Operations Analytics MOOC: Solutions to Practice Problems for Week 2

1. Making Turkish Delight at the Rahat Industries.

Questions and Answers:

(a) Suppose that RI decides to make 250 boxes of *Ortaköy* and 350 boxes of *Beyoglu*. What is the weekly profit that it will earn?

Answer:

RI earns a profit of \$5 for each box of *Ortaköy* and \$5.3 for each box of *Beyoglu*. The total profit for this production plan will be \$5*250+\$5.3*350 = \$1250 + \$1855 = \$3105.

(b) How many gallons of fruit juice will the production plan $N_O = 250$ and $N_B = 350$ require?

Answer:

Each box of *Ortaköy* and *Beyoglu* requires 0.03 and 0.02 gallons of fruit juice, respectively. The total usage of fruit juice for this production plan is 0.03*250+0.02*350 = 7.5 + 7 = 14.5.

(c) RI is considering setting its production for the next week at the maximum levels that market will bear - in other words, it is considering a production plan $N_O = 250$ and $N_B = 400$. Is this plan feasible?

Answer:

We need to check if this production plan satisfies all the resource constraints. The amount of starch required by this plan is 0.1*250+0.3*400 = 25 + 120 = 145 lbs. This amount is less than the available amount of 150 lbs. The amount sugar required by this plan is 0.05*250+0.1*400 = 12.5 + 40 = 52.5 lbs. This amount is also less than the available amount of 130 lbs. Finally, the amount of fruit juice required is 0.03*250+0.02*400 = 7.5 + 8 = 15.5 gallons. This exceeds 15 gallons available. Thus, this production plan is infeasible.

(d) Write down an algebraic expression for RI's weekly profit, in \$, as a function of the decision variables N_O and N_B.

Answer:

RI earns a profit of \$5 for each box of *Ortaköy* and \$5.3 for each box of *Beyoglu*. The total profit, in \$, for any production plan can be calculated as $5*N_O + 5.3*N_B$.

(e) Using the decision variables N_O and N_B, write down a linear algebraic expression for the following constraint: "The amount of fruit juice used in the production during the coming week cannot exceed 15 gallons".

Answer:

Each box of *Ortaköy* and *Beyoglu* requires 0.03 and 0.02 gallons of fruit juice, respectively. The total usage of fruit juice for any production plan can be expressed as $0.03*N_O + 0.02*N_B$. Thus, the constraint on the amount of the fruit juice used by any production plan can be written as $0.03*N_O + 0.02*N_B \leq 15$.

(f) Below is a picture of an excerpt from an Excel file set-up to model the RI's profit maximization problem, with the number in cell D3 hidden.

	A	B	C	D	E
1			**Ortaköy**	**Beyoglu**	
2	**Profit Contribution ($/box)**		5	5.3	
3	**Units to Make**		100		
4					
5			**Resource requirements**		
6			**Ortaköy**	**Beyoglu**	**Required**
7	**Starch**		0.1	0.3	100
8	**Sugar**		0.05	0.1	
9	**Fruit Juice**		0.03	0.02	
10					
11					

The formula in cell E7 is =SUMPRODUCT($C$3:$D$3,C7:D7). What is the value in cell D3?

Answer:

If the value in the cell D3 is X, then the value of SUMPRODUCT($C$3:$D$3,C7:D7) is 100*0.1+X*0.3. Equating this to 100 and solving for X, we get X=300.

(g) If the formula in cell E7 is copied and pasted into the cell E8, what value will appear in cell E8?

Answer:

After this copy-and-paste operation, the cell E8 will contain the formula SUMPRODUCT($C$3:$D$3,C8:D8). Then, the value in E8 will be 100*0.05+300*0.1 = 35.

(h) Use the file Zooter.xlsx as a template to set up and solve an optimization problem RI faces using Excel and Solver. What are the optimal values of the decision variables? What is the optimal objective function value?

Answer:

For completeness, here's the algebraic formulation of RI's problem.

Objective function to be maximized: $5*N_O + 5.3*N_B$

Constraints:

$0.1*N_O + 0.3*N_B \leq 150$ (starch availability)

$0.05*N_O + 0.1*N_B \leq 130$ (sugar availability)

$0.03*N_O + 0.02*N_B \leq 15$ (fruit juice availability)

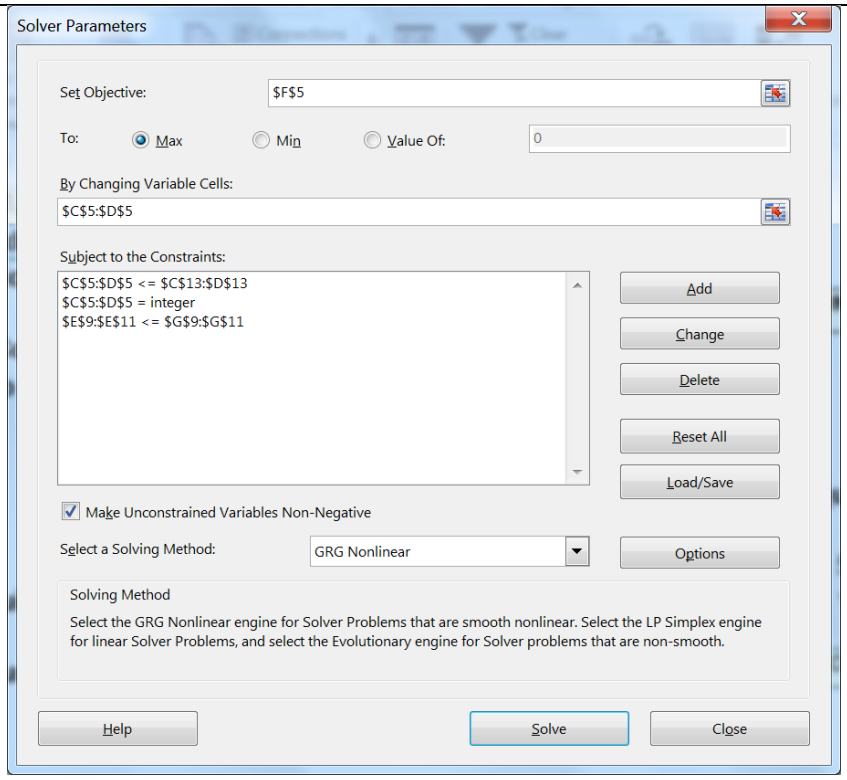
$N_O \leq 250$ (maximum number of Ortaköy boxes)

$N_B \leq 400$ (maximum number of Beyoglu boxes)

N_O, N_B = non-negative integers

See Rahat.xlsx for the spreadsheet implementation of the RI's problem. Below we show the optimized spreadsheet and the Solver parameters dialog box.

	A	B	C	D	E	F	G	H
1	**Rahat.xlsx**							
2	**Operations Analytics MOOC**							
3			**Ortaköy**	**Beyoglu**	**=SUMPRODUCT(C9:D9,C10:D10)**			
4	**Profit Contribution ($/box)**		5	5.3		**Total Profit ($)**		
5	**Units to Make**		233	400		3285		
6								
7			**Resource requirements**					
8			**Ortaköy**	**Beyoglu**	**Required**		**Available**	
9	**Starch**		0.1	0.3	143.3	<=	150	(lbs)
10	**Sugar**		0.05	0.1	51.65	<=	130	(lbs)
11	**Fruit Juice**		0.03	0.02	14.99	<=	15	(gallons)
12								
13	**Maximum Numbers**		250	400	=SUMPRODUCT($C$10:$D$10,C14:D14)			
14								
15								



The optimal solution is to produce 233 boxes of *Ortaköy* and 400 boxes of *Beyoglu*. The optimal profit value is $3285.

2. *Transporting Fruits at Florida Logistics.*

Questions and Answers:

(a) Suppose that FL plans to put the same number of cartons for each fruit on each truck, in other words, suppose that FL sets $A_1 = A_2 = A_3 = 1300$ and $M_1 = M_2 = M_3 = 1200$. Is this a feasible plan?

Answer:

This plan satisfies the constraints on the numbers of cartons of each fruit that need to be shipped. In terms of capacity constraints for each truck, note that each ton of truck capacity corresponds to 2000 lbs, which is equivalent to 100 cartons of 20 lbs each. Thus, truck capacities expressed in the number of cartons are 2700, 3100, and 2800 for trucks 1, 2, and 3, respectively. This plan requires 2500 cartons to be shipped on each truck, and that requirement is within the trucks' capacities. So, the plan is feasible.

(b) Irrespective of whether or not the plan in part a) is feasible, what is the total revenue that FL would earn under this plan?

Answer:

In order to calculate the revenue associated with this plan, we need to calculate the revenue contributions for each carton of fruit shipped on each truck.

For a carton of avocados shipped on truck 1, the revenue contribution is \$20 multiplied by 1-0.05=0.95 (a coefficient reflecting spoiling losses occurring during travel), or \$20*0.95=\$19. In a similar way, the revenue contributions for each carton of avocados shipped on trucks 2 and 3 are \$20*(1-0.04)=\$20*0.96=\$19.2 and \$20*(1-0.03)=\$20*0.97=\$19.4, respectively. For a carton of mangos traveling on trucks 1, 2, and 3, the revenue contributions are \$32*(1-0.10) = \$28.8, \$32*(1-0.12) = \$28.16, and \$32*(1-0.11) = \$28.48, respectively.

Now, using these revenue contributions, we can calculate the total revenue, in \$, corresponding to this plan: 19*1300 + 19.2*1300 + 19.4*1300 + 28.8*1200 +28.16*1200 + 28.48*1200 = (19+19.2+19.4)*1300 + (28.8+28.16+28.48)*1200 = 57.6*1300+85.44*1200 = 74880 + 102528 = 177408.

(c) Write down an algebraic expression for the revenue as a function of decision variables A1, A2, A3, M1, M2, M3.

Answer:

Using the revenue contributions for each carton and the decision variables, we can express the total revenue, in \$, as

$19*A_1 + 19.2*A_2 + 19.4*A_3 + 28.8*M_1 + 28.16*M_2 + 28.48*M_3$.

(d) Using these decision variables, write down an algebraic expression for the constraint "the total number of fruit cartons shipped in truck 1 cannot exceed this truck's capacity".

Answer:

The total number of cartons assigned to truck 1 is the sum of avocado and mango cartons $A_1 + M_1$. The total capacity of truck 1, expressed in number of cartons, is 2700. Thus, the constraint in case can be expressed as

$A_1 + M_1 \leq 2700$.

(e) Use the file Keystone.xlsx as a template to set up and solve the FL's optimization problem. What is the optimal solution? What is the optimal objective function value?

Answer:

Here's the complete algebraic formulation of the FL's problem.

Objective function to be maximized:

$19*A_1 + 19.2*A_2 + 19.4*A_3 + 28.8*M_1 + 28.16*M_2 + 28.48*M_3$

Constraints:

$A_1 + M_1 \leq 2700$ (truck 1 capacity)

$A_2 + M_2 \leq 3100$ (truck 2 capacity)

$A_3 + M_3 \leq 2800$ (truck 3 capacity)

$A_1 + A_2 + A_3 = 3900$ (total amount of avocado to be shipped)

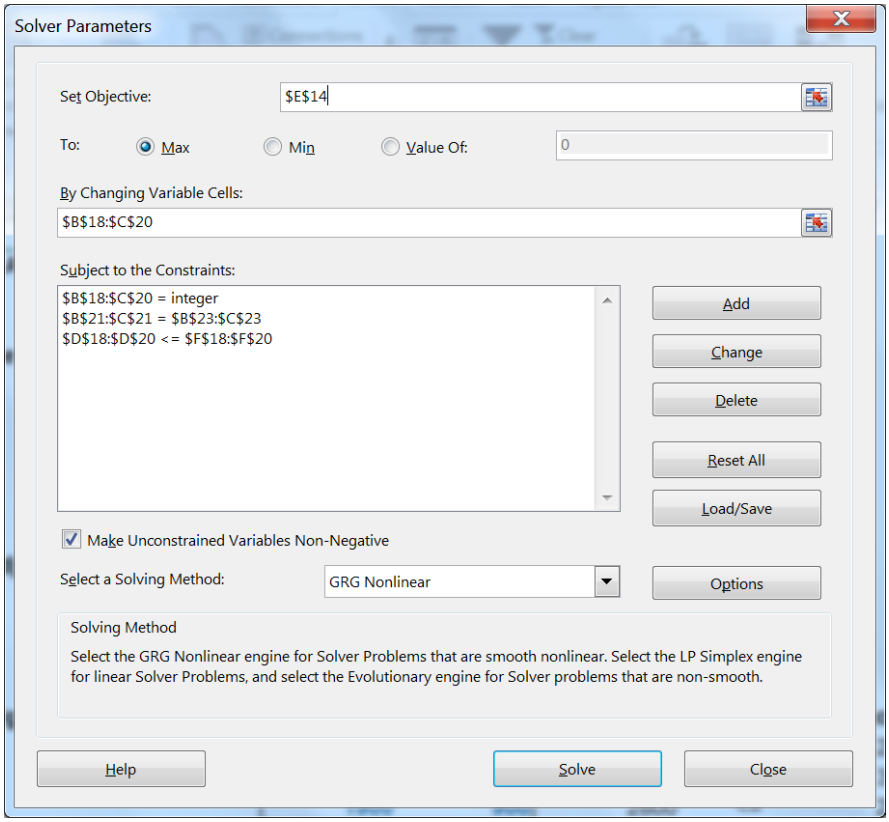
$M_1 + M_2 + M_3 = 3600$ (total amount of mango to be shipped)

$A_1, A_2, A_3, M_1, M_2, M_3$ = non-negative integers

See FL.xlsx for the spreadsheet implementation of this model. Below we show the optimized spreadsheet and the Solver parameters dialog box.

	A	B	C	D	E	F
2	**Operations Analytics MOOC**					
3						
4		**Avocados**	**Mangos**			
5	**Selling Price ($/carton)**	20	32			
6						
7	**Spoilage %**	**Avocados**	**Mangos**			
8	**Truck 1**	5	10			
9	**Truck 2**	4	12	=SUMPRODUCT(B13:C15,B18:C20)		
10	**Truck 3**	3	11			
11						
12	**Spoilage-Adjusted Price ($/carton)**	**Avocados**	**Mangos**			
13	**Truck 1**	19	28.8	**Total Revenue**		
14	**Truck 2**	19.2	28.16	178652		
15	**Truck 3**	19.4	28.48			
16				=SUM(B18:C18)		
17	**Carton Assignments**	**Avocados**	**Mangos**	**Total**		**Capacity**
18	**Truck 1**	0	2700	2700	<=	2700
19	**Truck 2**	2000	0	2000	<=	3100
20	**Truck 3**	1900	900	2800	<=	2800
21	**Total**	3900	3600			
22		=	=			
23	**Available**	3900	3600			

=SUM(B18:B20)



The optimal solution is to allocate truck 1 to mango cartons (entire capacity), and truck 2 to avocados (2000 cartons), while splitting the capacity of truck 3 between mangos (900 cartons) and avocados (1900 cartons). The optimal total revenue is $178652.