Date: / Discrete Mathematics Homework 5

§ $\boxed{9\text{-}6}$

12 reflexive : $aRa \rightarrow aR^{-1}a$

anti-symmetric : $aRb,\ bRa \rightarrow a=b$

$bR^{-1}a,\ aR^{-1}b \rightarrow a=b$

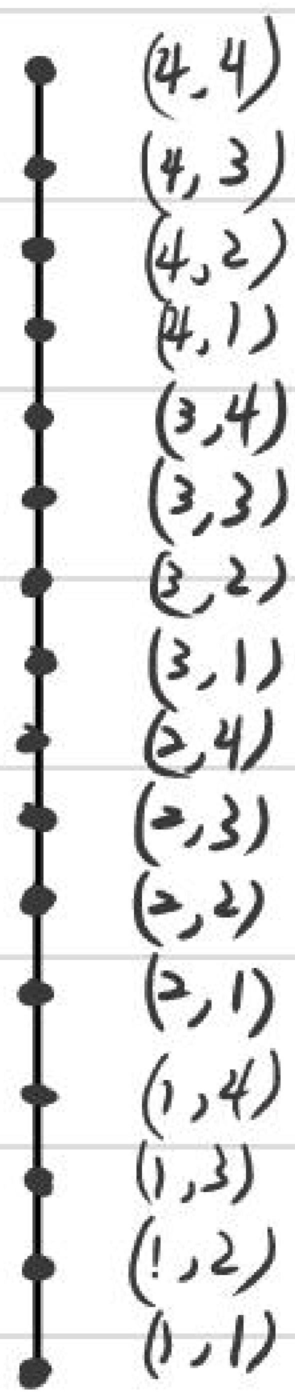
transitive : $aRb,\ bRc \rightarrow aRc$

$cR^{-1}b,\ bR^{-1}a \rightarrow cR^{-1}a$

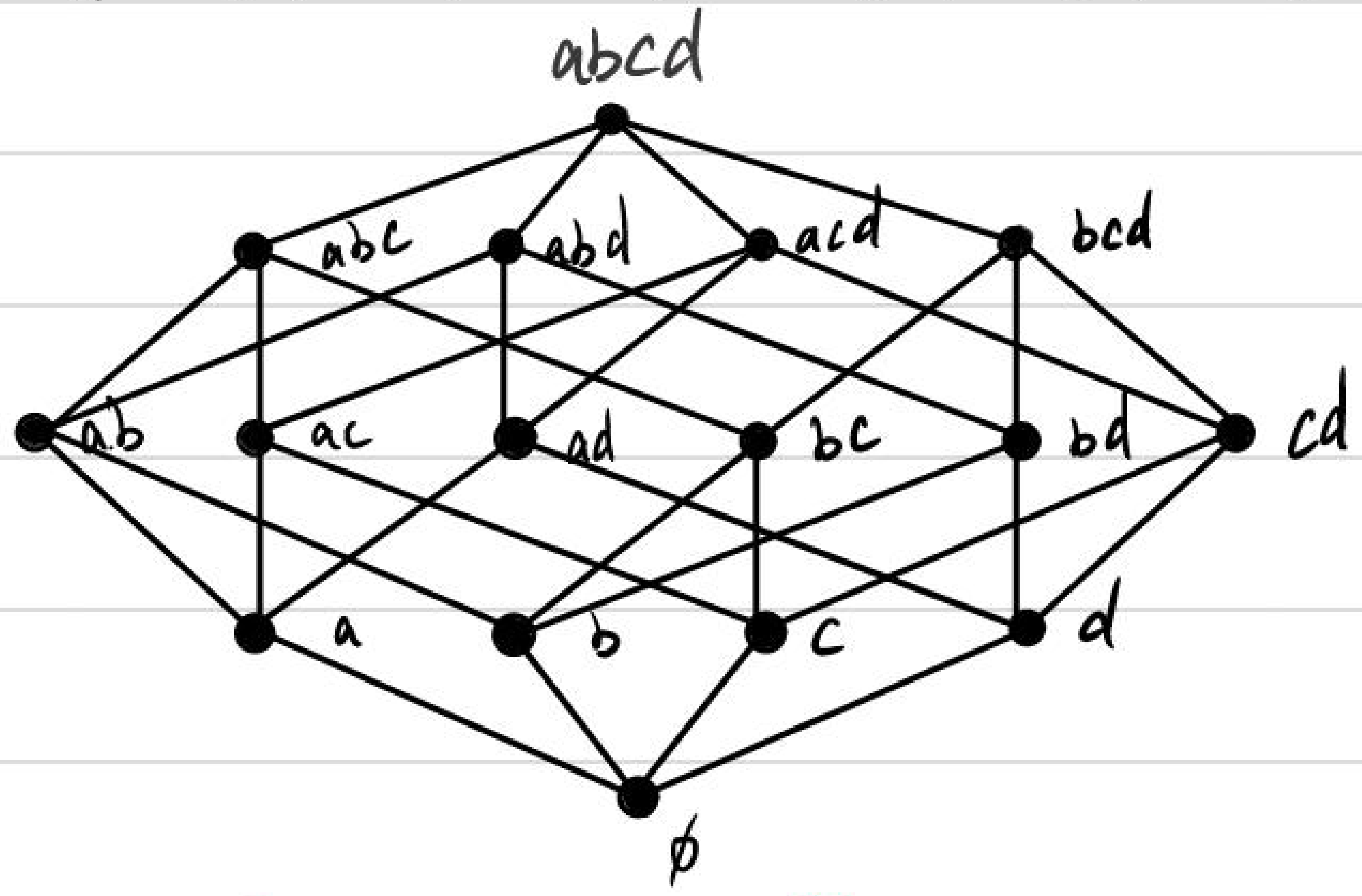
16 a $\{(1,1),(1,2),(1,3),(1,4),(2,2)\}$

b $\{(4,1),(4,2),(4,3),(4,4),(3,2),(3,3),(3,4)\}$

c



24



32 a l, m b a, b, c c d no, there's not e k, l, m

f k g h no lower bounds

36 a $(\mathbb{N}, \leq)$ b $(\mathbb{N}, \geq)$ c $(\mathbb{R}, \geq)$

44 a no, $(6, 9)$ have no upper bound

b yes c yes (both of b and c are linear)

d yes, g.l.b of A, B is $A \cup B$, and l.u.b is $A \cap B$

Date: /

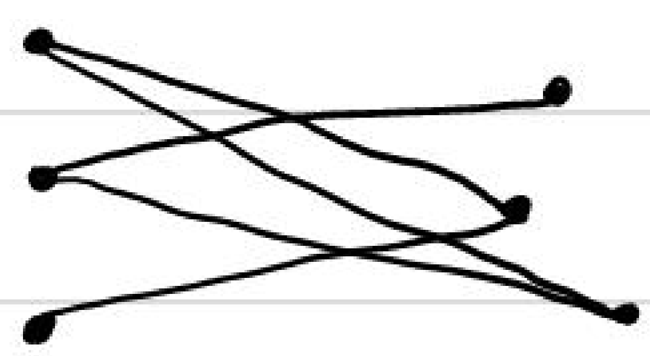
§ 10-1

(12) 1. G is a graph with a loop at every vertex, so uRu.

2. An edge indicates R, so uRv → vRu, thus symmetric.

(26) If you find a vertex with so many edegs going out from it, that should probably be that electronic mail mailing list.

(30)

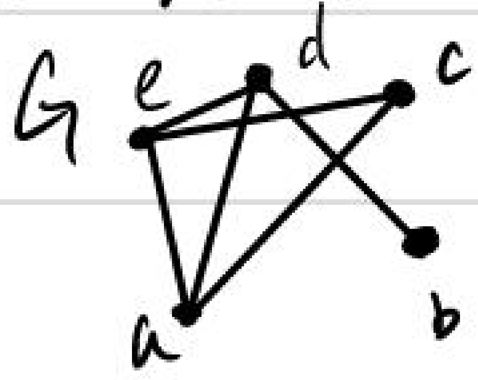
critics  movies

An edge means that a critic positively recommanded a movie.

§ 10-2

(6) S: The set of people at a party, R: relation of shaken hands

The relation is symmetric be cause if aRb, then bRa surely.



In graph G, each vertex represent a person, and each edge represents R. The degree of a vertex represents the number of people a person has shaken hands with.

The sum of the degrees of all the vertices will be even.

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

If we use an ajacency marix to represent such a symmetric relation, it will be a symmetric matrix of course. So the sum of it's 1s will be even.

(12) The degree represents how many people a person know.

The neighborhood represents people acquantant to each other.

Isolated vertices represents people that no one knows. They may be dead people. Pendant vertices represents people that only one

Date: /

of others knows. The study indicates that a person knows approximately 1,000 people in the world.

18 If an edge e connecting vertices a,b appears, that is, $\deg(a) += 1 \rightarrow \deg(b) += 1$. Same as exercise 6, an edge represents a symmetric relation.

§ $\boxed{10\text{-}3}$

28 Undirected Graph: The sum in the i^{th} row is the same as the corresponding column sum.

Directed Graph: The sum is the out degree of i.

36 Not, $\deg(v_2)=4$, but none of u_i is the same.

40 Not, $\deg(v_0)=4$, but none of u_i is the same.

46 An edge is in $\overline{G}$ $\Leftrightarrow$ it's not in G iff it's corresponding edge is in $\overline{H}$ $\Leftrightarrow$ it's not in H.

§ $\boxed{10\text{-}4}$

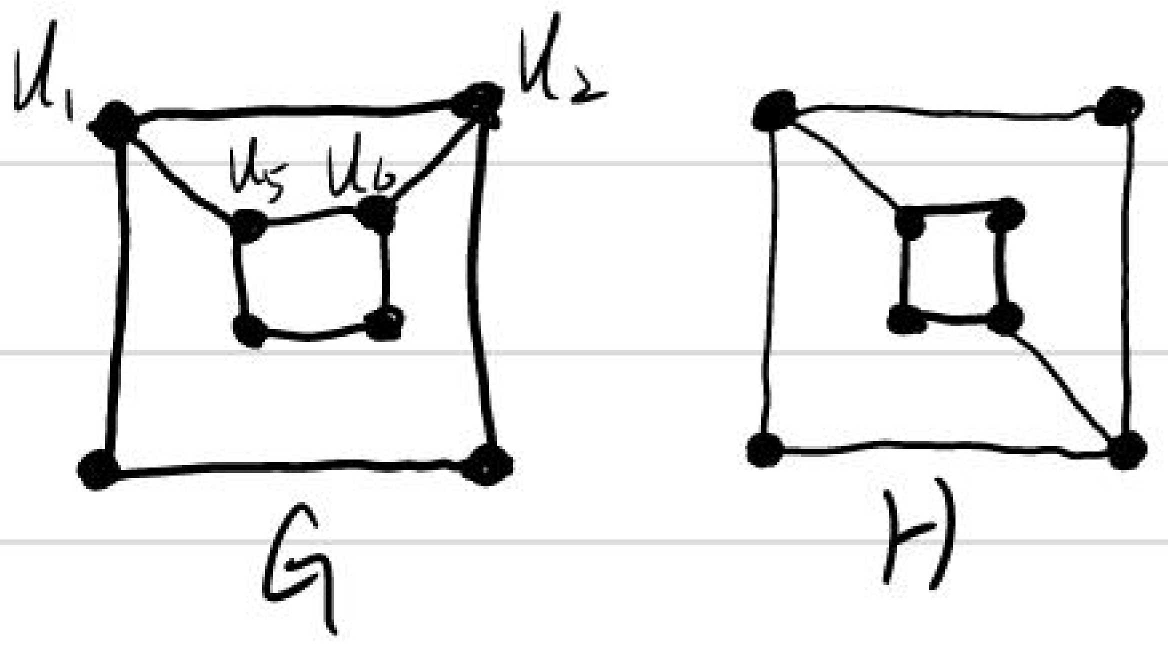
12 (a) Weakly connected (b) Strongly connected

(c) Neithe strongly nor weakly connected.

14 (a) $\{a,b,e\}, \{d\}, \{c\}$ (b) $\{a\}, \{b\}, \{c,d,e\}, \{f\}$

(c) $\{a,b,c,d,f,g,h,i\}, \{e\}$

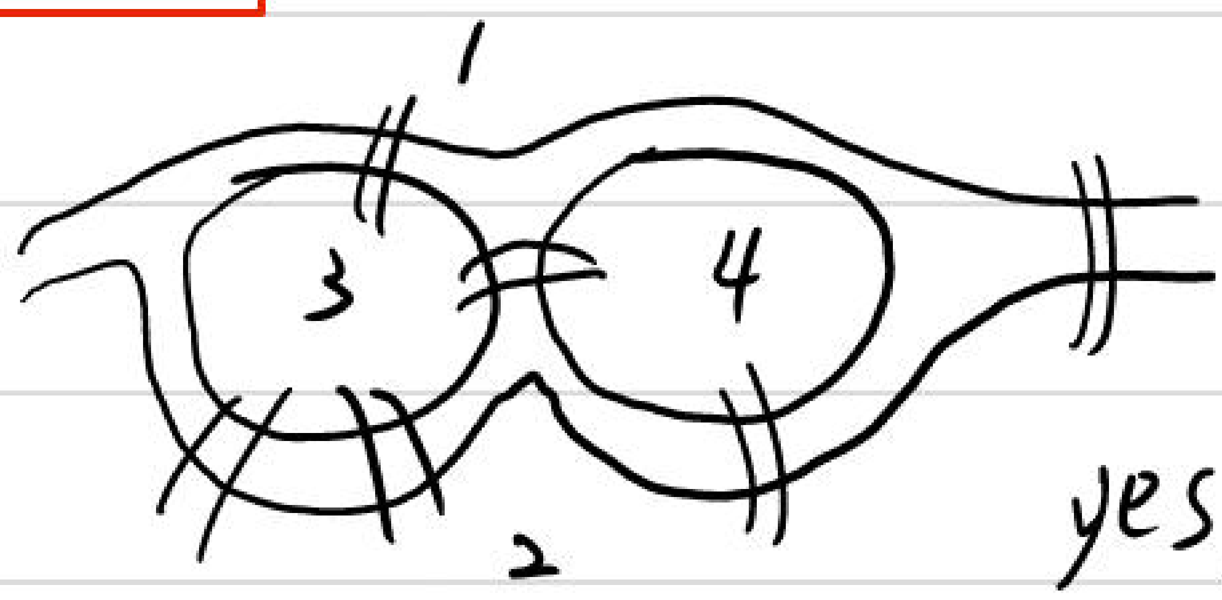
18

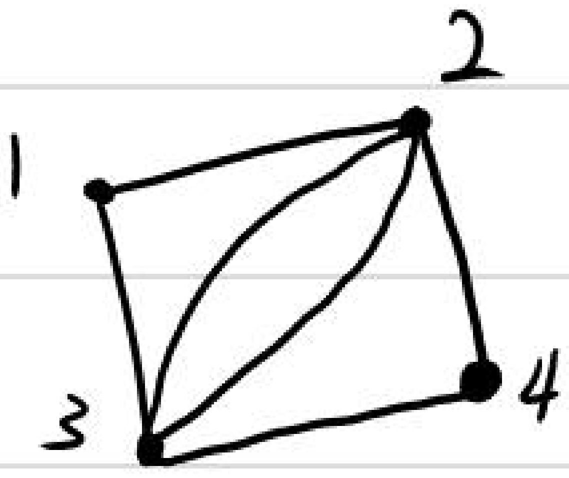
20 

G has a simple closed path containing exactly the vertices of degree 3, while H has not. Thus they're not isophormic.

Date: 32/ c, d

§ 10-5

10 



yes, 1243231

14 yes

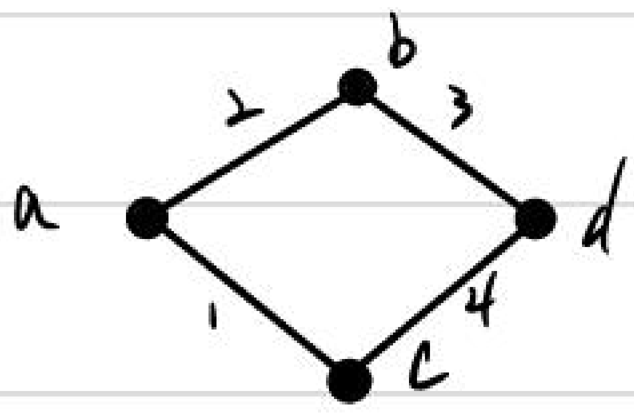
20 a, d, b, d, e, b, e, c, b, a

30 The graph doesn't have a Hamilton circuit.
Because in order to get to the starting point, both c and f must be traveled at least twice, since the cut edge {c, f}

46

§ 10-6

6 a 6 b 11 c 8 d 15

18 

No, counterexample

26

a b c d e a : 27	a c b d e a : 35
a b c e d a : 23	a c b e d a : 25
a b d c e a : 30	a c d b e a : 32
a b d e c a : 26	a c e b d a : 28
a b e c d a : 20	a d b c e a : 35
a b e d c a : 20	a d c b e a : 29

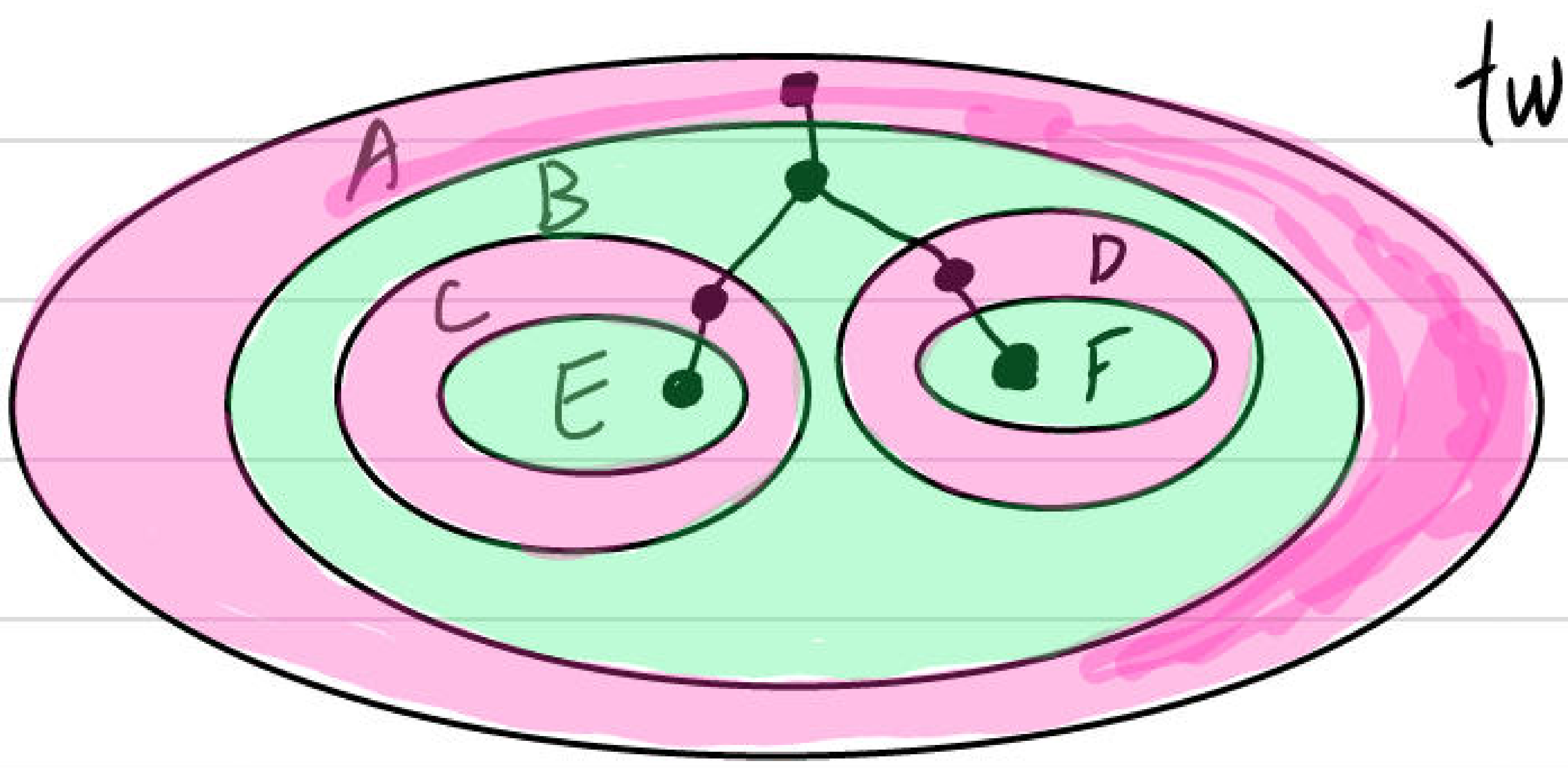
Date: / § 10-7

12 $v-e+r=2$, $v=8$, $e=8\times 3\div 2=12 \Rightarrow r=6$

18 $v-(e+k-1)+r=2 \Rightarrow r=e-v+k+1$

§ 10-8

4



two colors will be enough.

6 In each triangle, evere vertices must have different color. Therefore, the chromatic number is 3.

16 If we color the first vertex with color A, then color it's incidential vertices with color B, and so on. We will find the last two vertices connected to each other, since the graph is a circuit and has odd number of vertices.

24 If there's a vertex with degree D, then there are D edges incident with a common vertex. Thus, in any edge coloring each of those edges must have different color, so we need at least D colors.