NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Introduction to Data Science and Artificial Intelligence

Solving Problems by Search: Uninformed Search and Informed Search

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Lesson Outline

- Uninformed search strategies
- Informed search strategies
  - Greedy search
  - A * search

Review: Well-Defined Formulation

Definition of a problem	The information used by an agent to decide what to do

Specification	• Initial state
	• Action set, i.e. available actions (successor functions)
	• State space, i.e. states reachable from the initial state
	    • Solution path: sequence of actions from one state to another
	• Goal test predicate
	    • Single state, enumerated list of states, abstract properties
	• Cost function
	    • Path cost *g(n),* sum of all (action) step costs along the path

Solution	A path (a sequence of operators leading) from the Initial-State to a state that satisfies the Goal-Test

Search Algorithms

- Exploration of state space by generating successors of already-explored states
  - Frontier: candidate nodes for expansion
  - Explored set

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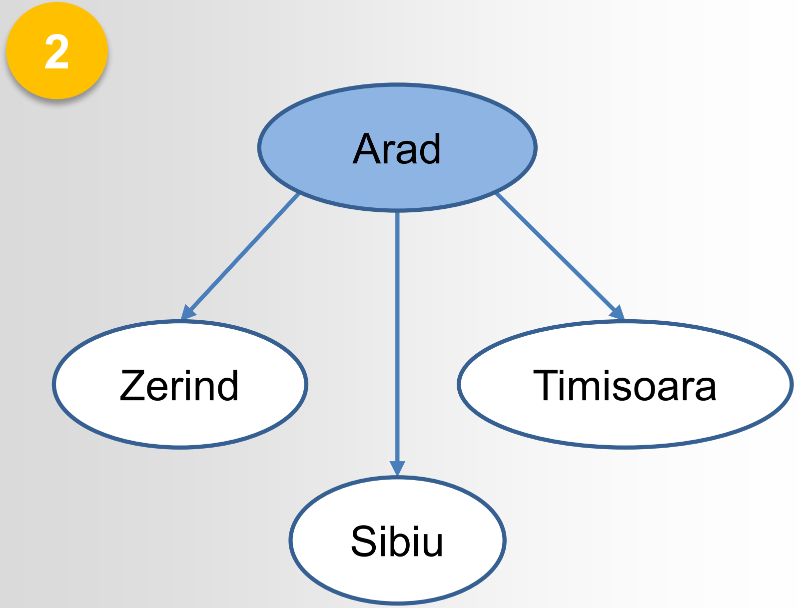

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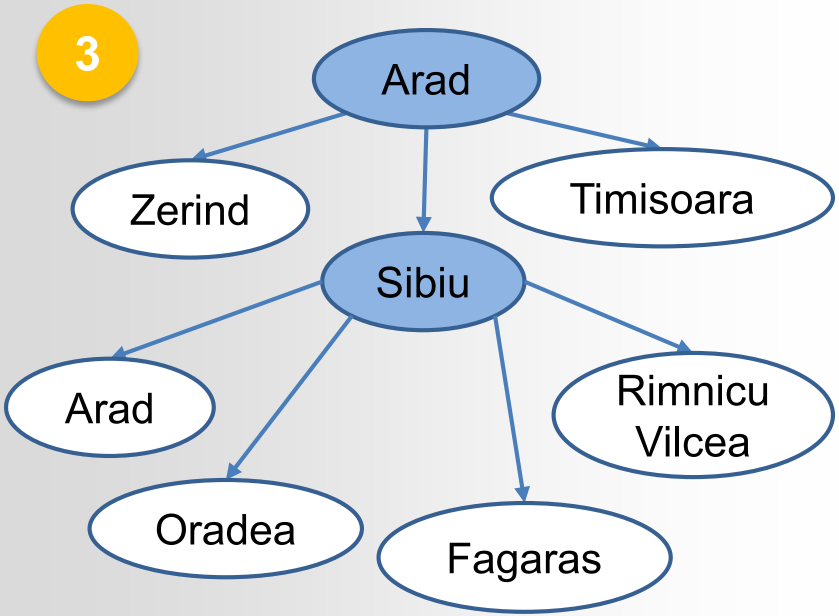

Search Algorithms

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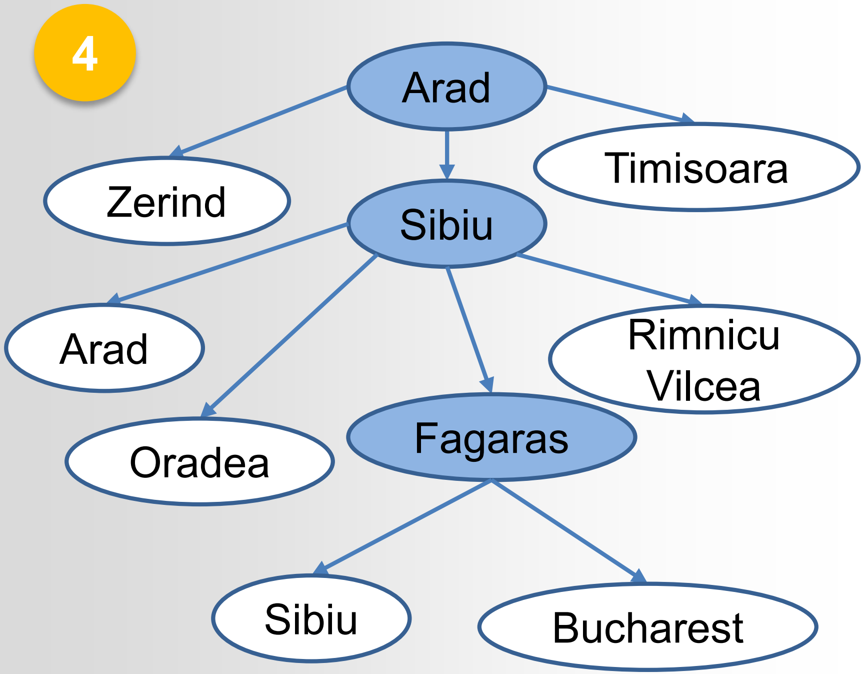


Search Strategies

- A strategy is defined by picking the order of node expansion.
- Strategies are evaluated along the following dimensions:

Completeness	Does it always find a solution if one exists?
Time Complexity	How long does it take to find a solution: the number of nodes generated
Space Complexity	Maximum number of nodes in memory
Optimality	Does it always find the best (least-cost) solution?

Search Strategies

- Branching factor
  - Maximum number of successors of any node
  - Or average branching factor

Uninformed vs Informed

Uninformed search strategies

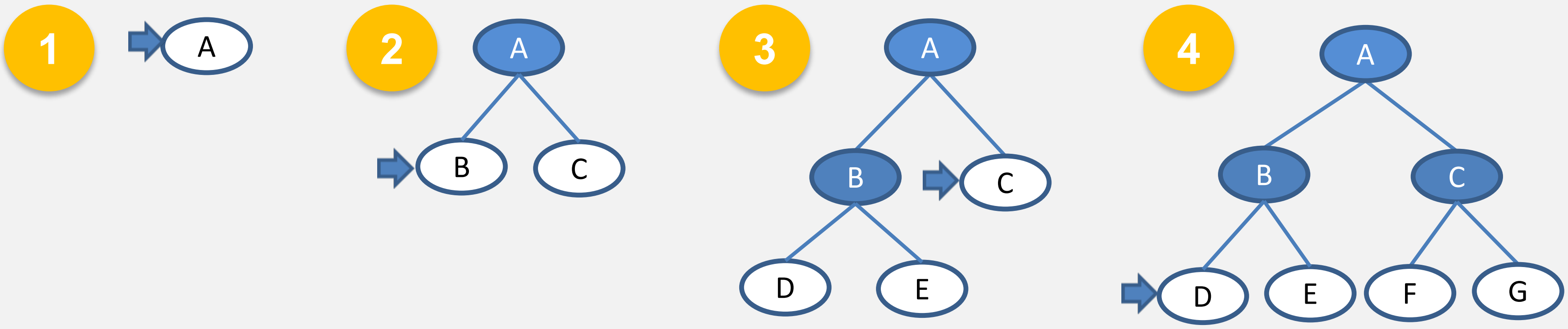
- Use only the information available in the problem definition
  1. Breadth-first search
  2. Uniform-cost search
  3. Depth-first search
  4. Depth-limited search
  5. Iterative deepening search

Informed search strategies

- Use problem-specific knowledge to guide the search
- Usually more efficient

Breadth-First Search

Expand shallowest unexpanded node which can be implemented by a First-In-First-Out (FIFO) queue



Denote
- b: maximum branching factor of the search tree
- d: depth of the least-cost solution
- Complete: Yes
- Optimal: Yes when all step costs equally

Complexity of BFS

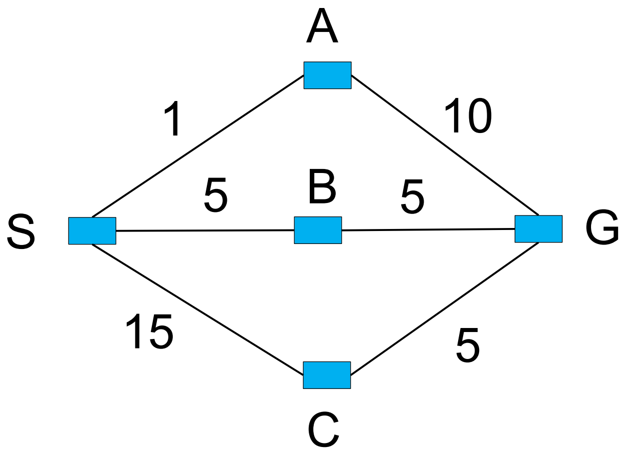
- Hypothetical state-space, where every node can be expanded into *b* new nodes, solution of path-length *d*
- Time: $1 + b + b^2$+b^3+ ... + b^d = $O(b^d)$
- Space: (keeps every node in memory) $O(b^d)$are equal

Depth	Nodes	Time		Memory	
0	1	1	millisecond	100	bytes
2	111	0.1	seconds	11	kilobytes
4	11111	11	seconds	1	kilobytes
6	10^6	18	minutes	111	megabyte
8	10^8	31	hours	11	gigabytes
10	10^{10}	128	days	1	terabyte
12	10^{12}	35	years	111	terabytes
14	10^{14}	3500	years	11111	terabytes

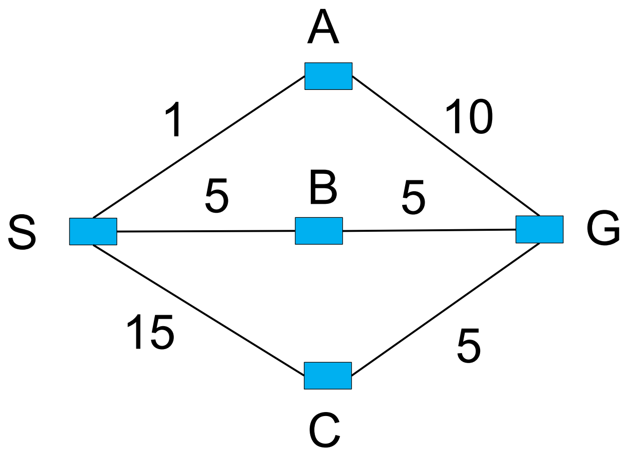
Uniform-Cost Search

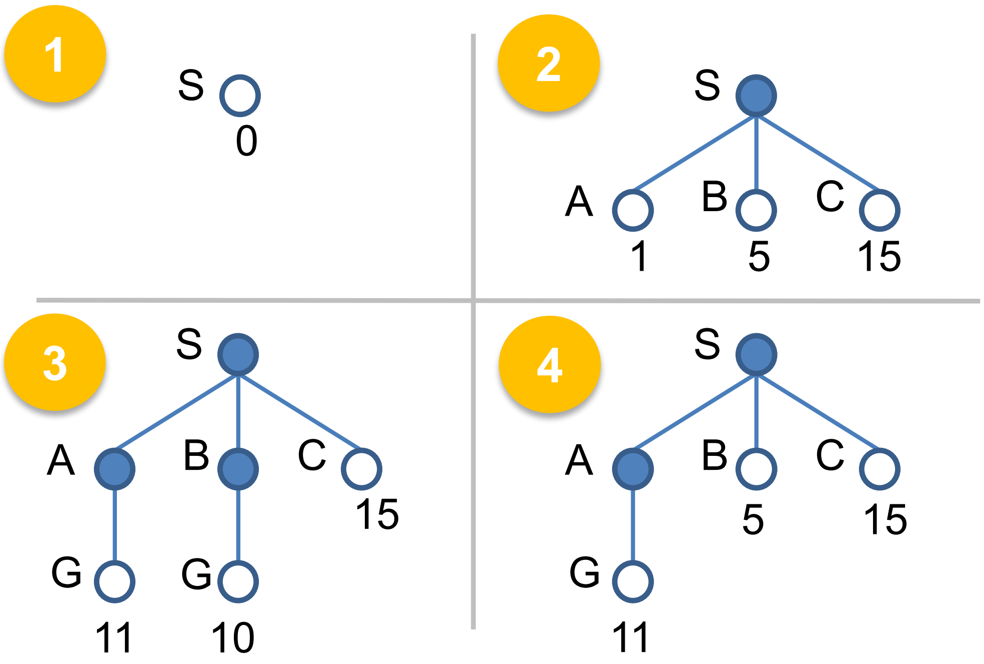
To consider edge costs, expand unexpanded node with the least path cost *g*

- Modification of breath-first search
- Instead of First-In-First-Out (FIFO) queue, using a priority queue with path cost *g(n)* to order the elements
- BFS = UCS with *g(n)* = Depth(n)



Uniform-Cost Search





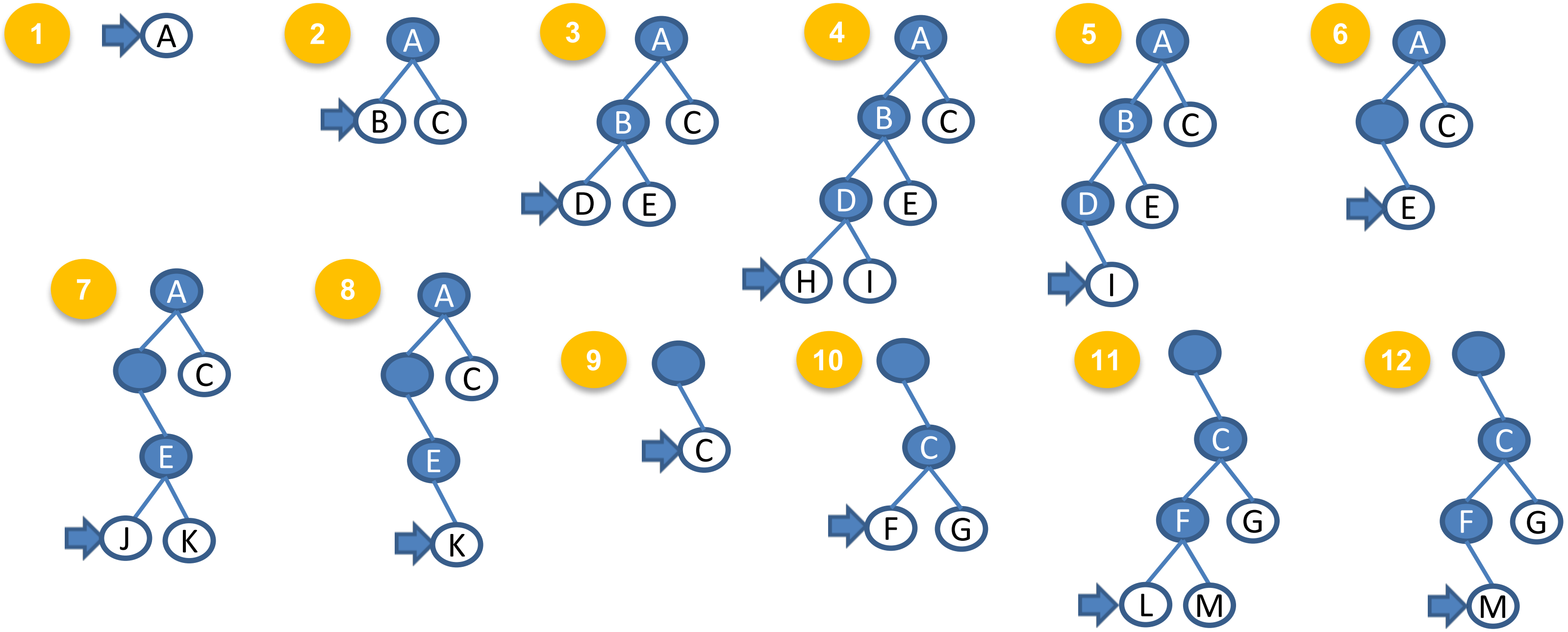
Here we do not expand notes that have been expanded.

Uniform-Cost Search

Complete	Yes
Time	# of nodes with path cost g <= cost of optimal solution (eqv. # of nodes pop out from the priority queue)
Space	# of nodes with path cost g <= cost of optimal solution
Optimal	Yes

Depth-First Search

Expand deepest unexpanded node which can be implemented by a Last-In-First-Out (LIFO) stack, Backtrack only when no more expansion



Depth-First Search

Denote

- m: maximum depth of the state space

Complete	• infinite-depth spaces: No • finite-depth spaces with loops: No • with repeated-state checking: Yes • finite-depth spaces without loops: Yes
Time	$O(b^m)$ If solutions are dense, may be much faster than breadth-first
Space	$O(bm)$
Optimal	No

Depth-Limited Search

To avoid infinite searching, Depth-first search with a cutoff on the max depth l of a path

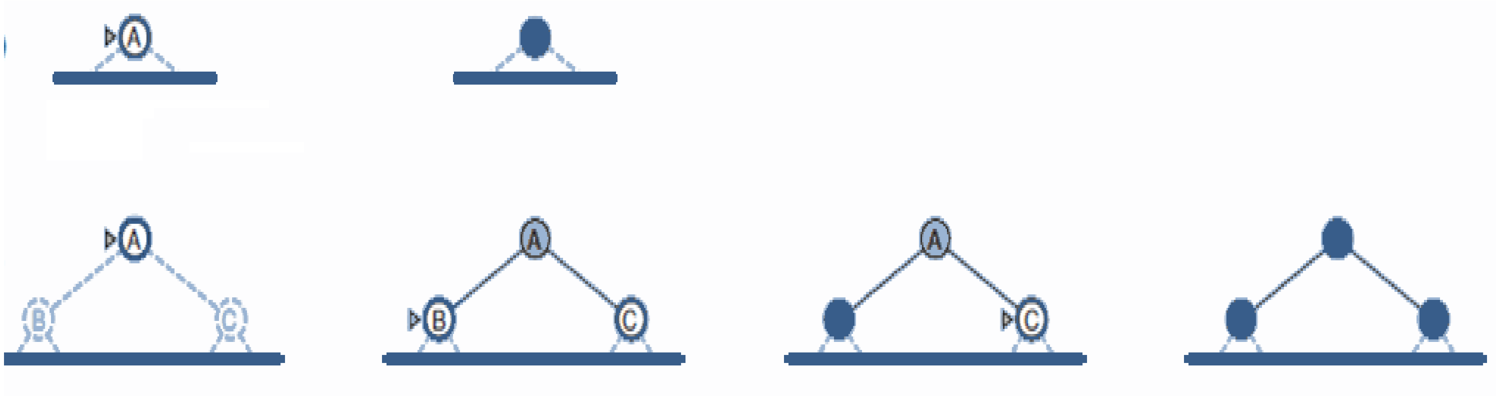
Complete	Yes, if $l \geq d$
Time	$O(b^l)$
Space	$O(bl)$
Optimal	No

Iterative Deepening Search

Iteratively estimate the max depth *l* of DLS one-by-one

Limit = 0

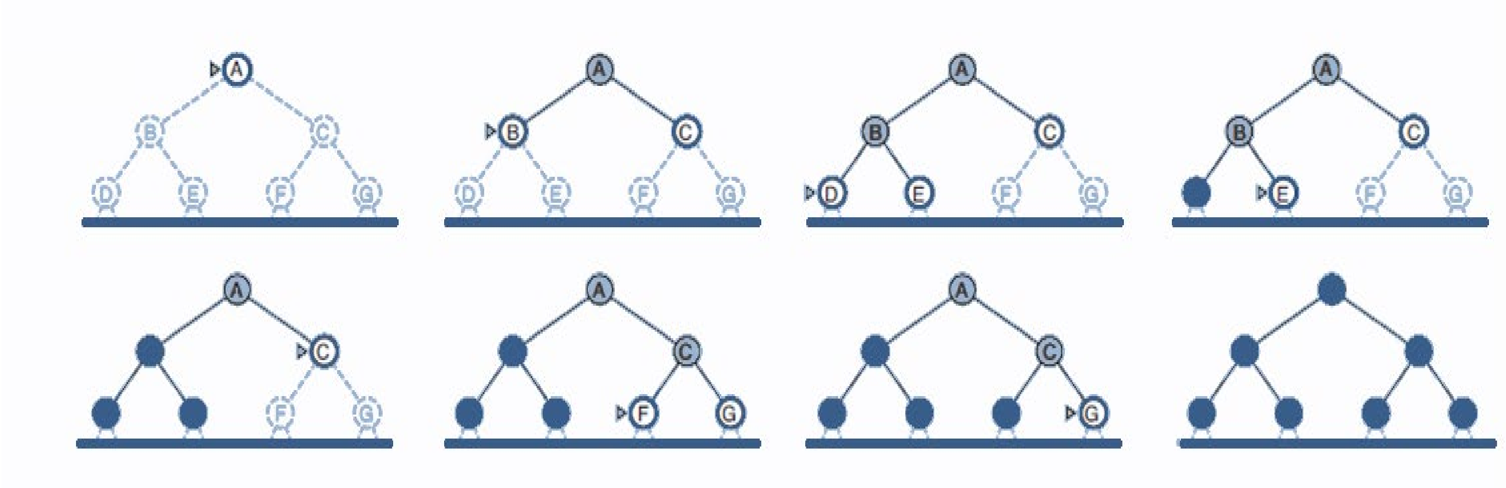
Limit = 1



Iterative Deepening Search

Iteratively estimate the max depth l of DLS one-by-one

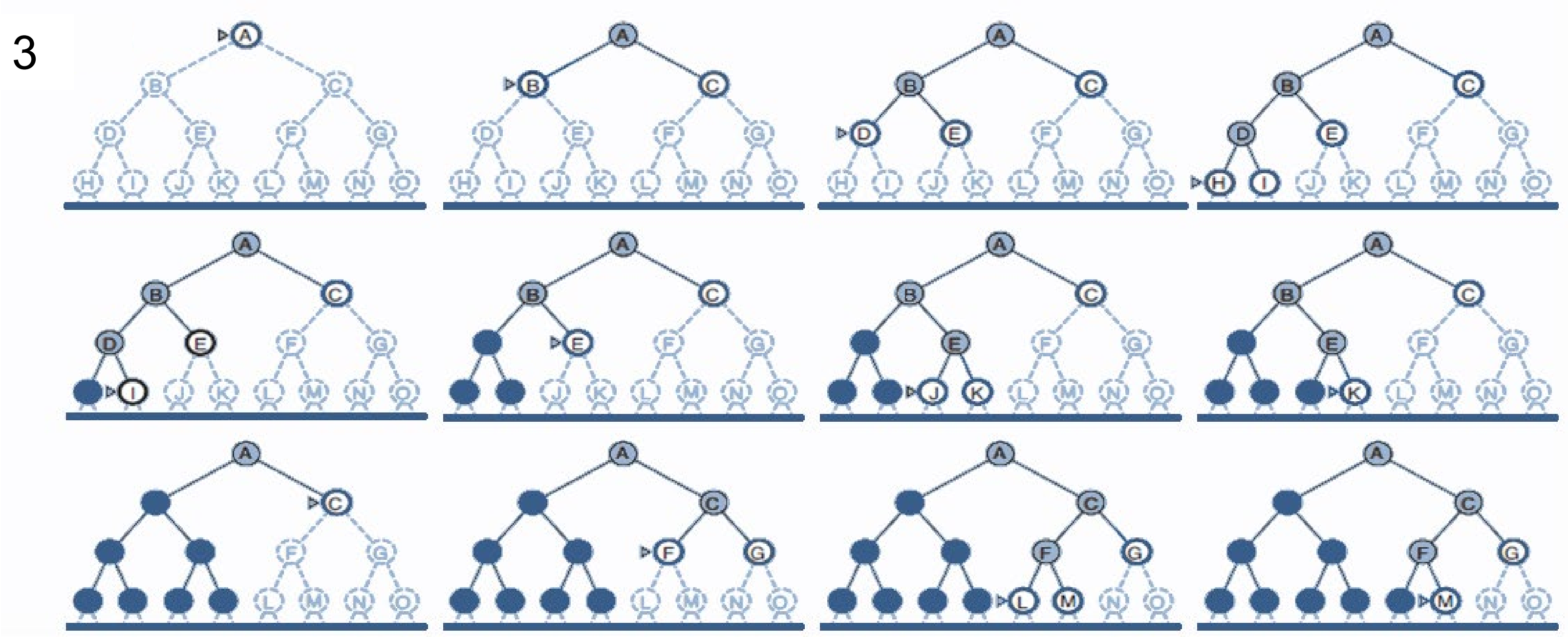
Limit = 2



Iterative Deepening Search

Iteratively estimate the max depth *l* of DLS one-by-one

Limit = 3



Iterative Deepening Search...

```
Function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution sequence
   inputs: problem, a problem

   for depth    0 to ∞ do
    if DEPTH-LIMITED-SEARCH(problem, depth) succeeds then return its result
   end
   return failure
```

Complete	Yes
Time	$O(b^d)$
Space	$O(bd)$
Optimal	Yes

Summary (we make assumptions for optimality)

Criterion	Breadth-first	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Time	b^d	b^d	b^m	b^l	b^d	$b^{d/2}$
Space	b^d	b^d	bm	bl	bd	$b^{d/2}$
Optimal	Yes	Yes	No	No	Yes	Yes
Complete	Yes	Yes	No	Yes, if $l \geq d$	Yes	Yes

Question to think:

- If a search strategy is optimal, is it also complete?

General Search

Uninformed search strategies

- Systematic generation of new states (→Goal Test)
- Inefficient (exponential space and time complexity)

Informed search strategies

- Use problem-specific knowledge
  - To decide the order of node expansion
- Best First Search: expand the most desirable unexpanded node
  - Use an evaluation function to estimate the "desirability" of each node

Evaluation function

- Path-cost function $g(n)$
  - Cost from initial state to current state (search-node) n
  - No information on the cost toward the goal
- Need to estimate cost to the closest goal
- "Heuristic" function $h(n)$
  - Estimated cost of the cheapest path from n to a goal state $h(n)$
    - Exact cost cannot be determined
  - depends only on the state at that node
  - $h(n)$ is not larger than the real cost (admissible)

Greedy Search

Expands the node that appears to be closest to goal

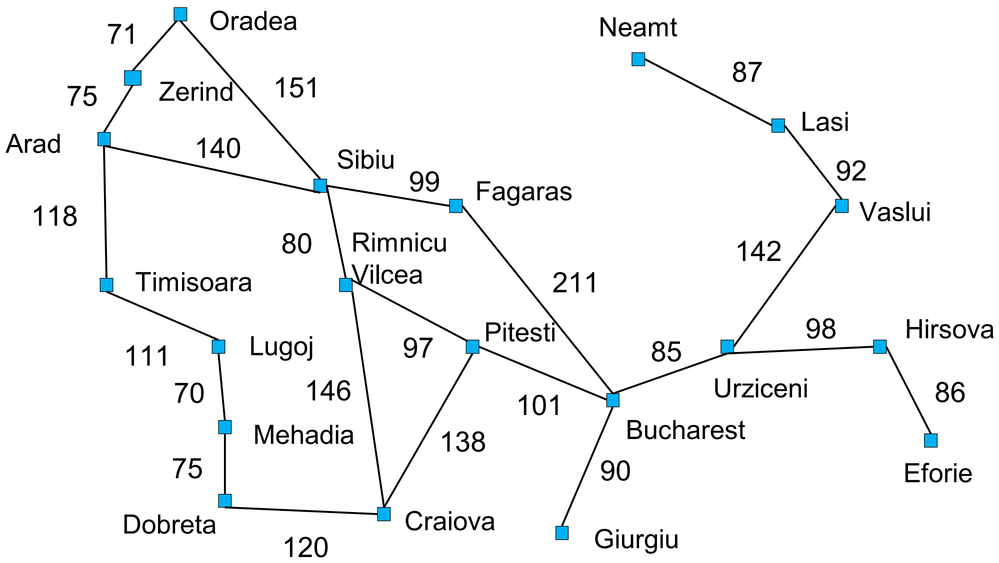
- Evaluation function $h(n)$:estimate of cost from n to $goal$
- Function Greedy-Search(problem) returns solution
  - Return Best-First-Search(problem, h)   // $h(goal) = 0$

Question: How to estimation the cost from n to goal?

Answer: Recall that we want to use problem-specific knowledge

Example: Route-finding from Arad to Bucharest

$h(n)$ = straight-line distance from n to Bucharest



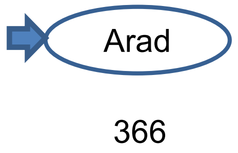
- Useful but potentially fallible (heuristic)
- Heuristic functions are problem-specific

Straight-line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Efoire	161
Fagaras	176
Giurgiu	77
Hirsova	151
Lasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Example

a) The initial state



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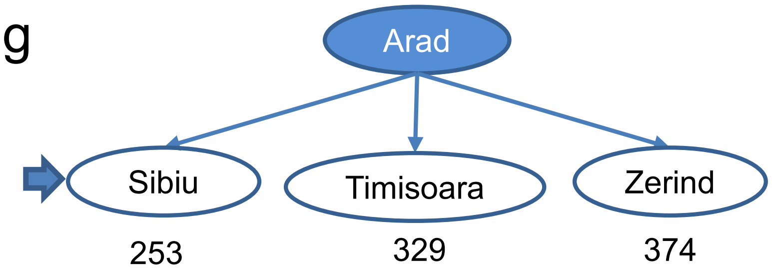
Example

b) After expanding Arad



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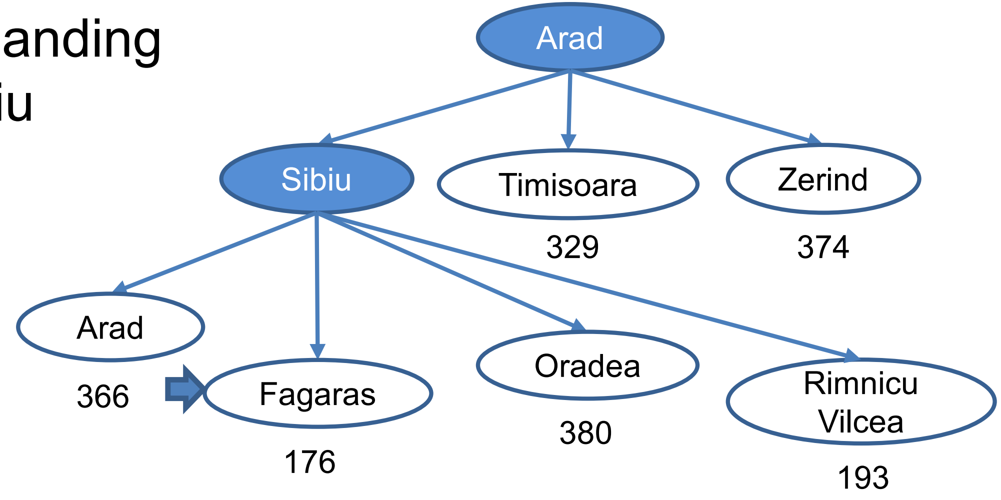
Example

c) After expanding Sibiu



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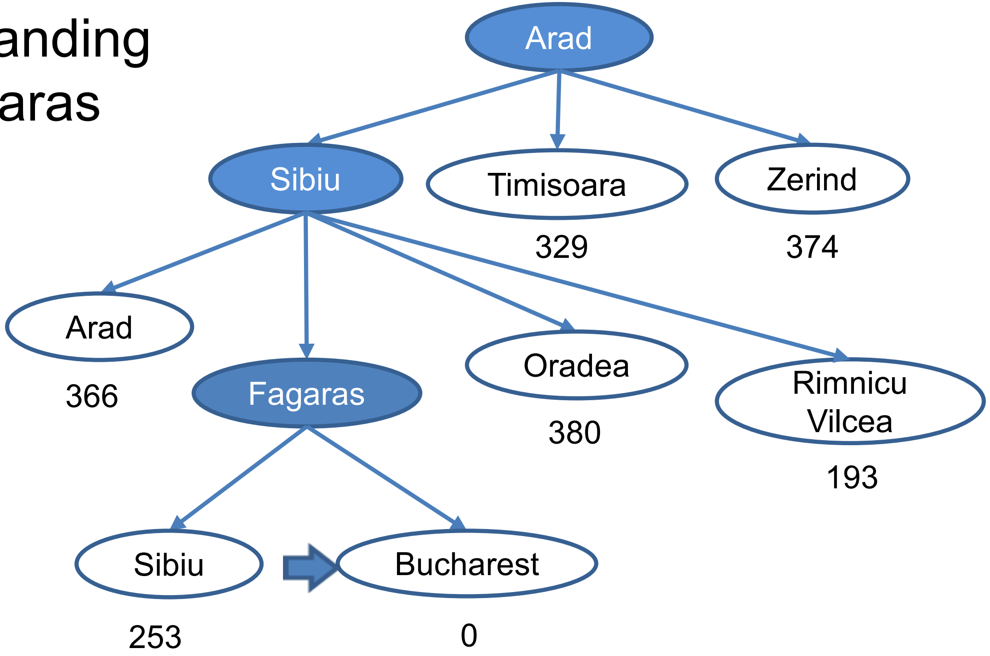
Example

d) After expanding Fagaras



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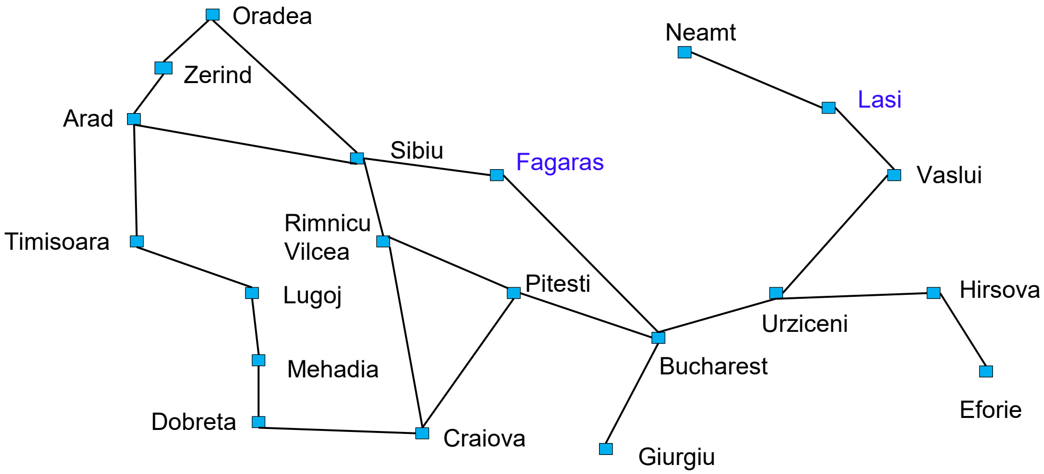
Complete?

Question: Is this approach complete?

Example: Find a path from Lasi to Fagaras



Answer: No

Greedy Search...

- m: maximum depth of the search space

Complete	No
Time	$O(b^m)$
Space	$O(b^m)$(keeps all nodes in memory)
Optimal	No

Question to think:

- Is it possible to combine functions g(n) and h(n) in one search strategy?

A * Search

- Uniform-cost search
  - *g(n):* cost to reach n (Past Experience)
  - optimal and complete, but can be very inefficient
- Greedy search
  - *h(n):* cost from n to goal (Future Prediction)
  - neither optimal nor complete, but cuts search space considerably

A * Search

Idea: Combine Greedy search with Uniform-Cost search

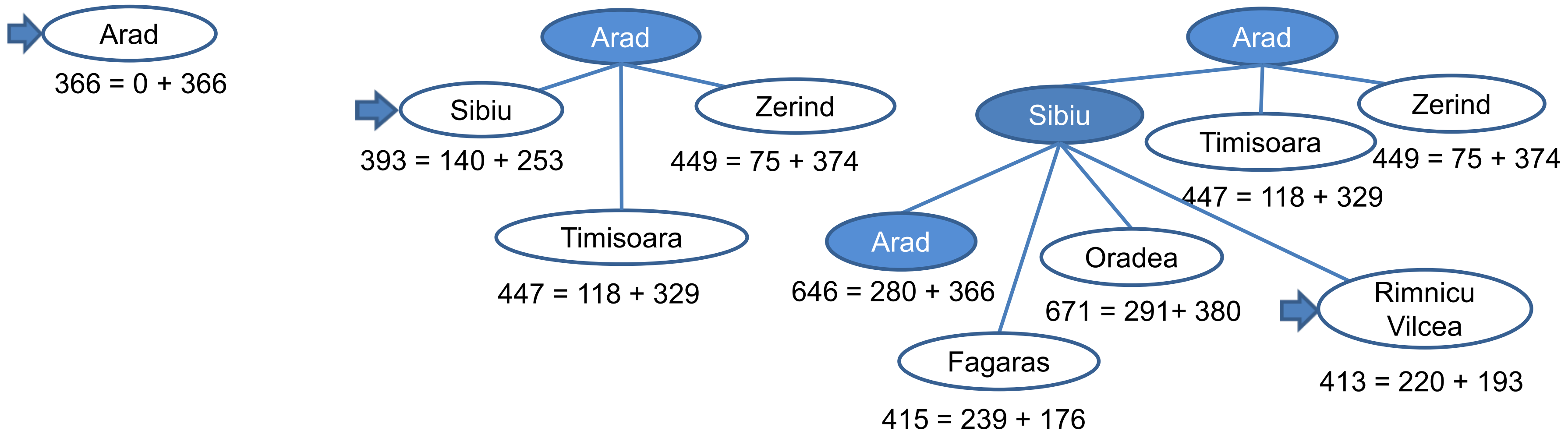
Evaluation function: $f(n) = g(n) + h(n)$

- *f (n):* estimated total cost of path through *n* to goal (Whole Life)
- If g = 0 → greedy search; If h = 0 → uniform-cost search
- Function A* Search(problem) returns solution
  - Return Best-First-Search(problem, $g + h$)

Example: Route-finding from Arad to Bucharest

Best-first-search with evaluation function $g + h$

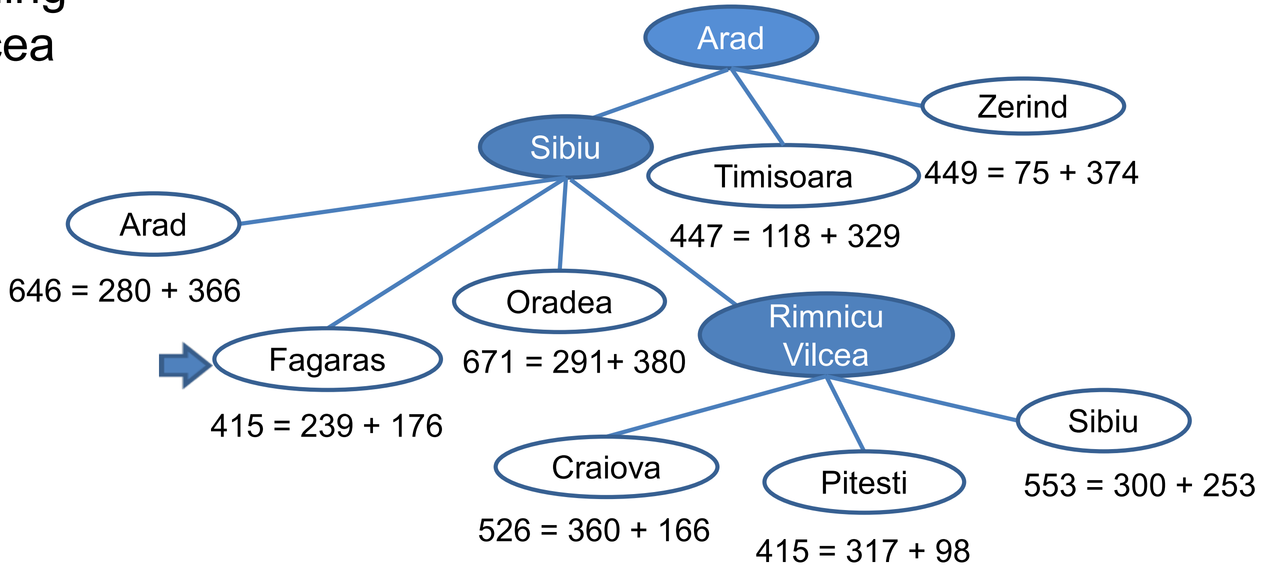
(a) The initial state (b) After expanding Arad (c) After expanding Sibiu



Example: Route-finding from Arad to Bucharest

Best-first-search with evaluation function $g + h$

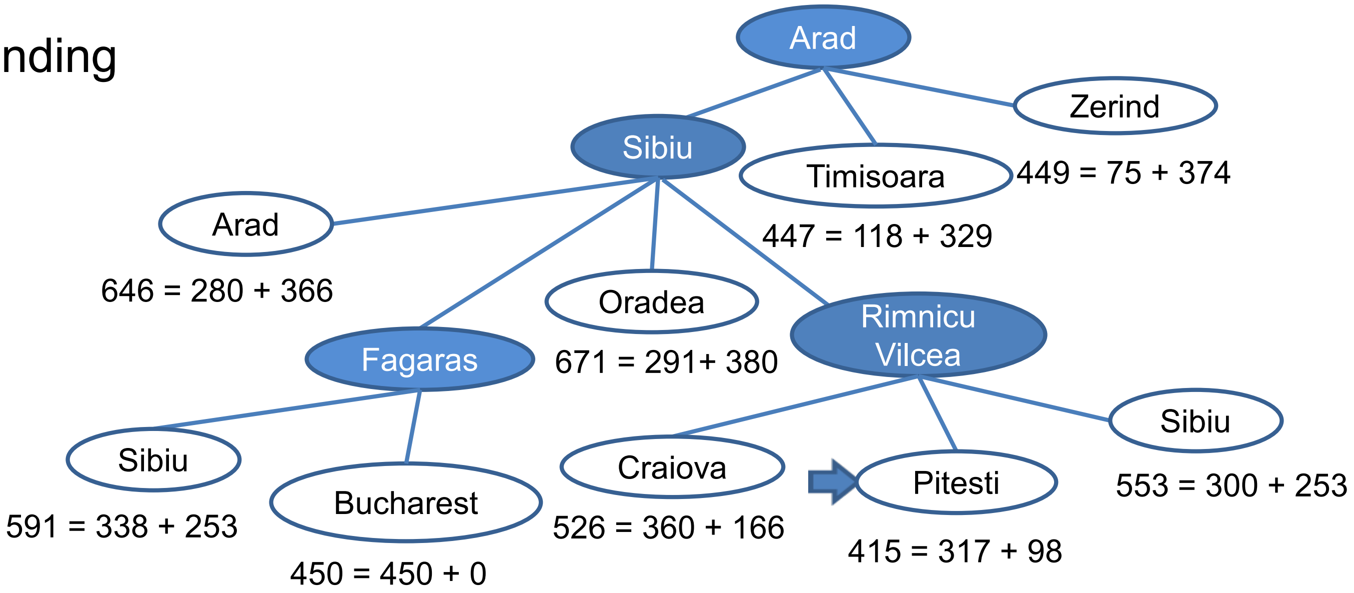
(d) After expanding
Rimnicu Vilcea



Example: Route-finding from Arad to Bucharest

Best-first-search with evaluation function $g + h$

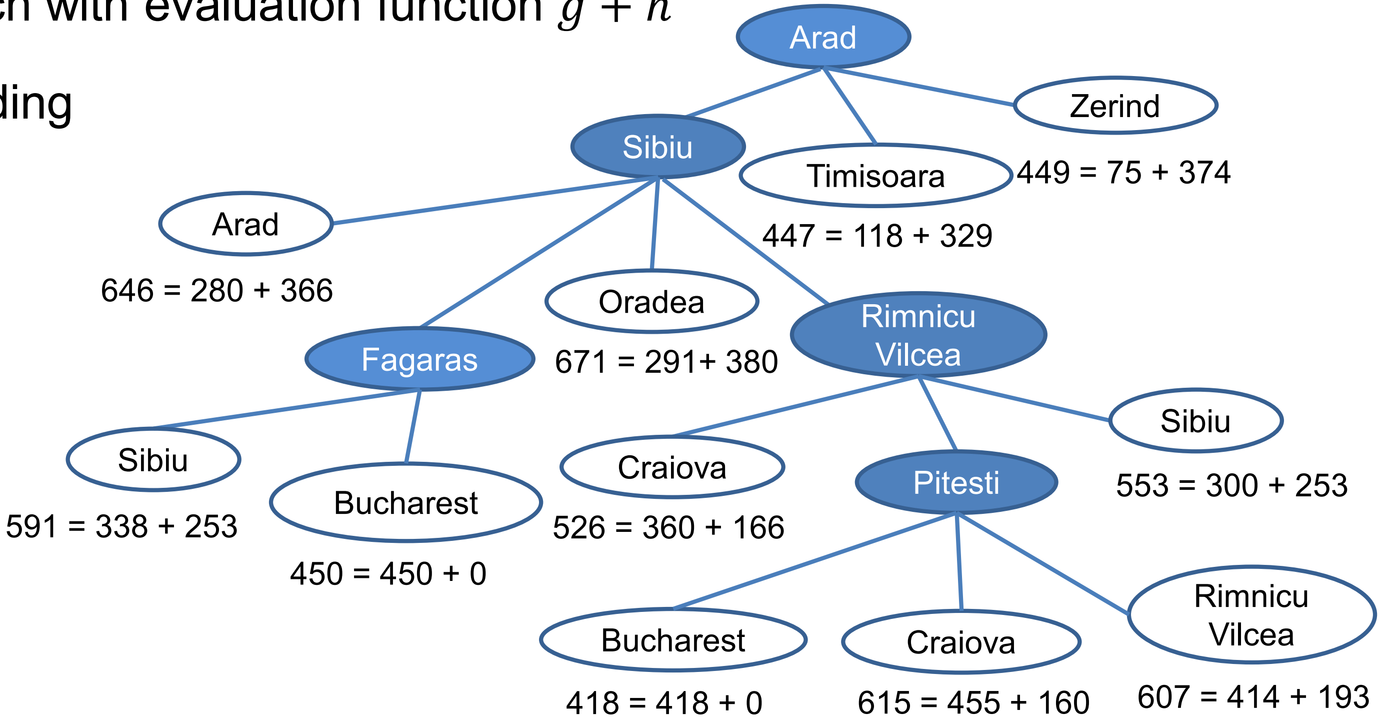
(e) After expanding Fagaras



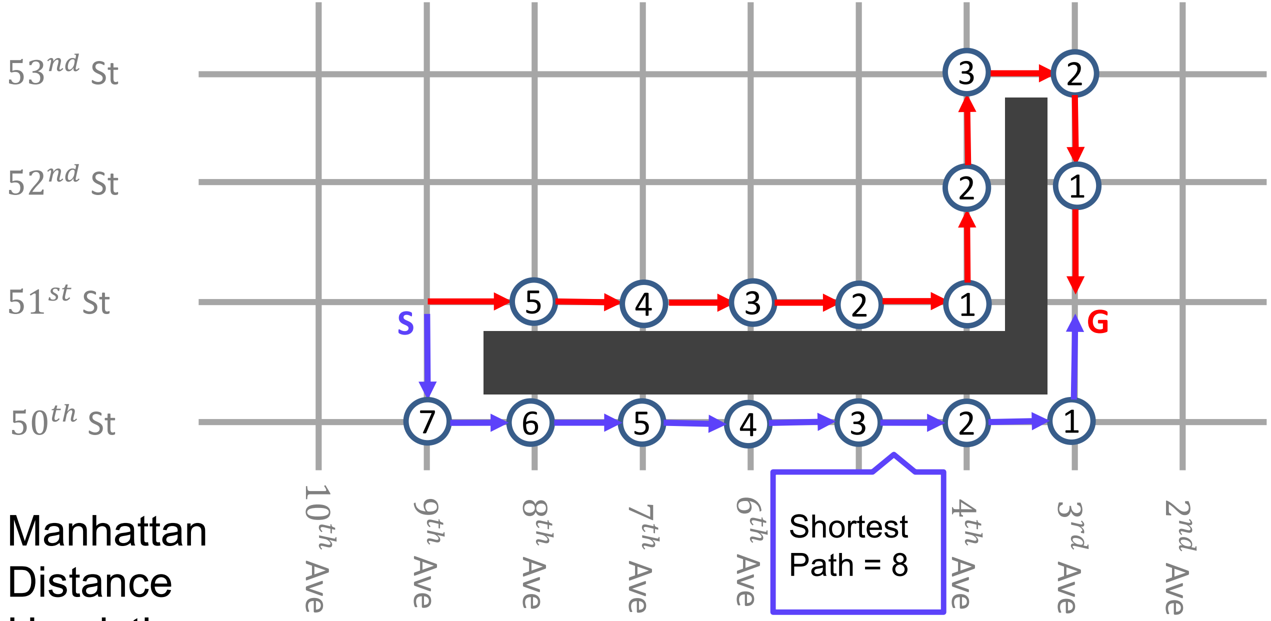
Example: Route-finding from Arad to Bucharest

Best-first-search with evaluation function $g + h$

(f) After expanding Pitesti

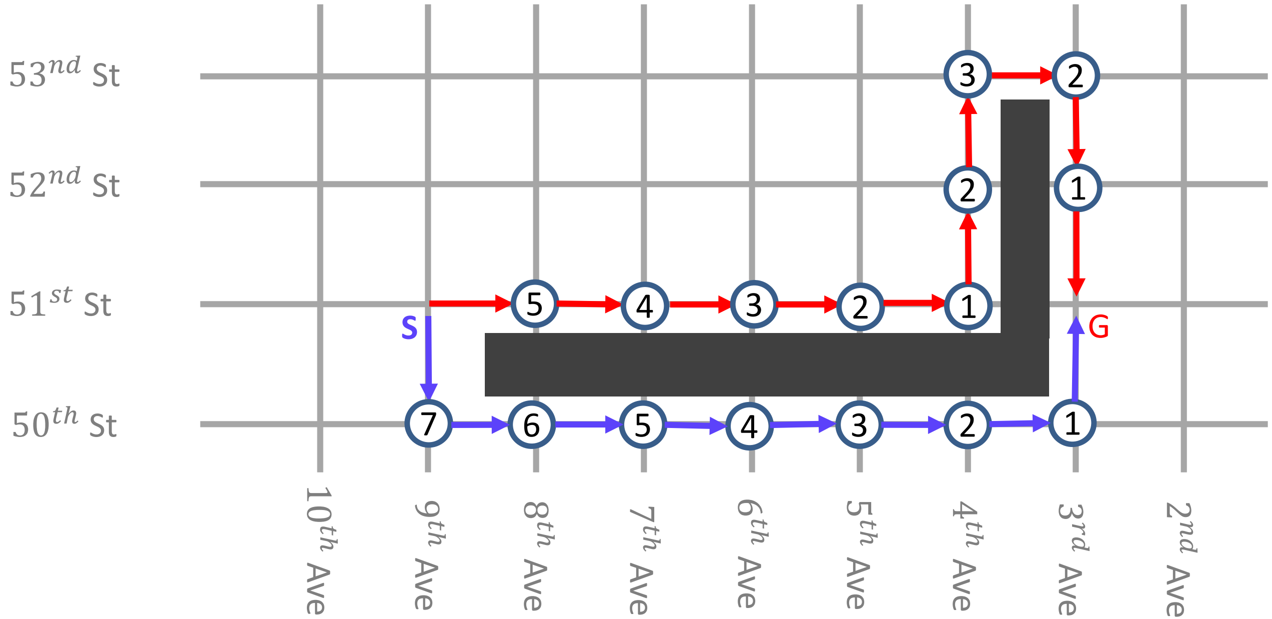


Example: Route-finding in Manhattan

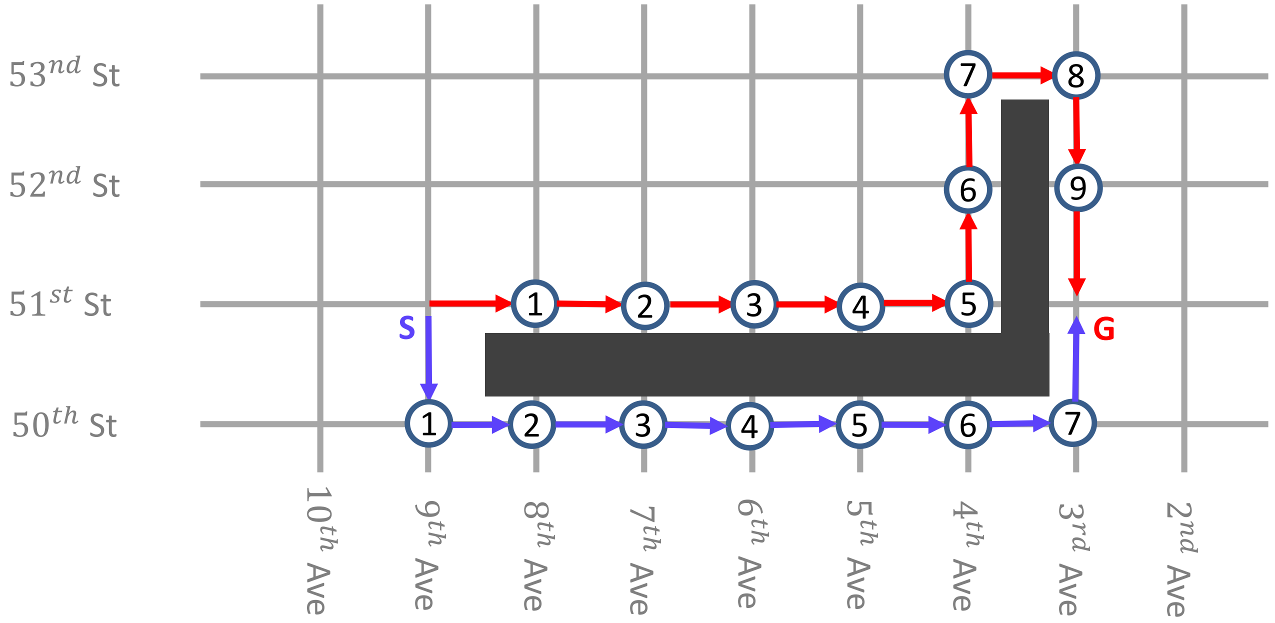


Manhattan Distance Heuristic

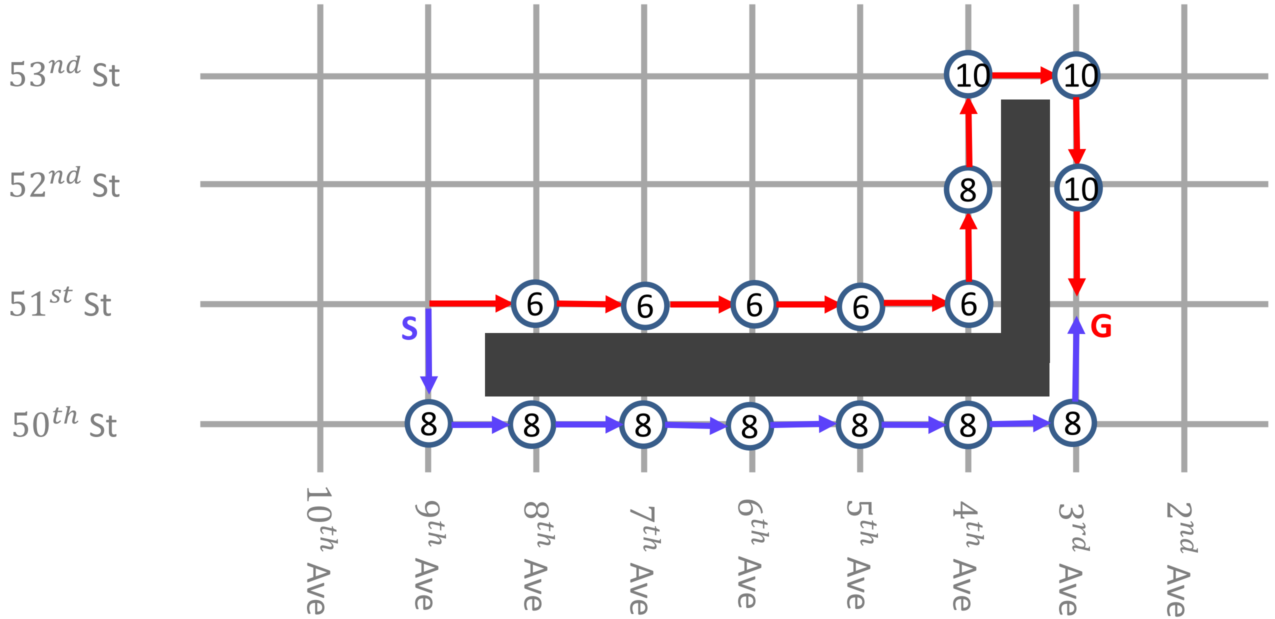
Example: Route-finding in Manhattan (Greedy)



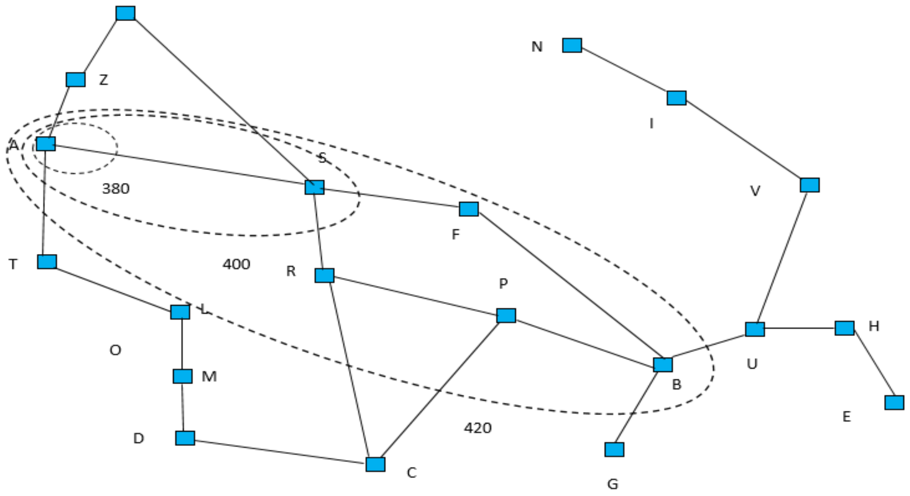
Example: Route-finding in Manhattan (UCS)



Example: Route-finding in Manhattan (A*)



Complexity of A*



Time	Exponential in length of solution
Space	(all generated nodes are kept in memory) Exponential in length of solution

With a good heuristic, significant savings are still possible compared to uninformed search methods