Example 5.5 (chp5ex5)

A three-phase, 60-Hz, 500-kV transmission line is 300 km long. The line inductance is 0.97 mH/km per phase and its capacitance is 0.0115 μF/km per phase. Assume a lossless line.

(a) Determine the line phase constant β, the surge impedance Z_C, velocity of propagation v and the line wavelength λ.
(b) The receiving end rated load is 800 MW, 0.8 power factor lagging at 500 kV. Determine the sending end quantities and the voltage regulation.

(a) For a lossless line, from (5.62) we have

$$\beta = \omega\sqrt{LC} = 2\pi \times 60\sqrt{0.97 \times 0.0115 \times 10^{-9}} = 0.001259 \text{ rad/km}$$

and from (5.63)

$$Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.97 \times 10^{-3}}{0.0115 \times 10^{-6}}} = 290.43 \ \Omega$$

Velocity of propagation is

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.97 \times 0.0115 \times 10^{-9}}} = 2.994 \times 10^5 \text{ km/s}$$

and the line wavelength is

$$\lambda = \frac{v}{f} = \frac{1}{60}(2.994 \times 10^5) = 4990 \text{ km}$$

(b) $\beta\ell = 0.001259 \times 300 = 0.3777 \text{ rad} = 21.641^\circ$

The receiving end voltage per phase is

$$V_R = \frac{500\angle 0^\circ}{\sqrt{3}} = 288.675\angle 0^\circ \text{ kV}$$

The receiving end apparent power is

$$S_{R(3\phi)} = \frac{800}{0.8}\angle \cos^{-1} 0.8 = 1000\angle 36.87^\circ = 800 + j600 \text{ MVA}$$

The receiving end current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V^*_R} = \frac{1000\angle -36.87^\circ \times 10^3}{3 \times 288.675\angle 0^\circ} = 1154.7\angle -36.87^\circ \text{ A}$$

From (5.71) the sending end voltage is

$$\begin{aligned} V_S &= \cos\beta\ell\, V_R + jZ_c \sin\beta\ell\, I_R \\ &= (0.9295)288.675\angle 0^\circ + j(290.43)(0.3688)(1154.7\angle -36.87^\circ)(10^{-3}) \\ &= 356.53\angle 16.1^\circ \ \text{kV} \end{aligned}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 617.53 \ \text{kV}$$

From (5.72) the sending end current is

$$\begin{aligned} I_S &= j\frac{1}{Z_c}\sin\beta\ell\, V_R + \cos\beta\ell\, I_R \\ &= j\frac{1}{290.43}(0.3688)(288.675\angle 0^\circ)(10^3) + (0.9295)(1154.7\angle -36.87^\circ) \\ &= 902.3\angle -17.9^\circ \ \text{A} \end{aligned}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 356.53\angle 16.1 \times 902.3\angle{-17.9^\circ} \times 10^{-3} \\ &= 800 \ \text{MW} + j539.672 \ \text{Mvar} \\ &= 965.1\angle 34^\circ \ \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{356.53/0.9295 - 288.675}{288.675} \times 100 = 32.87\%$$

The line performance of the above transmission line including the line resistance is obtained in Example 5.9 using the **lineperf** program. When a line is operating at the rated load, the exact solution results in $V_{S(L-L)} = 623.5\angle 15.57^\circ$ kV, and $I_s = 903.1\angle{-17.7^\circ}$ A. This shows that the lossless assumption yields acceptable results and is suitable for hand calculation.

5.7 COMPLEX POWER FLOW THROUGH TRANSMISSION LINES

Specific expressions for the complex power flow on a line may be obtained in terms of the sending end and receiving end voltage magnitudes and phase angles and the $ABCD$ constants. Consider Figure 5.2 where the terminal relations are given by (5.5) and (5.6). Expressing the $ABCD$ constants in polar form as $A = |A|\angle\theta_A$,