1. Breakeven

Q = Breakeven quantity : Number of item that need to sell before we can breakeven

- Breakeven : the state of having profit = 0
- Profit : Revenue − total cost
- Total Cost { Fixed cost / Variable cost : its total amount is dependent on Q

	A	B	C	D	E	F	G
1		x1	x2				
2		60	626.667	=	131333		
3		100	200				
4							
5		2	3	=	2000	<=	2000
6		1	0	=	60	>=	60
7		0	1	=	626.667	<=	720
8							
9							

→ value of x_1 & x_2 calculated

→ object value function

↳ obj func coeff.

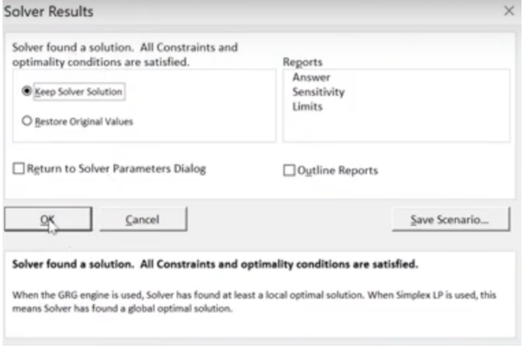
→ actual amount (cell reference)

2. Linear Programming Problem (LP Problem)

- Define Decision Variable : unit & time duration

* How to add solver

1. File → Options → Add-ins → Manage : Excel Add-ins → Go → Solver Add-in
2. Data → Solver : (Reset all)
   a) Set objective : object value function to max / min / value of
   (b) By Changing Variable Cells : B2 : C2
   (c) Add Constraint :
   (d) Make Unconstrained Variables Non-Negative
   (e) Solving method : Simplex LP



3. Sensitivity analysis / Post-optimality analysis

(a) Range of Optimality (Adjustable Cell)
- change in obj function coeff. (output of obj. func will change)
- guarantee : optimal solution not change in allowable range
- reduced cost :
  ↳ value = 0 in the optimal solution
  ↳ definition : amount the variable's objective function coeff. would have to be reduced by, before this variable could assume a positive value

(b) Range of Feasibility (Contraints)
- change in RHS value
- guarantee : shadow price not change in allowable range
- decision variable for optimal solution may change for different RHS value
- Shadow Price (Dual Price) :
  ↳ Numerical quantity of increase in optimal value (y) per unit increase in RHS (x)
  ↳ a constraint is said to be binding if LHS = RHS
  ↳ When LHS = RHS , shadow price is non-zero

4. Slack & Surplus Variables

- A linear program in which all the variables are non-negative and all the constraints are equalities is said to be in standard form.
- Standard form is attained by adding slack variables to "less than or equal to" constraints, and by subtracting surplus variables from "greater than or equal to" constraints.
- Slack and surplus variables are always non-negative, and represent the difference between the left and right sides of the constraints.
- Slack and surplus variables have objective function coefficients equal to 0.

4							
5	* If one of the objective fn coefficients is changed, will optimal solution be changed?						
6	Variable Cells						
7			Final	Reduced	Objective	Allowable	Allowable
8	Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9	B10	D	15	0	10	12.5	2.5
10	C10	P	17.5	0	15	5	8.33333333
11							
12	Constraints						
13			Final	Shadow	Constraint	Allowable	Allowable
14	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
15	D4	Aluminum LHS Value	100	3.125	100	60	46.6666667
16	D5	Steel LHS Value	80	1.25	80	70	30

(Guarantee : Optimal Solution will not be changed) → but obj func value will change

→ can decrease objective coeff. by 2.5 and increase by 12.5 with no change in optimal solution

* If one of the RHS value is changed, how much will the optimal value change?
(Guarantee : Shadow Price is still VALID)

5. Simultaneous Change (100% Rule)

Simultaneous Changes

- 100 Percent Rule for Objective Function Coefficients
  - **Range of Optimality: Validity of the current optimal solution**
  - The 100% rule states that simultaneous changes in **objective function coefficients** will not change the optimal solution as long as the sum of the percentages of the change divided by the corresponding maximum allowable change in the range of optimality for each coefficient does not exceed 100%.
- 100 Percent Rule for Constraint Right-Hand Sides
  - **Range of Feasibility: Validity of existing dual/shadow prices**
  - The 100% rule states that simultaneous changes in right-hand sides **will not change the dual prices (or shadow prices)** as long as the sum of the percentages of the changes divided by the corresponding maximum allowable change in the range of feasibility for each right-hand side does not exceed 100%.

To change an objective function coeff. & RHS value simultaneously, use solver again

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6. Applications of Linear Programming

(a) Marketing Applications

- Total number of evening interviews must be at least as great as the number of daytime interviews
- At least 40% of interviews for households with children must be conducted during the evening
- At least 60% of interviews for households without children must be conducted during the evening

$EC + ENC \geq DC + DNC$

$-DC - DNC + EC + ENC \geq 0$

$\frac{EC}{DC + EC} \geq 0.4 \Rightarrow -0.4DC + 0.6EC \geq 0$

$\frac{ENC}{DNC + ENC} \geq 0.6 \Rightarrow -0.6DNC + 0.4ENC \geq 0$

Special: If the number of TV ads cannot exceed the number of radio ads by more than 4, and if the advertising budget is $10,000, develop the model that will maximize the number reached and achieve an exposure quality of at least 1000.

$\cancel{x_1 \leq x_2 - 4 \Rightarrow x_1 - x_2 \leq -4}$ $x_1 \leq x_2 + 4$

(b) Financial Applications

- objective function: maximize total accured interests earned 累积
- assumption: end of this month = start of next month
- example:

Formulate a linear program that will help Winslow Savings determine how to invest over the next four months if at no time does it wish to have more than $8 million in either government bonds or construction loans.

↳ not accumulated

6

- Define the Objective Function

  Maximize total interest earned in the 4-month period:

  Max (interest rate on investment) X (amount invested)

  Max $0.02G_1 + 0.02G_2 + 0.02G_3 + 0.02G_4$
  $+ 0.06C_1 + 0.06C_2 + 0.06C_3 + 0.06C_4$
  $+ 0.0075L_1 + 0.0075L_2 + 0.0075L_3 + 0.0075L_4$

  → will only get the interest on the 6th both, but as the ques asks for accrued interests, it should be included.

7

- Define the Constraints

  Month 1's total investment limited to $20 million:

  (1) $G_1 + C_1 + L_1 = 20,000,000$

  Month 2's total investment limited to principle and interest invested locally in Month 1:

  (2) $G_2 + C_2 + L_2 = 1.0075L_1$ (0.0075 is the interest of liquid for 1 month)

  or $G_2 + C_2 - 1.0075L_1 + L_2 = 0$

  Month 3's total investment amount limited to principle and interest invested in government bonds in Month 1 and locally invested in Month 2:

  (3) $G_3 + C_3 + L_3 = 1.02G_1 + 1.0075L_2$ → the month of investing

  or $-1.02G_1 + G_3 + C_3 - 1.0075L_2 + L_3 = 0$

8

locally invested in Month 3: → Consider whether (C, G, L)'s investment period has ended

(4) $G_4 + C_4 + L_4 = 1.06C_1 + 1.02G_2 + 1.0075L_3$

or $-1.02G_2 + G_4 - 1.06C_1 + C_4 - 1.0075L_3 + L_4 = 0$

$10 million must be available at start of Month 5:

(5) $1.06C_2 + 1.02\boxed{G_3} + 1.0075L_4 \geq 10,000,000$

↳ Period = 2 month

when counting money available at start of month 5

contribution of G = (1+ interest rate) G5-period

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No more than $8 million in government bonds at any time:

(6)	G_1	$\leq$ 8,000,000
(7)	$G_1 + G_2$	$\leq$ 8,000,000
(8)	$G_2 + G_3$	$\leq$ 8,000,000
(9)	$G_3 + G_4$	$\leq$ 8,000,000

No more than $8 million in construction loans at any time:

(10)	C_1	$\leq$ 8,000,000
(11)	$C_1 + C_2$	$\leq$ 8,000,000
(12)	$C_1 + C_2 + C_3$	$\leq$ 8,000,000
(13)	$C_2 + C_3 + C_4$	$\leq$ 8,000,000

(C) Operations Management Applications

National Wing Company (NWC) is gearing up for the new B-48 contract. NWC has agreed to produce 20 wings in April, 24 in May, and 30 in June. Currently, NWC has 100 fully qualified workers. A fully qualified worker can either be placed in production or can train new recruits. A new recruit can be trained to be an apprentice in one month. After another month, the apprentice becomes a qualified worker. Each trainer can train up to two recruits.

The production rate and salary per employee type is listed below:

Type of Employee	Production Rate (Wings/Month)	Wage Per Month
Production	0.60	\$3,000
Trainer	0.30	\$3,300
Apprentice 学徒	0.40	\$2,600
Recruit	0.05	\$2,200

At the end of June, NWC wishes to have no recruits, but have at least 140 full-time workers.

13

- Define the Decision Variables

 unit, at the start of, duration

 P_i = number of producers ~~in~~ at the start of month i
 (where i = 1, 2, 3 for April, May, June)

 T_i = number of trainers ~~in~~ at the start of month i
 (where i = 1, 2 for April, May) (June doesn't have!)

 A_i = number of apprentices ~~in~~ at the start of month i
 (where i = 2, 3 for May, June) (April doesn't have!)

 R_i = number of recruits ~~in~~ at the start of month i
 (where i = 1, 2 for April, May) (June doesn't have!)

- Define the Constraints

 Total production in Month 1 (April) must equal or exceed contract for Month 1:

 (1) $0.6P_1 + 0.3T_1 + 0.05R_1 \geq 20$

 1. allow overproduction
 2. When we put " = ", it is highly possible to get fraction or can't be solved by solver

 * Total production in Months 1-2 (April, May) must equal or exceed total contracts for Months 1-2:

 (2) $0.6P_1 + 0.3T_1 + 0.05R_1 + 0.6P_2 + 0.3T_2 + 0.4A_2 + 0.05R_2 \geq 44$

 Total production in Months 1-3 (April, May, June) must equal or exceed total contracts for Months 1-3:

 (3) $0.6P_1+0.3T_1+0.05R_1+0.6P_2+0.3T_2+0.4A_2+0.05R_2 +0.6P_3+0.4A_3 \geq 74$

15

The number of producers and trainers in a month must equal the number of producers, trainers, and apprentices in the previous month:

(4) $P_1 - P_2 + T_1 - T_2 = 0$ ($P_1 + T_1 = P_2 + T_2$)

(5) $P_2 - P_3 + T_2 + A_2 = 0$ ($P_3 = P_2 + T_2 + A_2$) → Don't have trainers in third month.

The number of apprentices in a month must equal the number of recruits in the previous month:

(6) $A_2 - R_1 = 0$ ($A_2 = R_1$)

(7) $A_3 - R_2 = 0$ ($A_3 = R_2$)

Define the Constraints (continued)

Each trainer can train up to two recruits:

(8) $2T_1 - R_1 \geq 0$ $\frac{R_1}{T_1} \leq 2 \Rightarrow R_1 \leq 2T_1$ (Ratiometric Constraint)

(9) $2T_2 - R_2 \geq 0$

Wrong : $T_1 \leq 2R_1$
Reason : eg. Let $T_1 = 1$, $R_1 = 5$ (not feaseble)
but $T_1 = 1 \leq 2R_1 = 10$
use ratio

7. Integer Programming

(a) All-Integer Linear Program (ILP)

- LP Relaxation : LP that dropping the integer requirement
- cannot round up / round down

(b) 0-1 / Binary Integer Linear Program

eg. Tina's Tailoring

Excel coding & solution

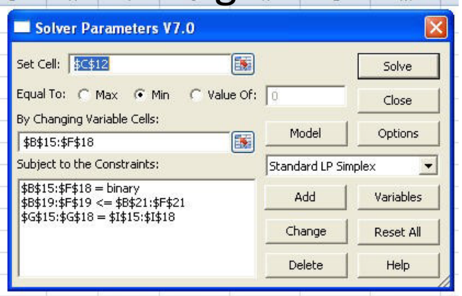
	A	B	C	D	E	F
1	Lecture 6: Tina's Taitoring (ILP)					
2						
3						
4	Garment	Tailor 1	Tailor 2	Tailor 2	Tailor 4	Tailor 5
5	Wedding gown	19	23	20	21	18
6	Clown costume	11	14	1000	12	10
7	Admiral's uniform	12	8	11	1000	9
8	Bullfigher's outfit	1000	20	20	18	21
9						
10	Model					
11						
12	Total Servcie Time (min)		55			
13						
14	Garment	Tailor 1	Tailor 2	Tailor 2	Tailor 4	Tailor 5
15	Wedding gown	1	0	0	0	0
16	Clown costume	0	0	0	0	1
17	Admiral's uniform	0	1	0	0	0
18	Bullfigher's outfit	0	0	0	1	0
19		1	1	0	1	1
20		<=	<=	<=	<=	<=
21		1	1	1	1	1

↳ to ensure the tailor won't get unacceptable assignment, we use very high hour

↳ sumproduct

	G	H	I
15	1	=	1
16	1	=	1
17	1	=	1
18	1	=	1

↳ sum (not sumproduct)



eg. Bank Location : Ohio Trust

Excel Solution (using incidence matrix)

Lecture 6: Ohio Trust (Bank Location Problem)

The Incidence Matrix

County	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12	#13	#14	#15	#16	#17	#18	#19	#20	LHS		RHS
1-Ashtabula	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	1	>=	1
2-Lake	1	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	>=	1
3-Cuyahoga	0	1	1	1	0	0	0	0	1	1	0	1	1	0	0	0	0	0	0	0	1	>=	1
4-Lorain	0	0	1	1	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	>=	1
5-Huron	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	>=	1
6-Richland	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	>=	1
7-Ashland	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	0	0	1	>=	1
8-Wayne	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	1	0	0	2	>=	1
9-Medina	0	0	1	1	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	>=	1
10-Summit	0	0	1	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	2	>=	1
11-Stark	0	0	0	0	0	0	0	1	0	1	1	0	1	1	1	0	0	1	1	1	1	>=	1
12-Geauga	1	1	1	0	0	0	0	0	0	1	0	1	1	0	0	1	0	0	0	0	1	>=	1
13-Portage	0	0	1	0	0	0	0	0	0	1	1	1	1	0	0	1	0	0	0	0	2	>=	1
14-Columbiana	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0	1	1	>=	1
15-Mahoning	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1	0	0	0	0	1	>=	1
16-Trumbull	1	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	0	0	0	0	1	>=	1
17-Knox	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	>=	1
18-Holmes	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	1	1	1	0	2	>=	1
19-Tuscarawas	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	1	>=	1
20-Carroll	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	1	1	1	>=	1

sumproduct (number of accessible bank in each country)

= SUMPRODUCT (B6:U6, B31: U31)

Objective value (min): 3 → sum (B31 : U31)

Decision variable	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12	#13	#14	#15	#16	#17	#18	#19	#20
	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0

(Trick)

* Modeling Flexibility Provided by 0-1 Variables

- When x_i and x_j represent binary variables designating (指定) whether projects i and j have been completed, the following special constraints may be formulated:
  - At most <u>k out of n</u> projects will be completed:

$$\sum_{j=1}^{n} x_j \leq k$$

eg. $n=5$, $k=3$
At most : $x_1 + x_2 + x_3 + x_4 + x_5 \leq 3$
At least : $x_1 + x_2 + x_3 + x_4 + x_5 \geq 3$
Exactly : $x_1 + x_2 + x_3 + x_4 + x_5 = 3$

  - Project j is <u>conditional</u> on project i:

$$x_j - x_i \leq 0$$

Project i is dominant. ($x_i \geq x_j$)
- If i is not choosen, j is definitely not choosen. (if $x_i = 0$, $x_j = 0$)
- If i is choosen, j may or may not choosen. (if $x_i = 1$, $x_j \leq 1 \Rightarrow x_j = 0$ or $x_j = 1$)
eg. $x_i = 1$ build condo
$x_j = 1$ build garden in condo

  - Project i is a <u>corequisite</u> (并存条件) for project j:

$$x_j - x_i = 0$$

eg. x_i = output rate of department A
x_j = input rate of department B
(x_i & x_j must be same)

  - Projects i and j are <u>mutually exclusive</u>:

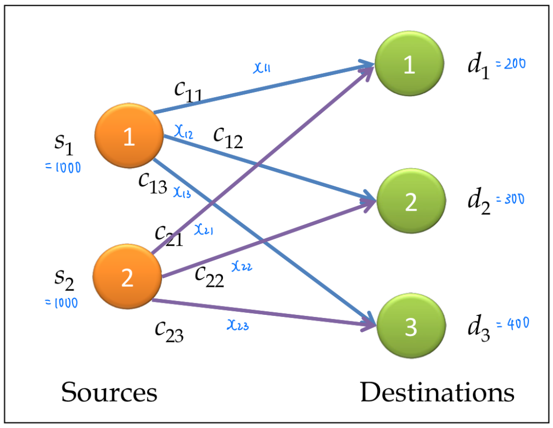
$$x_j + x_i \leq 1$$

↘ x_i & x_j are not allowed to be 1 simultaneously

- Shadow Prices CANNOT be used for integer programming
- For ILP, small change in constraint coeff. cause large change in optimal solution, so re-run

8. Transportation Problem
- network model : represented by nodes, arcs and function associated with arcs & nodes

- Network Representation



Question : how many products should be sent from S_i to d_j ?

x_{ij} = number of units shipped from origin i to destination j
c_{ij} = cost per unit of shipping from origin i to destination j
s_i = supply or capacity in units at origin i
d_j = demand in units at destination j

$$\text{Min} \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \quad \text{(shipping cost)}$$

s. t.

Supply Constraint $\sum_{j=1}^{n} x_{ij} \le s_i \quad i = 1, \ldots, m$ Supply $\begin{cases} x_{11} + x_{12} + x_{13} \le S_1 \\ x_{21} + x_{22} + x_{23} \le S_2 \end{cases}$

Demand Constraint $\sum_{i=1}^{m} x_{ij} = d_i \quad j = 1, \ldots, n$ Demand $\begin{cases} C_{11} + C_{21} = d_1 \\ C_{12} + C_{22} = d_2 \\ C_{13} + C_{23} = d_3 \end{cases}$

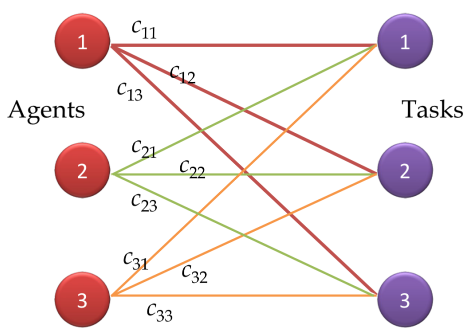
Non-negative Constraint $x_{ij} \ge 0$ for all i and j

9. Assignment Problem

- Purpose : minimize total cost assignment
- special case of transportation problem (all suplies & all demands = 1)

Number of agents = Number of tasks

- Network Representation



– c_{ij} = cost of assigning agent i to task j
– x_{ij} = 1 if agent i is assigned to task j; = 0 otherwise.
↳ Binary Variable

$$\text{Min} \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$
$$\sum_{j=1}^{n} x_{ij} \le 1 \quad i = 1,2,\ldots,m \quad \text{Agents}$$
$$\sum_{i=1}^{m} x_{ij} = 1 \quad j = 1,2,\ldots,n \quad \text{Tasks}$$
$$x_{ij} \ge 0 \text{ for all } i \text{ and } j$$

x_{ij} = Binary

※ Special Case

- Number of tasks exceeds the number of agents: eg. which tasks should be perform first
  - Add enough dummy agents 假的 to equalize the number of agents and the number of tasks. The objective function coefficients for these new variable would be zero.

9. Shortest-route Problem

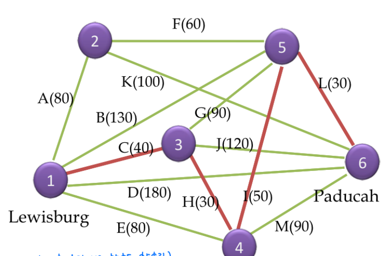
- Purpose : find shortest path in a network from one node to another node
  ↳ distance / time / cost

Shortest Route: Winslow

- ※ Solution

	B Arc Start Node	C End Node	D Cost	E Decision Variable	F On Shortest Route?
A	1	2	80	0	
B	1	5	130	0	$x_{13} = 1$
C	1	3	40	1	Yes
D	1	6	180	0	
E	1	4	80	0	
F	2	5	60	0	
G	3	5	90	0	$x_{34} = 1$
H	3	4	30	1	Yes
I	4	5	50	1	Yes $x_{45} = 1$
J	3	6	120	0	
K	2	6	100	0	
L	5	6	30	1	Yes
M	4	6	90	0	$x_{56} = 1$
				0	
A-Rev	2	1	80	0	
B-Rev	5	1	130	0	
C-Rev	3	1	40	0	
D-Rev	6	1	180	0	
E-Rev	4	1	80	0	
F-Rev	5	2	60	0	
G-Rev	5	3	90	0	
H-Rev	4	3	30	0	
I-Rev	5	4	50	0	
J-Rev	6	3	120	0	
K-Rev	6	2	100	0	
L-Rev	6	5	30	0	
M-Rev	6	4	90	0	

↳ copy from upper half of the excel.



= SUMIF (\$B\$5 : \$B\$31, H9, \$E\$5 : \$E\$31)

Out - In

Node	Arcs Used Out	In	LHS Net		RHS Balance
1	1	0	1	=	1
2	0	0	0	=	0
3	1	1	0	=	0
4	1	1	0	=	0
5	1	1	0	=	0
6	0	1	-1	=	-1
	SUMIF (range, criteria, [sum_range])				
	Total Costs		150	→ SUMPRODUCT	

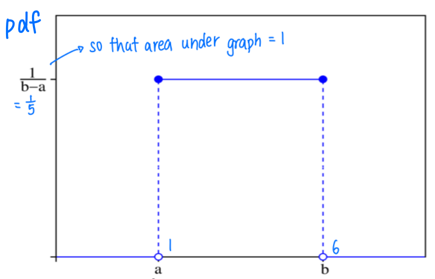
Minimum total cost = \$150

10. Simulation

- Purpose : understand dynamic behaviors
- random variable (r.v.)
  - ↳ discrete r.v. : probability
  - ↳ continuous r.v. : probability density function (pdf)

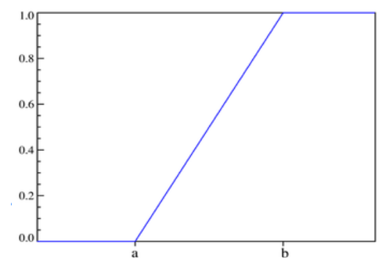
(a) Continuous Probability Distributions

① Uniform Distribution



$f(x) = 1/(b-a)$ for all $a \le x \le b$

Cumulative Distribution Function (cdf)



$F(x) = (x - a)/(b - a)$ for all $a \le x \le b$

↳ CDF Function of Continuous Uniform Distribution

$F(x) = P(X \le x)$

X ↓ r.v.

x ↳ certain value of X

② Normal Distribution

- pdf in excel : NORM.DIST (x, μ, σ, 0)
- cdf in excel : NORM.DIST (x, μ, σ, 1)
- Standard normal distribution : $\mu = 0$, $\sigma = 1$
- invert cdf in excel : NORM.INV (x, μ, σ)

→ calculate invert CDF of a random number → simulation
Calculate invert CDF of a constant number ⇒ statistics

* excel function :

- RAND(X) : Generate random number from 0 - 1
- AVERAGE(G:G) : average of all numbers in column G
- STDEV.P (G:G)

③ Exponential Distribution (Mean = $\frac{1}{\lambda}$, Std = $\frac{1}{\lambda}$)

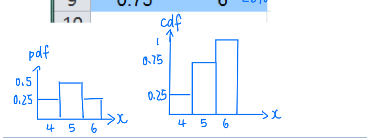
- r.v. → only positive value
- parameter : rate λ
- pdf : $f(x) = \lambda e^{-\lambda x}$
- cdf : $F(x) = 1 - e^{-\lambda x}$
- inv. cdf = $F^{-1}(x) = -\frac{\ln(1-y)}{\lambda}$ (manually calculate)

(b) Discrete Event Simulation

- given : a table of cumulative probabilities & resulting random events
- EXCEL formula : VLOOKUP (RAND() , cumulative table , 2, 1)
  - ↳ dictionary that define our allowed symbol

	J	K
4		
5		Number
6	Cumulative	of Days
7	0	4 25%
8	0.25	5 50%
9	0.75	6 25%

→ always start with zero



K	L	M	N	O	P
Prob	cdf	Weapons	Game On!	Weapons	RelFreq
0.5	0	Mud	Stone	Mud	0.5005
0.3	0.5	Stone	Mud	Stone	0.301
0.15	0.8	Spear	Mud	Spear	0.152
0.05	0.95	Sword	Mud	Sword	0.0465
	↳ = L4 + K5		Stone		
			Mud		
	~~0.99~~	~~Sword~~	Mud		
			Stone		
			Stone		

→ COUNTIF (n:n , O2 /2000)

→ = VLOOKUP (rand() , L2 : M5, 2, 1)

↳ 2000 weapons are generated

10. Time-Series Forecasting

- Long Range, Medium Range, Short Range (most accurate)

(a) Last-value method (Naïve method)

- use the last-month's actual value ($F_{t+1} = X_t$)
- when conditions tend to change so quickly

(b) Averaging method

- $F_{t+1} = (X_1 + \cdots + X_t) / t$
- when conditions tend to remain stable (null data)

(c) Moving Average Method

- if period = 3, $F_{t+1} = (X_{t-2} + X_{t-1} + X_t) / 3$
- when conditions tend to change occasionally but not extremely rapidly

(d) Exponential Smoothing (ES) Method

- $F_{t+1} = \alpha X_t + (1-\alpha) F_t$
  - ↳ α : smoothing constant
  - ↳ more recent periods are given more weight

(e) ES with Trend Method

- Forecast = Baseline Estimate + Trend Estimate

  $F_{t+1} = B_{t+1} + T_{t+1}$

  ↳ $B_{t+1} = \alpha X_t + (1-\alpha) F_t$

  ↳ $T_{t+1} = \beta (B_{t+1} - B_t) + (1-\beta) T_t$ (can be +ve or -ve)

- Given: α, β, B_1, F_1, (Set $T_1 = 0$)

(f) Forecast Accuracy

Positive & Negative Error will cancel out

- Cumulative error
  - Deviations/errors (actual value – forecast) are added up

$$\text{Cumulative Error} = \sum_i (X_i - F_i)$$

- Bias
  - It is the average cumulative error

$$Bias = \frac{\sum_i (X_i - F_i)}{N}$$

- A large positive value of bias (or cumulative error) indicates the forecast is biased low; a large negative value indicates forecast is biased high.

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Better Estimates of Forecast Accuracy

- Mean Absolute Deviation (MAD)
  - Deviations are linearly (proportionately) penalized.

$$MAD = \frac{\sum_i |X_i - F_i|}{N}$$ → SUM(ABS(range))/N

- Mean Square Error (MSE)
  - Additional deviations are penalized more.

$$MSE = \frac{\sum_i (X_i - F_i)^2}{N}$$ EXCEL function: SUMSQ(range)/N

- Root Mean Square Error (RMSE)

$$RMSE = \sqrt{\frac{\sum_i (X_i - F_i)^2}{N}}$$ Square root in EXCEL: SQRT(Number)

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11. Causal Forecasting

(a) Linear Regression

- **Linear regression** approximates the relationship between the dependent and the independent variables by a straight line.
- This **linear regression line** is drawn on a graph with the independent variable on the horizontal axis and the dependent variable on the vertical axis.
- The linear regression line has the form: $Y = a + bX$
  - Y = Estimated value of the dependent variable
  - a = Intercept of the linear regression line with the y-axis
  - b = Slope of the linear regression line
  - X = Value of the independent variable
- Our goal is to find estimates for a and b, such that the squared deviations of the predicted values of Y from the actual values X are minimized.
- This is called the **Method of Least Squares (MLS)**.

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- Given the n pairs of past data, a and b are given by:

$$b = \frac{SS_{XY}}{SS_{XX}}, \quad a = \bar{Y} - b\bar{X}$$

where $SS_{XY} = \sum_{i=1}^{n}\{(X_i - \bar{X})(Y_i - \bar{Y})\}, SS_{XX} = \sum_{i=1}^{n}(X_i - \bar{X})^2$

and $\bar{Y}$ = Mean value of Y, $\bar{X}$ = Mean value of X

BU8401-Lect12-Forecasting.xlsx

=INTERCEPT(D5:D16,C5:C16)

	A	B	C	D	E	F	G	H	I	J
1		Linear Regression of Call Volume vs. Sales Volume for CCW								
2										
3		Time	Independent	Dependent		Estimation	Square		Linear Regression Line	
4		Period	Variable	Variable	Estimate	Error	of Error		y = a + bx	
5		1	4,894	6,809	6,765	43.85	1,923		a =	-1,223.86
6		2	4,703	6,465	6,453	11.64	136		b =	1.63
7		3	4,748	6,569	6,527	42.18	1,780			
8		4	5,844	8,266	8,316	49.93	2,493			
9		5	5,192	7,257	7,252	5.40	29		Estimator	
10		6	5,086	7,064	7,079	14.57	212		If x =	5,000
11		7	5,511	7,784	7,772	11.66	136			
12		8	6,107	8,724	8,745	21.26	452		then y=	6,938.18
13		9	5,052	6,992	7,023	31.07	965			
14		10	4,985	6,822	6,914	91.70	8,408			
15		11	5,576	7,949	7,878	70.55	4,977			
16		12	6,647	9,650	9,627	23.24	540			

=SLOPE(D5:D16,C5:C16)

We can then use Y = a + bX to estimate Y given any X.

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Seminar 8 : Waiting Line Models I

1. Structure of Queues

(a) 3 components :

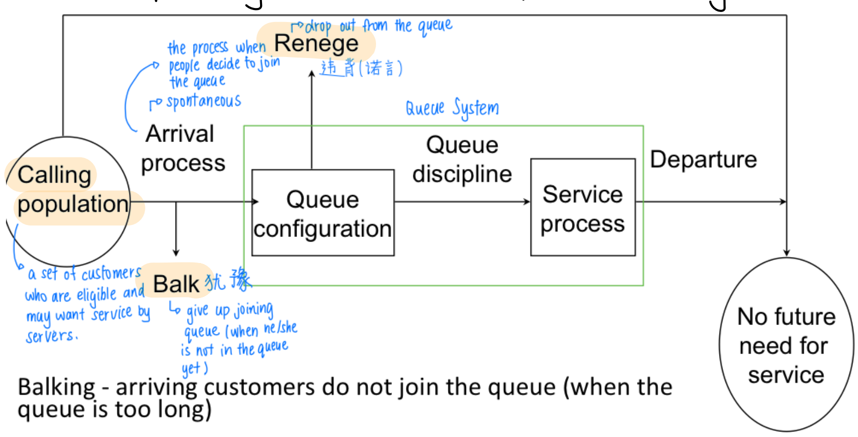
- Queue Boundary
- Queue Component
- Servers

2. Parameters

- λ : (average) rate of arrival
  $\frac{1}{\lambda}$: (average) inter-arrival time
- μ : (average) rate of service
  $\frac{1}{\mu}$: (average) service time (per customer)

$\lambda < \mu$

- L_q : average number of customers in queue line
  L_w : average number of customers served
  L : average number of customers in the system
- W_q : average time in queue
  W_s : average service time $= \frac{1}{\mu}$
  W : average time in system
- ρ : service utilisation factor ($\rho = \frac{\lambda}{\mu}$)
  ↳ propotion of time that server is busy
- P_0 : probability that no one is in the system
  P_n : probability that n customers present in the system



Balking - arriving customers do not join the queue (when the queue is too long)

Reneging - customers give up and leave after waiting for a while

Queues study Steady State Equilibrium (SSE) behaviors

3. Service Pattern

- Exponential service time → Poisson service rate

4. Kendall-Lee Notation (A/B/k)

- A : arrival distribution (λ)
- B : service distribution (μ)
- k : number of servers

↳ M → Markov distributions (Poisson / exponential)
  D → Deterministic (constant)
  G → General distributions (eg. Normal distribution)

5. Little's Law :

$L = \lambda W$, $L_q = \lambda W_q$, $L_s = \lambda W_s$

6. Single Server : M/M/1 Queue

Assumptiom

- Infinite calling population
- Infinite queue length

} to simplify the formula

- Poisson Arrivals Rates, Exponential inter-arrival times
- FCFS queue discipline, infinite queue size
- Exponential service time
- Customer are patient (i.e., no reneging)

(a) Steady State Measurements

- $P_0 = 1 - \rho = 1 - \frac{\lambda}{\mu}$ (propotion of time that server is busy)
- $P_n = P_0 \left(\frac{\lambda}{\mu}\right)^n = P_0 (\rho)^n$
- $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\rho^2}{1-\rho}$
- $L = L_q + \frac{\lambda}{\mu} = \frac{\rho}{1-\rho}$
- $W_q = \frac{L_q}{\lambda} = \frac{\lambda}{\mu(\mu-\lambda)}$
- $W = \frac{L}{\lambda} = \frac{1}{\mu-\lambda}$

7. Single Server : M/G/1 Queue

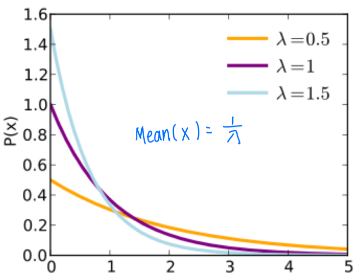
- service time is generally distributed (mean $= \frac{1}{\mu}$, standard deviation $= \sigma$)
- $P_0 = 1 - \rho$
- $P_n = P_0 \rho^n$
- ★ $L_q = \frac{(\lambda\sigma)^2 + \rho^2}{2(1-\rho)}$
- $L = L_q + \rho$
- $W_q = L_q / \lambda$
- $W = L / \lambda$

8. Single Queue : M/D/1 Queue

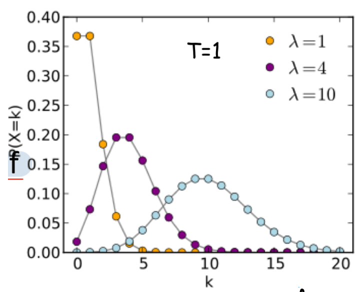
- No variation in service times ($\sigma = 0$)
- $P_0 = 1 - \rho$
- $P_n = P_0 \rho^n$
- $L_q = \frac{\rho^2}{2(1-\rho)}$
- $L = L_q + \rho$
- $W_q = L_q / \lambda$
- $W = L / \lambda$

9. Exponential Distribution (arrival rate)

- Probability density function : $f(t) = \lambda e^{-\lambda t}$



Exponential Distribution of inter-arrival time



Poisson Distribution of probability distribution function

10. Poisson Distribution

- inter-arrival time → exponential distribution
- Probability Distribution Function $P(X=n) = \frac{(\lambda T)^n e^{-\lambda T}}{n!}$

Seminar 9: Waiting Line Models II

1. Multiple server: M/M/K Queue

↱ Exponential inter-arrival time

- Poisson arrival rate/Exponential service times
- k multiple identical servers with single queue
- Each server services at μ customers per unit time
- Every server is identical in performance
- Customer does not choose which server for service
- $\frac{\lambda}{\mu}$ is called the "implied utilization", and is often greater than 1.
  ↳ indicate how severe would be if all but one server fail to operate" the higher the value, the worse the situation
- $\rho = \frac{\lambda}{k\mu}$ is our usual "utilization" on a per-server basis. In other words, each server is ρ percent of the time busy.

(a) Formulae

k: number of servers

kμ: mean effective service rate for the system ($\lambda < k\mu$)

Probability of no customers in the system (all servers idle) is	$P_0 = \dfrac{1}{\left(\sum_{n=0}^{k-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right) + \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k \boxed{\frac{k\mu}{k\mu-\lambda}}}$ $\quad \frac{k}{k-\frac{\lambda}{\mu}}$
Probability of j customers in the system ($1 \le j \le k$):	$P_j = \left(\frac{\lambda}{\mu}\right)^j \frac{1}{j!} \cdot P_0$
Probability of j customers in the system ($j \ge k$):	$P_j = P_k\left(\frac{\lambda}{k\mu}\right)^{j-k} = \left(\frac{\lambda}{k\mu}\right)^j \frac{k^k}{k!} \cdot P_0$ 5

Average number of customers in queue:

$$L_q = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^k}{(k-1)!\,(k\mu-\lambda)^2} P_0$$

Average time a customer spends in waiting to be served:

$$W_q = \frac{L_q}{\lambda}$$

Average number of customers in the system:

$$L = L_q + \boxed{\frac{\lambda}{\mu}}$$

$= \frac{\frac{1}{\mu}}{\frac{1}{\lambda}} = \frac{\text{serving time}}{\text{inter-arrival time}}$

$= L_s$ (average number of customers stuck in or is served by the servers in the serving area)

Average waiting time in the system (waiting and being served):

$$W = \frac{L}{\lambda}$$

$= W_s + \underline{W_q} \to \frac{1}{\mu}$

↳ not affected by k

(b) Special question:

服务员

What is the LEAST number of attendants that is feasible for this system?

λ = 10.4/minute, μ = 4/minute, k = ?

The LEAST number of attendants is the smallest value of k such that λ < kμ. ∴ $k_{min} = 3$

k	λ	kμ	System OK?
2	10.4	8.0	No
3	10.4	12.0	YES!
4	10.4	16.0	Yes
5	10.4	20.0	Yes

test one by one

2. Economic Analysis of Queues

Total Cost per hour (TC) = Cost of servers + Cost of Waiting

$= C_s \times k + C_w \times L$

C_s: cost per server per hour

C_w: cost of waiting per customer per hour

L: expected customers in system

Example:

American Weavers operates a cloth manufacturing plant which has several weaving machines that jam frequently at the rate of 25 per hour. Machines can be repaired on a FCFS basis by one of the 7 available repair persons. Repair time is exponentially distributed with an average of 15 min. The manager has estimated that the company loses $100 for every hour that a machine is out of operation. Adding an extra repair person costs $50 per hour. Determine the optimal number of repair persons.

must start with $k=7$ as $\frac{\lambda}{\mu}=6.25$, so we need at least 7 servers.

k	No. of Repair Persons				
	7	8	9	10	11
Utilization $\frac{\lambda}{k\mu}$	0.89	0.78	0.69	0.63	0.57
Exp. Queue Length (Lq)	5.85	1.49	0.54	0.21	0.08
Exp. No. in System (L)	12.10	7.74	6.79	6.46	6.33
Exp. Time in Queue (Wq)	0.23	0.06	0.02	0.01	0.003
Exp. Time in System (W)	0.48	0.31	0.27	0.26	0.25
Hourly Cost ($)	50x7 + 100 x 12.1	50x8 + 100 x 7.74	50x9 + 100 x 6.79	50x10 + 100 x 6.46	50x11 + 100 x 6.33
=	1560	1174	1129	1146	1183

k = 9 is optimal in reducing the total cost of operations

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3. Comparing M/M/k with k x M/M/1
- server cost is the same
- waiting cost (M/M/k < k x M/M/1)

Example:

A new tire shop is designed to have 5 workstations in which a car can have 1 to 4 of its tires changed. Regardless of the number of tires to be changed, each workstation takes 15 mins (exponentially distributed) on average to complete its work. Patronizing cars are expected to drive in at a rate of 12 every hour in a Poisson distributed fashion. The shop owner is considering whether to layout the shop in M/M/5 or 5xM/M/1.

- If the owner wants to save space allocated for waiting cars, which queuing system should the owner choose?

For M/M/5,
λ = 12/hr,
μ = 4/hr,
k = 5
L_q = 0.3542 cars (1 car lot)
W_q = 0.0295 hr = 1.77 min
L = 3.3542 cars
W = 0.2795 hr = 16.78 min

For each queue of 5xM/M/1,
Effective λ = 12/hr / 5 = 2.4/hr,
μ = 4/hr, effective arrival rate that each queue system sees
k = 1
Lq = 0.9 cars
W_q = 0.375 hr = 22.5 min
L = 1.5 cars
W = 0.625 hr = 37.5 min

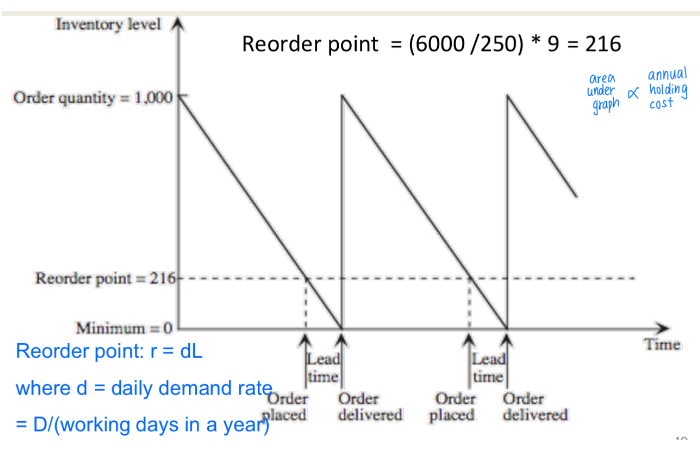
For 5xM/M/1,
L_q = 0.9x5 = 4.5 cars (5 car lots)
L = 1.5x5 = 7.5 cars

Seminar 10 : Inventory Management I (Deterministic Demand)

- objective : minimize total cost
- only discuss non-perishable goods

1. Parameters
   - D : Annual Demand Rate
   - S : Setup Cost / Ordering Cost per order
   - H : Annual Holding Cost per unit per year
   - C : Purchase Cost per unit
   - B : Shortage Cost per unit per year
   - d : daily demand
   - L : Lead Time
   - Q : Order Quantity

Assume : demand rate is constant
no shortages are allowed
a constant lead time



2. Economic Order Quantity (EOQ) Model
- Annual Purchase Cost = D·C
- Annual Ordering Cost = $S \cdot \frac{D}{Q}$ → number of order per year
- Annual Holding Cost = $H \cdot \frac{Q}{2}$ → average inventory level

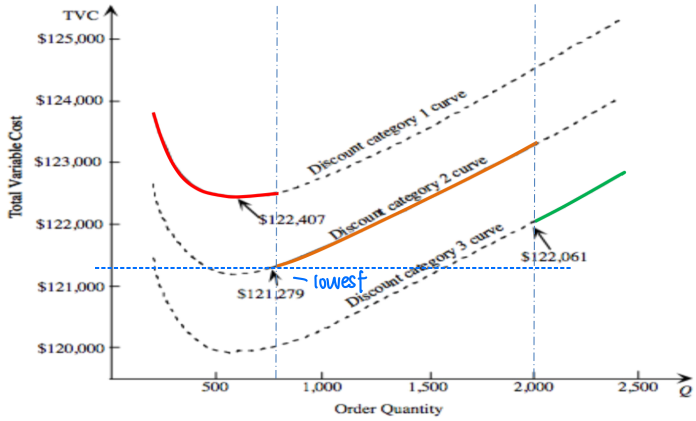
- TVC (Total Variable Annual Cost) = Ordering Cost + Holding Cost
  $= S \cdot \frac{D}{Q} + H \cdot \frac{Q}{2}$
- value of Q that minimizes TVC : $Q^* = \sqrt{\frac{2DS}{H}}$
- Number of orders per year $= \frac{D}{Q^*}$
- Time between orders (cycle time) $= \frac{Q^*}{D}$ years
- Minimum annual relevant cost $TVC^* = \sqrt{2DSH}$

3. EOQ with Quantity Discounts
- let Holding Cost $H = CI$ (I is the inventory holding cost rate)

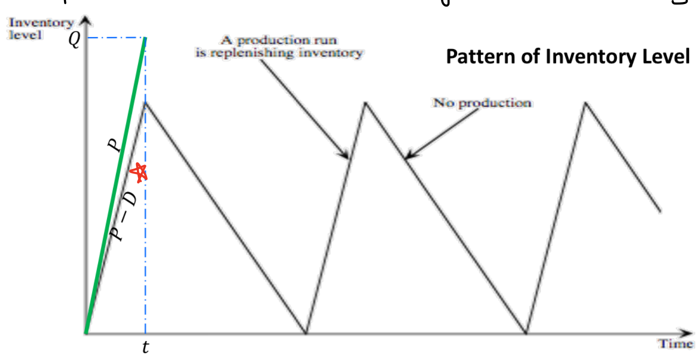
Discount Category		Price (C)	Unit Holding Cost $H = I*C = 0.21C$
1	0 - 749	\$20.00	0.21(\$20) = \$4.20
2	750 - 1999	\$19.80	0.21(\$19.80) = \$4.158
3	≥ 2000	\$19.60	0.21(\$19.60) = \$4.116

Discount Category	Order Quantity	Annual Costs Purchase	Ordering $S \cdot \frac{D}{Q}$	Holding $H \cdot \frac{Q}{2}$	Total
1 EOQ = 573	573	\$120,000 = 20 x 6000	\$1,204	\$1,204	\$122,407
2 EOQ = 576	750	\$118,800 = 19.8 x 6000	\$920	\$1,559	\$121,279
3 EOQ = 579	2,000	\$117,600 = 19.6 x 6000	\$345	\$4,116	\$122,061

$EOQ = \sqrt{\frac{2DS}{H}}$ — variable



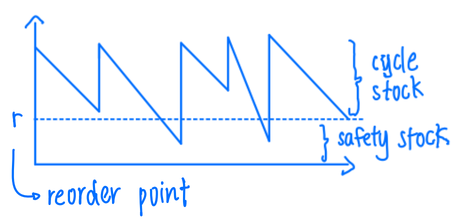
4. Economic Production Lot Size Model
- the item is produced internally at a constant rate P
- a production run is scheduled to begin when the inventory level drops to 0



Number of production runs per year $= \frac{D}{Q}$
Production rate (per year) $= P$
Maximum inventory level $= Q(1 - \frac{D}{P})$
Average inventory level $= \frac{Q}{2}(1 - \frac{D}{P})$
$TVC = H \times \frac{Q}{2}(1 - \frac{D}{P}) + S \cdot \frac{D}{Q}$
Optimal production lot size $Q^* = \sqrt{\frac{2DS}{H(1-\frac{D}{P})}}$
Minimum annual cost : $TVC^* = \sqrt{2DSH(1-\frac{D}{P})}$

Seminar 11 : Inventory Management II (Stochastic Demand)

1. Fixed Order Quantity Model (Continuous-Review Policy)
- Determine order quantity $Q = \sqrt{\frac{2DS}{H}}$
- Set service level $= 1 - a$ (probability of stockout $= a$)
  ↳ $P(d_L \le r) = 1 - a$
  ↳ lead time demand
- Determine re-order point r



- Total Cost (TC) $= S(\frac{D}{Q^*}) + H(\frac{Q^*}{2}) + H(Z\sigma_{dL})$

Ordering Cost — Holding Cost — Holding Cost for safety stock

- Example

Demand:
Weekly average = 120 (cases) (calculate from the data)
Average annual demand rate D = 120 * 52 = 6240 (cases)

Ordering cost:
S = $12 per order

Holding cost:
Unit purchase price = $10,
Annual inventory holding cost rate = 14%
H = 10*0.14 = $1.4 per unit per year

$Q^* = \sqrt{2*(6240)*12/1.4} = 327.0649$

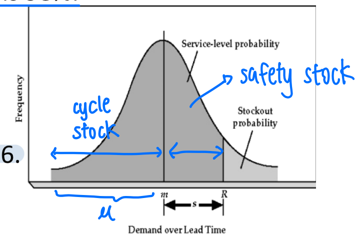
$Q^* = \sqrt{\frac{2DS}{H}}$

Mean $(d_L) = L \times d$

Lead time demand is normally distributed with $\mu = 80$, $\sigma = 10$. Since Roberts wants at most a 2% probability of selling out of *Comfort*, the service level is 98%.

Reorder point: $r = \mu + z\sigma$
Safety stock: $z\sigma$

The corresponding z value is 2.06.
That is, $P(z > 2.06) = 0.0197$.



Hence Roberts should reorder *Comfort* when supply reaches $r = \mu + z\sigma = 80 + 2.06(10) = 101$ cases. The safety stock is $z\sigma = 21$ cases.

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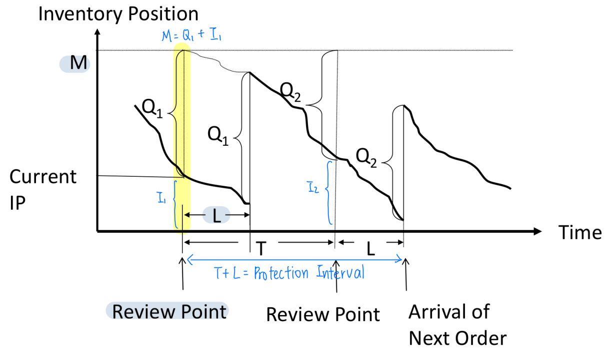
Work in excel : r = NORM.INV (0.98, 80, 10)
$= \bar{d}_L + Z\sigma_{dL}$

2. The Periodic-Review Order-up-to Policy

- Inventory position is reviewed in a constant interval of T time units
- Ordering decisions are made only at points of review
- Replenishment policy : order stock to raise the inventory position to a fixed replenishment level M
- Easier to implement
- Stock M is to meet the demand in an interval of L+T (protection interval)



(a) Steps

- Define service level $= 1-\alpha$ — probability of stockout
- Determine M : $P(d_{L+T} \le M) \ge 1-\alpha$
  demand in protection interval
- $M = \mu_{L+T} + Z\sigma_{L+T}$

Example :
$d \sim N(20, 8^2) \rightarrow \bar{d} = 20,\ \sigma_d = 8$
T = 7 days , L = 3 days
T + L = 10 days

Need to know $d_{T+L} = d_1 + d_2 + \cdots + d_{10}$
$= N((T+L)\bar{d},\ (T+L)\sigma_d^2)$
$= N(200,\ (8\sqrt{10})^2)$

service level

M = NORM.INV (SL , 200, $8\sqrt{10}$)
If SL = 0.95 , M = 241.6119
$M = \bar{d}_{T+L} + Z \times \sigma_{T+L}$

Total Cost = Annual Setup Cost + Annual Holding Cost for cycle stock + Annual Holding Cost for safety stock
$= S \times \frac{\bar{D}}{(\bar{d} \cdot T)} + H\frac{(\bar{d} \cdot T)}{2} + HZ \cdot \sigma_{T+L}$

★ Number of stockouts / year $= \alpha \times \frac{D}{Q}$
Prob of stockout
↳ number of orders

T: review period
$\bar{d}$: mean daily / weekly demand = 20
↳ based on question

Seminar 12 : Decision Analysis

1. Criteria
   - MaxiMax (optimistic) - Choose max payoff from each state of nature, choose decision with max payoff.
   - MaxiMin (pessimistic) - Choose min payoff from each state of nature, choose decision with max payoff.
   - MiniMax Regret - Choose max regret value from each state of nature, choose decision with min regret value.
2. Decision with Probabilities

① Profit in $millions

State of nature

Decisions	S1	S2	S3	Max	Min
D1	0.5	0.4	0.3	0.5	0.3
D2	1.0	0.6	0.1	1.0	0.1
D3	2.0	0.5	-0.5	2.0	-0.5
				max = 2.0	Max = 0.3

Hence, choose D3 using MaxiMax algorithmn

Hence, choose D1 using MaxiMin algorithmn

For regret table, (idea in mind: If Sxx was to happen)

	S1	S2	S3	Max
D1	1.5 = 2−0.5	0.2	0	1.5
D2	1.0	0	0.2	1.0
D3	0	0.1	0.8	0.8

min = 0.8 ↳ to minimize the worst outcome

4

* regret values are always non-negative

② Expected Value

	S1	S2	S3	EV
D1	0.5	0.4	0.3	0.38
D2	1.0	0.6	0.1	0.49
D3	2.0	0.5	-0.5	0.35
Prob.	0.1	0.6	0.3	

EV: has the same unit as payoff; Expected value = sumproduct of payoffs with probabilities

= 2(0.1) + 0.5(0.6) − 0.5(0.3)

Max = 0.49

Hence, choose D2

State of nature has probability, decisions has expected value.

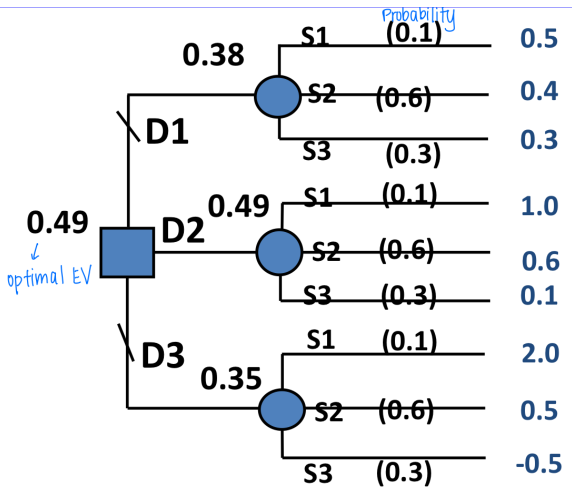
3. Decision Tree

– Consists of Decision Event, Chance (uncertain) event, Value (payoff), link

□ ○ ▭ →

leaving a decision node for different decision alternative

leaving a chance event for different outcome



- for each chance node, calculate EV by sumproduct of probabilities & payoff
- for each decision node, select & mark the decision with max EV

4. Decision with Research / Survey

(a) Bayes Theorem

- S_j state of nature j = 1, … ,n
- I_k market research indicator k = 1, …, L
  ↳ consultant's predict
- $P(I_k \mid S_j)$: reliability of the marketing research

I_k: Consultant recommends S_k

eg. I_1: Consultant says "S_1 will happen"

$$P(S_j \mid I_k) = \frac{P(I_k \mid S_j)P(S_j)}{\sum_{t=1}^{n} P(I_k \mid S_t)P(S_t)} = \frac{P(I_k \mid S_j)\, P(S_j)}{P(I_k)}$$

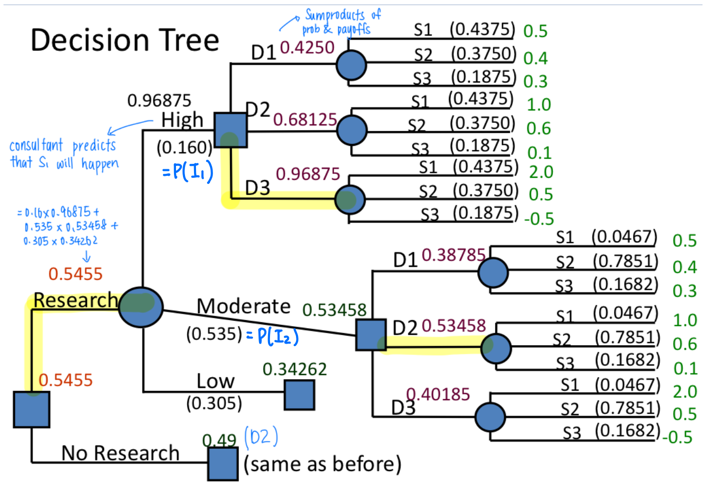
Calculation:

Given

	$P(S_j)$	$P(I_k \mid S_j)$ I_1	I_2	I_3	$P(I_k . S_j)$ $= P(S_j) \cdot P(I_k \cdot S_j)$ I_1	I_2	I_3	$P(S_j \mid I_k)$ I_1	I_2	I_3
S_1	0.1	0.70	0.25	0.05	0.07 $= 0.1 \times 0.7$	0.025	0.005	0.4375 $= \frac{0.07}{0.16}$	0.0467	0.0164
S_2	0.6	0.10	0.70	0.20	0.06 $= 0.6 \times 0.1$	0.420	0.120	0.3750 $= \frac{0.06}{0.16}$	0.7851	0.3934
S_3	0.3	0.10	0.30	0.60	0.03	0.090	0.180	0.1875 $= \frac{0.03}{0.16}$	0.1682	0.5902
				$P(I_k)$	0.16	0.535	0.305			

Sum

Probability of predicting S_1 = 0.07 + 0.06 + 0.03



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(same as above)

Survey outcome

- If I_1: then D_3 market the product internationally
- If I_2: then D_2 market the product in Asia
- If I_3: then D_1 market the product in Singapore

Expected Value of Perfect Information

* EVPI requires EVwPI : Expected Value with Perfect Information

- Expected Value with Perfect Information or
  EVwPI = 2*0.1 + 0.6*0.6 + 0.3*0.3 = 0.65
- Expected Value of Perfect Information or
  EVPI = 0.65 – 0.49 = 0.16

EV

EVPI = EVwPI − EV (Expected Value)

Had you known in advance that S1 is going to happen, what action will you take?

① : max payoff for state of nature S_1

② : $P(S_1)$

Expected Value of Sampled Information

- Expected value with sample information:
  EVwSI = 0.5455 (from the decision tree)
- Expected value of sample information:
  EVSI = 0.5455 – 0.49 = \$0.0555 million = \$55,000
- Efficiency of Information
  EVSI/EVPI = 0.0555/0.16 = 34.69%

EVSI = EVwSI (without cost) − EV

↳ charges of consultant

↳ measure the quality of consultant's information

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EVSI = 0.0555 → Budget that we are willing to pay to this consultant with this reliability

0.49 — 0.5455 — 0.65

EV — EVwSI (without cost) — EVwPI

EVPI = 0.16

↳ Budget that we are willing to pay to any consultant

- If consultant fee > \$55000, don't implement the survey
- If consultant fee < \$55000, implement the survey

5. Work in Excel (Maximin)

Profit in \$millions

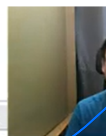
Get minimum along selected decision axis

	S1	S2	S3	Min
D1	0.5	0.4	0.3	0.3
D2	1.0	0.6	0.1	0.1
D3	2.0	0.5	0.2	0.2

	S1	S2	S3	
D1	X_{11}	X_{12}	X_{13}	= 1
D2	X_{21}	X_{22}	X_{23}	= 1
D3	X_{31}	X_{32}	X_{33}	= 1

X_{ij} is binary

Results After Excel



Min payoff for each state of nature

	B	C	D	E	F	G	H	I	J	K	L	M
1			s1	s2	s3			Pi = Mins	= I2	= I2		V
2		d1	0.5	0.4	0.3		>=	0.3	0.3	0.3	<=	0.3
3		d2	1	0.6	0.1			0.1	0.1	0.1		0.3 = V2
4		d3	2	0.5	0.2			0.2	0.2	0.2		0.3 = V2
5								↳ sumproduct of payoff & Xij				
6					X13			LHS		RHS		
7		X11 ←	0	0	1			1	=	1		
8			0	0	1			1	=	1		
9			0	0	1			1	=	1		
10												
11												
12		Obj Fn:	0.3		d1 → Decision							

For ease of copy-and-pasting, columns J2:J4 and K2:K4 are just formula-equals of I2:I4. Also, V3 and V4 are formula-equals of V2.

max payoff for each decision

constraints :

① D2 : F4 >= I2 : K4

② I2 : I4 <= M2 : M4

③ I7 : I9 = K7 : K9

④ D7 : F9 = binary

The result is that our optimal value gives MaxiMin payoff. Each decision's minimum payoff is given by the 1's in D7:F9. If the payoffs are unique, we can match the optimal payoff against I2:I4 to arrive at which decision we would choose to get MaxiMin payoff.