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Question #3

Soit la récurrence suivante $C(n) = \begin{cases} 0 & \text{si } n \leq 1 \\ 2C(\lfloor \frac{n}{3} \rfloor) + n & \text{si } n > 1 \end{cases}$

Supposons que $n = 3^K$ (puissance de 3)

Nous avons
$$\begin{aligned}
C(3^K) &= 2 \cdot C(3^{K-1}) + 3^K \\
&= 2(2 \cdot C(3^{K-2}) + 3^{K-1}) + 3^K \\
&= 2(2(2 \cdot C(3^{K-3}) + 3^{K-2}) + 3^{K-1}) + 3^K \\
&= 8 \cdot C(3^{K-3}) + 3^K + 2 \cdot 3^{K-1} + 4 \cdot 3^{K-2} \\
&\vdots \\
\langle \forall i \geq 0 \rangle \quad &= 2^i \cdot C(3^{K-i}) + 2^0 3^K + 2^1 3^{K-1} + \ldots + 2^{i-1} 3^{K-(i-1)} \\
\langle \text{avec } i = k \rangle \quad &= 2^K \cdot \underbrace{C(3^0)}_{0} + 2^0 3^K + 2^1 3^{K-1} + \ldots + 2^{K-1} 3^{K-(K-1)} \\
&= 0 + \sum_{j=0}^{K-1} 3^{K-j} 2^j \\
&= \sum_{j=0}^{K-1} \frac{3^K}{3^j} 2^j \\
&= 3^K \sum_{j=0}^{K-1} \left(\frac{2}{3}\right)^j \\
&= 3^K \left(\frac{\left(\frac{2}{3}\right)^K - 1}{\frac{2}{3} - 1} \right) \\
&= 3^K \left(\frac{\left(\frac{2}{3}\right)^K}{\frac{-1}{3}} - \frac{1}{\frac{-1}{3}} \right) \\
&= 3^K \left(3 - 3 \times \left(\frac{2}{3}\right)^K \right) \\
&= 3^{K+1} - 3 \times 2^K
\end{aligned}$$

$$\langle \text{awee } k=\log_3 n\rangle = 3^{\log_3 n+1} - 3\times 2^{\log_3 n}$$

$$\langle \text{awee } 3^{\log_3 2}=2\rangle = 3\cdot n - 3\times\left(3^{\log_3 2}\right)^{\log_3 n}$$

$$= 3\cdot n - 3\times 3^{(\log_3 2)(\log_3 n)}$$

$$= 3\cdot n - 3\times\left(3^{\log_3 n}\right)^{\log_3 2}$$

$$= 3\cdot n - 3\times n^{\log_3 2}$$

$$= 3\cdot n - 3n^{0.6309}$$