AIM: The objective of Learning MPC is to design a reference-free iterative control strategy, able to learn from previous iterations.
An infinite horizon optimal control problem can be approximated as a finite horizon optimal control problem for MPC fomrulation:

$$\begin{aligned} \min_{U_k} \quad & \sum_{i=0}^{\infty} (h(x_k, u_k)) \\ \text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k \\ & x_0 = x_s \\ & x_k \in X_c \\ & u_k \in U_c \end{aligned} \Longrightarrow \begin{aligned} \min_{U_k} \quad & \sum_{i=0}^{N_p-1} \left(h(x_k, u_k) + \boxed{Q^{j-1}(x_{N_p|k})} \right) \\ \text{s.t.} \quad & \boxed{x_{i+1|k} = f(x_{i|k}, u_{i|k})}, \quad i \in [0, 1, ..., N_p - 1] \\ & x_0 = x^j_{0|k} \\ & x_{i|k} \in X_c, \quad i \in [0, 1, ..., N_p - 1] \\ & u_{i|k} \in U_c, \quad i \in [0, 1, ..., N_p - 1] \\ & x_{N_p|k} \in \boxed{SS^{j-1}} \end{aligned} \tag{1}$$

where: SS^{j-1} terminal constraint (Safe Set)
Q^{j-1} terminal cost function (Iteration Cost)

Assumption: There exists a initial feasible trajectory x^0, which starts from the initial state x_0 and converging to the final goal x_f.

Safe Set

Terminal constraint is needed to ensure:

- Recursive feasibility
- Iterative feasibility

It is defined the Safe Set SS at j-th iteration, as the set of all the successful trajectories performed in the first j iterations:

$$SS^j = \left\{ \bigcup_{i \in M^j} \bigcup_{t=0}^{\infty} x_t^i \right\} \tag{2}$$

where: M^j set of trajectories of successful iterations

Particular case: Linear System For linear systems it is possible to relax the Safe Set to its convex hull, defined Convex Set:

$$CS^j = Conv(SS^j) \tag{3}$$

Iteration Cost

Terminal cost is needed to ensure:

- Asymptotical stability
- Performance improvement

It is defined the Iteration cost:

$$Q^j(x) = \begin{cases} \min_{(i,t)\in F^j(x)} \quad J^i_{t\to\infty}(x) & \text{if } x \in SS^j \\ +\infty, \quad \text{if } x \notin SS^j \end{cases} \tag{4}$$

where: $\quad J^i_{t\to\infty}(x) = \sum_{i=0}^{\infty} \left(h(x^j_{i|k}, u^j_{i|k}) \right)$ terminal cost (cost-to-go)

Particular case: Linear System For linear systems it is possible to relax the iteration cost, to its convex function:

$$P^j(x) = \begin{cases} p^j(x) & \text{if } x \in SS^j \\ +\infty, & \text{if } x \notin SS^j \end{cases} \tag{5}$$

which is a LP:

$$\begin{aligned} p^j(x) = \min_{\lambda^j_t \geqslant 0} \quad & \sum_{k=0}^{j} \sum_{t=0}^{t_j} \left(\lambda^k_t J^i_{t\to\infty}(x) \right) \\ \text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k, \quad x_0 = x_s \\ & x_k \in X_c \\ & u_k \in U_c \end{aligned} \tag{6}$$