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DAA

Tutorial-1

Q1 What do you understand by asymptotic notation. Define different asymptotic notation with examples.

Solⁿ i) **Big O(n)**

$f(n) = O(g(n))$

iff

$f(n) \leq c\, g(n)$

$\forall\ n \geq n_0$

for some constant, $c > 0$

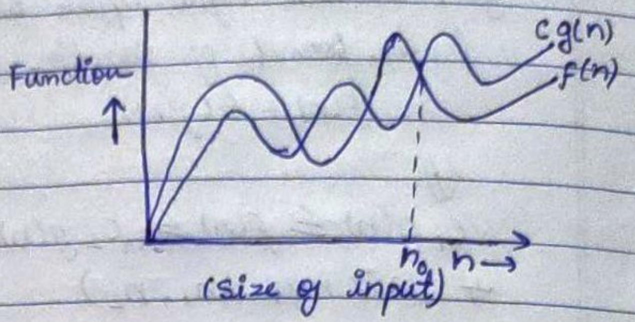
g(n) is "tight" upper bound of f(n)

Ex:- $f(n) = n^2 + n$

$g(n) = n^3$

$n^2 + n \leq c.n^3$

$n^2 + n = O(n^3)$



ii) **Big Omega (Ω)**

$f(n) = \Omega(g(n))$

g(n) is "tight" lower bound of function f(n)

$f(n) = \Omega(g(n))$

iff

$f(n) \geq c\, g(n)$

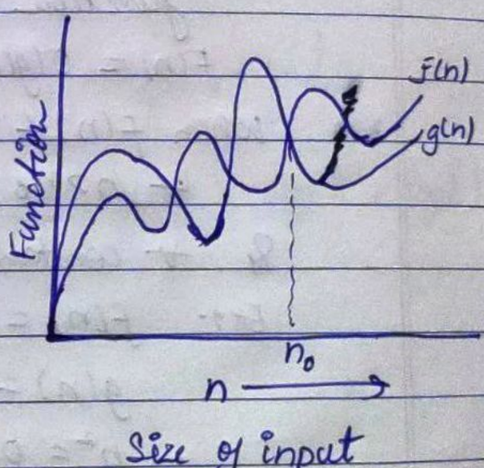
$\forall\ n \geq n_0$

For some constant $c > 0$

Ex:- $f(n) = n^3 + 4n^2$

$g(n) = n^2$

$n^3 + 4n^2 = \Omega(n^2)$



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(iii) **Big Theta (Θ)**

$f(n) = \Theta(g(n))$

g(n) is both "tight" upper bound and lower bound of function f(n)

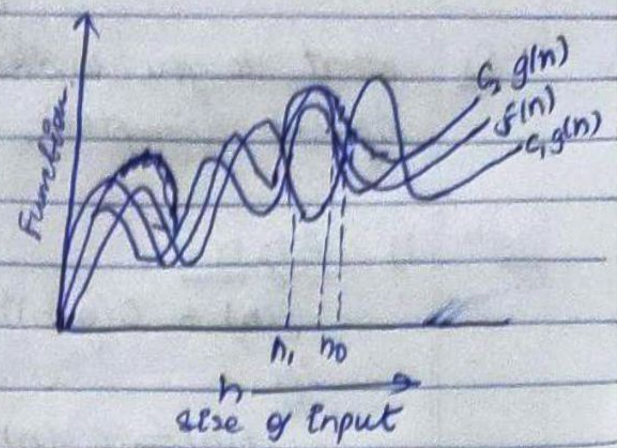
$f(n) = \Theta(g(n))$

iff

$c_1 g(n) \le f(n) \le c_2 g(n)$

$\forall\ n \geq \max(n_1, n_2)$

for some constant $c_1 > 0$ & $c_2 > 0$



Ex:- $3n+2 = \Theta(n)$ as $3n+2 \geq 3n$ &

$3n+2 \le 4n$ for n, $K_1 = 3$, $K_2 = 4$, & $n_0 = 2$

(iv) **Small o(θ)**

$f(n) = o(g(n))$

g(n) is upper bound of function f(n)

$f(n) = o(g(n))$

when $f(n) < c\,g(n)$

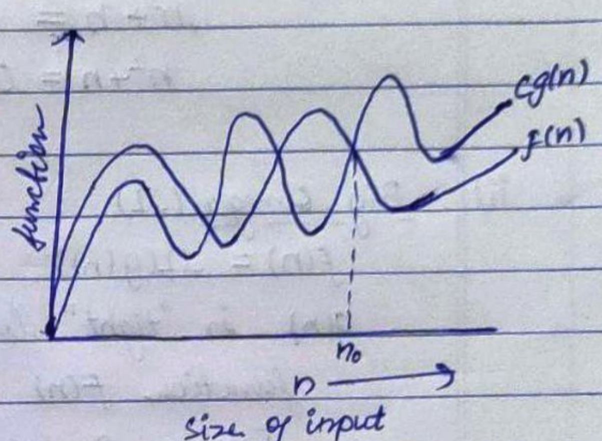
$\forall\ n > n_0$

& $\forall$ constants, $c > 0$

Ex:- $f(n) = n^2$

$g(n) = n^3$

$n^2 = o(n^3)$



v) **Small Omega (ω)**

$f(n) = \omega(g(n))$

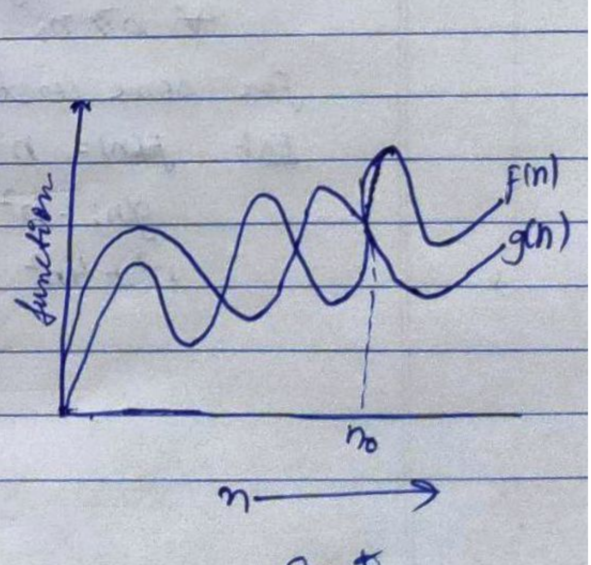
g(n) is lower bound of func^n f(n)

$f(n) = \omega(g(n))$ when

$f(n) > c \cdot g(n)$

$\forall\ n > n_0$

& $\forall$ constants, $c > 0$



Q2 What should be time complexity of -

for (i = 1 to n) { i = i * 2; }

Solⁿ

```
for (i=1 to n)
{
    i = i * 2;        O(1)
}
```

$$i = \overbrace{1, 2, 4, \ldots\ldots n}^{k}$$

$a = 1$, $r = \frac{b_2}{b_1} = 2$

G.P k^{th} value, $t_k = ar^{k-1}$

$t_k = 2^{k-1}$

$t_k = \frac{2^k}{2}$ $\{t_k = n\}$

$2n = 2^k$

$\log_2(2n) = k \log_2 2$ $\{\log_a a = 1\}$

$k = \log_2 2n$

$k = \log_2 2 + \log_2 n$ $\{\log ab = \log a + \log b\}$

$k = 1 + \log_2 n$

$O(1 + \log_2 n)$

$\Rightarrow O(\log n)$

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Q3 $T(n) = \{3T(n-1)$ if $n>0$, otherwise $1\}$

Solⁿ $T(n) = 3T(n-1)$ —— ①

Put $n = n-1$ in ①

$T(n-1) = 3T(n-2)$ —— ②

Put value of $T(n-1)$ from ② to ①

$T(n) = 3[3T(n-2)] \Rightarrow 9T(n-2)$ —— ③

Put $n = n-2$ in ①

$T(n) = 3T(n-3)$ —— ④

Put value of $T(n-2)$ from ④ to ③

$T(n) = 9[3T(n-3)]$

$T(n) = 27T(n-3)$

By generalizing, $T(n) = 3^k T(n-k)$ —— ⑤

Let $n-k = 1$

$k = n-1$

Put value of k in ⑤

T(n) value of k in ⑤

$T(n) = 3^{n-1} T(n-n+1)$

$T(n) = 3^{n-1} T(1)$

$T(n) = \frac{3^n}{3} \times 1$

$\underline{O(3^n)}$

Q4 $T(n) = \{2T(n-1) - 1$ if $n>0$, otherwise $1\}$

Solⁿ $T(n) = 2T(n-1) - 1$ —— ①

Put $n = n-1$ in ①

$T(n-1) = 2T(n-2) - 1$ —— ②

Put value of $T(n-1)$ from ② to ①

$T(n) = 2[2T(n-2) - 1] - 1$

$T(n) = 4T(n-2) - 2 - 1$ —— ③

Put $n = n-2$ in ①

$T(n-2) = 2T(n-3) - 1$ —— ④

Put value of $T(n-2)$ from ④ to ③

$T(n) = 4[2T(n-3) - 1] - 2 - 1$

$T(n) = 8T(n-3) - 4 - 2 - 1$ —— ⑤

$T(n) = 8T(n-3) - 7$

By generalizing, we get.

$T(n) = 2^k T(n-k) - (2^k - 1)$ —— ⑥

Let $n-k = 1$

$k = n-1$

Put k in ⑥

$T(n) = 2^{n-1} T(n-n+1) - (2^{n-1} - 1)$

$= 2^{n-1} T(1) - 2^{n-1} - 1$

$= 2^{n-1} - 2^{n-1} + 1$

$= 1$

$= \underline{O(1)}$

Q5 What should be time complexity of -

```
void function (int n) {
  int i, count = 0;
  for (i=1; i*i<=n; i++)
    count++
}
```

Solⁿ

As $i^2 <= n$

$O <= \sqrt{n}$

$i = 1, 2, 3, 4, ----, \sqrt{n}$

$\sum_{i=1} 1 + 2 + 3 + 4 + ----- + \sqrt{n}$

$T(n) = \frac{\sqrt{n} \times (\sqrt{n}+1)}{2}$

$T(n) = \frac{n \times \sqrt{n}}{2}$

$T(n) = \underline{O(n)}$

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Q6 What should be time complexity of -

```
int i = 1, s = 1;
while (s <= n) {
    i++; s = s + i;
    printf("#");
}
```

Sol^n $i = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \; \ldots$

$s = 1 + 3 + 6 + 10 + 15 + \ldots$

sum of $s = 1 + 3 + 6 + 10 + \ldots + n$ — ①

also $s = 1 + 3 + 6 + 10 + \ldots + T_{n-1} + T_n$ — ②

$0 = 1 + 2 + 3 + 4 + \ldots + n - T_n$

$T_k = 1 + 2 + 3 + 4 + \ldots + k$

$T_k = \frac{1}{2} k(k+1)$

for k iterations.

$1 + 2 + 3 + \ldots + k <= n$

$\Rightarrow \frac{k(k+1)}{2} <= n$

$\frac{k^2 + k}{2} <= n$

$O(k^2) <= n$

$k = O(\sqrt{n})$

$k = O(\sqrt{n})$

$\underline{T(n) = O(\sqrt{n})}$

Upand

Q7 Time complexity of -

```
void function (int n) {
  int i, j, k, count = 0;
  for (i = n/2 ; i <= n; i++)
    for (j = 1 ; j <= n ; j = j * 2)
      for (k = 1 ; k <= n ; k = k * 2)
        count ++
}
```

Sol^n For $k = k^2$

$k = 1, 2, 4, 8, \dots, n$

G.P $\Rightarrow$ $a = 1$, $r = 2$

$$= \frac{a(r^n - 1)}{r-1}$$

$$= \frac{1(2^k - 1)}{1}$$

$n \Rightarrow 2^k - 1$

$\log n = k$

i	j	k
1	$\log n$	$\log n * \log n$
2	$\log n$	$\log n * \log n$
⋮	⋮	⋮
n	$\log n$	$\log n * \log n$

$\Rightarrow O(n * \log n * \log n)$

$\Rightarrow O(n \log^2 n)$

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Q8 Time complexity of-

```
function (int n) {
    if (n==1) return;
    for (i= 1 to n) {
        for (j= 1 to n) {
            printf("*");
        }
    }
    function (n-3);
}
```

Sol^n For (i= 1 to n)

we get j= n times every turn.

$\because i \times j = n^2$

Now, $\left.\begin{aligned} T(n) &= n^2 + T(n-3); \\ T(n-3) &= (n^2 3)^2 + T(n-6); \\ T(n-6) &= (n^3 6)^2 + T(n-5); \end{aligned}\right\}$ K times.

$T(1) = 1;$

Now subs each value in T(n)

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \text{----} + 1$

Let,

$n - 3K = 1$

$K = (n-1)/3$

Total terms $= K + 1$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \text{-----} + 1$

$T(n) \approx kn^2$

$T(n) \approx \frac{(n-1)}{3} \times n^2$

$\therefore T(n) = O(n^3)$

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Q9 Time complexity of-

```
void function (int n) {
  for (i=1 to n) {
    for (j=1; j<=n ; j= j+1]
        printf("*")
  }
}
```

Sol^n For :-

$i=1$ $\quad j = 1+2+ \text{------} (n \geq j+1)$

$i=2$ $\quad j = 1+3+5+ \text{----} (n \geq j+1)$

$i=3$ $\quad j = 1+4+7+ \text{-----} (n \geq j+1)$

⋮

M^{th} term of A.P is

$T(m) = a + d \times m$

$T(m) = 1 + d \times m$

$(n-1)/d = m$

for $i=1$ $\quad (n-1)/1$ times

$i=2$ $\quad (n-2)/2$ times

$i=3$ $\quad (n-1)/3$ times

⋮

$i=n-1$ $\quad 1$

We get,

~~$T(m) = a+d \times m$~~ $\quad T(n)$

~~$T(m) = 1+d \times m$~~

$T(n) = i_1 j_1 + i_2 j_2 + \text{----} i_n + j_{n-1}$

$= \frac{(n-1)}{1} + \frac{(n-2)}{2} + \frac{(n-3)}{3} + \text{-----} + 1$

$= n + \frac{n}{2} + \frac{n}{3} + \text{----} + \frac{n}{n-1} - n*1$

$= n\left[1 + \frac{1}{2} + \frac{1}{3} + \cdot \frac{1}{n-1}\right] - n + 1$

Upon

$= n \times \log n - n + 1$

since $\int \frac{1}{x} = \log x$

$T(n) = O(n \log n)$

Q10 For the functions n^k and c^n, what is the asymptotic relationship between these functions?
Assume that k>=1 and c>1 are constants. Find out the value of c and no for which relation holds.

Soln as given n^k & c^n

relation between n^k and c^n is

$n^k = O(c^n)$

$n^k \le a(c^n)$

$\forall\ n \geq n_0$ &

constant, $a > 0$

for $n_0 = 1$

$c = 2$

$\Rightarrow 1^k \le a^{2 \cdot 1}$

$\Rightarrow n_0 = 1$ & $c = 2$