- An Operational Decision Problem
- Forecasting with Past Historical Data
- Moving Averages
- Exponential Smoothing
- Thinking about Trends and Seasonality
- Forecasting for new Products
- Fitting distributions

An Operational Decision Problem

**Session 1** 

- Forecasting with Past Historical Data
- Moving Averages
- Exponential Smoothing

Thinking about Trends and Seasonality

- Forecasting for New Products
- Fitting distributions

- An Operational Decision Problem
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**Session 2** 

◆ Thinking about Trends and Seasonality

- Forecasting for New Products
- Fitting distributions

- An Operational Decision Problem
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Thinking about Trends and Seasonality

**Session 3** 

- Forecasting for New Products
- Fitting distributions

- An Operational Decision Problem
- Forecasting with Past Historical Data
- Moving Averages
- Exponential Smoothing

Thinking about Trends and Seasonality

- Forecasting for New Products
- Fitting distributions

**Session 4** 

An Operational Decision Problem

**Session 1** 

- Forecasting with Past Historical Data
- Moving Averages
- Exponential Smoothing

Thinking about Trends and Seasonality

- Forecasting for New Products
- Fitting distributions

#### **Descriptive Analytics**

- Before we dive into analyzing data, let us a look at a fundamental problem that firms face
- An Operations problem:
  - How much to produce?
  - We need to know or estimate the cost of the product, price of the product, and some data on the demand of the product.
- Let us explore a problem to get started.

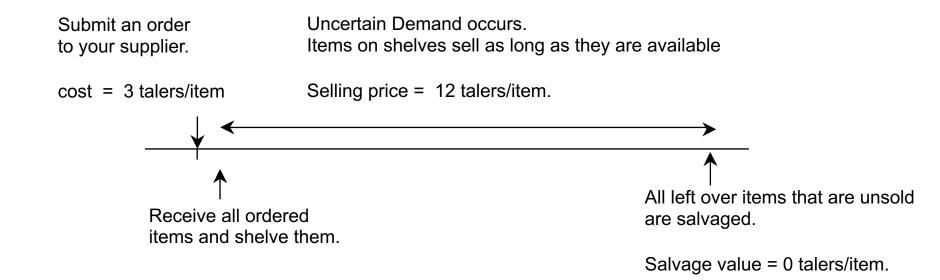
#### A Fundamental Operations Problem: An example

- Suppose that you are making operations decisions for a retailer who orders a product from a supplier and sells it to customers.
- The ordered product items are received and placed on store shelf.
- There is a large customer population
  - Each customer may choose to buy or not buy the product.
  - If the customer chooses to buy, he arrives at the store to buy the product.
  - He buys it as long as it is available on the shelf.
- However, you have to order the product before you see the customer demand, since you have to have the items available on shelf.
- You get only one chance to order (i.e., you can cannot change your purchase order after your decision).

#### An Operations Problem: Costs

- You order the product from the supplier at cost = 3 talers/item.
   (Talers are the currency units).
- After your order is received and placed on shelves, demand occurs.
- ◆ The product on the shelf sells at price = 12 talers/item.
- All unsold items are salvaged. Salvage value =0 talers/item.
- Let us look at timeline of events.

#### Timeline of Events



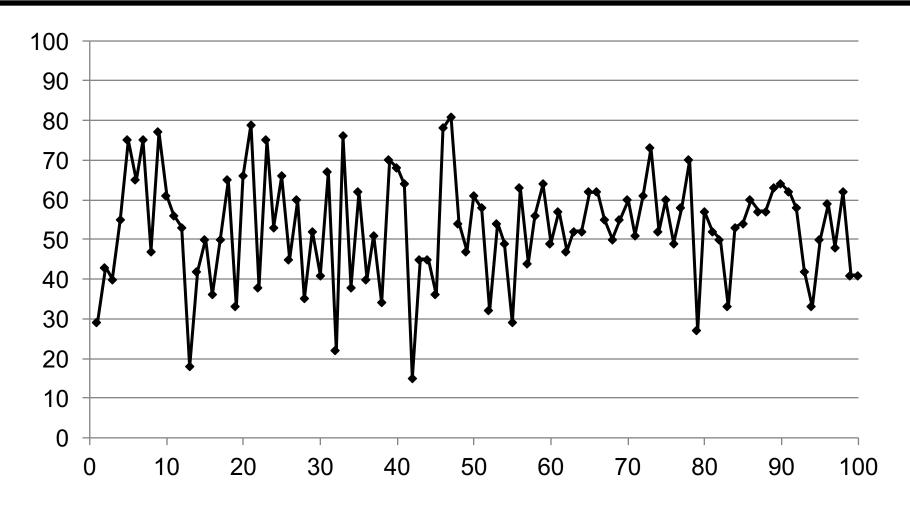
- ◆ Demand is uncertain. Suppose you bought 10 items
  - A High Demand Scenario: Demand is 100. You will sell all 10 items, and make a profit of 10\*(12-3)=90 talers.
  - A Low demand Scenario: No demand (i.e., demand = 0). You sell nothing and lose 10\*3=30 talers.

#### Problem Recap

- You don't know what the demand is going to be...
- You have to decide on the number of units to order from supplier before seeing the customer demand.
- What could help?
  - Past demand data...

Fortunately, we have the demand data from past 100 periods.

#### **Past Demand Data**



◆ The chart shows the demands (y-axis) observed in past 100 periods (x-axis).

#### **Past Demand Data**

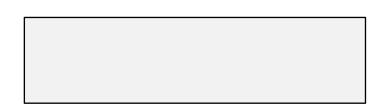
- Some more information from past demand data
- From the observations over the past 100 such periods.
  - Maximum Demand observed was 81.
  - Minimum Demand observed was 15.
  - The arithmetic average of those 100 observations is 52.8
- Based on the data, I am going to ask you to go through an exercise
  - on deciding how much to order.

## Before you make your decision

- ◆ There is no penalty for a wrong answer, or conversely, no extra course credit for the right answer.
- You get one attempt at making your decision.
- ◆ The objective of the exercise is not to test or grade you, but to set a baseline "initial thinking" as we start the course.
- Write down your answer on a sheet of paper and keep the sheet through the course.
- We will see the best answer and you will then get a chance to compare your answers and calibrate learning progress.

#### Question: How much would you order...

- Suppose you are a manager contemplating the question of how many items to order from the supplier.
- Choose the quantity (Q) that you will order.
- Once you select Q, the market will produce 50 random demand instances from the distribution of demand similar to the Figure I showed you.
- ◆ Each random demand instance will correspond to the demand value you may face in the coming selling season.
- Your objective is to select Q to maximize total profit that you will earn when faced with these 50 random demand values.



#### **Newsvendor Problem**

- The problem you just saw is called a Newsvendor problem.
  - Its characteristics are:
    - » You have an objective (usually maximize profits, minimize costs, improve market share, etc.)
    - » You have to make one decision (usually, how much to buy, or plan for).
    - » ... before you see the future demand
    - » Demand occurs, and profits and costs are realized.
- This is called the newsvendor problem:
  - because it is similar to a vendor who sells newspapers:
    - » Buy too much and you may be left with unsold newspapers,
    - » or buy too little, and you will forgo revenue opportunity.
- In this course, we will show you how to think about and analyze this problem.

#### A Business Application at *Time Inc.*

- Time Magazine Supply chain:
  - Stores were either selling out inventories (too little inventory)
  - or sold only a small fraction of allocation (too much inventory).
- Time Magazine evaluated and adjusted for every issue:
  - National print order (total number of copies printed and shipped),
  - Wholesale allotment structure (How those copies are allotted to wholesalers).
  - Store distribution (Final distribution to stores).
- Note: above three decisions are made before the actual demand is realized
  - Need to analyze past data
  - Forecast future demand.
- Time Magazine reports saving \$3.5M annually from tackling the newsvendor problem.
  - Koschat et al, Interfaces, Volume 33, No 3. May-June 2003, pages 72-84.

#### Broader applications of the Newsvendor problem

- Governments order flu vaccines before the flu season begins, and before the extent or the nature of the flu strain is known
  - How many vaccines to order?
- Smart phone users buy mobile data plans before they know their actual future usage
  - What is the right plan for you?
- Consumers buy health insurance plans, before they know their actual health expenditures
  - How to think about the right plans?
- For all the above examples: some forecast of future demand is essential

## Introduction to Forecasting

- What is forecasting?
  - Primary Function is to Predict the Future
- Why are we interested?
  - Dictates the decisions we make today
- Examples: who uses forecasting in their jobs?
  - forecast demand for products and services
  - forecast inventory and capacity needs daily
- What makes a good forecast?
  - It should be timely, reliable.
  - It should be as accurate as possible, and
  - It should be in meaningful units
  - The method should be easy to use and be understood in practice.

#### **Characteristics of Forecasts**

- Point forecasts are usually wrong! Why?
  - Examples: In December 2015, there will be 37cms of snow.
  - We will sell 314 umbrellas during the rains next week.
  - Demand could be a random variable.

- Therefore, a good forecast should be more than a single number
  - mean and standard deviation
  - range (high and low) (e.g. weather forecasts).

## Modeling Uncertain Future: Probability Distributions

- ♦ We often do not control purchasing behavior as a result, we cannot predict future demand with certainty
- How do we describe uncertain future demand?
- We can try to decide what future demand scenarios are possible, for each scenario, estimate the likelihood of its realization
- Where do scenarios come from?
  - Past data
  - Expert estimates

## An Example of a Model of Future Demand

- ◆ Let's start by looking at a small number of scenarios, say, three: "high demand", "ordinary demand" and "low demand".
- ◆ Let's say that "high demand" scenario corresponds to the demand value of 80, "ordinary demand" scenario to the value of 50, and "low demand" scenario to a value of 20
- For each scenario, a likelihood of its occurring must be estimated

## Example of a Model of Future Demand: How Likely is Each Scenario

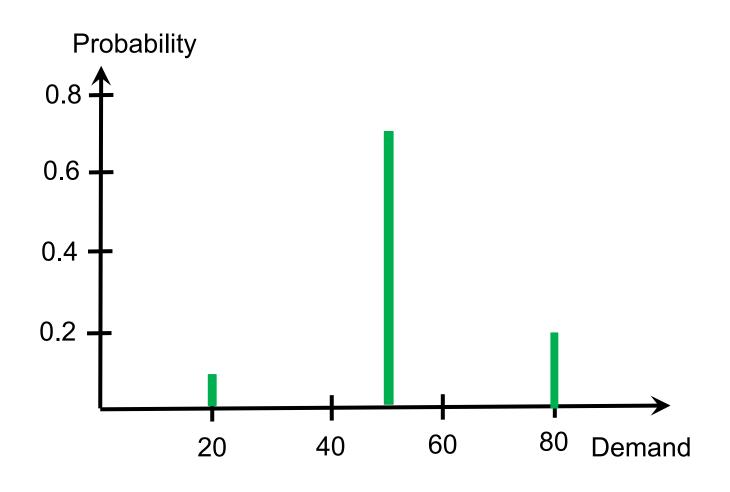
- ◆ For each scenario, a likelihood of its coming true must be estimated
- Where do estimates of likelihood come from?
  - Statistical analysis of past data

- Suppose that after analyzing the past data and using subjective inputs, we estimate that scenarios have the following likelihoods of being realized in the next selling season:
  - Likelihood of "high" demand is 20%
  - Likelihood of "normal" demand is 70%
  - Likelihood of "low" demand is 10%

#### Three Scenarios and Probability Distribution

- In other words, we project that the demand is not equal to a certain number with probability 1, but, rather can take one of three values with those probabilities
- We have just created a probability distribution for the future demand:
  - $D_1$  = 80 with probability  $p_1$  = 0.2
  - $D_2$  = 50 with probability  $p_2$  = 0.7
  - $D_3$  = 20 with probability  $p_3 = 0.1$
- Probability distributions like that one, described by a number of distinct scenarios with attached probabilities, are called discrete
- Note that the probabilities are
  - greater than zero, and
  - they sum upto 1.

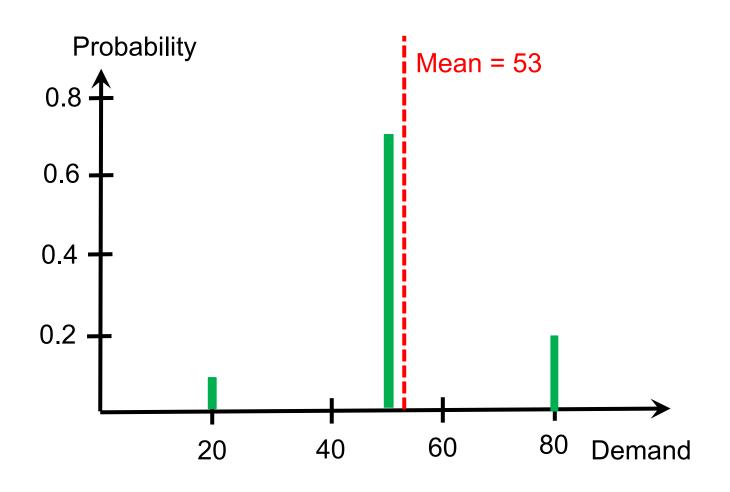
# Three Scenarios Probability Distribution: Scenarios and Their Probabilities



## Describing Probability Distribution: Mean and Standard Deviation

- ◆ For any probability distribution, including a simple one reflecting three demand scenarios, two useful descriptive quantities are often calculated: mean (also called expected value) and standard deviation
- ◆ For a discrete probability distribution, the mean is defined as a sum of the products of scenario values and their probabilities
- For our demand distribution, the mean  $\overline{D} = p_1D_1 + p_2D_2 + p_3D_3 = 0.2 * 80 + 0.7 * 50 + 0.1 * 20 = 53.$
- Mean reflects the demand value that we will get, on average, in a selling season, if we keep observing the demand realizations over infinite number of selling seasons

## Three Scenarios Probability Distribution: Mean



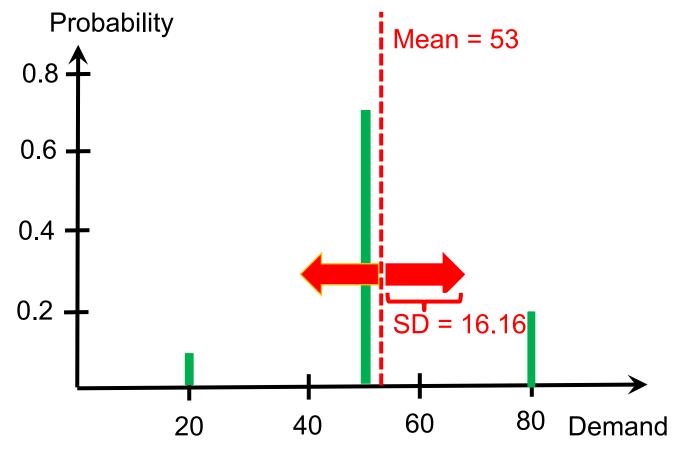
## Describing Probability Distribution: Mean and Standard Deviation

- Standard deviation describes, roughly speaking, how far away actual random variable values are from the mean, on average. In other words, it describes how, in a colloquial sense, "spread out" the distribution is around its mean
- Standard deviation is defined as a square root of the sum of products of scenario probabilities and the squares of the difference between scenario value and the mean value
- ◆ For example, for the three-scenario demand probability distribution we consider, the standard deviation is calculated as

$$SD = \sqrt{p_1 * (D_1 - \overline{D})^2 + p_2 * (D_2 - \overline{D})^2 + p_3 * (D_3 - \overline{D})^2}$$
  
=  $\sqrt{0.2 * (80 - 53)^2 + 0.7 * (50 - 53)^2 + 0.1 * (20 - 53)^2} \approx 16.16$ 

## Three Scenarios Probability Distribution: Mean and Standard Deviation

 Knowledge of mean and standard deviation values helps to support a general intuition about the nature of a random variable



## Mean and Standard Deviation: More than three scenarios

- What if we have more than three scenarios?
  - $D_1$  with probability  $p_1$
  - $D_2$  with probability  $p_2$
  - $D_3$  with probability  $p_3$

. . . . . .

-  $D_n$  with probability  $p_n$ 

and 
$$p_1 + p_2 + p_3 + \dots + p_n = 1$$

What about mean and standard deviation of this demand distribution for n scenarios?

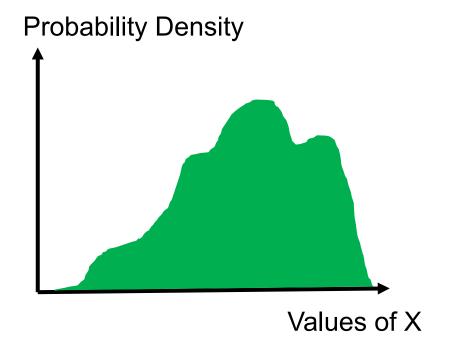
Mean = 
$$\overline{D} = p_1D_1 + p_2D_2 + p_3D_3 + \cdots + p_nD_n$$
  
Standard Deviation = 
$$\sqrt{p_1 * (D_1 - \overline{D})^2 + p_2 * (D_2 - \overline{D})^2 + \cdots + p_n * (D_n - \overline{D})^2}$$

#### Discrete vs. Continuous Probability Distributions

- So far, we have looked at a discrete probability distributions with a number of future scenarios with an "attached" probability for each scenario
- But what will happen to a discrete probability picture when
  - a) random variable being modeled has a really large number of scenarios on any small interval of possible interval of values and
  - b) the probability that any one scenario is realized is really small
- Think of examples such as stock prices, or the amount of rainfall in a region.
- ◆ In such cases, it makes sense to describe such probability distribution using groups of scenarios rather than focusing on individual scenarios

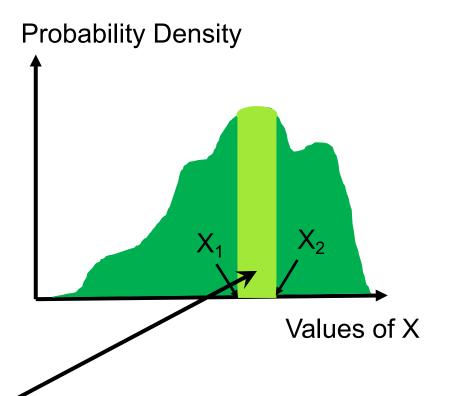
#### Continuous Distribution: Random Variable X

Distributions like this are called continuous



#### Continuous Distribution: Random Variable X

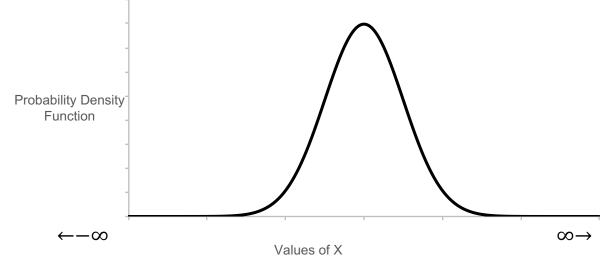
Distributions like this are called continuous



- The area is equal to probability to that the random variable X takes values in the interval between X₁ and X₂
- The area under the entire curve is equal to 1

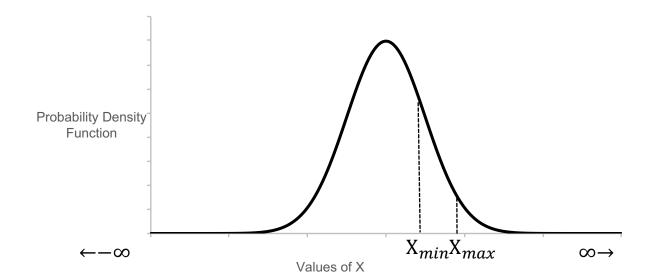
#### Normal Distribution

- One of the most popular examples of a continuous probability distribution is normal distribution
- Normal distribution:
  - Allows the underlying random variable to take any value from negative infinity to positive infinity, and
  - is completely characterized by two parameters mean  $\mu$  and standard deviation  $\sigma$ .



#### **Normal Distribution**

 There exist statistical formulas (also implemented in Excel) that calculate a probability that a normal random variable X with given mean μ and standard deviation σ produces a value within a specified interval of values [X<sub>min</sub>, X<sub>max</sub>]



## Other Continuous Probability Distributions

- ◆ There exist a large number of other "popular" continuous probability distribution: exponential, beta, etc. with easily computable mean and variance/standard deviation
- Each of those distributions is often used to describe specific uncertain setting/quantity
- ◆ For example, normal distribution is used to describe a distribution of future relative (percentage) changes in the values of stocks, FX rates
- Another example: exponential distribution can be used in characterizing time between successive arrivals of customers in service systems (e.g. call centers).

#### Returning back: Characteristics of Forecasts

- Point forecasts are usually wrong! Why?
  - Demand could be a random variable
- Therefore, a good forecast should be more than a single number
- Forecasts should include some distribution information
  - mean and standard deviation
  - range (high and low)
- Aggregate forecasts are usually more accurate
- Accuracy of forecasts erodes as we go further into the future
- Don't exclude known information

## Subjective Forecasting Methods

- Composites
  - Sales Force Composites: Aggregation of sales personnel estimates.
  - Election Polling Composites: sites that aggregate polls.
- Customer Surveys
- Jury of Executive Opinion
- The Delphi Method
  - Individual opinions are compiled and reconsidered. Repeat until overall group consensus is (hopefully) reached.
- We will return to subjective forecasting methods at the end of the Week 1 (Last Session).

#### How to forecast with past data, *objectively*?

- We can leverage past data to come up with forecasts:
  - Two primary methods: causal models and time series methods

#### ◆ Causal Models

- Let D be the demand or future outcome to be predicted and assume that there
  are n variables (or root causes) that influence the demand.
- A causal model is one in which demand D is formulated as a theoretical function of all those n causes.
- Causal models are generally intricate and complex, and need advanced tools in addition to domain expertise.
- In this course, we will focus mainly on time series based models.

#### **Time Series Methods**

- A time series is just collection of past values of the variable being predicted.
- Can be considered as a "naïve" method. Goal is to isolate patterns in past data.
- Past data might have characteristics such as:
  - Trend
  - Seasonality/Cycles
  - Randomness

#### Next...

- An Operational Decision Problem
- Forecasting with Past Historical Data
- Moving Averages
- Exponential Smoothing

**Session 2** 

- Thinking about Trends and Seasonality
- Forecasting for new products
- Fitting distributions