

Operations Analytics MOOC: Solutions to Practice Problems for Week 3

1. Simulating an Outcome of a Bidding Process.

Department of Health in a large US city is accepting bids for the contract to design and deploy a new citywide healthcare information platform. The IT company Rapid Deployment (RD) plans to submit a bid and is considering a decision on how much to bid. Since RD does not know exactly how much its main competitor will bid, it treats its main competitor's bid B as a random variable (RD believes that bids from other potential competitors are unlikely to win). RD estimates, based on past experience, that B will be normally distributed with mean \$12 million and the standard deviation of \$1 million. RD estimates that its own cost of designing and deploying the new platform will be \$11 million, and it plans to submit a bid amount $A = \$11.5$ million. The company that submits the lowest bid will win the contract, and if the bids are tied, RD's competitor will get the contract as it has more experience with similar projects. Thus, if B turns out to be less than or equal to \$11.5 million, RD will lose this contract and RD's profit will be \$0. On the other hand, if B turns out to be greater than \$11.5 million, then RD will win this contract, and RD's profit will be equal to \$11.5 million - \$11 million = \$0.5 million. RD would like to use simulation to model the distribution of its profit.

Questions:

- (a) Write down an algebraic expression for RD's profit as a function of its competitor's bid B .

Answer:

RD's profit (in \$millions) can be expressed as $\text{IF}(B \leq 11.5, 0, 0.5)$.

- (b) Suppose that Excel's Random Number Generation tool has generated the following sequence of 5 random values from the normal distribution with mean of 12 and standard deviation of 1: 10.5, 12.3, 11.4, 13.8, 12.5. These values reflect 5 random values of competitor's bid, in \$ millions. What is the sample mean of the RD's profit values corresponding to these competitor's bid values?

Answer:

The RD's profit is 0 for the bid values of 10.5 and 11.4, and 0.5 for the bid values of 12.3, 13.8, and 12.5. The sample mean of numbers 0, 0.5, 0, 0.5, 0.5 is $(0 + 0.5 + 0 + 0.5 + 0.5) / 5 = 0.3$.

- (c) Use Excel to set up and run a simulation of the RD's profit using $n=100$ simulation trials and the random seed of 123. Based on the results of this simulation, what are the estimates for the expected value and the standard deviation of the RD's profit?

Answer:

See RD.xlsx for the details of the simulation set-up. Based on the results of the simulation with $n=100$ simulation trials and the random seed of 123, the estimate for RD's expected profit is \$0.345 million, and the estimate for the standard deviation of RD's profit is \$0.2324 million.

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2. Setting Price for Hotel Rooms.

Bharatiya Ghar (BG) is an Indian real-estate development company that have just finished building a hotel in Goa. BG's hotel has 150 luxury rooms, and the company needs to decide on the long-term price levels to be charged for the hotel rooms. In particular, BG considers a potential price level of ₹10000 per room per day. Based on the analysis of the market research and past occupancy data, the company estimates that if it uses the ₹10000 price level, the daily demand for rooms at the hotel D will be distributed as a Poisson random variable with parameter 147¹. The company assumes that if demand for rooms on a particular day exceeds 150, all extra room requests (above 150) will be lost to another hotel.

Questions:

- (a) Suppose the demand on a given day turns out to be 137. What is the total revenue that BG will receive on that day?

Answer:

If the demand for rooms turns out to be 137, BG's revenue for that day is $137 * ₹10000 = ₹1370000$.

- (b) Suppose the demand on a given day turns out to be 153. What is the total revenue that BG will receive on that day?

Answer:

If the demand for rooms turns out to be 153, the first 150 room requests will be satisfied, and the 3 remaining room requests will be lost to competitors. Thus, BG's revenue for that day will be $150 * ₹10000 = ₹1500000$.

- (c) What is the algebraic expression for the daily total revenue BG will receive, R , as a function of the daily demand D ?

Answer:

BG's daily revenue (in ₹) can be calculated as

$$R = \text{IF}(D \leq 150, 10000 * D, 1500000).$$

¹ Poisson random variable with parameter λ can only take integer non-negative values 0,1,2,..., and has both the mean and the variance equal to λ . It can be used to describe random quantities that cannot take negative or fractional values, such as demand values. In particular, a Poisson random variable with parameter 147 has the expected value of 147 and the standard deviation (i.e., square root of variance) of $\sqrt{147} \approx 12.12$. Excel's "Random Number Generation" utility includes the ability to generate instances of Poisson random variables.

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- (d) Suppose that Excel's Random Number Generation tool has generated the following sequence of 5 random values from the Poisson distribution with parameter 147: 171, 148, 156, 157, 140. These values reflect 5 random daily demand values. What is the sample standard deviation of the BG's daily total revenue values corresponding to these demand values?

Answer:

BG's total daily revenue values, in ₹, corresponding to the demand values $D = 171, 148, 156, 157, \text{ and } 140$ are $R = 1500000, 1480000, 1500000, 1500000, 1400000$. The sample standard deviation for these five numbers can be calculated using STDEV() function in Excel. The result is 43358.96678 or about 43359.

- (e) Use Excel to set up and run a simulation of the total daily revenue R using $n=100$ simulation trials and the random seed of 123. Based on the results of this simulation, what are the estimates for the expected value and the standard deviation of the total daily revenue?

Answer:

See BG.xlsx for the details of the simulation set-up. Based on the results of the simulation with $n=100$ simulation trials and the random seed of 123, the estimate for BG's expected total daily revenue is ₹1442700, and the estimate for the standard deviation of BG's total daily revenue is ₹71602.9315.

- (f) An analyst working for BG believes that the daily demand for rooms can be modeled in a simpler way. In particular, she estimates that the daily demand can be "high" and equal to 159 or "low" and equal to 135. Each scenario, "high" or "low", is equally likely to be observed on any particular day. Note that under this demand model, the expected demand is equal to $0.5 \cdot 159 + 0.5 \cdot 135 = 147$, and the standard deviation of the daily demand is equal to $\sqrt{0.5 \cdot (159 - 147)^2 + 0.5 \cdot (135 - 147)^2} = 12$, which is close to the standard deviation of the "Poisson model" described above. Calculate the expected value (mean) and the standard deviation of BG's daily total revenue values under this alternative model.

Answer:

If the demand value on a particular day is "high", i.e., $D=159$, then the total revenue value on that day, in ₹, is $R = 1500000$. On the other hand, if the demand value on a particular day is "low", i.e., $D=135$, then the total revenue value on that day, in ₹, is $R = 1350000$. The expected value of the total daily revenue is then equal to $0.5 \cdot 1500000 + 0.5 \cdot 1350000 = 1425000$. The standard deviation of the total daily revenue values is calculated as

$$\sqrt{0.5 \cdot (1500000 - 1425000)^2 + 0.5 \cdot (1350000 - 1425000)^2} = \sqrt{0.5 \cdot (75000)^2 + 0.5 \cdot (75000)^2} = 75000.$$