve
4	1.50	1.625	1.5625	-0.0538	-ve
5	1.5625	1.625	1.5937	-0.0201	-ve
6	1.5937	1.625	1.5609	-0.0201	-ve
7	1.609	1.625	1.609	-0.0023	-ve
8	1.617	1.625	1.617	0.0066	ve
9	1.617	1.625	1.619	0.0021	ve
10	1.617	1.619	1.618	-0.00007	-ve

$\boxed{\text{Root, } m = 1.618}$

(ii) Regula Falsi method :-

$f(x) = x^2 - x - 1 = 0$

Let $a = 1.000$ and $b = 2.000$

$\left.\begin{array}{l} f(1) < 0 \\ f(2) > 0 \end{array}\right\}$

$f(1) \cdot f(2) < 0$ => Root will lie b/w 1 & 2

Iteration	a	b	$m = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)}$	f(m)
1	1.000	2.000	1.500	-ve
2	1.5000	2.000	1.600	-ve
3	1.600	2.000	1.6153	-ve
4	1.6153	2.000	1.6176	-ve
5	1.6176	2.000	1.6179	-ve
6	1.6179	2.000	1.6180	-ve
7	1.6180	2.000	1.61803	-ve

$\boxed{\text{Root} = 1.6180}$

(iii) Secant Method :-

$f(x) = x^2 - x - 1 = 0$

Let $a = 1, b = 2$, $f(1) < 0$, $f(2) > 0$, $f(1) \cdot f(2) < 0$

Iteration	a	b	m	f(m)
1	1.000	2.000	1.500	-ve
2	2.000	1.500	1.600	-ve
3	1.500	1.600	1.6190	+ve
4	1.600	1.6190	1.6180	-ve
5	1.6190	1.61803	1.61803	-ve

$\boxed{\text{Root} = 1.6180}$

(iv) Newton Raphson Method :-

$f(x) = x^2 - x - 1 = 0$

Let $b = 2$.

$m = b - \frac{f(b)}{f'b}$

PAGE NO.:
DATE: / /

Iteration	b	f(b)	f'(b)	$m = b - \frac{f(b)}{f'(b)}$
1	2.00	1.000	3.000	1.6667
2	1.6667	0.1111	2.333	1.6190
3	1.619	0.0023	2.2381	1.6180
4	1.6180	0	2.2361	1.6180

$\boxed{\text{Root} = 1.6180}$

(v) Fixed Point Iteration Method :-

$f(x) = x^2 - x - 1 = 0$

$x = g(x)$

$x = (x+1)^{1/2}$

Iteration	b	$m = \sqrt{b+1}$	$\lvert b - m\rvert$
1	1.000	1.4142	0.4142
2	1.4142	1.5538	0.1396
3	1.5538	1.5979	0.0446
4	1.5979	1.6118	0.0139
5	1.6118	1.6161	0.0043
6	1.6161	1.6174	0.0013
7	1.6174	1.6178	0.0004
8	1.6178	1.6179	0.0001
9	1.6179	1.6179	0.0000

$\boxed{\text{Root} = 1.6179}$

PAGE NO.:
DATE: / /

Q2 Apply Interpolation using data (5, 150), (7, 392), (11, 1452), (13, 2366), (17, 5202) and evaluate f(9).

Ans::

1) Langrange Interpolation :-

x	5	7	11	13	17
y	150	392	1452	2366	5202

$$f(9) = \frac{(9-7)(9-11)(9-13)(9-17)}{(5-7)(5-11)(5-13)(5-17)} \times 150 +$$

$$\frac{(9-5)(9-11)(9-13)(9-17)}{(7-5)(7-11)(7-13)(7-17)} \times 392 + \frac{(9-5)(9-7)(9-13)(9-17)}{(11-5)(11-7)(11-13)(11-17)} \times 1452$$

$$+ \frac{(9-5)(9-7)(9-11)(9-17)}{(13-5)(13-7)(13-11)(13-17)} \times 2366 + \frac{(9-5)(9-7)(9-11)(9-13)}{(17-5)(17-7)(17-11)(17-13)} \times 5202$$

$$f(9) = \frac{-50}{3} + \frac{8 \times 392}{5 \times 3} + \frac{3 \times 1452}{3 \times 3} + \frac{5202}{15 \times 3}$$

$$f(9) = 810$$

2) Divided Difference Interpolation :-

x	5	7	11	13	17
y	150	392	1452	2366	5202

Divided Difference Table :-

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
5	150				
~~7~~		121			
7	392		24		
		265		1	
11	1452		32		0
		457		1	
13	2366		42		
		709			
17	5202				

By divided difference formula

$$f(9) = f(5) + (9-5)\,\Delta f(5) + (9-5)(9-7)\,\Delta^2 f(5) + (9-5)(9-7)(9-11)\,\Delta^3 f(5)$$

$$f(9) = 150 + 4\times 121 + 8\times 24 + (-16)\times 1$$

$$f(9) = 810$$

(3) Forward Interpolation / Backward Interpolation :-
$\because$ the data is not equidistant, so this method cannot be used to determine values.

Q3 Apply Numerical Differentiation technique on function from (5,150), (7, 392), (11, 1452), (13, 2366), (17, 5202).

Ans:

(i) Method based on Interpolation:

Divided Difference Table.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
5	150				
		121			
7	392		24		
		265		1	
11	1452		32		0
		457		1	
13	2366		42		
		709			
17	5202				

By Newton's Divided Difference Interpolation formula

$y = 150 + (x-5)121 + (x-5)(x-7)24 + (x-5)(x-7)(x-11) \times 1$

$y = x^3 + x^2$

Now differentiating with respect to x

$y' = 3x^2 + 2x$

$y'' = 6x + 2$

So, first order derivative of function is $3x^2 + 2x$ and second order derivative is $6x+2$

PAGE NO.:
DATE: / /

• By Langrange's Interpolation :-

$$f(x) = \frac{(x-7)(x-11)(x-13)(x-17)}{(5-7)(5-11)(5-13)(5-17)} \times 150 +$$

$$\frac{(x-5)(x-11)(x-13)(x-17)}{(7-5)(7-11)(7-13)(7-17)} \times 392 + \frac{(x-5)(x-7)(x-13)(x-17)}{(11-5)(11-7)(11-13)(11-17)} \times 1452$$

$$\frac{(x-5)(x-7)(x-11)(x-17)}{(13-5)(13-7)(13-11)(13-17)} \times 2366 + \frac{(x-5)(x-7)(x-11)(x-13)}{(17-5)(17-7)(17-11)(17-13)} \times 5202$$

$f(x) = x^3 + x^2$

Now differentiating with respect to x

$y' = 3x^2 + 2x$

$y'' = 6x + 2$

∴ First order derivative is $3x^2 + 2x$ and second order derivative is $6x + 2$.

2) Method based on Finite Difference :-

The method based on finite difference i.e. forward and backward interpolation methods are not applicable because the interval are not equidistant and finite difference uses that are equidistant. Hence not applicable.

Scanned by TapScanner

Q4) Derive formulas for Algorithm for Numerical Quadrature and obtain Trapezoidal, Simpson's 1/3, 3/8 and Weddle's rule from it.

Ans:-

(i) General Quadrature Formula :-

$$I = \int_a^b f(x) \cdot dx$$

expressing in term of n putting $x = a + hu$
at $x = a$, $u = 0$ & $x = a + nh$, $u = n$
$dx = h \cdot du$

$$\int_a^{a+nh} f(x) \cdot dx = \int_0^n f(a+hu) \cdot h\,du$$

$$\int_a^{a+nh} f(x) \cdot dx = h \cdot \int_0^n f(a+hu) \cdot du$$

Expanding using Newton's Forward difference Interpolation Formula :-

$$= h \int_0^n \left[f(a) + \frac{u}{\underline{|1}} \Delta f(a) + \frac{u(u-1)}{\underline{|2}} \Delta^2 f(a) + \frac{u(u-1)(u-2)}{\underline{|3}} \Delta^3 f(a) + \dots \right] du$$

$$= h \left[f(a) \cdot u + \Delta f(a) \frac{u^2}{2} + \frac{\Delta^2 f(a)}{\underline{|2}} \left(\frac{u^3}{3} - \frac{u^2}{2} \right) + \left(\frac{u^4}{4} - u^3 + u^2 \right) \frac{\Delta^3 f(a)}{\underline{|3}} \right.$$

$$\Rightarrow \int_a^{a+nh} f(x) \cdot dx = h \left[n f(a) + \frac{n^2}{2} \Delta f(a) + \left(\frac{n^3}{3} - \frac{n^2}{2} \right) \frac{\Delta^2 f(a)}{\underline{|2}} + \right.$$

$$\left. \left(\frac{n^4}{4} - n^3 + n^2 \right) \frac{\Delta^3 f(a)}{\underline{|3}} \dots \right] \quad \text{---} \quad (1)$$

This is the required General Quadrature Formula.

Scanned by TapScanner

PAGE NO.:
DATE: / /

- Derivation of Trapezoidal from general Quadrature formula :-

Put $n = 1$ in eqn. (1) and ignore higher power terms.

$$\int_a^{a+h} f(x).dx = h\left[f(a) + \frac{1}{2}\Delta f(a)\right]$$

$$\int_a^{a+h} f(x).dx = h\left[f(a) + \frac{1}{2}\left(f(a+2h) - 2f(a+h) + f(a)\right)\right]$$

$$\int_a^{a+h} f(x).dx = h \cancel{h}\left[f(a) + \frac{f(a+h) - f(a)}{2}\right]$$

$$\int_a^{a+h} f(x).dx = \frac{h}{2}\left[f(a) + f(a+h)\right]$$

- Derivation of simpson's 1/3 rule from general Quadrature formula :-

Putting $n = 2$ in eqn (1) and ignoring higher terms

$$\int_a^{a+2h} f(x)dx = h\left[2f(a) + \frac{2^2}{2}\Delta f(a) + \left(\frac{2^3}{3} - \frac{2^2}{2}\right)\frac{\Delta^2 f(a)}{2}\right]$$

$$= h\left[2f(a) + 2\Delta f(a) + (8/3 - 2)\frac{\Delta^2 f(a)}{2}\right]$$

$$= \frac{h}{3}\left[6f(a+h) + \Delta^2 f(a)\right]$$

$$= \frac{h}{3}\left[6f(a+h) + f(a+2h) - 2f(a+h) + f(a)\right]$$

$$\int_a^{a+2h} f(x).dx = \frac{h}{3}\left[f(a) + 4f(a+h) + f(a+2h)\right]$$

PAGE NO. :
DATE : / /

- Derivation of Simpson's 3/8 Rule from General Quadrature formula :-

Putting $n = 3$ in eqn. ① and ignoring higher terms

$$\int_a^{a+3h} f(x)\cdot dx = h\Big[3f(a) + \frac{9}{2}\{f(a+h) - f(a)\} + \left(\frac{27}{3} - \frac{9}{2}\right)\{f(a+2h) - 2f(a+h) + f(a)\} + \left(\frac{81}{4} - 27 + 9\right)\{f(a+3h) - 3f(a+2h) + 3f(a+h) - f(a)\}$$

$$\int_a^{a+3h} f(x)\cdot dx = \frac{3h}{8}\Big[f(a) + 3f(a+h) + 3f(a+2h) + f(a+3h)\Big]$$

- Derivation of Weedle's Rule from General Quadrature formula :-

Putting $n = 6$ in eqn ① and ignoring higher terms of Δ^6

$$\int_a^{a+6h} f(x)\cdot dx = h\Big[6f(x) + \frac{6^2}{2}\big(f(a+h) - f(a)\big) + \left(\frac{6^3}{3} - \frac{6^2}{2}\right)\big(f(a+2h) - 2f(a+h) + f(a)\big) + \left(\frac{6^4}{4} - 6^3 + 6^2\right)\big(f(a+3h) - 3f(a+2h) + 3f(a+h) - f(a)\big) + \left(\frac{6^5}{5} - \frac{3}{4}(6^4) + \frac{8}{3}(6)^3 - 3(6^2)\right)\big(f(a+4h) -$$

$$\Rightarrow \int_a^{a+6h} f(x)\cdot dx = \frac{3h}{10}\Big[f(a) + 5f(a+h) + f(a+2h) + 6f(a+3h) + f(a+4h) + 5f(a+5h) + f(a+6h)\Big]$$

2) Newton Cotes Formula :-

$$I = \int_a^b f(x) \cdot dx = \int_a^b y\, dx$$

Put $x = a + hu$

at $x = a$ $\quad u = 0$

$x = a + nh$ $\quad u = n$

$dx = du$

$$y = L_0(x) * y_0 + L_1(x) * y_1 + \dots + L_i(x) * y_i$$

$$y = \sum_{i=0}^{n} L_i(x) * y_i$$

$$I = \sum_{i=0}^{n} y_i \int_a^b L_i(x) \cdot dx$$

This is Newton Cotes Formula

$$I = \sum_{i=0}^{n} y_i \int_a^n L_i (a + hu)\, h \cdot du$$

$$I = h \sum_{i=0}^{n} y_i \int_a^n L_i (a + hu)\, du$$

multiplying and dividing by n

$$I = nh \sum_{i=0}^{n} y_i \frac{1}{n} \int_0^n L_i(u) \cdot du$$

$$I = nh \sum_{i=0}^{n} y_i \, {}^{n}C_i$$

Scanned by TapScanner

PAGE NO.:
DATE: / /

- Derivation of Trapezoidal Rule from Newton cotes formula.

Put $n=1$

$I = h \sum_0^1 {}^1C_i y_i$ where ${}^1C_i = \int_0^1 L_i(u) \cdot du$

$I = h \left[{}^1C_0 y_0 + {}^1C_1 y_1 \right]$

$I = h \left[\int_0^1 L_0(u) \cdot du * y_0 + \int_0^1 L_1(u) \cdot du * y_1 \right]$

$I = h \left[\int_0^1 \frac{u-1}{0-1} * du * y_0 + \int_0^1 \frac{u-0}{1-0} du * y_1 \right]$

$I = h \left[\frac{1}{2} y_0 + \frac{1}{2} y_1 \right]$

$I_1 = \frac{h}{2} [y_0 + y_1]$

$I_2 = \frac{h}{2} [y_1 + y_2]$

$I_3 = \frac{h}{2} [y_2 + y_3]$

⋮

$I_n = \frac{h}{2} [y_{n-1} + y_n]$

$I = I_1 + I_2 + I_3 + \cdots I_n$

$I = \frac{h}{2} [y_0 + y_n + 2(y_1 + y_2 + y_3 + \cdots y_{n+1})]$

This is the required Trapezoidal Rule

PAGE NO.:
DATE: / /

Q.5 Apply pivoting and solve system of linear equations

$x+2y+3z=14$, $3x+2y+z=10$, $2x+3y+2z=14$.

Ans:-

(a) Direct Method - Gauss Elimination Method

→ convert equations in augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 14 \\ 3 & 2 & 1 & 10 \\ 2 & 3 & 2 & 14 \end{array}\right]$$

by Row transformation, convert augmented matrix into upper triangular matrix.

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 10 \\ 0 & 4/3 & 8/3 & 32/3 \\ 0 & 0 & -2 & -6 \end{array}\right]$$

Now modified equations are

$3x+2y+z=10$ — (1)

$4/3y+\frac{8}{3}z=\frac{32}{3}$ — (2)

$-2z=-6$ — (3)

from eqn (3) $z=\frac{-6}{-2} \Rightarrow \boxed{z=3}$

from eqn (2)

$\frac{4}{3}y+\frac{8}{3}\times 3=\frac{32}{3}$

$\boxed{y=2}$

PAGE NO.:
DATE: / /

Putting y & z in eqn (i)

$$3x+2\times2+3=10$$

$$\boxed{x=1}$$

(b) Iterative Method :-

(i) Jacobi Method -

Arrange given equation such that it should be in diagonally Row dominant

$$3x+2y+z=10$$
$$2x+3y+2z=14$$
$$x+y+3z=14$$

$$x=\tfrac{1}{3}(10-2y-z)$$
$$y=\tfrac{1}{3}(14-2x-2z)$$
$$z=\tfrac{1}{3}(14-x-2y)$$

Initially $x=y=z=0$

→ 1st Iteration

$$x_1=\tfrac{1}{3}(10-2\times0-0)=3.33$$
$$y_1=2/3(14-2\times0-2\times0)=4.667$$
$$z_1=\tfrac{1}{3}(14-0-0)=4.667$$

→ 2nd Iteration

$$x_2=\tfrac{1}{3}(10-2(4.667)-4.667)=-1.333$$
$$y_2=\tfrac{1}{3}(14-2(3.33)-2(4.667))=-0.667$$
$$z_2=\tfrac{1}{3}(14-(3.33)-2(4.667))=0.444$$

$\rightarrow$ 3rd iteration:-

$x_3 = \frac{1}{3}(10-2(-0.667)-(0.447)) = 3.63$

$y_3 = 1/3\,[14-2(-1.333)-2(0.444)] = 5.259$

$z_3 = 1/3\,[14-(-1.333)-2(-0.667)] = 5.556$

$\rightarrow$ 4th Iteration:-

$x_4 = 1/3\,[10-2(5.259)-(5.556)] = -2.025$

$y_4 = 1/3\,[14-2(3.63)-2(5.556)] = -1.457$

$z_4 = 1/3\,[14-(3.63)-2(5.259)] = -0.049$

$\rightarrow$ 5th Iteration:-

$x_5 = 1/3\,[10-2(-1.457)-(-0.049)] = 4.321$

$y_5 = 1/3\,[14-2(-2.025)-2(-0.049)] = 6.049$

$z_5 = 1/3\,[14-2(-2.025)-2(-0.049)] = \cancel{6.049}\ 6.313$

$\rightarrow$ 6th Iteration:-

$x_6 = 1/3\,[10-2(6.049)-(6.313)] = -2.804$

$y_6 = 1/3\,[14-2(4.321)-2(6.313)] = -2.422$

$z_6 = 1/3\,[14-(4.321)-2(6.049)] = -0.807$

Hence the solution is diverging for the given system for Jacobi's Method.

PAGE NO. :
DATE : / /

(ii) Gauss Seidel Method :-

$x = 1/3\ (10 - 2y - z)$

$y = 1/3\ (14 - 2x - 2z)$

$z = 1/3\ (14 - x - 2y)$

Initially $x = y = z = 0$

→ First Iteration

$x_1 = 1/3\ (10 - 2\times0 - 0) = 3.333$

$y_1 = 1/3\ (14 - 2\times(3.333) - 0) = 2.444$

$z_1 = 1/3\ (14 - 3.333 - 2(2.444)) = 1.926$

→ 2nd Iteration

$x_2 = 1/3\ [10 - 2(2.444) - (1.926)] = 1.062$

$y_2 = 1/3\ [14 - 2(1.062) - 2(1.926)] = 2.675$

$z_2 = 1/3\ [14 - (1.0625) - 2(2.675)] = 2.529$

→ 3rd Iteration

$x_3 = 1/3\ (10 - 2(2.675) - 2.529) = 0.707$

$y_3 = 1/3\ (14 - 2(0.707) - 2(2.529)] = 2.509$

$z_3 = 1/3\ [14 - 0.707 - 2(2.509)] = 2.758$

→ 4th Iteration

$x_4 = 1/3\ [10 - 2(2.509) - 2.758] = 0.741$

$y_4 = 1/3\ [14 - 2(0.741) - 2(2.758)] = 2.334$

$z_4 = 1/3\ [14 - (0.741) - 2(2.334)] = 2.864$

we will iter upto 8 iteration, and we will get

$x = 0.986 \simeq 1$

$y = 2.018 \simeq 2$

$z = 2.99 \simeq 3$

Scanned by TapScanner

PAGE NO.:
DATE: / /

Q.7) Solve ordinary differential equation $dy/dx = x+y$; $y(0)=1$ using

(i) Euler Method :-

Let $h = 0.25$ and we have to find value at $x=1$
we know formula $y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$

$y_1 = y_0 + h[x_0 + y_0] = 1 + 0.25(0+1) = 1.25$
$y_2 = y_1 + h[x_1 + y_1] = 1.25 + 0.25(1.5) = 1.625$
$y_3 = y_2 + h[x_2 + y_2] = 1.625 + 0.25(2.125) = 2.1562$
$y_4 = y_3 + h[x_3 + y_3] = 2.1562 + 0.25(2.906) = 2.882$
hence $y(1) = 2.882$

x	0	0.25	0.5	0.75	1
y	1	1.25	1.625	2.1562	2.882

(ii) Runge Kutta Method (4th order)

$\frac{dy}{dx} = x+y$

$y(0) = 1$

Let $h = 0.25$, and we have to find $y(1)$

$k_1 = hf(x_0, y_0) = 0.25 \times 1 = 0.25$
$k_2 = hf(x_0 + h/2, y_0 + k_1/2) = 0.25(1.25) = 0.3125$
$k_3 = hf(x_0 + h/2, y_0 + k_2/2) = 0.25(1.2812) = 0.3203$
$k_4 = hf(x_0 + h/2, y_0 + k_3) = 0.25(1.5703) = 0.3925$

$k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6} = \frac{0.25 + 2\times0.3125 + 2\times0.3203 + 0.392}{6}$

$k = 0.3180$

PAGE NO. :
DATE : / /

Now $y_1 = y_0 + K = 1 + 0.3180 = 1.318$

$y(0.25) = 1.318$

Again repeat the same Process again

$K_1 = 0.392$

$K_2 = 0.4723$

$K_3 = 0.482$

$K_4 = 0.5752$

$K = \frac{K_1 + 2K_2 + 2K_3 + K_4}{6} = \cancel{0.479}$

$K = \cancel{0.7}\ 0.4794$

$y_2 = y_1 + K = 1.318 + 0.4794 = 1.7974$

$y(0.5) = 1.7974$

Again

$K_1 = 0.5743$

$K_2 = 0.6774$

$K_3 = \cancel{K_4}\ 0.6903$

$K_4 = 0.809$

$K = \frac{K_1 + 2K_2 + 2K_3 + K_4}{6}$

$y_3 = y_2 + K = 2.4839$

$y(0.75) = 2.4839$

Again

$k_1 = 0.808$

$k_2 = 0.9408$

$k_3 = 0.957$

$k_4 = 1.1203$

$y_4 = y_3 + k = 3.436$

$y(1) = 3.436$

Hence the value at $x = 1$ is 3.436.

(iii) Picard's method :-

$$\frac{dy}{dx} = x + y \quad ; \quad y(0) = 1$$

$$dy = (x+y)\,dx$$

Integrating both sides.

$$\int_{y_0}^{y} dy = \int_{x_0}^{x} (x+y)\,dx$$

$$y - y_0 = \int_{x_0}^{x} (x+y)\,dx$$

$$y_1 = y_0 + \int_{x_0}^{x} (x + y_0)\,dx$$

$$y_1 = 1 + \int_{0}^{x} (x+1)\,dx$$

$$y_1 = 1 + x + \frac{x^2}{2}$$

Now

$$y_2 = y_0 + \int_{x_0}^{x} (x + y_1)\,dx$$

$$y_2 = 1 + \int_{0}^{x} x + 1 + x + \frac{x^2}{2}\,dx$$

$$y_2 = 1 + \int_{0}^{x} 1 + 2x + \frac{x^2}{2}\,dx$$

PAGE NO.:
DATE: / /

$$y_2 = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$$

Now,

$$y_3 = y_0 + \int_{x_0}^{x} (x + y_2)$$

$$y_3 = 1 + \int_0^x \left(x + 1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right) dx$$

$$y_3 = 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{24}$$

Now,

$$y_4 = 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{120} + \cdots$$

$y(1)$

$1 + 1 + 1 + 1/3 + 1/24$

$= 3.3756.$

(iv) Taylor series Method :-

$$\frac{dy}{dx} = x + y \quad ; \quad y(0) = 1$$

$$h = 0.25 \quad ; \quad y(1) = ?$$

$y' = x + y$ $\qquad$ $y_0' = x_0 + y_0 = 1$

$y'' = 1 + y'$ $\qquad$ on substituting $\qquad$ $y_0'' = 1 + y_0' = 2$

$y''' = y''$ $\qquad$ $y_0''' = y'' = 2$

$y''' = y^{iv}$ $\qquad$ $y_0''' = y''' = 2$

Putting the above obtained values in ~~eqn~~ taylor series.

PAGE NO.:
DATE: / /

$$y_1 = y_0 + h y_0' + \frac{h^2}{\lfloor 2} y_0'' + \frac{h^3}{\lfloor 3} y_0''' + \frac{h^4}{\lfloor 4} y_0^{iv} + \ldots$$

$$y_1 = 1 + 0.25 + \frac{(0.25)^2}{2} \times 2 + \frac{(0.25)^3}{6} \times 2 + \frac{(0.25)^4}{24} \times 2$$

$y_1 = 1.318$

$y(0.25) = 1.318$

Repeat the same Process with x_1, y_1 in place of x_0, y_0

$y_1' = x_1 + y_1 = 1.568$
$y_1'' = 1 + y_1' = 2.568$
$y_1''' = y_1'' = 2.568$
$y_1^{iv} = y_1''' = 2.568$

Putting in taylor series

$$y_2 = 1.318 + 0.25 \times 1.568 + \frac{(0.25)^2}{2}(2.568) + \frac{(0.25)^3}{6}(2.568) + \frac{(0.25)^4}{24} \times 2.568$$

$y_2 = 1.7974$
$y(0.5) = 1.7974$

Again
$y_2' = x_2 + y_2 = 2.2974$
$y_2'' = 1 + y_2' = 3.2974$
$y_2''' = y_2'' = 3.2974$
$y_2^{iv} = y_3'' = 3.2974$

Scanned by TapScanner

PAGE NO.:
DATE: / /

$$y_3 = 1.7974 + 0.25(2.2974) + \frac{(0.25)^2}{2} \times 3.2974$$

$$+ \frac{(0.25)^3}{6} \times 3.2974 + \frac{(0.25)^4}{24}(3.2974)$$

$y_3 = 2.4839$

$y(0.75) = 2.4839 \simeq 2.484$

Again

$y_3' = 3.234$

$y_3'' = 4.234$

$y_3''' = 4.234$

$y_3^{iv} = 4.234$

$$y_4 = 2.484 + 0.25(3.234) + \frac{(0.25)^2}{2}(4.234) + \frac{(0.25)^3}{6}(4.234)$$

$$+ \frac{(0.25)^4}{24}(4.234)$$

$y_4 = 3.436$

$y(1) = 3.436$