14.1-3

rod-cutting problem with cut cost

```
BOTTOM-UP-CUT-ROD-WITH-COST(p, n, c)
    let r[0..n] be a new array
    r[0] = 0
    for j = 1 to n
        q = -∞
        for i = 1 to j
            q = max(q, p[i] + r[j - i] - (c if i != j else 0))
        r[j] = q
    return r[n]
```

14.1-5

```
MEMOIZED-CUT-ROD-WITH-SOLUTION(p, n)
    let r[0..n] be a new array
    let s[0..n] be a new array
    for i = 0 to n
        r[i] = -∞
    return MEMOIZED-CUT-ROD-AUX-WITH-SOLUTION(p, n, r, s)

MEMOIZED-CUT-ROD-AUX-WITH-SOLUTION(p, n, r, s)
    if r[n] ≥ 0
        return r[n], s[n]
    if n == 0
        return 0, []
    q = -∞
    for i = 1 to n
        temp_revenue, temp_cuts = MEMOIZED-CUT-ROD-AUX-WITH-SOLUTION(p, n - i, r, s)
        if p[i] + temp_revenue > q
            q = p[i] + temp_revenue
            s[n] = [i] + temp_cuts
    r[n] = q
    return q, s[n]
```

14.2-1

optimal parenthesization of <5, 10, 3, 12, 5, 50, 6>

	A	B	C	D	E	F
A	0	150	330	405	1655	2010
B		0	360	330	2430	1950
C			0	180	930	1770
D				0	3000	1860
E					0	1500

$$=> ((AB))(((CD)(EF)))$$

14.2-6

Show that a full parenthesization of an n-element expression has exactly $n-1$ pairs of parentheses.

We will have $n-1$ ways to break $A_1 A_2 \ldots A_n$ into 2 pices.
So $n-1$ pairs of parentheses.

14.3-1

1. enumerate all the ways

$$P(n) = \begin{cases} 1 & \text{, if } n = 1 \\ \sum_{i=1}^{n-1} (P(n-i) - P(i)) & \text{, if } n \geq 2 \end{cases}$$

$$\begin{aligned} P(n) &= 2(P(1) + P(n-1) + P(2) + P(n-2) + \cdots + P(n-1) + P(1)) + \underline{2(n-1)}_{\text{counting}} \\ &= 2(P(1) + P(2) + \cdots + P(n-1)) + (2(n-1)) \end{aligned}$$

$$P(n+1) = 2(P(1) + P(2) + \cdots + P(n)) + (2(n))$$

$$P(n+1) - P(n) = 2P(n) + 2 \Rightarrow P(n+1) = 2 + 3P(n)$$

$$
\begin{aligned}
P(n) + 1 &= 3 + 3P(n-1) = 3(P(n-1)+1) = 9(P(n-2)+1) \\
&= 3^{n-2}(P(2)+1) = 3^{n-1}
\end{aligned}
$$

$$P(n) = 3^{n-1} - 1 = \Omega(3^{n-1})$$

2. recursive time complexity

$$T(n) = \begin{cases} C & \text{, if } n = 1 \\ C + \sum_{i=1}^{n-1} \left(T(i) + T(n-i) + C\right) & \text{, if } n \geq 2 \end{cases}$$

$$\Rightarrow T(n) \leq Cn + 2\sum_{i=1}^{n-1} T(i)$$

we assume that $T(n) \leq Cn3^{n-1}$

$$\begin{aligned}
T(n) &\leq Cn + 2\sum_{i=1}^{n-1} Ci3^{i-1} = C\left(n + 2\sum_{i=1}^{n-1} i3^{i-1}\right) \\
&= C\left(n + n3^{n-1} + \frac{1-3^n}{2}\right) = Cn3^{n-1} + C\left(n + \frac{1-3^n}{2}\right) \\
&\leq Cn3^{n-1}
\end{aligned}$$

$2n + 1 - 3^n \leq 0, \forall n \geq 1 \Rightarrow T(n) = O(n3^{n-1})$

so running recursively is faster.

14.3-4

We assume that there are A_1, A_2, and A_3

to find that $P_0 P_1 P_3 < P_0 P_2 P_3$ and
$P_0 P_1 P_2 + P_0 P_2 P_3 < P_1 P_2 P_3 + P_0 P_1 P_3$

Let $P_0, P_1, P_2, P_3 = 1, 2, 3, 3$ in this case $P_0 P_1 P_3 < P_0 P_2 P_3$
choosing $A_1 (A_2 A_3)$ we get
$A_1 (A_2 A_3) : 2 \cdot 3 \cdot 3 + 1 \cdot 2 \cdot 3 ; (A_1 A_2) A_3 : 2 \cdot 3 \cdot 3 + 1 \cdot 2 \cdot 3$

$(A_1 A_2) A_3$ is optimal solution but Caupulet didn't choose it.

14.4-2

```
FIND-LCS(X, Y, C, i, j)
    if i == 0 or j == 0 return
    if X[i] == Y[j]
        FIND-LCS(X, Y, C, i-1, j-1)
        print(X[i])
    else if C[i-1, j] == C[i, j]
        FIND-LCS(X, Y, C, i-1, j)
    else
        FIND-LCS(X, Y, C, i, j-1)
```

initial call : `FIND-LCS(X, Y, C, m, n)` to print LCS

time complexity $= O(m+n)$

14.4-5

find the LCS of the array and the sorted copy of that array

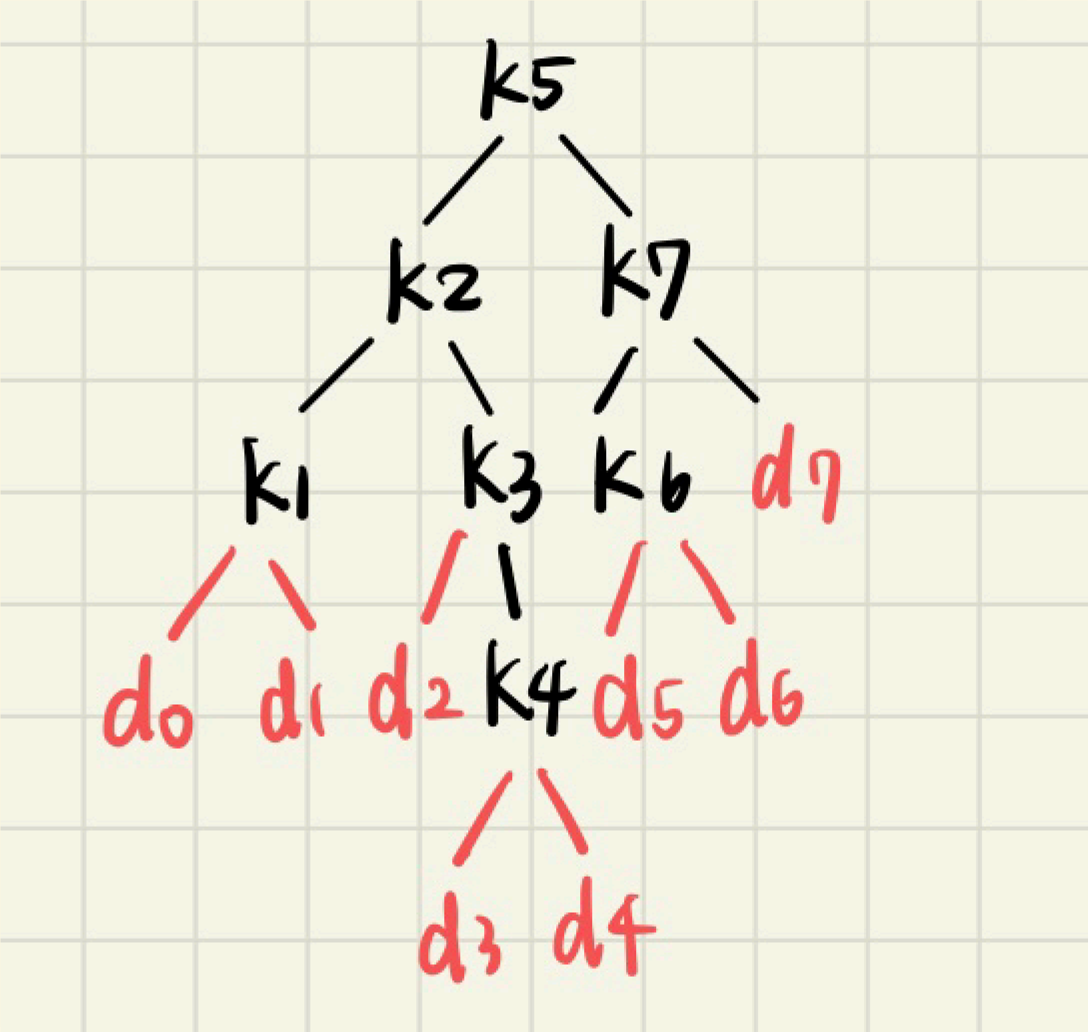
time complexity $= \text{sorting } O(n \lg n) + \text{LCS } O(n^2) = O(n^2)$

14.5-1

```
CONSTRUCT-OBST-SUBTREE(root[], i, j, r, dir)
    if j < i
        print('d', j, ' is the ', dir, ' child of ', 'k', r)
    else
        rt = root[i, j]
        print('k', rt, ' is the ', dir, ' child of ', 'k', r)
        CONSTRUCT-OBST-SUBTREE(root, i, rt-1, rt, 'left')
        CONSTRUCT-OBST-SUBTREE(root, rt+1, j, rt, 'right')

CONSRTUCT-OPTIMAL-BST(rot[], n)
    r = root[1, n]
    print('k', r, ' is the root')
    CONSTRUCT-OBST-SUBTREE(root, 1, r-1, r, 'left')
    CONSTRUCT-OBST-SUBTREE(root, r+1, n, r, 'right')
```

14.5-2



`root[1, 7] = 5`

14-2

```
Let dp[1:n][1:n] and p[1:n][1:n] be new array
Let n be the length of the word
for i  =  1 to n
    for j  =  1 to n
        dp[i][j] = -1
Find_LPS(i, j, w[])
    if i  =  j return 1
    if i  >  j return 0
    if dp[i][j] != -1 return dp[i][j]
    if w[i] = w[j]
        dp[i][j] = Find_LPS(i+1, j-1, w) + 2
        p[i][j] = ""
    else
        temp1 = Find_LPS (i, j-1, w)
        temp2 = Find_LPS (i+1, j, w)
        if temp1 < temp2
            dp[i][j] = tep
            p[i][j] = ""
        else
        dp[i][j] = temp2
            p[i][j] = ""
Return_LPS(i, j, P[][], w[])
    if(i > j) return ""
    if(i = j) return w[i]
    if p[i][j] = "left" return Return_LPS(i, j-1, p, w)
    if p[i][j] = "down" return Return_LPS(i+1, j, p, w)
    if p[i][j] = "" return w[i] + Return_LPS(i+1, j-1, p, w) + w[j]
```

15.1-2

$S = \{a_1, a_2, \ldots, a_n\}$
$S_t = \{S_1, S_2, \ldots, S_n\}$ is the optimal set of starting time
$F_i = \{f_1, f_2, \ldots, f_n\}$ is the optimal set of finisish time
$a_i = [S_i, f_i)$ $\quad a_i$ strats at S_i and finish at f_i

creat a $S' = \{a'_1, a'_2, \ldots, a'_n\}, a'_i = [f_i, S_i)$

$\{a_{i1}, a_{i2}, \ldots, a_{ik}\} \subseteq S \Leftrightarrow \{a'_{i1}, a'_{i2}, \ldots, a'_{ik}\} \subseteq S'$

i.e. selecting the first activity to finish is compatable with selecting the last activity.

To porve that it yields an optimal solution, we let S_{ij} be the set of activities that start after a_i and finish before a_j. To find the maximum set of S_{ij}, let the maximum set be A_{ij}, including some activity a_k.

$A_{ik} = A_{ij} \cap S_{ik}$ and $A_{ki} = A_{ij} \cap S_{kj} \Rightarrow A_{ij} = A_{ik} \sqcup a_k \sqcup A_{kj}$

S_{ij} contains $|A_{ij}| = |A_{ik}| + |A_{kj}| + 1$ activities.

A_{ij} includes optimal solutions to S_{ik} and S_{kj}.

If I could find a set A'_{kj} in S_{kj} where $|A'_{kj}| > |A_{kj}|$, then I could use A'_{kj} rather than A_{kj}, A_{ik} is similary.

so $|A_{ik}| + |A'_{kj}| + 1 > |A_{ik}| + |A_{kj}| + 1 = |A_{ij}|$

$\Rightarrow A_{ij}$ is an optimal solutioin.

15.1-4

```
Hull_Assign(S[], f[], n)
Let STACK be the DSA of stack and create n Hulls variable
create t[1:2n] and a[1:n]
for i = n ddownto 1
    STACK.push(Hulli)
for i = 1 to n
    t[i] = {s[i], activity = i, type = "start}
    t[n+i] = {f[i], activity = i, type = "finish"}
    sort(t)
    for i = 1 to 2n-1
        if t[i].type == "finish"
            idx = t[i].activity
            STACK.push(a[idx])
        else
            idx = t[i].activity
            a[idx] = STACK.pop()
```

15.2-1

Let there be items a, b where $\frac{V_a}{W_a} > \frac{V_b}{W_b}$

Let $n = min(W_a, W_b)$. If we take n weight of b, we get $n \times \frac{V_b}{W_b}$

However, if we take n weight of a, we get $n \times \frac{V_a}{W_a}$

The total value increase since $n \times \frac{V_a}{W_a} - n \times \frac{V_a}{W_a} > 0 \Rightarrow$ it has the greedy-choice property.

15.2-2

```
Solve-Knapsack(w[], n, W)
    creat dp[0:w] and p[0:w]
    max_val = 0
    for i = 1 to w
        dp[i] = 0
    dp[0] = 1
    for n = 1 to n
        for j = w to W-wi
            if dp[j-wi] = 1
                dp[j] = 1
                p[j] = i
                max-val = max(j,max-val)
Find_Item (p[],w[],n)
    if n = 0 return
        get one item w[n]
        Find_Item (p[],d[],n-d[p[n]])
```

15.3-1

$$\text{a.freq} \leq \text{b.freq and x.freq} \leq \text{y.freq}$$
$$\text{a.freq} = \text{b.freq} \Rightarrow \text{a.freq} \leq \text{b.freq} = \text{x.freq} \leq \text{y.freq}$$

Since x and y are two characters having the lowest frequency

we have $\text{b.freq} \leq \text{y.freq} \Rightarrow \text{b.freq} = \text{y.freq}$

and $\text{a.freq} \leq \text{x.freq} \Rightarrow \text{a.freq} = \text{x.freq}$

Thus, the freq of a, b, x, y are the same.

15.3-3

$\{a: 00000000, b: 0000001, c: 000001, d: 00001, e: 0001, f: 001, g: 01, h: 1\}$

the character with frequency F_i will be

$$\begin{cases} i = 1 : 0\underbrace{\ldots}_{n-1}0 \\ i \geq 1 : 0\underbrace{\ldots}_{n-i}01 \end{cases}$$

15.4-1

```
Furthset_In_Future(b[], n, k)
    Let S be a set, i = 1
    while i <= n
        if b[i] in S
            print b[i] "cache hit"
        else
            if |S| < k
                print b[i]
                S = S U b[i]
            else
                break
        i = i + 1
    Let priority[1:n] be the new array
        for j = 1 to n
            priority[j] = 0
        for j = i to n
            priority[b[j]] = n+1-j
    Let Q be a min-priority queue, sort by the second key
        for v in S
            ENQUEUE(Q,{v,priority[v]})
```

Cont.

```
m = i
while i <= n
    if b[i] in S
        print b[i] "cache hit"
    else
        print b[i] "cache miss"
        if PQ is not empty
            {Z, P} = DEQUEUE(PQ)
            S = (S-{Z}) U b[i]
            if priority [b[i]] = n+1-i
                priority[b[i]] = 0
            ENQUEUE(PQ,{b[i], priority[b[i]]})
        else
            Z = b[m]
            S = (S-{Z}) U b[i]
        m = i
    i = i+1
```

15.4-3

x: evict block x when b_i is requested

y: evict block x when b_j is requested

Note that $j = n+1, \ldots m > i$

If $C_{s'j} = D_j \sqcup x$

be changed into $C_{s'j} = D_j \sqcup y$,

meaning that when b_{j-1} is requested, the y is evicted.

However, when b_j is requested, y is evicted again!

y is evicted twice is not true, since the value in cache

15-1

a.

```
Exchange(n)
    nq = floor(n/25)
    n = n mod 25
    nd = floor(n/10)
    n = n mod 10
    nn = floor(n/5)
    n = n mod 5
    np = n
```

We exchange n cents into the lagest coin and solve the $n - c$ cents subproblem. Whict yields the optimal solution.

b.

Assume that we exchange n cents. by the algorithm above, we get $\lfloor n \mod c^{i+1}/c^i \rfloor$ of c^i where $i = 0 \sim k-1$ and get $\lfloor n/c^k \rfloor$ of C^k.

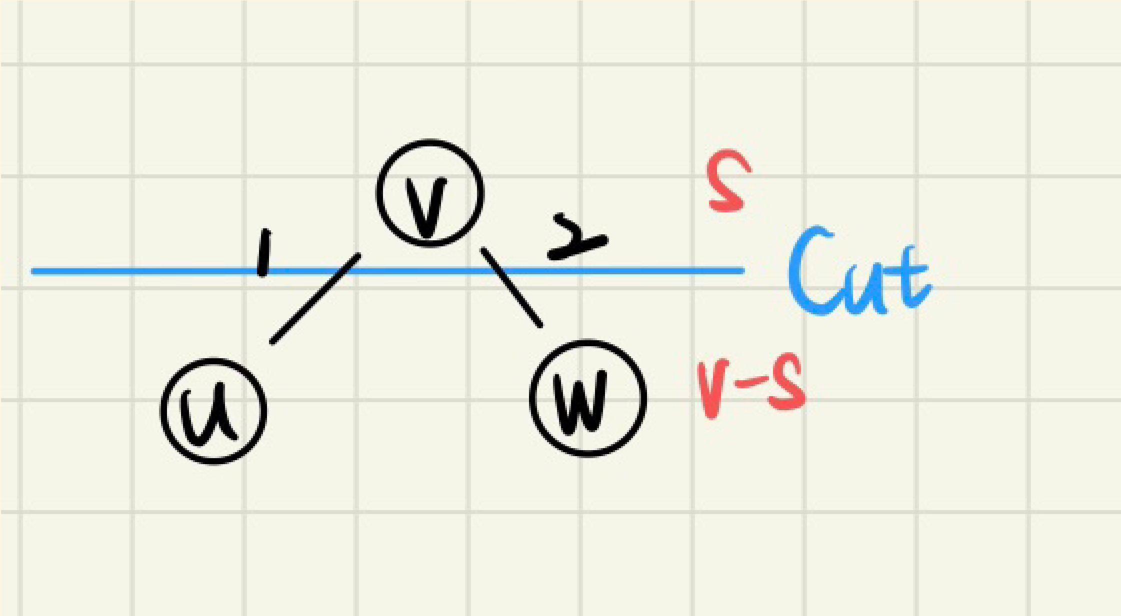
c.

If there are only penny, dime and quarter, 30 cents with the greedy algroithm would yield the 1 quarter and 5 pennies. However, we would just use 3 dimes, taking less coins than the algorithm. It doesn't yield optimal solution.

d.

```
Exchange(d[], n, k)
    create dp[0:n] and p[1:n]
    dp[0] = 0
    for i = 1 to n
        dp[i] = ∞
        for j = 1 to K
            if n >= d[j] and 1+dp[i-d[j]] < dp[i]
                dp[i] = dp[i-d[j]]+1
                p[i] = d[j]
        return dp[n] and p[]
    Find_Coin(p[], d[], n)
        if n = 0 return
            get one d[p[n]] coin
            Find_Coin(p[], d[], n-d[p[n]])
```

21.1-2



$G = (\{u, v, w\}, \{(u, w), (v, w)\})$
both (u, v) and (v, w) are safe edges crossing $(S, V - S)$. However, (u, w) is not a light edge for the cut.

21.1-3

Let T be the MST of G
$\{(u, v) \in T | (u, v)$ is the edge crossing some cut of $(s, v - s)$ and (u, v) is not a light edge$\} \Rightarrow \exists w(x, y)$ such that $w(x, y) < w(u, v)$

by ***Theorem 21.1***, we could construct spanning tree T', removing (u, v) from T and adding $(x, y) \Rightarrow w(T') = w(T) - w(u, v) + w(x, y) < w(T)$

$\Rightarrow$ (u, v) is a light edge crossing some cut.

21.2-4

1. The sorting time could be $O(|V|+|E|)$ by counting sort.
   Because $|V|=O(|E|) \Rightarrow O(|V|+|E|)=O(|E|)$
   $O(|V|+|E|)\alpha(|V|)$ = $O(|E|\alpha|V|)$
   Time Complexity : $O(|E|\alpha|V|)$

2. As above, sorting cost $O(w+|E|)$
   The operation of set cost $O(|v|+|E|)\alpha(|V|)=O(|E|\alpha|V|)$
   Time Complexity : $O(|E|\alpha|V|)$

21.2-5

```
for u in G.V
    INSERT(Q, u)
```

the time for priority queue insert |V| elements could be

$O(|V|\lg\lg|V|)$

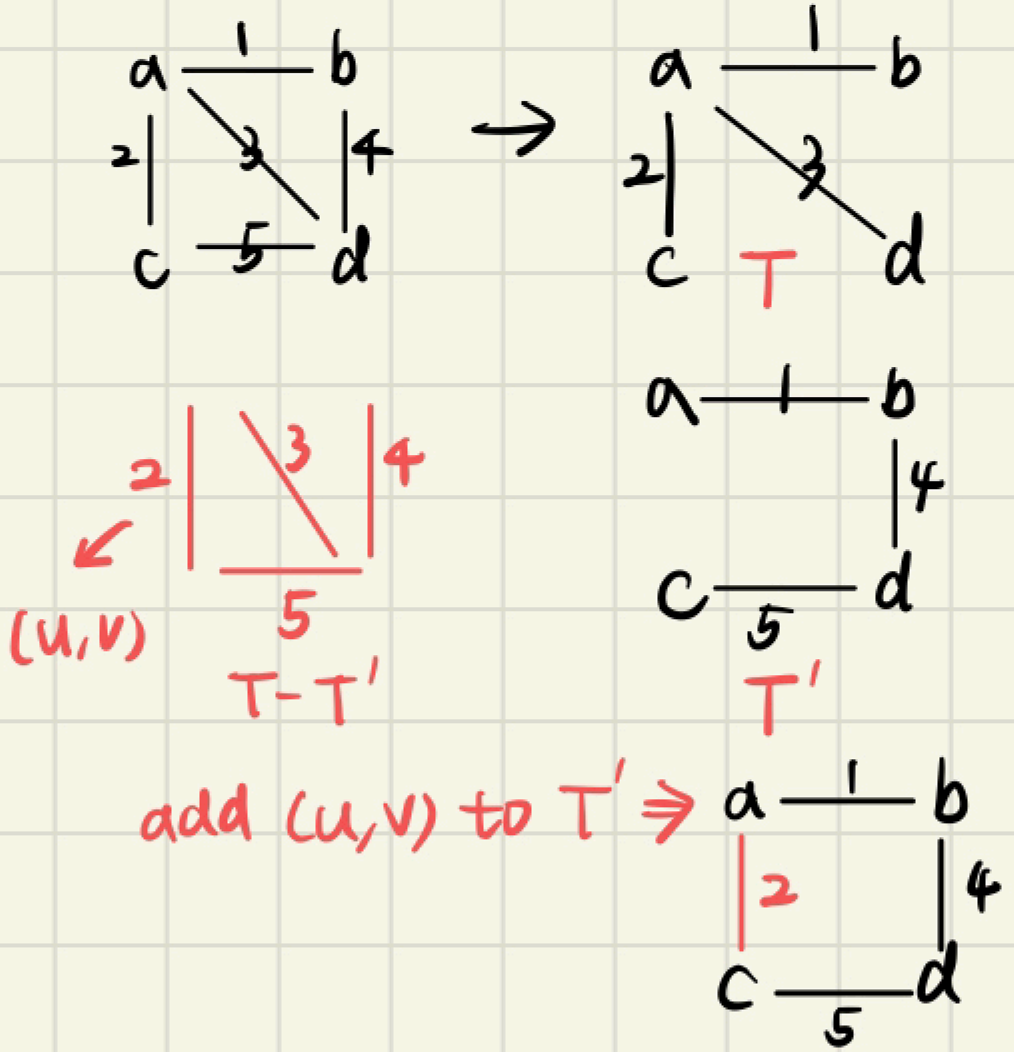
```
for v in G.adj[u]
    if v in Q and w(u,v) < v.key
        v.pi = u
        v.key = w(u,v)
        DECREASE-KEY(Q, u, w(u,v))
```

$|V|$ is constant $\Rightarrow O(|E|)$

21-1 and 21.1-6

a.

Assuming G has no two different MST T and T'. Let $(u, v) \in T \text{ and } \notin T'$
If we remove(u, v) from T to make T not connected.
(u, v) is a unique light edge for some cut$(s, v - s)$.
Let (x, y) be the edge crossing $(s, v - s)$ and $(x, y) \in T'$.
We have $w(x, y) > w(u, v)$. By ***Thm.21.1***, we could construct $T'' \text{ s.t. } w(T'') < w(T') \Rightarrow T'$ is not MST (*) so MST is unique.


a b c d
1 2 3 4 5
T
(u,v)
T-T'
T'
add (u,v) to T' ⇒

b.

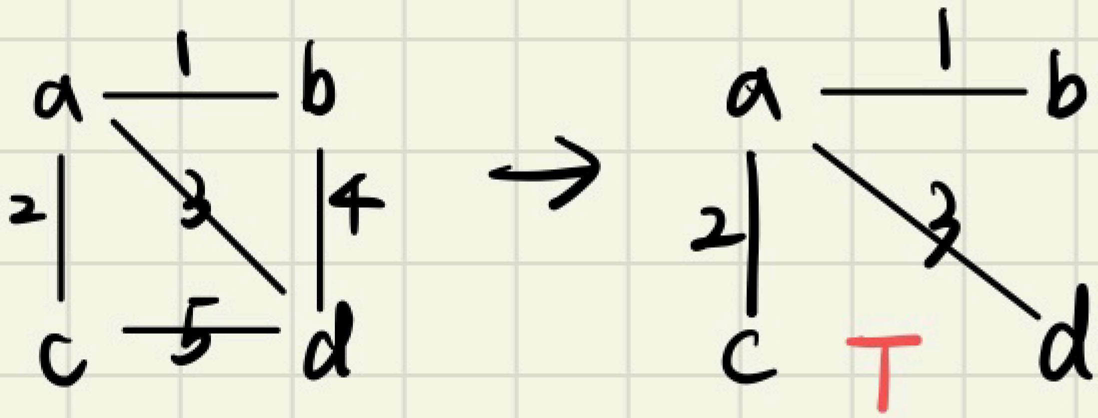
Let T be the MST of G , suppose T' is the 2nd-best MST which is different from T by two or more edges.
Let (u, v) be the minimum weight edge in $T - T'$
If we add (u, v) to T' , we could get a cycle C.
This cycle contains some edge (x, y) in $T' - T$

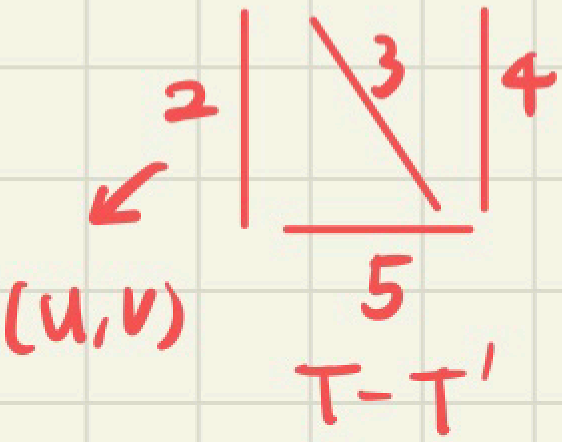
claim that $w(x, y) > w(u, v)$

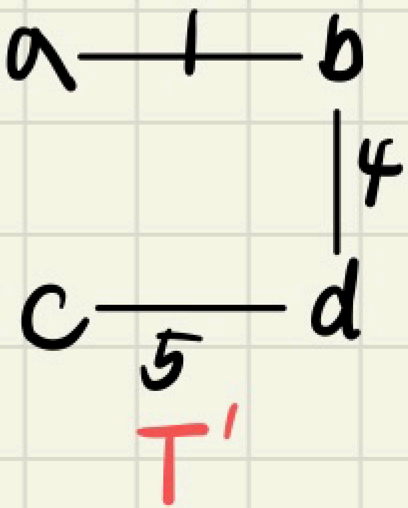
assume w(x,y) < w(u,v), if we add (x,y) to T,
we get a cycle C', which contains some edge (u', v') in $T - T'$.

The set $T'' = T - \{(u', v')\} \cup \{x, y\}$
forms a spanning tree, we have $w(u', v') < w(x, y)(*)$
It contradict with (u, v) is the edge with minimum weight in $T - T'$

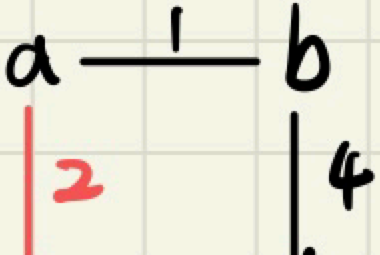
so we get $w(x, y) > w(u, v)$, and we could get $T''' = T' - \{(x, y)\} \cup \{(u, v)\}$ which is also a spanning tree.
It's neither T' nor T. $w(T''') < w(T')$ which yields a better solution than T'. So T' is not the 2nd-best solution.
i.e. 2nd-best MST and MST will be different with 1 edge.







add (u,v) to T' ⇒



c.

```
BFS(G, T, W)
    create max [1:|V|][1:|V|]
    for u in G.V
        for v in G.V
            max[u,v] = w[u,v]
        Let Q be a new empty queue
        ENQUEUE(Q,U)
        while Q is not empty
            X = DEQUEUE(Q)
            for v in G.adj[x]
                if max[u,v] == NULL and v != u
                    if x == u or w(x,v) > w(max([u,x]))
                        max[u,v] = (x,v)
                    else
                        max[u,v] = max[u,x]
                    ENQUEUE(Q, V)
    return max
```

d.

1. make MST: $O(|E| + |V|\lg|V|)$.
2. compute max array in (C.): $O(|V|^2)$
3. find an edge $(u, v) \notin T$ that minimize $w[u, v] - w(max[u, v])$: $O(|V|^2)$
4. Get the $T' = T - \{max[u, v]\} \cup \{u, v\}$

time complexity: $O(|V^2|)$