

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. Suppose $\lim_{x \rightarrow 2} \frac{f(x) - 5}{x - 2} = 3$, then $\lim_{x \rightarrow 2} f(x) = 5$.
2. If $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = 4$, then $\lim_{x \rightarrow 0} \frac{f(x)}{x} = \lim_{x \rightarrow 0} f(x)$.
3. There exists a function $y = f(x)$ which has three vertical asymptotes.
4. The function $y = x^2 \sin \frac{1}{x} + 1$ has a horizontal asymptote.
5. $f(x) = \frac{x}{x+1}$ is symmetric with respect to the point $(-1, 1)$.
6. $\lim_{x \rightarrow \infty} \frac{\sin(x)}{x} = 1$.
7. If $f'(a)$ does not exist, then it is still possible that f is continuous at a .
8. $x^{100} - 10x + 1$ has at least one root in $[0, 2]$.
9. If $f(x)$ is an even function and $f'(c) = 1$, then $f'(-c) = 1$.
10. If $f(x) = \begin{cases} \frac{1 - \cos x}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $f'(0) = 0$.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. If there is a number k that makes

$$f(x) = \begin{cases} \frac{\sin^2 x}{4x^2}, & x > 0 \\ k + \cos x, & x \leq 0 \end{cases}$$

continuous at $x = 0$, then find k .

Answer : _____.

2. Evaluate $\lim_{x \rightarrow 0} \frac{\tan(2x)}{3x}$.

Answer : _____.

3. Find an equation of the normal line to the parabola $y = x^2 - 2x + 3$ that is parallel to $y = 4x + 5$.

Answer : _____.

4. Find the linearization of

$$y = \cos(x^2 + x) + \frac{1}{2x + 1}$$

at $x = 0$. Answer : _____.

5. Evaluate $\lim_{x \rightarrow 0} \frac{8x}{x - 3 \sin x}$.

Answer : _____.

6. Let $f(x) = \frac{1}{\sqrt{x-1}}$. Find the domain of the composite function $f \circ f$.

Answer : _____.

7. Find $\lim_{x \rightarrow 0^+} \left[\left(\sin \frac{1}{x} \right) (\cos x) \right]$.

Answer : _____.

8. If $\lim_{x \rightarrow 0^+} f(x) = A$, $\lim_{x \rightarrow 0^-} f(x) = B$. Find $\lim_{x \rightarrow 0^-} f(x^4 - x^2)$.

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Find the tangent line to the curve $x^2 \cos^2 y - \sin y = 0$ at $(0, \pi)$.
2. (10 points) A rock is thrown into a still pond and causes a circular ripple (漣漪). If the radius of the ripple is increasing at 2 feet per second. How fast is the area changing when the radius is 10 feet?

3. (10 points) Find the derivative

$$y = \tan \left(2 \sec \sqrt{x^2 + \frac{1}{3x}} \right).$$

Please no need to simplify your answer.

4. (10 points) Let $f(x) = |x^2 - 3x|$.
 - a. At which points is $f(x)$ continuous?
 - b. At which points is $f(x)$ differentiable?

Please explain why.

5. (10 points) Use $\epsilon - \delta$ definition of limit to prove that

$$\lim_{x \rightarrow 0} \sqrt{4 - x} = 2.$$

6. (10 points) Find all asymptotes of the graph of $y = \frac{2x^2}{1 - x}$.

(試題結束)

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(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $f'(c) = 0$, then f has a local maximum or minimum value at c .
2. If $f''(c) = 0$, then $(c, f(c))$ is a point of inflection of the curve $y = f(x)$.
3. There exists a function having a point of inflection where neither the first nor the second derivative exists.
4. Suppose that f has a positive derivative for all values of x and that $f(1) = 0$.
Let $g(x) = \int_0^x f(t)dt$. Then g has a local maximum at $x = 1$.
5. $\int_{-\pi}^{\pi} x^2 \sin(x^3) dx = 0$.
6. $\sum_{k=-1}^4 (2k+1) = \sum_{k=0}^5 (2k-1)$.
7. If $f(x)$ is continuous on $[0, 1]$, then $\int_0^1 f(x) dx = \int_0^1 f(1-x) dx$.
8. $\int x \sin x dx = \frac{x^2}{2} \sin x + C = -x \cos x + \sin x + C$.
9. The region bounded by the curve $y = \sqrt{x}$, the x -axis, and the line $x = 4$ is revolved about the x -axis to generate a solid. Let V be the volume of that solid.

$$V = \int_0^4 \pi(\sqrt{x})^2 dx = \int_0^2 2\pi y(4-y^2) dy.$$

(下頁還有試題)

10. Let $x \geq 0$ and $y = \sqrt{x}$. $\int_0^{a^2} \sqrt{1 + \frac{1}{4x}} dx = \int_0^a \sqrt{1 + 4y^2} dy$, for any positive number a .

填充題 **Short answer questions** (40 points), 每題 5 分。

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1. Use integral to evaluate

$$\lim_{n \rightarrow \infty} \frac{1 + 2^5 + 3^5 + \dots + n^5}{n^6}.$$

Answer : _____.

2. Find the average value of $f(x) = 2 - |x|$ on $[-1, 1]$.

Answer : _____.

3. Find the area of the region enclosed by the curves $x + y^2 = 1$ and $2x + y^2 = 0$.

Answer : _____.

4. Find the length of the graph of $f(x) = \frac{x^3}{12} + \frac{1}{x}$ from $x = 1$ to $x = 2$.

Answer : _____.

5. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \leq x \leq 3$, about the x -axis.

Answer : _____.

6. Suppose that $F(x)$ is an antiderivative of $f(x) = \frac{\sin x}{x}, x > 0$. Express

$$\int_1^3 \frac{\sin 2x}{x} dx \text{ in terms of } F.$$

Answer : _____.

7. Evaluate the definite integral $\int_0^\pi \sin^2 x \cdot \cos^2 x dx$.

Answer : _____.

8. Let a be the absolute maximum value of $f(x) = x^2 - 1$ on $-1 \leq x \leq 2$, and b be the absolute minimum value. Find $a + b$.

Answer : _____.

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1. (10 points)

- Please describe the Mean Value Theorem.
- Prove the inequality

$$|\cos x - \cos y| \leq |x - y|$$

for all x and y .

2. (10 points) A wire 10 m long is cut into two pieces. One piece is bent into an equilateral triangle and the other is bent into a circle. If the sum of the areas enclosed by each part is a minimum, what is the length of each part?

3. (10 points)

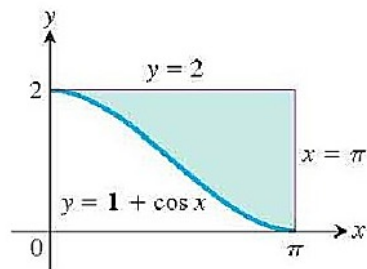
- Find $\int x^3 \sqrt{x^2 + 1} dx$.
- Find $\frac{d}{dx} \int_0^{\sqrt{x}} |\cos t| dt, x > 0$.

4. (10 points) The region in the first quadrant (象限) bounded by the curve $y = x^2$, the x -axis and the line $x = 2$ is revolved about the line $x = -1$ to generate a solid. Compute the volume of the solid.

5. (10 points) Let $f(x) = \frac{x^2 - 3}{x - 2}, x \neq 2$. Find **a.** critical numbers, **b.** intervals of increasing/decreasing, **c.** local maximum/minimum values, **d.** intervals of concavity, **e.** inflection points, **f.** oblique, horizontal and vertical asymptotes. **g.** Then sketch the graph of $f(x)$.

6. (10 points) Let $f(x) = 1 + \cos x$. Find **a.** the area of the shaded region, and **b.** the average value of f on $[0, \pi]$.

(試題結束)



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1. There exists a function $y = f(x)$ which has three vertical asymptotes.
2. Let $x \geq 0$ and $y = \sqrt{x}$. $\int_0^{a^2} \sqrt{1 + \frac{1}{4x}} dx = \int_0^a \sqrt{1 + 4y^2} dy$, for any positive number a .
3. $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x = e^2$.
4. $\frac{d}{dx}(8^x) = x8^{x-1}$.
5. $\cos(\tan^{-1} x) = \frac{x}{\sqrt{1+x^2}}$.
6. $\int_0^6 \frac{x}{x^2-4} dx = \ln(2\sqrt{2})$.
7. $\frac{x^2+4}{x(x^2-4)}$ can be put in the form $\frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$.
8. $\ln(\ln x) = o(\ln x)$.
9. A function has an inverse if and only if it is increasing or decreasing.
10. $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) = \frac{3\pi}{4}$.

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1. If $f(x) = \begin{cases} \frac{1 - \cos x}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then find $f'(0)$.
Answer : _____.

2. Let $y = \sqrt[3]{\frac{x(x+2)}{x^2+1}}$. Use logarithmic differentiation to find $\frac{dy}{dx}\big|_{x=1}$.
Answer : _____.

3. Let $y = \int_{e^{\sqrt{x}}}^{e^x} \ln t \, dt$, $x > 0$. Find $\frac{dy}{dx}$.
Answer : _____.

4. Order(排序) the following functions from slowest growing to fastest growing as $x \rightarrow \infty$. a. \sqrt{x} b. $\ln x^3$ c. x^x d. e^x
Answer : _____.

5. Let $f(x) = x^3 + 3 \sin x + 2 \cos x$. Find the value of $\frac{df^{-1}}{dx}$ at the point $x = 2 = f(0)$.
Answer : _____.

6. Evaluate $\int_0^1 x\sqrt{1-x} \, dx$.
Answer : _____.

7. Evaluate $\int_{\pi/3}^{\pi/2} \frac{\sin^2 x}{\sqrt{1-\cos x}} \, dx$.
Answer : _____.

8. Evaluate $\int_{-\pi}^{\pi} \sin 3x \sin 3x \, dx$.
Answer : _____.

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1. (10 points) Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \leq x \leq 3$, about the x -axis.

2. (10 points) Evaluate the integrals.

a. $\int \sec^3 \theta \, d\theta$

b. $\int \sqrt{x^2 - 9} \, dx$

3. (10 points) Evaluate the integrals.

a. $\int \frac{x^3 e^{x^2}}{(x^2 + 1)^2} \, dx$

b. $\int_1^e x^3 \ln x \, dx$

4. (10 points) Find the limits.

a. $\lim_{x \rightarrow \infty} x^{1/\ln x}$

b. $\lim_{x \rightarrow 0} \frac{\tan^{-1} x^2}{x \sin^{-1} x}$

5. (10 points) Evaluate the integral.

$$\int \frac{x^2 - x + 2}{x^3 - 1} \, dx$$

6. (10 points) Investigate the convergence:

a. $\int_0^\infty \frac{dx}{\sqrt{x^6 + 1}}$

b. $\int_1^\infty \frac{1 - e^{-x}}{x} \, dx$

(試題結束)