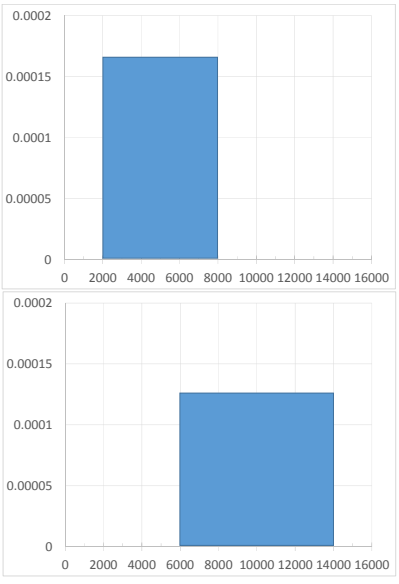
Week 4: Decisions in Settings with High Uncertainty

- Session 1 – Decision Trees
  - Example: Furniture maker IDEA Chooses a Supplier
- Session 2 – Using Simulation within Decision Trees
  - Example: More Complex Demand Distributions for IDEA
- Session 3 – Using Optimization Together with Simulation
  - Example: IDEA Chooses Order Quantities
- Session 4 – Wrap Up
  - Example: Back to the Newsvendor Problem

Updated demand model for IDEA's Krusbär tent

- 50%-50% chance that demand will be strong or weak
- If weak, demand is uniformly distributed: 2,000-8,000 units
- If strong, demand is uniformly distributed: 6,000-14,000 units
- In either case, call the random variable for demand, **D**



Other elements of IDEA's problem remained the same

- Price of 150€ per unit
- Order quantity, fixed charge, and unit cost for each supplier

	Sweden (S)	Poland (P)
Order Quantity	5,000 units	10,000 units
Fixed Charge	0€	50,000€
Unit Cost	120€	100€

But suppose IDEA could choose the order quantity

- In the original problem, IDEA had to use 100% of a supplier's capacity
  - All 5,000 units from supplier S
  - All 10,000 units from supplier P
- Supplier P now offers IDEA the following contract
  - The up-front charge would increase from 50,000€ to 100,000€
  - IDEA would need to place a first order of at least 4,000 units
  - A second order could vary anywhere from 0 to 6,000 units
- IDEA would place the second order in response to market conditions
  - The timing would be shortly after the start of the summer selling season
  - IDEA would know with certainty if the market were weak or strong
- Call the total quantity IDEA orders Q
  - With the new contract IDEA chooses a Q between 4,000 and 10,000 units

If IDEA chooses supplier P and the market is weak…

- IDEA pays a fixed cost of 100,000€
- IDEA decides on a total order quantity Q between 4,000 and 10,000
  - Order cost = 100€ * Q
- Demand D is uniformly distributed between 2,000 and 8,000 uniits
- IDEA earns 150€ revenue for each unit sold
  - If D ≤ Q then revenue = 150€ * D
  - If D > Q then revenue = 150€ * Q
  - Revenue = 150€ * min{D , Q}
- IDEA's Profit = -100,000€ - 100€*Q + 150€ * min{D , Q}
- This is almost identical to the profit formula from last session
  - But last time the fixed cost was 50,000€
  - And last time Q was fixed at 10,000 units

We can use Session 2's simulations to find a good Q

	A	B	C	D	E	F
1	**IDEA Optimization.xlsx**					
2						
3	**Order Quantity (Q) =**	**10000**	units			
4	**Fixed Cost =**	100000	euros			
5	**Price =**	150	euros per unit			
6	**Unit Cost =**	100	euros per unit			
7						
8	**Sample**	**Demand**				
9	**Number**	**Sample (D)**	**Revenue**	**Fixed Cost**	**Variable Cost**	**Profit**
10	1	2,744.90	411,734.37	100,000.00	1,000,000.00	(688,265.63)
11	2	2,039.00	305,850.40	100,000.00	1,000,000.00	(794,149.60)
12	3	4,336.68	650,502.03	100,000.00	1,000,000.00	(449,497.97)
13	4	3,603.69	540,553.00	100,000.00	1,000,000.00	(559,447.00)
14	5	6,221.81	933,271.28	100,000.00	1,000,000.00	(166,728.72)
15	6	3,413.07	511,960.20	100,000.00	1,000,000.00	(588,039.80)
16	7	4,796.84	719,525.74	100,000.00	1,000,000.00	(380,474.26)
17	8	6,487.69	973,152.87	100,000.00	1,000,000.00	(126,847.13)
18	9	2,742.70	411,404.77	100,000.00	1,000,000.00	(688,595.23)
19	10	4,436.11	665,416.43	100,000.00	1,000,000.00	(434,583.57)
20						
21	**average =**	4,082.25			**Average Profit =**	**(487,662.89)**
22	**std. deviation =**	1,471.92				220,788.48
23						

=$B$5*MIN($B$3,B10)

=$B$3*$B$6

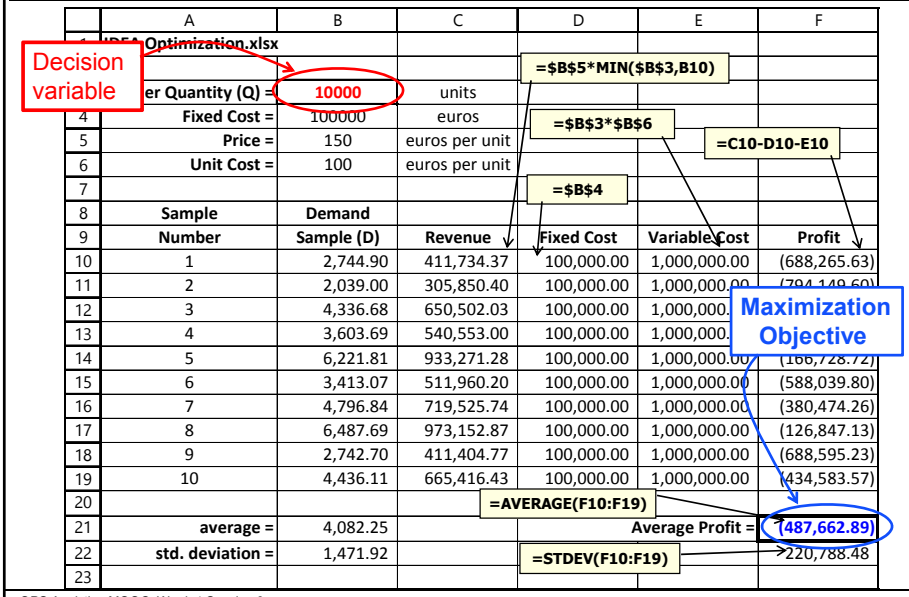
=C10-D10-E10

=$B$4

=AVERAGE(F10:F19)

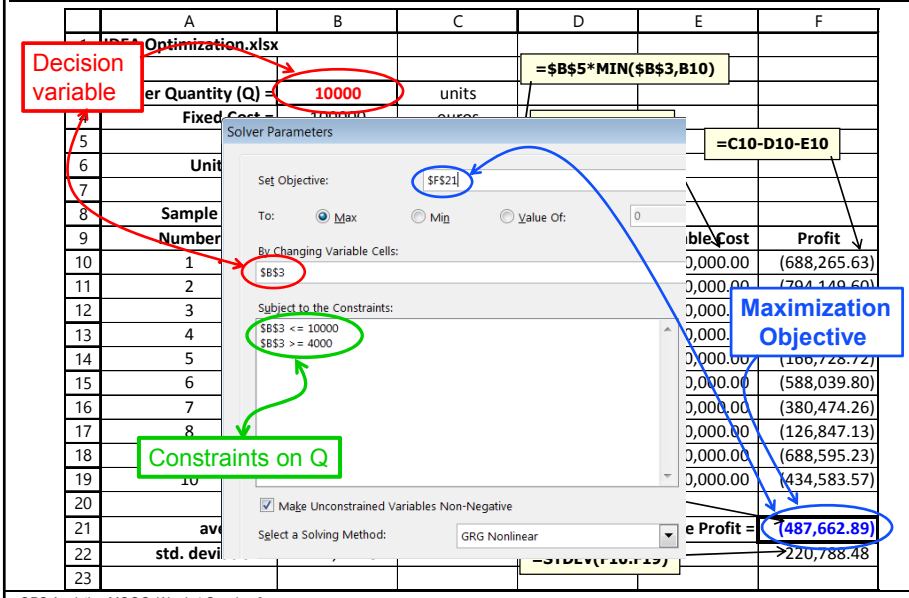
=STDEV(F10:F19)

We'll optimize Q using Sergei's Solver from Week 2



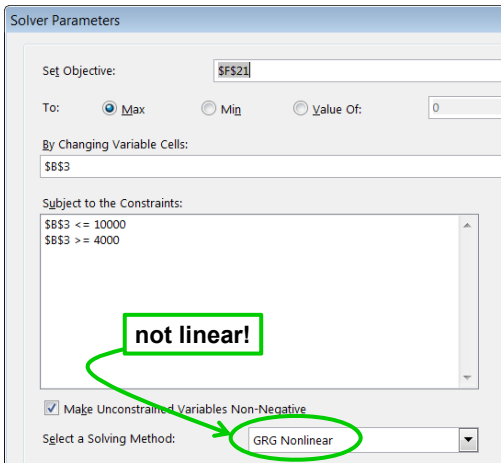
	A	B	C	D	E	F
1	RP54 Optimization.xlsx					
2						
3	er Quantity (Q) =	10000	units			
4	Fixed Cost =	100000	euros			
5	Price =	150	euros per unit			
6	Unit Cost =	100	euros per unit			
7						
8	Sample	Demand				
9	Number	Sample (D)	Revenue	Fixed Cost	Variable Cost	Profit
10	1	2,744.90	411,734.37	100,000.00	1,000,000.00	(688,265.63)
11	2	2,039.00	305,850.40	100,000.00	1,000,000.00	(794,149.60)
12	3	4,336.68	650,502.03	100,000.00	1,000,000.	
13	4	3,603.69	540,553.00	100,000.00	1,000,000.	
14	5	6,221.81	933,271.28	100,000.00	1,000,000.00	(166,728.72)
15	6	3,413.07	511,960.20	100,000.00	1,000,000.00	(588,039.80)
16	7	4,796.84	719,525.74	100,000.00	1,000,000.00	(380,474.26)
17	8	6,487.69	973,152.87	100,000.00	1,000,000.00	(126,847.13)
18	9	2,742.70	411,404.77	100,000.00	1,000,000.00	(688,595.23)
19	10	4,436.11	665,416.43	100,000.00	1,000,000.00	(434,583.57)
20						
21	average =	4,082.25			Average Profit =	(487,662.89)
22	std. deviation =	1,471.92				220,788.48
23						

Here's the Solver optimization dialog box



The problem is not linear…but we could solve it

- The objective is to the maximize the average of the profits across the samples
- The revenue in each sample is 150 * min{D , Q}, which is not linear (in a not very nice way)
- But we are lucky that Excel's solver can solve it….it might not always work.
- In an optional advanced session we will "clean up" the formulation to eliminate the problem with non-linearity.



With 10 samples, Q = 4,000 maximizes average profit

	A	B	C	D	E	F
1	**IDEA Optimization.xlsx**					
2				=$B$5*MIN($B$3,B10)		
3	**Order Quantity (Q) =**	4,000.00	units			
4	**Fixed Cost =**	100000	euros	=$B$3*$B$6		
5	**Price =**	150	euros per unit		=C10-D10-E10	
6	**Unit Cost =**	100	euros per unit			
7				=$B$4		
8	**Sample**	**Demand**				
9	**Number**	**Sample (D)**	**Revenue**	**Fixed Cost**	**Variable Cost**	**Profit**
10	1	2,744.90	411,734.37	100,000.00	400,000.00	(88,265.63)
11	2	2,039.00	305,850.40	100,000.00	400,000.00	(194,149.60)
12	3	4,336.68	600,000.00	100,000.00	400,000.00	100,000.00
13	4	3,603.69	540,553.00	100,000.00	400,000.00	40,553.00
14	5	6,221.81	600,000.00	100,000.00	400,000.00	100,000.00
15	6	3,413.07	511,960.20	100,000.00	400,000.00	11,960.20
16	7	4,796.84	600,000.00	100,000.00	400,000.00	100,000.00
17	8	6,487.69	600,000.00	100,000.00	400,000.00	100,000.00
18	9	2,742.70	411,404.77	100,000.00	400,000.00	(88,595.23)
19	10	4,436.11	600,000.00	100,000.00	400,000.00	100,000.00
20				=AVERAGE(F10:F19)		
21	**average =**	4,082.25			**Average Profit =**	18,150.27
22	**std. deviation =**	1,471.92		=STDEV(F10:F19)		106,367.95
23						

Note that Q = 4,000 is only optimal for that sample

- If we were to run the same optimization on a different set of 10 samples
  - Then we might get a different optimal Q.
- Why? As Sergei showed you in Week 3…
  - A set of 10 samples is quite small
  - We need more samples for more precise, stable estimates from the simulation
- This is (typically) true for an optimal solution too
  - With 1,000 samples, the optimal Q's tend not to change much across samples
- The spreadsheet's optimal solution is just an estimate of the optimal Q
  - To find the truly optimal Q using simulation
  - We'd need to include an infinite number of samples

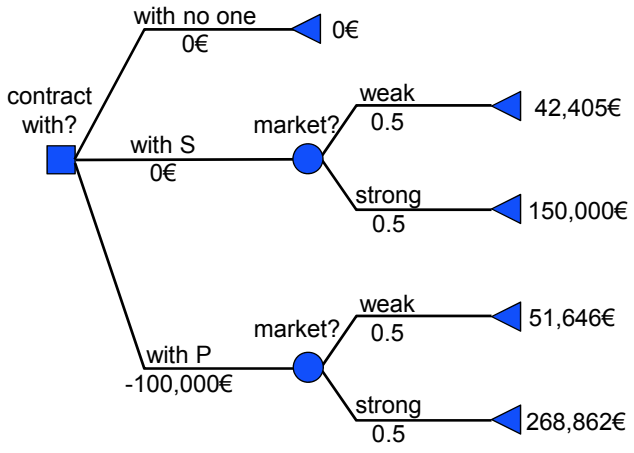
Optimal solution for 1000 samples – weak market

	A	B	C	D	E	F
1	**IDEA.xlsx**					
2						
3	**Order Quantity (Q) =**	**4,137.82**	units			
4	**Fixed Cost =**	100,000	euros			
5	**Price =**	150	euros per unit			
6	**Unit Cost =**	100	euros per unit			
7						
8	**Sample**	**Demand**				
9	**Number**	**Sample (D)**	**Revenue**	**Fixed Cost**	**Variable Cost**	**Profit**
10	1	2,744.90	411,734.37	100,000.00	413,781.82	(102,047.45)
11	2	2,039.00	305,850.40	100,000.00	413,781.82	(207,931.42)
12	3	4,336.68	620,672.73	100,000.00	413,781.82	106,890.91
13	4	3,603.69	540,553.00	100,000.00	413,781.82	26,771.17
14	5	6,221.81	620,672.73	100,000.00	413,781.82	106,890.91
15	6	3,413.07	511,960.20	100,000.00	413,781.82	(1,821.62)
16	7	4,796.84	620,672.73	100,000.00	413,781.82	106,890.91
17	8	6,487.69	620,672.73	100,000.00	413,781.82	106,890.91
1008	999	2,642.54	396,380.50	100,000.00	413,781.82	(117,401.32)
1009	1000	3,713.92	557,087.92	100,000.00	413,781.82	43,306.10
1010						
1011	**average =**	5,044.06			**Average Profit =**	**51,646.29**
1012	**std. deviation =**	1,712.70				94,281.34

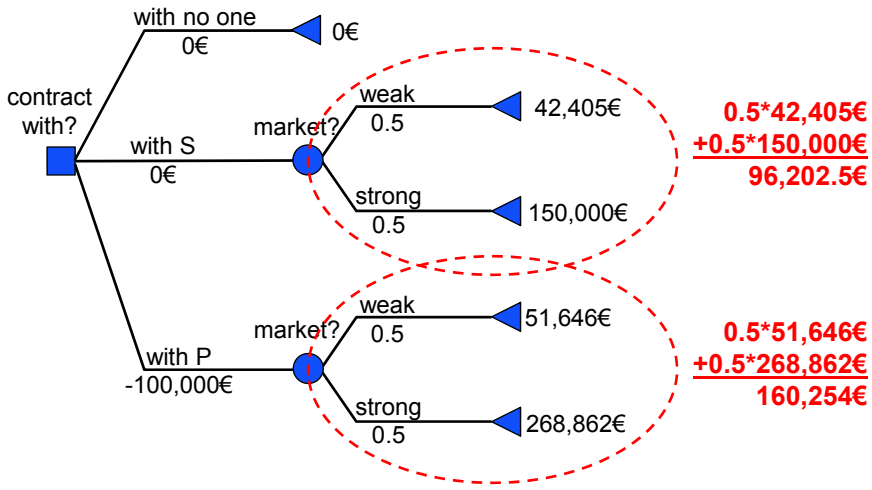
Optimal solution for 1000 samples – strong market

	A	B	C	D	E	F
1	**IDEA.xlsx**					
2						
3	**Order Quantity (Q) =**	**8,850.43**	units			
4	**Fixed Cost =**	100000	euros			
5	**Price =**	150	euros per unit			
6	**Unit Cost =**	100	euros per unit			
7						
8	**Sample**	**Demand**				
9	**Number**	**Sample (D)**	**Revenue**	**Fixed Cost**	**Variable Cost**	**Profit**
10	1	6,993.19	1,048,979.16	100,000.00	885,042.53	63,936.62
11	2	6,052.00	907,800.53	100,000.00	885,042.53	(77,242.00)
12	3	9,115.57	1,327,563.80	100,000.00	885,042.53	342,521.27
13	4	8,138.25	1,220,737.33	100,000.00	885,042.53	235,694.79
14	5	11,629.08	1,327,563.80	100,000.00	885,042.53	342,521.27
15	6	7,884.09	1,182,613.61	100,000.00	885,042.53	197,571.07
16	7	9,729.12	1,327,563.80	100,000.00	885,042.53	342,521.27
17	8	11,983.58	1,327,563.80	100,000.00	885,042.53	342,521.27
1008	999	6,856.72	1,028,507.34	100,000.00	885,042.53	43,464.80
1009	1000	8,285.23	1,242,783.90	100,000.00	885,042.53	257,741.36
1010						
1011	**average =**	10,058.75			**Average Profit =**	**268,861.72**
1012	**std. deviation =**	2,283.61				125,708.51

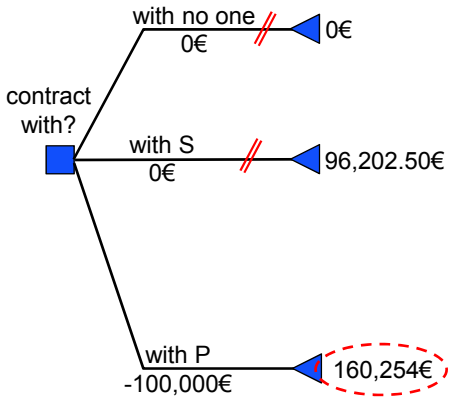
Tree with simulated expected profits and flexible Q



We can then roll back the event nodes



Supplier P's new contract offer makes it preferred



- The ability for IDEA to change order quantity in response to market conditions added value for supplier P
- IDEA's expected profit from contracting with P increases from ~6,500€ to ~160,000€
- The new contact with supplier P is now favored over IDEA's other options

Wrap-up for Session 3 of Week 4

- How did IDEA's problem change from last session to this one?
- As before, we simulated the outcomes for weak and strong markets
  - Demand model had a 50%/50% chance the market would be weak or strong
  - For each case we simulated uniformly distributed demand
- This time the structure of the decision problem became more complex
  - First IDEA needed to decided on a supplier: S, P, or none
  - For supplier P, IDEA could then decide on an order quantity
- Rather than running a separate simulation for each possible Q
  - We used a common set of simulated demands for all possible Q's
  - We optimized to find an "approximately optimal" Q
- In fact, we essentially solved Senthil's newsvendor problem from Week 1
  - In the next session we'll go back to it to see