1. Simulating an Outcome of a Bidding Process.

Department of Health in a large US city is accepting bids for the contract to design and deploy a new citywide healthcare information platform. The IT company Rapid Deployment (RD) plans to submit a bid and is considering a decision on how much to bid. Since RD does not know exactly how much its main competitor will bid, it treats is main competitor's bid B as a random variable (RD believes that bids from other potential competitors are unlikely to win). RD estimates, based on past experience, that B will be normally distributed with mean $12 million and the standard deviation of $1 million. RD estimates that its own cost of designing and deploying the new platform will be $11 million, and it plans to submit a bid amount A=$11.5 million. The company that submits the lowest bid will win the contract, and if the bids are tied, RD's competitor will get the contract as it has more experience with similar projects. Thus, if B turns out to be less than or equal to $11.5 million, RD will lose this contract and RD's profit will be $0. On the other hand, if B turns out to be greater than $11.5 million, then RD will win this contract, and RD's profit will be equal to $11.5 million - $11million = $0.5 million. RD would like to use simulation to model the distribution of its profit.

Questions:

(a) Write down an algebraic expression for RD's profit as a function of its competitor's bid B.

(b) Suppose that Excel's Random Number Generation tool has generated the following sequence of 5 random values from the normal distribution with mean of 12 and standard deviation of 1: 10.5, 12.3, 11.4, 13.8,12.5. These values reflect 5 random values of competitor's bid, in $ millions. What is the sample mean of the RD's profit values corresponding to these competitor's bid values?

(c) Use Excel to set up and run a simulation of the RD's profit using n=100 simulation trials and the random seed of 123. Based on the results of this simulation, what are the estimates for the expected value and the standard deviation of the RD's profit?

2. Setting Price for Hotel Rooms.

Bharatiya Ghar (BG) is an Indian real-estate development company that have just finished building a hotel in Goa. BG's hotel has 150 luxury rooms, and the company needs to decide on the long-term price levels to be charged for the hotel rooms. In particular, BG considers a potential price level of ₹10000 per room per day. Based on the analysis of the market research and past occupancy data, the company estimates that if it uses the ₹10000 price level, the daily demand for rooms at the hotel D will be distributed as a Poisson random variable with parameter 147[1]. The company assumes that if demand for rooms on a particular day exceeds 150, all extra room requests (above 150) will be lost to another hotel.

Questions:

(a) Suppose the demand on a given day turns out to be 137. What is the total revenue that BG will receive on that day?

(b) Suppose the demand on a given day turns out to be 153. What is the total revenue that BG will receive on that day?

(c) What is the algebraic expression for the daily total revenue BG will receive, *R*, as a function of the daily demand *D*?

(d) Suppose that Excel's Random Number Generation tool has generated the following sequence of 5 random values from the Poisson distribution with parameter 147: 171, 148, 156, 157, 140. These values reflect 5 random daily demand values. What is the sample standard deviation of the BG's daily total revenue values corresponding to these demand values?

(e) Use Excel to set up and run a simulation of the total daily revenue *R* using *n*=100 simulation trials and the random seed of 123. Based on the results of this simulation, what are the estimates for the expected value and the standard deviation of the total daily revenue?

(f) An analyst working for BG believes that the daily demand for rooms can be modeled in a simpler way. In particular, she estimates that the daily demand can be "high" and equal to 159 or "low" and equal to 135. Each scenario, "high" or "low", is equally likely to be observed on any particular day. Note that under this demand model, the expected demand is equal to 0.5*159+0.5*135=147, and the standard deviation of the daily demand is equal to $\sqrt{0.5 * (159 - 147)^2 + 0.5 * (135 - 147)^2} = 12$, which is close to the standard deviation of the "Poisson model" described above. Calculate the expected value (mean) and the standard deviation of BG's daily total revenue values under this alternative model.

[1] Poisson random variable with parameter λ can only take integer non-negative values 0,1,2,…, and has both the mean and the variance equal to λ. It can be used to describe random quantities that cannot take negative or fractional values, such as demand values. In particular, a Poisson random variable with parameter 147 has the expected value of 147 and the standard deviation (i.e., square root of variance) of $\sqrt{147} \approx 12.12$. Excel's "Random Number Generation" utility includes the ability to generate instances of Poisson random variables.