Calculus Homework Assignment 5

Class: CSIE 2-B

Student Number: 110502567

Name: 蔡淵丞

1. Identify the symmetries of the curve $r = \sin\left(\frac{\theta}{2}\right)$. Then sketch the curves in the xy-plane. [§10.4 − 7]

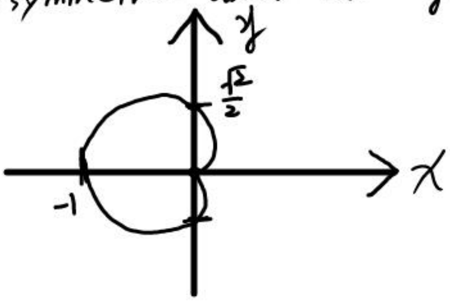
$\sin\left(-\frac{\theta}{2}\right) = -\sin\left(\frac{\theta}{2}\right) = -r$

$\Rightarrow$ symmetric about x-axis

$\sin\left(\frac{2\pi-\theta}{2}\right) = \sin\left(\frac{\theta}{2}\right) = r$

$\Rightarrow$ symmetric about y-axis

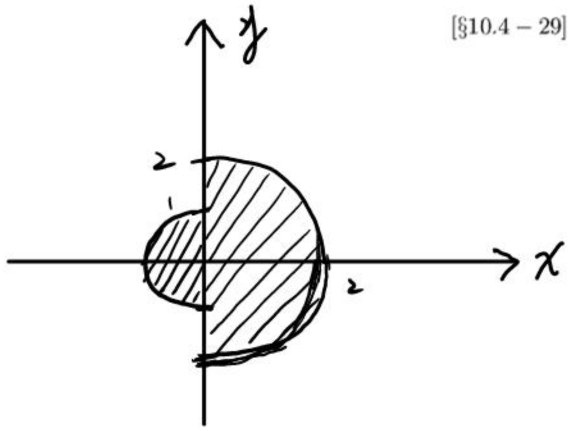
$\Rightarrow$ symmetric about the origin



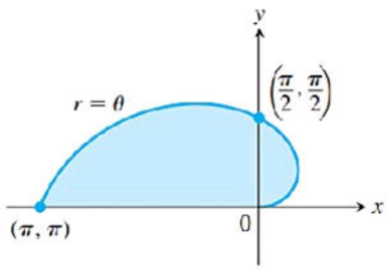
2. Sketch the region defined by the inequalities

$$-1 \le r \le 2 \text{ ,and } -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}.$$

[§10.4 − 29]



3. Find the areas of the blue regions. Which bounded by the spiral $r = \theta$ for $0 \le \theta \le \pi$.



[§10.5 − 1]

$$\frac{1}{2}\int_0^{\pi} \theta^2 \, d\theta = \frac{1}{6}\theta^3\Big]_0^{\pi}$$

$$= \underline{\underline{\frac{1}{6}\pi^3}}$$

4. Find the lengths of polar curve

$$r = \theta^2,\ 0 \le \theta \le \sqrt{5}. \int_0^{\sqrt{5}} \sqrt{r^2 + (r')^2}\, d\theta$$

[§10.5 − 21]

$$\int_0^{\sqrt{5}} \sqrt{\theta^4 + (2\theta)^2}\, d\theta$$

$$= \int_0^{\sqrt{5}} \theta\sqrt{\theta^2 + 4}\, d\theta \ \geq 0, \forall\, 0 \le \theta \le \sqrt{5}$$

let $\theta^2 + 4 = u \Rightarrow du = 2\theta\, d\theta$

$$\int_0^{\sqrt{5}} \theta\sqrt{\theta^2+4}\, d\theta = \frac{1}{2}\int_4^9 u^{\frac{1}{2}}\, du = \frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_4^9$$

$$= \frac{1}{3}(27-8) = \underline{\underline{\frac{19}{3}}}$$

5. Give the position vectors of particles moving along various curves in the xy-plane. In each case, find the particle's velocity and acceleration vectors at the stated times and sketch them as vectors on the curve. Motion on the circle $x^2 + y^2 = 1$.

$$\mathbf{r}(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j}, \quad t = \frac{\pi}{4} \text{ and } \frac{\pi}{2}.$$

[§12.1 − 9]

velocity $v(t) = (\cos t)\,i - (\sin t)\,j$

acceleration $a(t) = (-\sin t)\,i - (\cos t)\,j$

$v\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$, $a\left(\frac{\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$v\left(\frac{\pi}{2}\right) = (0, -1)$, $a\left(\frac{\pi}{2}\right) = (-1, 0)$

6. As mentioned in the text, the tangent line to a smooth curve $r(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$ at $t = t_0$ is the line that passes through the point $(f(t0), g(t0), h(t0))$ parallel to $v(t_0)$, the curve's velocity vector at t_0. Find parametric equation for the line that is tangent to the given curve at the given parameter value $t = t_0$.

$$\mathbf{r}(t) = (\sin t)\mathbf{i} + (t^2 - \cos t)\mathbf{j} + e^t\mathbf{k}, \; t_0 = 0$$

[§12.1 − 23]

$v(t) = (\cos t)\,i + (2t + \sin t)\,j + e^t k$

$v(0) = i + k$, $r(0) = (0, -1, 1)$

tangent line : $(t, -1, 1+t)$

7. Evaluate the integral

$$\int_0^{\frac{\pi}{2}} \left[\cos t\mathbf{i} - \sin 2t\mathbf{j} + \sin^2 t\mathbf{k}\right] \, dt.$$

[§12.2 − 9]

$\sin^2 t = \frac{1 - \cos 2t}{2} = \frac{1}{2} - \frac{1}{2}\cos 2t$

$\int_0^{\frac{\pi}{2}} \cos t\, i - \sin 2t\, j + \left(\frac{1}{2} - \frac{1}{2}\cos 2t\right) dt$

$= \sin t\Big]_0^{\frac{\pi}{2}} + \frac{1}{2}\cos 2t\Big]_0^{\frac{\pi}{2}} + \frac{1}{2}t\Big]_0^{\frac{\pi}{2}} - \frac{1}{4}\sin 2t\Big]_0^{\frac{\pi}{2}}$

$= 1 + \frac{1}{2}(-1-1) + \frac{\pi}{4} - 0$

$= i - j + \frac{\pi}{4}k$

#

8. At time $t = 0$, a particle is located at the point $(1, 2, 3)$. It travels in a straight line to the point $(4, 1, 4)$, has speed 2 at $(1, 2, 3)$ and constant acceleration $3\mathbf{i} - \mathbf{j} + \mathbf{k}$. Find an equation for the position vector $\mathbf{r}(t)$ of the particle at time t.

[§12.2 − 21]

$a(t) = 3i - j + k$

$v(t) = 3t\,i - t\,j + t\,k + C_1$

$v(0) = \frac{2}{\sqrt{9+1+1}}(3i - j + k) = C_1$

$\Rightarrow v(t) = \left(3t + \frac{6}{\sqrt{11}}\right)i - \left(t + \frac{2}{\sqrt{11}}\right)j + \left(t + \frac{2}{\sqrt{11}}\right)k$

$r(0) = i + 2j + 3k = C_2$

$r(t) = \left(\frac{3}{2}t^2 + \frac{6}{\sqrt{11}}t\right)i - \left(\frac{1}{2}t^2 + \frac{2}{\sqrt{14}}t\right)j + \left(\frac{1}{2}t^2 + \frac{2}{\sqrt{14}}t\right)k + C_2$

$\Rightarrow r(t) = \left(\frac{3}{2}t^2 + \frac{6}{\sqrt{14}}t + 1\right)i - \left(\frac{1}{2}t^2 + \frac{2}{\sqrt{11}}t - 2\right)j + \left(\frac{1}{2}t^2 + \frac{2}{\sqrt{14}}t + 3\right)k$

#

(The end)