

AN INTRODUCTION TO ANALYSIS OF SURPLUS

By Shauna Ferris (1997)

Introduction : What is an analysis of surplus ?

When we do a triennial review of a defined benefit superannuation fund, we usually do a projection of financial position of the fund over the next three years. In this projection, we make numerous assumptions about the future experience of the fund, e.g. assumptions about investment earning rates, salary increases, rates of decrement, crediting rates, expenses, new entrants, etc. Hence we can project the value of the liabilities and assets at the end of three years, and determine the expected surplus.

When we come to do the next review, which is usually three years later, we will almost invariably find that the actual surplus does not match our expected surplus - because the experience of the fund did not match our assumptions. Our task is to examine the relationship between the actual and expected surplus, to account for the difference. We look at each item of experience, compare the actual experience to the expected experience, and determine the financial effect on the fund of the any deviations.

The purpose : Why do we do an analysis of surplus ?

Firstly, this serves as a check on the valuation process, by reconciling the previous valuation and the current valuation. This will not be a perfect check, for the following reasons :

(1) Approximations

As is explained below, in the process of the analysis we make many approximations and assumptions about the experience. For example, we might assume that cash flows from contributions and benefits are spread evenly over the year. Since this is not exactly correct, the analysis of surplus will not balance precisely - it is not like an accounting exercise, where everything must balance to the last cent. The actuary will usually be satisfied if the analysis of surplus is within 5% of the actual surplus.

(2) Data Errors

In the analysis, we generally use the same data that was used for the valuation. If this data contains errors, it is likely that both the valuation and the analysis will be wrong, and the analysis will not reveal these data errors. (NB this highlights the need for checks on the data before starting the valuation)

(3) Non-independence

Some of the results from the valuation may be used in the analysis, so the analysis is not an independent check.

Secondly, the analysis reveals the discrepancies between our assumptions and the experience. This may reveal the need to change the valuation basis, to make it more realistic (although judgement is required in deciding whether changes are needed - see the module on setting valuation assumptions).

Finally, this analysis gives us a greater understanding of the financial significance of deviations from our assumptions. We can see which elements of experience have relatively little impact on the fund's financial position - and in this case, we might decide not to waste much time and effort in refining these assumptions. More importantly, we can see which elements have the greatest financial impact - not only will we spend extra time in setting the assumptions for these elements, we will also gain greater insight into areas of potential vulnerability of the fund. For example,

- * if resignation rates are high and withdrawal surplus is a significant element of surplus, how will the fund be affected by an improvement in vesting scales?
- * if the most significant risk factor is changes in interest rates, what would be the impact of a proposed change in asset allocation to include more equities ?
- * if the fund is small and variations in mortality rates have a significant effect, this may point to the need for greater scrutiny of the insurance arrangements

The Method

How do we do an analysis of surplus ?

Theoretically, it is quite simple. The surplus at the end of the year will be a function of the variables which affect the fund. Suppose that for a given fund , these variables are

- i = investment earnings during the year
- f = salary inflation during the year
- d = deaths during the year
- w = resignations during the year

(Note that in practice, other variables may be needed, depending on the benefit design and circumstances of the fund)

Then we can say the surplus S is a function of these variables, i.e.

$$S = f(i, f, d, w)$$

Let's call this the surplus function.

Now let the expected surplus assuming that the experience is i'', f', d'', w'' , as expected in the valuation basis, be

$$\text{Expected surplus} = f(i'', f', d'', w'')$$

and let the actual surplus assuming that the actual experience is i', f', d', w'

$$\text{Actual Surplus} = f(i', f', d', w')$$

We want to analyse the difference between the actual surplus and expected, to see the effect of each variable.

$$\text{Actual - expected surplus} = f(i', f', d', w') - f(i'', f', d'', w'')$$

Let us break this down by going from the actual surplus to the expected surplus in steps, changing one variable at a time

$$\begin{aligned} \text{Actual - expected surplus} &= f(i', f', d', w') \\ &\quad - f(i'', f', d', w') \\ &\quad + f(i'', f', d', w') \\ &\quad - f(i'', f', d', w') \\ &\quad + f(i'', f', d', w') \\ &\quad - f(i'', f', d'', w') \\ &\quad + f(i'', f', d'', w') \\ &\quad - f(i'', f', d'', w'') \end{aligned}$$

Note that the net effect of each pair of lines inserted is nil, since we are adding and subtracting the same amount. So the overall value of the right hand side is unchanged.

Now we simply regroup these lines (to give equation 1)

$$\begin{aligned} S' - S'' &= f(i', f', d', w') && \text{interest surplus} \\ &\quad - f(i'', f', d', w') \\ &\quad + f(i'', f', d', w') && \text{salary increase surplus} \\ &\quad - f(i'', f', d', w') \\ &\quad + f(i'', f', d', w') && \text{mortality surplus} \\ &\quad - f(i'', f', d'', w') \\ &\quad + f(i'', f', d'', w') && \text{withdrawal surplus} \\ &\quad - f(i'', f', d'', w'') \end{aligned}$$

If we calculate the difference between the first two lines, we will have the interest surplus - basically the first line is the actual surplus, and the second line is the surplus which would have arisen if the valuation rate of interest had been earned on the actual cash flows.

The second pair of lines gives us the surplus arising from the difference between actual and expected salary increases, based on actual cash flows and expected investment returns, i.e. the surplus in relation to salary increases.

Similarly, the remaining lines give us the mortality surplus and the withdrawal surplus.

The total of these four components should add up to the total surplus to be analysed, i.e. the difference between actual and expected surplus.

THE SURPLUS FUNCTION DERIVED : AN EXAMPLE

Our first step is to determine the "surplus function", i.e. a formula which will allow us to calculate the surplus at the end of the period, given the initial assets and liabilities and the details of experience during the intervaluation period.

We will use a simple example to illustrate the method. Suppose we have a fund that pays death benefits of G times salary at death, a flat resignation benefit of F , and the contribution rate is $k\%$ per annum paid at mid year on average. We will assume that the intervaluation period is one year instead of three years, just for simplicity. We will assume that we have $l(x)$ members, all age x , with salary S at the start of the year.

We can start by saying that the surplus at the end of the period is

$$\text{Surplus} = A(0) * (1+i) + C * (1+i/2) - B * (1+i/2) - \text{PVNL}(1) \quad (\text{eqn 2})$$

where

$A(0)$ is the assets at the start of the year

C is the contribution income, which is assumed to occur at mid year on average

B is the benefit payments during the year, which is assumed to occur at mid year

$\text{PVNL}(1)$ is the present value of net liabilities on our valuation basis at $t = 1$

$(1+i/2)$ is an approximation for $(1+i)^{.5}$

[Note that if we are using the projected unit credit method, PVNL is the present value of accrued liabilities. If we are using a projected method of valuation, then PVNL is the present value of total service liabilities less the present value of future contributions on our valuation basis. The method of analysis of surplus works either way.]

However, to do the analysis we need to go a bit further and express C , B , and $\text{PVNL}(1)$ as functions of the salary and decrements during the intervaluation period.

Looking firstly at contributions - the contributions payable during the year depend on the rate of contribution, the number of people in the fund from time to time during the year, and the salaries of these members during the year. We can express C as follows

$$C = K * S * (1+f/2) * l(x+.5) \quad (\text{equation 3})$$

where K is the contribution rate
 S is the initial salary per person
 f is the rate of increase of salaries during the year
 $l(x+.5)$ is the number of members remaining in the fund at mid-year
 assuming that contributions occur evenly throughout the year
 assuming that salary increases occur evenly throughout the year
 assuming that exits occur evenly throughout the year

If we next consider the benefit payments B - If death benefits are G times salary at death and deaths occur at mid year, and resignation are F per person and resignations occur at mid year, then

$$B = G * S * (1+f/2) * d(x) + F * w(x) \quad (\text{equation 4})$$

where $d(x)$ is the number of deaths and $w(x)$ is the number of resignations during the year.

Looking at the present value of net liabilities at the end of the year, PVNL - this will be proportional to the number of members at the end of the year, and will be approximately proportional to the salary levels at the end of the year (assuming that the value of resignations are small relative to the value of retirements). So let us say that

$$PVNL = S(1+f) * l(x+1) * V \quad (\text{equation 5})$$

where V is some constant.

Now substituting back into our original equation (equation 1), we get

$$\begin{aligned} \text{Surplus} = & A(0) * (1+i) \\ & + K * S * (1+f/2) * l(x+.5) * (1+i/2) \\ & - [G * S * (1+f/2) * d(x) + F * w(x)] * (1+i/2) \\ & - S(1+f) * l(x+1) * V \end{aligned} \quad (\text{equation 6})$$

And we can simplify our task even further by expressing membership in terms of original members less exits during the year

$$l(x+1) = l(x) - d(x) - w(x)$$

$$l(x+.5) = l(x) - .5 * d(x) - .5 * w(x)$$

and substituting again

$$\begin{aligned}
 \text{Surplus} = & A(0) * (1+i) \\
 & + K * S * (1+f/2) * [l(x) - .5 d(x) - .5w(x)] * (1+i/2) \\
 & - [G * S * (1+f/2) * d(x) + F * w(x)] * (1+i/2) \\
 & - S(1+f) * [l(x) - d(x) - w(x)] * V
 \end{aligned} \tag{eqn 4}$$

We now have a surplus function which expresses the surplus in terms of each item of experience (i, f, d, and w) and some constants (A(0), K, S, G, F, and V). We can now proceed with our analysis.

INTEREST SURPLUS

Using the formula given above for interest surplus

$$\text{interest surplus} = f(i', f', d', w') - f(i'', f', d', w')$$

In this case we will use the form of the surplus function given in equation 2, since it is simpler.

$$\begin{aligned}
 \text{interest surplus} = & A(0) (1+i') + C' (1+i'/2) - B' (1+i'/2) - \text{PVNL}'(1) \\
 & - [A(0) (1+i'') + C' (1+i''/2) - B' (1+i''/2) - \text{PVNL}''(1)] \\
 = & A(0) * (i' - i'') + [C' - B'] (i' - i'') / 2
 \end{aligned}$$

i.e. the interest surplus is the difference between the actual and expected interest rates, applied to the average amount of assets held during the year, based on actual cash flows. That is, we get a whole year's extra interest on the original assets plus half a year's extra interest on the net cash flow, which is expected to occur mid way through the year on average.

This seems reasonable; and as a logical check, if actual interest earnings match expected interest earnings, then the interest surplus is zero.

SALARY SURPLUS

Using the formula given in equation 1 above, for salary surplus,

$$\text{salary surplus} = f(i'', f', d', w') - f(i'', f', d', w')$$

and using the equation 3 this gives

$$\begin{aligned} \text{salary surplus} = & A(0) * (1+i'') \\ & + K * S * (1+f'/2) * l'(x+.5) * (1+i''/2) \\ & - [G * S * (1+f'/2) * d'(x) + F * w'(x)] * (1+i''/2) \\ & - S(1+f') * l'(x+1) * V \\ & - \{ A(0) * (1+i'') \\ & + K * S * (1+f'/2) * l'(x+.5) * (1+i''/2) \\ & - [G * S * (1+f'/2) * d'(x) + F * w'(x)] * (1+i''/2) \\ & - S(1+f') * l'(x+1) * V \} \end{aligned}$$

which looks complicated, but it can be simplified to

$$\begin{aligned} \text{salary surplus} = & K * S * (f' - f'') / 2 * [l(x) - .5 d'(x) - .5 w'(x)] * (1+i''/2) \\ & - G * S * (f' - f'') / 2 * d'(x) * (1+i''/2) \\ & - S * (f' - f'') * l'(x+1) * V \end{aligned}$$

Expressing this in words, and looking at each term separately, when salary increases are higher than expected, the salary surplus at the end of the year consists of

- (1) the value of extra contributions received as a result of higher than expected salaries

offset by

- (2) the value of higher death benefits payable as a result of higher than expected salaries at death, and
- (3) higher reserves required at the end of the year for each survivor

And of course vice versa if the salary increases are lower than expected

Note that in practice, the net effect of these three effects is a loss when salary increases are higher than expected, and a surplus if salary increases are lower than expected.

Note that the cash flows are accumulated at the valuation expected interest rate, not the actual interest rate earned. This is because we have already allowed for the effect of higher-than-expected interest earnings when calculating the interest surplus above, and we do not want to double-count this extra interest by including it here as well.

MORTALITY SURPLUS

Using the formula given in equation 1 above, for mortality surplus,

$$\text{mortality surplus} = f(i'', f', d', w') - f(i'', f'', d'', w')$$

and using the equation 4 this gives

$$\begin{aligned} \text{mortality surplus} &= A(0) * (1+i'') \\ &\quad + K * S * (1+f'/2) * [l(x) - .5 d'(x) - .5 w'(x)] * (1+i''/2) \\ &\quad - [G * S * (1+f'/2) * d'(x) + F * w'(x)] * (1+i''/2) \\ &\quad - S(1+f'') * [l(x) - d'(x) - w'(x)] * V \\ &\quad - \{ A(0) * (1+i'') \\ &\quad \quad + K * S * (1+f'/2) * [l(x) - .5 d''(x) - .5 w'(x)] * (1+i''/2) \\ &\quad \quad - [G * S * (1+f'/2) * d'(x) + F * w'(x)] * (1+i''/2) \\ &\quad \quad - S(1+f) * [l(x) - d''(x) - w'(x)] * V \} \end{aligned}$$

which looks complicated, but it can be simplified to

$$\begin{aligned} \text{mortality surplus} &= - K * S * (1+f'/2) * .5 [d'(x) - d''(x)] * (1+i''/2) \\ &\quad - G * S * (1+f'/2) * (d'(x) - d''(x)) * (1+i''/2) \\ &\quad + S * (1+f'/2) * (d'(x) - d''(x)) * V \end{aligned}$$

Expressing this in words, and looking at each term separately, when the number of deaths is higher than expected, the mortality surplus at the end of the year consists of

- (1) the reduction in contributions received (since we don't get any contributions after the date of death for those who died)
- (2) the cost of paying out more death benefits than we expected

offset by

- (3) lower reserves required at the end of the year, since we don't need to keep reserves for those who died

Usually the net effect of these three items is to cause a loss when mortality rates are higher than expected (and vice versa if mortality rates are lower than expected).

Again, the mortality surplus is calculated using the valuation interest rate and the expected salary increases according to the valuation basis. This is because we have already allowed for the effect of higher-than expected interest earnings and salary increases when calculating the interest and salary surplus above, and we do not want to double-count by including these amounts here as well.

WITHDRAWAL SURPLUS

The withdrawal surplus is calculated using the same method as the mortality surplus. It is left as an exercise for the student to find the withdrawal surplus.

A numerical example, using the aggregate funding method, is provided for the students as an appendix.

CHECKING

The total of all components of the analysed surplus should be approximately equal to the difference between the actual and the expected surplus. In practice, this is unlikely to happen, because we have made numerous assumptions which are unlikely to be true in practice, for example

- * salary increases are spread evenly over the year
- * salary increase rates are the same for those who exit and those who remain
- * benefit payments occur at mid year on average
- * contributions are paid mid year on average
- * the reserves are proportional to the salary increases and number of survivors

In a realistic example, of course, the benefit design is more complex, and we would be grouping data to simplify the calculations - so even more approximations are required ! We would also have complications caused by new entrants, insurance claims and rebates, tax, employer and employee contributions, rule changes, etc

So in a realistic example, we would probably not expect the total of the components from the analysis to be closer than plus or minus 5% of the actual amount to be analysed (or even worse depending on the thoroughness of the analysis).

SOURCES OF SURPLUS

In practice, you would have many more sources of surplus, including:

- Interest on Surplus brought forward
- Interest
- Salary
- Mortality
- Ill health retirements
- Withdrawal (resignation / Dismissal)
- Redundancies
- Age Retirement
- Pensioner Mortality
- New Entrants
- Change in Basis
- Change in Fund Rules
- Insurance Premiums and Rebates

THE ORDER OF ANALYSIS

It is clear from observation of the surplus function that the elements of the basis are inter-related, i.e. there are interactions between interest, salary increases, and decrement rates. This means that we would get a different answer if we changed the order of analysis.

We have analysed firstly interest, then salaries, then decrements. However, it is equally possible to perform the analysis in a different order - there is no theoretical reason why one method is "more correct" than another method. It is common practice to analyse the most significant elements of the basis first.

THE VALUATION BASIS

The analysis requires us to compare actual to expected experience, where "expected" experience is the experience assumed in the valuation basis. But what is the valuation basis has changed? Should we do the valuation on the old valuation basis or the new valuation basis?

In practice, we usually perform the analysis on the old valuation basis, i.e. calculating PVNL(1) on the old basis. Then we calculate PVNL(1) on the new basis, and the difference between these two values is another component of the analysis of surplus, i.e. this is the surplus arising from the change in basis.

FURTHER READING

Any student who wants to obtain further enlightenment may read the section on analysis of surplus in the IAA Superannuation textbook, on Occupational Superannuation in Australia (chapter 17). This gives a long and rather complex worked example, (which is much more realistic than the above example) and shows the types of approximations which are required in real life.

ACKNOWLEDGEMENTS

These notes are based on the approach developed by Jim Farmer in his work on analysis of surplus for life insurance.

ANALYSIS OF SURPLUS EXAMPLE

Scheme design:

Retirement benefits at age 65 equal to 5 times salary

Death benefits at any time equal to 3 times salary.

Membership:

1000 members all aged 30 with salary 20,000

Assets of fund :

\$10,000,000

VALUATION at 30/6/96 AGGREGATE FUNDING METHOD

Valuation basis

Interest on assets 8.00%

Salary increases 6.00%

Mortality $q(x)$, all x 0.003 paid at mid year on average

No withdrawals except death and normal retirement

Contributions paid at mid year on mid year salary and survivors

AGE	LIVES	EXPECTED SALARY	EXPECTED DEATHS	BENEFIT PAYMENTS	PRES VAL PV BENEFITS of 1%	CONTS
30	1000	20000	3.0000	185400	178401	197926.26
31	997	21200	2.9910	195934	174572	193678.17
32	994	22472	2.9820	207067	170825	189521.27
33	991	23820	2.9731	218833	167159	185453.58
34	988	25250	2.9642	231267	163571	181473.19
35	985	26765	2.9553	244408	160061	177578.24
36	982	28370	2.9464	258295	156625	173766.89
37	979	30073	2.9376	272971	153263	170037.33
38	976	31877	2.9288	288481	149974	166387.83
39	973	33790	2.9200	304873	146755	162816.65
40	970	35817	2.9112	322196	143605	159322.13
41	967	37966	2.9025	340503	140523	155902.60
42	965	40244	2.8938	359850	137507	152556.47
43	962	42659	2.8851	380297	134556	149282.16
44	959	45218	2.8764	401906	131668	146078.12
45	956	47931	2.8678	424742	128842	142942.85
46	953	50807	2.8592	448876	126076	139874.87
47	950	53855	2.8506	474381	123371	136872.74
48	947	57087	2.8421	501335	120723	133935.05
49	945	60512	2.8335	529821	118132	131060.40
50	942	64143	2.8250	559925	115596	128247.46
51	939	67991	2.8166	591740	113115	125494.89
52	936	72071	2.8081	625363	110687	122801.40
53	933	76395	2.7997	660896	108312	120165.72
54	930	80979	2.7913	698448	105987	117586.60
55	928	85837	2.7829	738134	103712	115062.85
56	925	90988	2.7746	780075	101486	112593.26
57	922	96447	2.7662	824399	99308	110176.67
58	919	102234	2.7579	871241	97177	107811.95
59	917	108368	2.7497	920745	95091	105497.99
60	914	114870	2.7414	973062	93050	103233.69
61	911	121762	2.7332	1028351	91053	101017.99
62	908	129068	2.7250	1086782	89099	98849.84
63	906	136812	2.7168	1148533	87186	96728.23
64	903	145021	2.7087	1213793	85315	94652.16
65	900	153722		691888043	45029021	
					\$49,451,403	\$4,906,388

Valuation at 30/6/90

Assets
Liabilities
PV Contributions

Aggregate funding method

\$10,000,000
\$49,451,403
\$39,451,403 at cont rate

8.04%

Surplus

0.00

ANALYSIS OF SURPLUS EXAMPLE : 1 YEAR LATER

If experience matches the assumptions, there will be no surplus of deficiency.

VALUATION at 30/6/97 Using same basis as 1996 valuation
Experience from 1/7/96 to 30/6/97

Deaths	3.00
interest	8.00%
salary increases	6.00%

ACCOUNTS

Assets at start	\$10,000,000
Less payments	\$185,400
Plus contributio	\$1,653,925
Plus interest	\$857,611

assets at end \$12,326,136

AGE	MEMBERS	SALARY	DEATHS	BENEFIT PAYMENTS	PRES VAL BENEFITS	PV CONTS of 1%
31	997	21200	2.99100	195934	188538	209172.43
32	994	22472	2.98203	207067	184491	204682.97
33	991	23820	2.97308	218833	180532	200289.86
34	988	25250	2.96416	231267	176657	195991.05
35	985	26765	2.95527	244408	172865	191784.50
36	982	28370	2.94640	258295	169155	187668.24
37	979	29975	2.93753	272182	165266	183377.99
38	976	31580	2.92866	286069	161377	179087.74
39	973	33185	2.91979	300000	157488	174797.49
40	970	35817	2.91121	322196	155094	172067.90
41	967	37966	2.90247	340503	151765	168374.81
42	965	40244	2.89376	359850	148508	164760.99
43	962	42659	2.88508	380297	145320	161224.73
44	959	45218	2.87643	401906	142201	157764.37
45	956	47931	2.86780	424742	139149	154378.28
46	953	50807	2.85920	448876	136163	151064.86
47	950	53855	2.85062	474381	133240	147822.56
48	947	57087	2.84207	501335	130380	144649.85
49	945	60512	2.83354	529821	127582	141545.24
50	942	64143	2.82504	559925	124844	138507.26
51	939	67991	2.81656	591740	122164	135534.48
52	936	72071	2.80811	625363	119542	132625.51
53	933	76395	2.79969	660896	116977	129778.97
54	930	80979	2.79129	698448	114466	126993.53
55	928	85837	2.78292	738134	112009	124267.87
56	925	90988	2.77457	780075	109605	121600.72
57	922	96447	2.76624	824399	107253	118990.81
58	919	102234	2.75795	871241	104951	116436.91
59	917	108368	2.74967	920745	102698	113937.83
60	914	114870	2.74142	973062	100494	111492.39
61	911	121762	2.73320	1028351	98337	109099.43
62	908	129068	2.72500	1086782	96226	106757.83
63	906	136812	2.71682	1148533	94161	104466.49
64	903	145021	2.70867	1213793	92140	102224.33
65	900	153722	2.70055	691888043	48631343	

Valuation at 30/6/97

Assets	\$12,326,136	
Liabilities	\$53,214,842	
PV future conts	\$40,888,706	at cont rate 8.04%
Surplus	0	

\$53,214,842	\$5,085,138
Reserve =	\$12,326,136

ANALYSIS OF SURPLUS EXAMPLE : 1 YEAR LATER

If experience does not match assumptions
there will be a surplus or deficiency

VALUATION at 30/6 Using same basis as 1996 valuation
Experience from 1/7/96 to 30/6/97

Deaths	1.00
Interest	9.00%
salary increases	5.00%

ACCOUNTS

Assets at start	\$10,000,000
Less payments	\$61,500
Plus contributions	\$1,647,545
Plus interest	\$969,835
assets at end	\$12,555,880

AGE	MEMBERS	SALARY	DEATHS	BENEFIT PAYMENTS	PRES VAL BENEFITS	PV CONTS of 1%
31	999	21000	2.99700	194475	187134	207614.75
32	996	22260	2.98801	205525	183118	203158.72
33	993	23596	2.97904	217203	179187	198798.33
34	990	25011	2.97011	229545	175341	194531.53
35	987	26512	2.96120	242588	171578	190356.31
36	984	28103	2.95231	256371	167895	186270.70
37	981	29789	2.94346	270938	164292	182272.78
38	978	31576	2.93463	286333	160766	178360.66
39	975	33471	2.92582	302603	157315	174532.51
40	972	35479	2.91705	319797	153939	170786.53
41	969	37608	2.90829	337967	150635	167120.94
42	967	39864	2.89957	357171	147402	163534.03
43	964	42256	2.89087	377465	144238	160024.11
44	961	44791	2.88220	398913	141142	156589.52
45	958	47479	2.87355	421579	138113	153228.64
46	955	50328	2.86493	445533	135149	149939.90
47	952	53347	2.85634	470848	132248	146721.75
48	949	56548	2.84777	497602	129410	143572.66
49	946	59941	2.83922	525876	126632	140491.17
50	944	63538	2.83071	555756	123914	137475.81
51	941	67350	2.82221	587334	121255	134525.17
52	938	71391	2.81375	620706	118652	131637.86
53	935	75674	2.80531	655975	116105	128812.53
54	932	80215	2.79689	693247	113613	126047.83
55	929	85028	2.78850	732637	111175	123342.47
56	927	90129	2.78013	774266	108789	120695.17
57	924	95537	2.77179	818260	106454	118104.70
58	921	101269	2.76348	864753	104169	115569.82
59	918	107345	2.75519	913888	101933	113089.35
60	916	113786	2.74692	965816	99746	110662.12
61	913	120613	2.73868	1020693	97605	108286.98
62	910	127850	2.73047	1078689	95510	105962.82
63	907	135521	2.72227	1139980	93460	103688.54
64	905	143652	2.71411	1204754	91454	101463.08
65	902	152272	2.70596	686735643	48269192	

Valuation at 30/6/97

Assets	\$12,555,880
Liabilities	\$52,818,558
PV future conts	\$40,584,213
Surplus	321534

\$52,818,558	\$5,047,270
Reserve =	\$12,234,345

229743.236	
at cont rate	8.04%

ANALYSIS OF SURPLUS : CALCULATIONS

Experience	Expected	Actual
Interest	0.08	0.09
Salary increases	0.06	0.05
Deaths	3	1

Note no change in basis, so no item of surplus caused by change in basis

ACTUAL SURPLUS

Assets	\$12,555,880
Liabilities	\$52,818,558
PV future conts	\$40,584,213

Surplus	\$321,534

INTEREST SURPLUS

Allow for interest at expected rate on actual cash flows

Actual cash flows

Assets at start	\$10,000,000
Death benefits	\$61,500
Conts paid	\$1,647,545

Interest at actual rates	\$10,969,835
Interest at expected rates	\$10,862,221

Interest surplus \$107,613

SALARY SURPLUS

When salaries are lower than expected the effects are

- (i) Lower contributions
- (ii) Lower death benefits payable
- (iii) Extra reserves required at year end

(i) Extra contributions received on actual number of members

Since salaries were lower than expected the contributions for any member are lower than expected and this reduces the surplus

Actual contributions	1647544.94
Expected contributions	1655581.75
Difference	-8036.80

This must be accumulated to the end of the year at the expected interest rate

Salary surplus (i) = -8352.09

(ii) Extra death benefits paid for actual deaths

Since salaries were lower than expected, the death benefits for each person are lower than expected and we make a gain.

Actual death benefits	61500.00
Expected death benefits	61800.00
Difference	300.00

This must be accumulated to the end of the year at the expected interest rate

Salary surplus (ii) = 311.77

(iii) Change in reserves required at year end

Since all surviving members have lower salaries, then the reserves required at year end are lower than expected and we make a gain
The reserve per member per \$1 salary is the value of liabilities less future contributions

Reserve per surviving member per \$1 salary	0.583171037
Check	0.583171037
Actual Reserve	\$12,234,345
Expected reserve for actual survivors	\$12,350,863
Difference	\$116,518

Total salary surplus = **\$108,477**

SURPLUS FOR DECREMENTS

The number of deaths was lower than expected, so the effect is

- (i) a surplus since we pay less in death benefits than expected
- (ii) a surplus since we get more contributions than expected
- (iii) a loss since we need a greater reserve at the end of the year

(i) Reduction in death benefits

We expected to pay

3 death benefits of \$61,800

If we only pay 1 death benefits

Then the surplus is **\$123,600**

This is accumulated to the end of the year at the expected interest rate giving a surplus of **\$128,449**

(ii) Increase in contributions received

We expected to lose contributions for half a year for 3 deaths **\$2,485**

We lose contributions for half a year for 1 deaths **\$828**

Difference **\$1,656**

Accumulate to the end of the year at the expected interest rate **\$1,721**

(iii) Increase in reserves required at year end

As calculated above the reserve per person at year end based on expected salary increases was **\$12,363**

Since we have extra survivors, we need extra reserves of **\$24,726**

Total decrement surplus **\$105,444**

ANALYSIS OF SURPLUS	
INTEREST	\$107,613
SALARY	\$108,477
DECREMENTS	\$105,444
 TOTAL	 \$321,534
 Compared to	 \$321,534
 Discrepancy	 \$