

Date:

Discrete Mathematics Homework 5

§ 9-6

12

reflexive: $aRa \rightarrow aR^{-1}a$

anti-symmetric: $aRb, bRa \rightarrow a=b$
 \downarrow
 $bR^{-1}a, aR^{-1}b \rightarrow a=b$

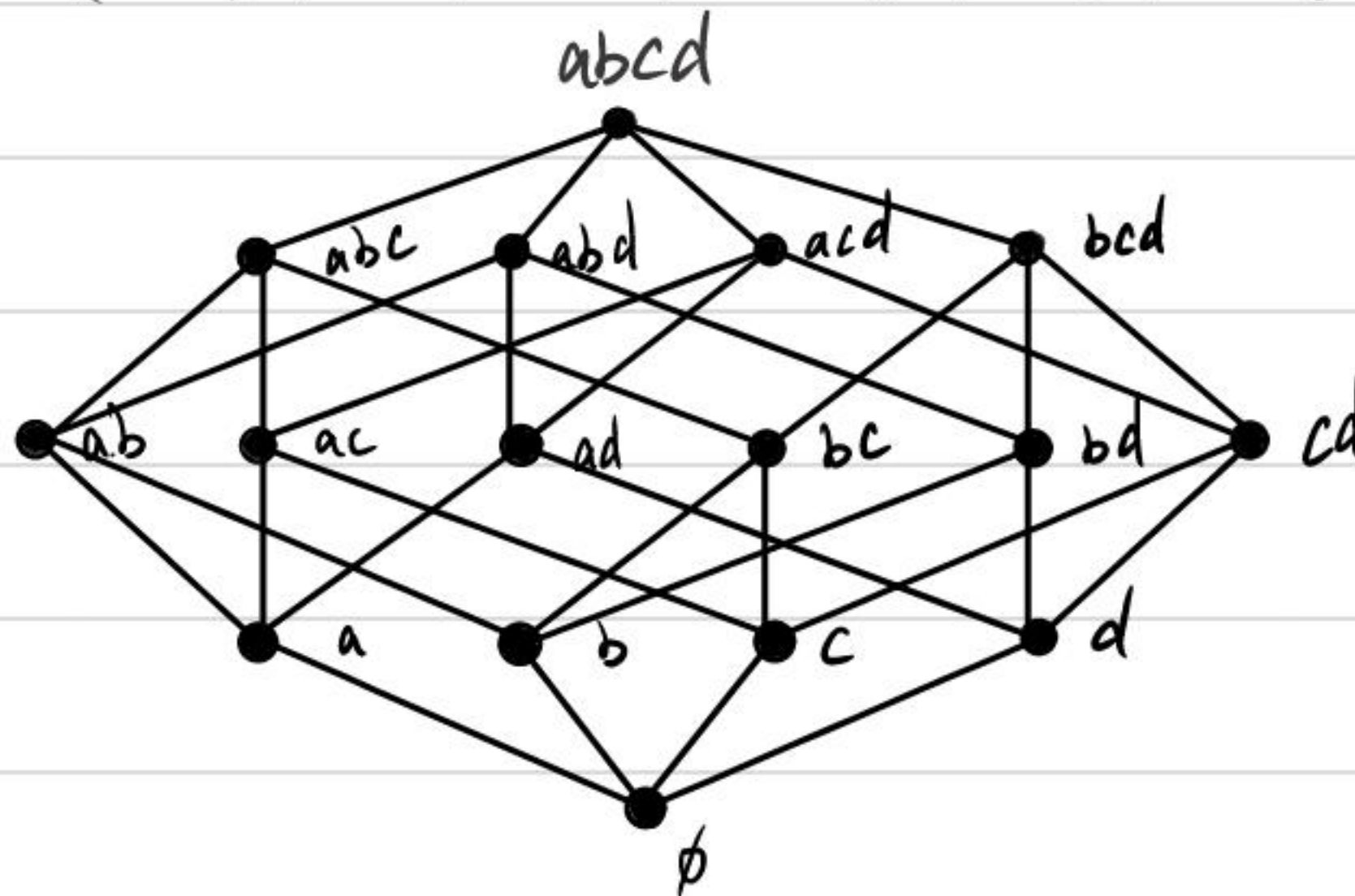
transitive: $aRb, bRc \rightarrow aRc$
 $cR^{-1}b, bR^{-1}a \rightarrow cR^{-1}a$

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a $\{(1,1), (1,2), (1,3), (1,4), (2,2)\}$

b $\{(4,1), (4,2), (4,3), (4,4), (3,2), (3,3), (3,4)\}$

24



(4,4)
(4,3)
(4,2)
(4,1)
(3,4)
(3,3)
(3,2)
(3,1)
(2,4)
(2,3)
(2,2)
(2,1)
(1,4)
(1,3)
(1,2)
(1,1)

32

a l, m b a, b, c c d no, there's not e k, l, m

f k g h no lower bounds

36

a (\mathbb{N}, \leq) b (\mathbb{N}, \geq) c (\mathbb{R}, \geq)

44

a no, (6,9) have no upper bound

b yes c yes (both of b and c are linear)

d yes, g.l.b of A, B is $A \cup B$, and l.u.b is $A \cap B$

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/ § 10-1

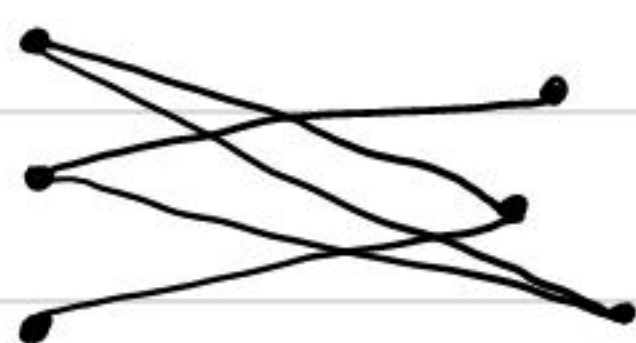
(12) 1. G is a graph with a loop at every vertex, so uRu .

2. An edge indicates R , so $uRv \rightarrow vRu$, thus symmetric.

(26) If you find a vertex with so many edges going out from it, that should probably be that electronic mail mailing list.

(30)

critics



movies

An edge means that a critic positively recommended a movie.

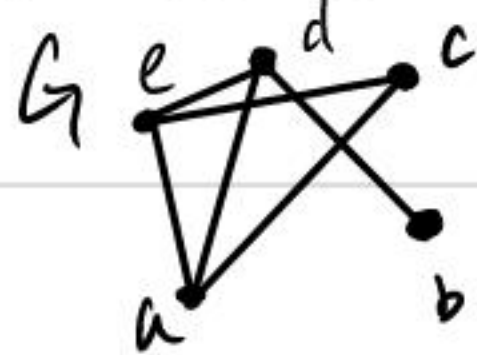
§

10-2

(6)

S = The set of people at a party, R = relation of shaken hands

The relation is symmetric because if aRb , then bRa surely.



In graph G , each vertex represents a person, and each edge represents R . The degree of a vertex

represents the number of people a person has shaken hands with.

The sum of the degrees of all the vertices will be even.

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

If we use an adjacency matrix to represent such a symmetric relation, it will be a symmetric matrix of course. So the sum of its 1s will be even.

(12)

The degree represents how many people a person knows.

The neighborhood represents people acquainted to each other.

Isolated vertices represent people that no one knows. They may be

dead people. Pendant vertices represent people that only one

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of others knows. The study indicates that a person knows approximately 1,000 people in the world.

- (18) If an edge e connecting vertices a, b appears, that is, $\deg(a) \neq 1 \rightarrow \deg(b) \neq 1$. Same as exercise (6), an edge represents a symmetric relation.

⚡ 10-3

- (28) Undirected Graph: The sum in the i^{th} row is the same as the corresponding column sum.

Directed Graph: The sum is the out degree of i .

- (36) Not, $\deg(v_2) = 4$, but none of u_i is the same.

- (42) Not, $\deg(v_6) = 4$, but none of u_i is the same.

- (46) An edge is in $\bar{G} \Leftrightarrow$ it's not in G iff its corresponding edge is in $\bar{H} \Leftrightarrow$ it's not in H .

⚡ 10-4

- (12) (a) Weakly connected (b) Strongly connected

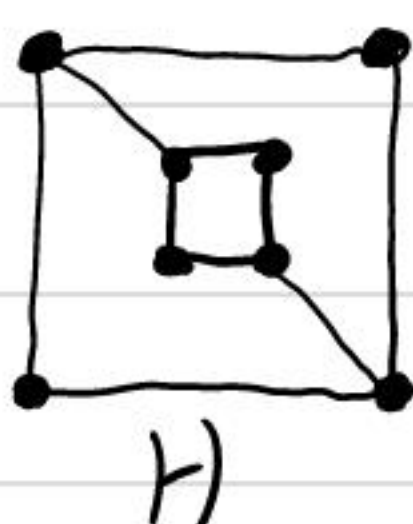
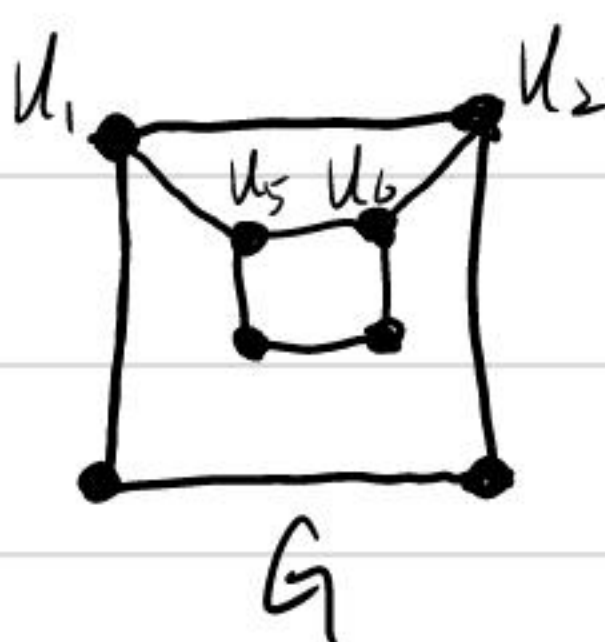
- (c) Neither strongly nor weakly connected.

- (14) (a) $\{a, b, e\}, \{d\}, \{c\}$ (b) $\{a\}, \{b\}, \{c, d, e\}, \{f\}$

- (c) $\{a, b, c, d, f, g, h, i\}, \{e\}$

(18)

(20)

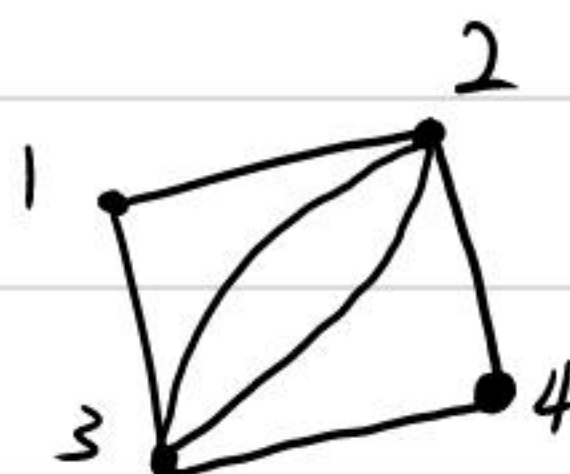
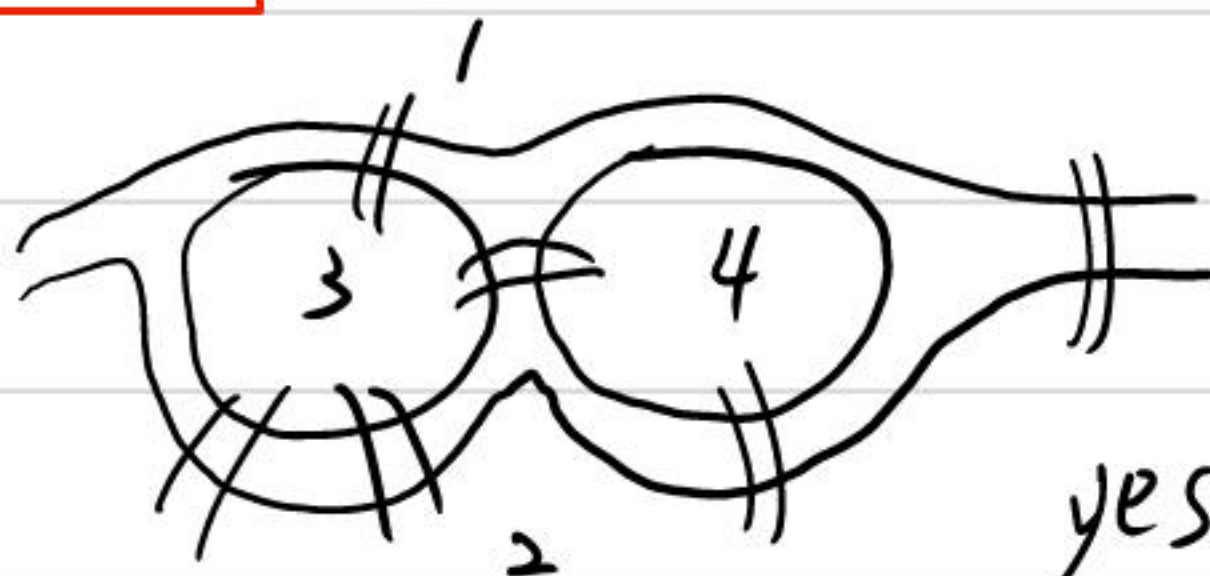


G has a simple closed path containing exactly the vertices of degree 3, while H has not. Thus they're not isomorphic.

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10-5

10



yes, 1243231

14

yes

20

a, d, b, d, e, b, e, c, b, a

30

The graph doesn't have a Hamilton circuit.

Because in order to get to the starting point, both c and f must be traveled at least twice, since the cut edge {c, f}

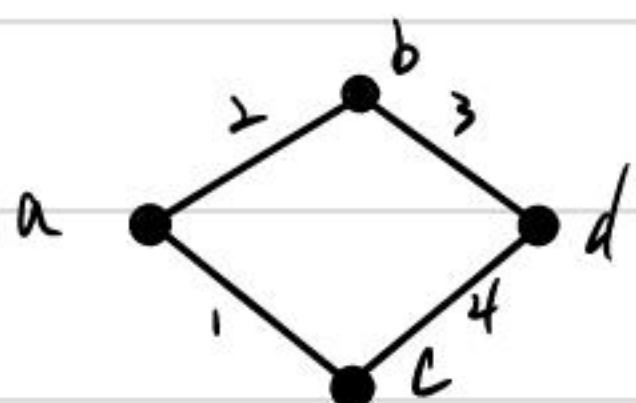
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10-6

6

a 6 b 11 c 8 d 15

18



No, counterexample

26

abcdea : 27

acbdea : 35

abcda : 23

acbeda : 25

abdcea : 30

acdbea : 32

abdeca : 26

acebda : 28

abecda : 20

adbcea : 35

abedca : 20

adcbea : 29

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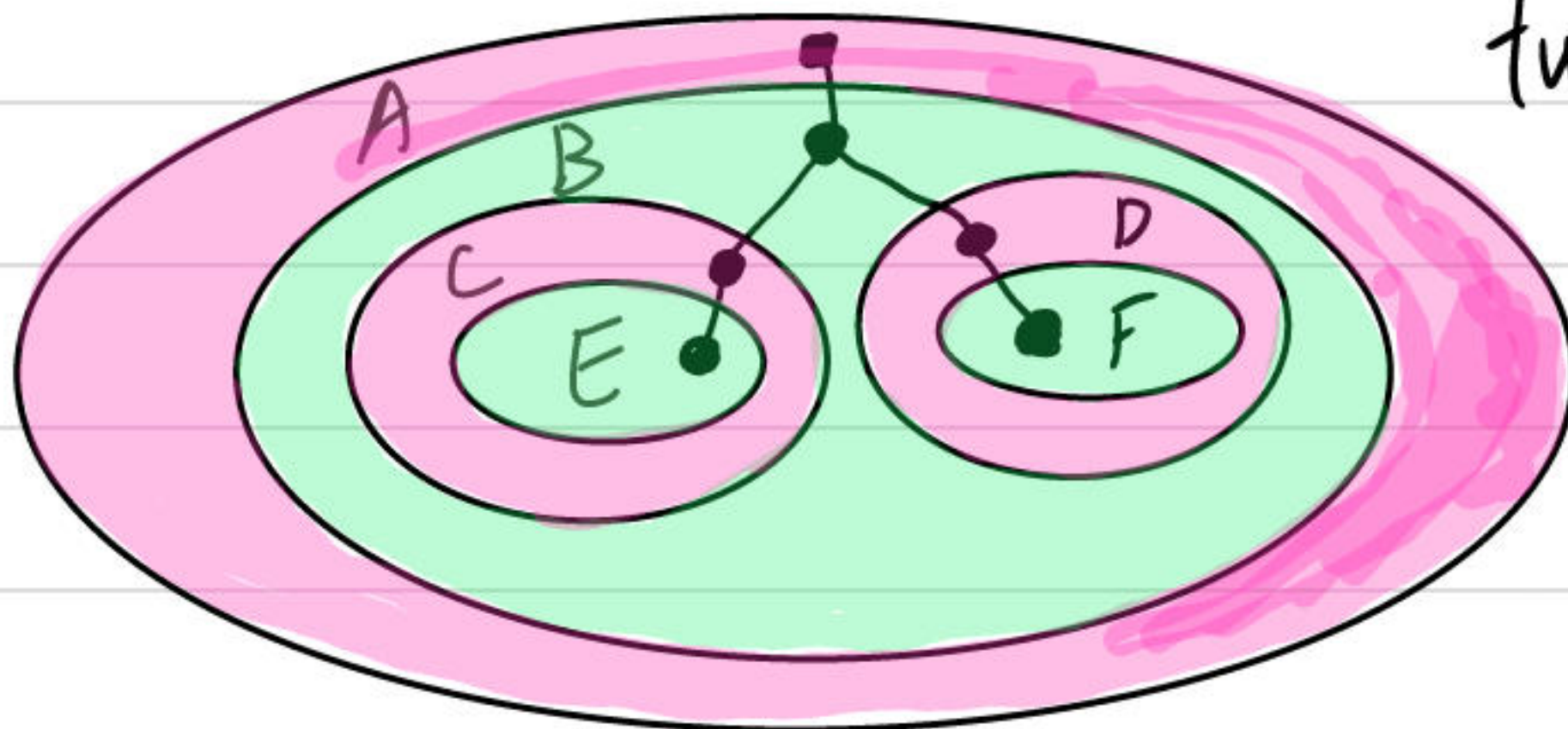
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⑫ $v - e + r = 2$, $v = 8$, $e = 8 \times 3 \div 2 = 12 \Rightarrow r = 6$

⑱ $v - (e + k - 1) + r = 2 \Rightarrow r = e - v + k + 1$

§ 10-8

④



two colors will be enough.

⑥ In each triangle, every vertex must have different color. Therefore, the chromatic number is 3.

⑩ If we color the first vertex with color A, then color its incident vertices with color B, and so on. We will find the last two vertices connected to each other, since the graph is a circuit and has odd number of vertices.

②④ If there's a vertex with degree D , then there are D edges incident with a common vertex. Thus, in any edge coloring each of those edges must have different color, so we need at least D colors.

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