微積分聯合教學　會考一　**Oct 17, 2017**

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $\lim_{x\to c}|f(x)| = 0$ then $\lim_{x\to c} f(x) = 0$.

2. $\lim_{x\to c}[f(x)g(x)] = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$ is true.

3. We can obtain the graph of $y = x^2 + 6x + 11$ by shifting the graph of $y = x^2$ 3 units to the left and then 2 units upward.

4. No point on the graph of $x^2 - 3xy + y^2 = 1$ has a horizontal tangent line.

5. If f is even then f' is odd, and if f is odd then f' is even.

6. If $f(x)$ and $g(x)$ are discontinuous at $x = c$ then $f(x) + g(x)$ is discontinuous at $x = c$.

7. There exists a differentiable function f such that $f(-2) = -2$, $f(2) = 6$, and $f'(x) < 1$ for all x.

8. Let $f(x)$ be a real-valued function on $\mathbf{R}$ and $c \in \mathbf{R}$. If $\lim_{x\to c}[f(x) - f(c)] = 0$, then the limit $\lim_{x\to c} \dfrac{f(x) - f(c)}{x - c}$ exists.

9. Let $g(x)$ be a real-valued function on $\mathbf{R}$ and $g'(c) < 0$ for some c in $\mathbf{R}$. Then there exists $\delta > 0$ such that $g(x) < g(c)$ for all $x \in (c, c + \delta)$.

(下頁還有試題)

10. Let $f(x)$ be continuous on $[a,b]$ and differentiable on (a,b). If $f(a)=f(b)$, then there is one number c in (a,b) such that $f(c)$ is a relative extremum of f on $[a,b]$.

填充題 **Short answer questions** (*40* points)，每題 **5** 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find $\lim_{x\to 0^+}\left[\left(\cos\frac{1}{x}\right)(\tan x)\right]$. Answer：_____.

2. Find the constant a such that the function is continuous on the entire real number line.

$$g(x)=\begin{cases}\dfrac{\sin(4x)}{x} & \text{, if } x<0\\ a-2x & \text{, if } x\geq 0\end{cases}.$$

Answer：_____.

3. Find the second derivative of the function $y=\sec x$.

Answer：_____.

4. Find the vertical asymptotes (if any) of the graph of the function

$$g(x)=\frac{x^3-8}{x-2}.$$

Answer：_____.

5. Suppose that f is a differentiable function such that $f(g(x))=x$, and $f'(x)=1+(f(x))^2$. Find $g'(x)$. Answer：_____.

6. If $f(x)=\dfrac{2}{x-1}$ and $g(x)=\dfrac{3}{x}$, then find the domain of the composite function $f\circ g$. Answer：_____.

7. Let $g(x)=\left[\cos x^2\right]^{1/3}(x-2)$ on $\mathbf{R}$. What is the value $g'(0)$?

Answer：_____.

8. Let $f(x)=x^4-2x^2$ on $[-2,2]$. What is the minimum of f on $[-2,2]$?

Answer：_____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find d^2y/dx^2 implicitly in terms of x and y.

$$xy - 1 = 2x + y^2.$$

2. (*10* points) Find the limit L. Then use the ϵ-δ definition to prove that the limit is L.

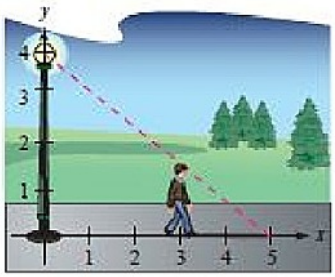
$$\lim_{x \to 3} \frac{1}{x}$$

3. (*10* points) Use the Intermediate Value Theorem and Rolle's Theorem to prove that the equation $2x - 2 - \cos x = 0$ has exactly one real solution.

4. (*10* points) A man 165 cm tall walks at a rate of 120 cm per second away from a light that is 4 meters above the ground (see figure).

a. When he is 3 meters from the base of the light, at what rate is the tip of his shadow moving?

b. When he is 3 meters from the base of the light, at what rate is the length of his shadow changing?



5. (*10* points) Define

$$f(x) = \begin{cases} x^3 \sin \dfrac{1}{x} & , \text{if } x \neq 0 \\ 0 & , \text{if } x = 0 \end{cases}.$$

a. For $x \neq 0$, find $f'(x)$.

b. Show that $f'(0) = 0$.

c. Show that $f'(x)$ is continuous for all real numbers x. (Hint: Use **a.** and **b.**.)

6. (*10* points) Prove or disprove that $\lim_{x \to 0} \dfrac{\sin x}{|x|}$ exists.

(試題結束)

微積分聯合教學　會考二　**Nov 28, 2017**

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. $\lim_{x\to\infty} x\sin\dfrac{1}{x}$ does not exist.

2. $\displaystyle\int \frac{1}{x}dx = \ln|Cx|, C \neq 0.$

3. Let $f(x)$ be a continuous function on $[a,b]$ and $F(x) = \displaystyle\int_a^x f(t)\,dt$ for $x \in [a,b]$. Then $F(x)$ is differentiable on (a,b).

4. If $f''(x) = 0$, then $(c, f(c))$ is a point of inflection of the graph of f.

5. $\displaystyle\lim_{h\to 0}\frac{1}{h}\int_x^{x+h}\sqrt{1+t^2}\,dt = \sqrt{1+x^2}.$

6. $\sin^{-1}(\sin\dfrac{3\pi}{4}) = \dfrac{3\pi}{4}.$

7. If $f'(x) > 0$ for all real numbers x, then f increases to infinity.

8. There exists no function f such that $f = f^{-1}$.

9. If the norm of a partition approaches zero, then the number of subintervals approaches infinity. That means $\|\triangle\| \to 0 \Rightarrow n \to \infty$.

10. If f is continuous on $[a,b]$, then f is integrable on $[a,b]$.

填充題 **Short answer questions** (*40* points)，每題 **5** 分。
(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

(下頁還有試題)

1. Find $\dfrac{d}{dx}\displaystyle\int_0^{x^2} \sin\theta^2 \, d\theta$. Answer：_____.

2. Use implicit differentiation to find dy/dx.

$$4y^2 + \ln x^2 y = 7$$

Answer：_____.

3. Evaluate the definite integral.

$$\int_0^4 |x^2 - 9| \, dx$$

Answer：_____.

4. Evaluate the definite integral.

$$\int_{-1}^{1} x\left(e^{-x} + e^{x}\right) dx$$

Answer：_____.

5. Let $f(x) = \displaystyle\int_2^x \frac{dt}{\sqrt{1+t^4}}$. Find $(f^{-1})'(0)$. Answer：_____.

6. Find an equation of the tangent line to the graph of the function at the given point.

$$y = (\ln x)^{\cos x}, \quad (e, 1).$$

Answer：_____.

7. Evaluate the integral.

$$\int \tan^3 x \, dx$$

Answer：_____.

8. Find $\lim\limits_{n\to\infty} \dfrac{1}{n^3}\left[1^2 + 2^2 + 3^2 + ... + n^2\right]$. Answer：_____.

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

(下頁還有試題)

1. (*10* points) A rectangular page is to contain 36 square inches of print. The margins on each side are $1\frac{1}{2}$ inches. Find the dimensions of the page such that the least amount of paper is used.

2. (*10* points) Use differentials to approximate $\sqrt[3]{26}$.

3. (*10* points) Use the limit process to find the area of the region bounded by the graph of the function and the x-axis over the given interval.

$$y = 27 - x^3, \quad [1, 3].$$

4. (*10* points) **a.** Find $F'(x)$.

$$F(x) = \int_1^{e^{\arctan(3x)}} \ln t \, dt$$

b. Find $\displaystyle\int \frac{dx}{\sqrt{e^{2x} - 1}}$.

5. (*10* points) Let $f(x) = \dfrac{x^3 - 1}{x^3 + 1}$. Find **a.** its domain, **b.** critical numbers, **c.** intervals of increasing/decreasing, **d.** relative(local) maximum/minimum values, **e.** intervals of concavity, **f.** inflection points, **g.** slant(oblique), horizontal and vertical asymptotes. **h.** Then sketch the graph of $f(x)$.

6. (*10* points) Let f be a differentiable function on $(0, \infty)$ and g be a differentiable function on $(-\infty, \infty)$. Suppose that $f : (0, \infty) \to \mathbf{R}$ and $g : \mathbf{R} \to (0, \infty)$ are inverse functions of each other and $f(ab) = f(a) + f(b)$ for all $a, b > 0$.

a. Show that $f'(x) = \dfrac{f'(1)}{x}$ for all $x > 0$.

b. Show that $f'(1)g'(0) = 1$.

Hint:

1. To show **a.**, you can differentiate both sides of the equation $f(cx) = f(c) + f(x)$ with respect to x.

2. To show **b.**, you can show that $f(1) = 0$ and $g(0) = 1$. Then apply the chain rule to the definition of the inverse function.

(試題結束)

微積分聯合教學　會考三　Jan 9, 2018

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

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(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. Let $f(x)$ be a real-valued function on $\mathbf{R}$ and $c \in \mathbf{R}$. If $\lim_{x\to c}[f(x)-f(c)]=0$, then the limit $\lim_{x\to c}\dfrac{f(x)-f(c)}{x-c}$ exists.

2. Let $f(x)$ be continuous on $[a,b]$ and differentiable on (a,b). If $f(a)=f(b)$, then there is one number c in (a,b) such that $f(c)$ is a relative extremum of f on $[a,b]$.

3. $\sin^{-1}(\sin\dfrac{3\pi}{4})=\dfrac{3\pi}{4}$.

4. $\lim_{x\to 0}\dfrac{e^{2x}-1}{e^x}=\lim_{x\to 0}\dfrac{2e^{2x}}{e^x}=\lim_{x\to 0}2e^x=2$.

5. Let the point masses $m_1=7, m_2=3, m_3=5$ be located at $x_1=-5, x_2=0, x_3=3$ on the x-axis. Then the center of mass of the linear system is $\bar{x}=-\frac{4}{3}$.

6. If f is continuous on $(-\infty,\infty)$, then $\displaystyle\int_{-\infty}^{\infty}f(x)\,dx=\lim_{t\to\infty}\int_{-t}^{t}f(x)\,dx$.

7. $\displaystyle\int_0^2\frac{x}{x^2-1}\,dx=\frac{1}{2}\ln 3$.

8. $\displaystyle\int_0^1\frac{1}{x^{\sqrt{2}}}\,dx$ diverges.

9. $\displaystyle\int_1^{\infty}\frac{1}{x}\sqrt{1+\frac{1}{x^4}}\,dx$ is convergent.

10. $\lim_{x\to\infty}\dfrac{x^3+x^2+x+1}{e^x}$.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 **5** 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Suppose that f is a differentiable function such that $f(g(x)) = x$, and $f'(x) = 1 + (f(x))^2$. Find $g'(x)$.

Answer : _____.

2. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ about the x-axis.

Answer : _____.

3. Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \sin^3 x \cos^4 x \, dx$.

Answer : _____.

4. Find $\displaystyle\lim_{x \to 1^+} (\ln x)^{x-1}$.

Answer : _____.

5. Evaluate $\displaystyle\int_0^1 \frac{x+3}{\sqrt{4-x^2}} \, dx$.

Answer : _____.

6. Evaluate $\displaystyle\lim_{x \to 1^+} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$.

Answer : _____.

7. Find

$$\int_0^{\frac{\pi}{4}} \sin\theta \sin 3\theta \, d\theta.$$

Answer : _____.

8. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2 + 1$, $y = 0$, $x = 0$ and $x = 1$ about the y-axis.

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。 **請詳列計算過程**，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find an equation of the tangent line to the graph of the function

$$y = (\ln x)^{\cos x}$$

at the point $(e, 1)$.

2. (*10* points)

a. Find the indefinite integral

$$\int \tan^3 2t \sec^3 2t \, dt.$$

b. Evaluate the definite integral

$$\int_0^2 \frac{3}{4x^2 + 5x + 1} \, dx.$$

3. (*10* points) Evaluate

$$\int x^2 \sin x \, dx.$$

4. (*10* points) Let $f(x) = \sqrt{9 - x^2}$, $0 \le x \le 3$.

a. Use the formula for arc length to find the length of the graph of f.

b. Use the formula for surface area of revolution to find the area of the surface generated by revolving the curve $y = f(x)$ about the x-axis.

5. (*10* points) Evaluate $\displaystyle\int_{-2}^{-\sqrt{3}} \frac{\sqrt{x^2 - 3}}{x} \, dx$.

6. (*10* points) Prove that the line $y = (1 - \sqrt[3]{0.5})x$ divides the region bounded by the graph of $y = x(1 - x)$ and the x-axis into two regions of equal area.

(試題結束)