微積分聯合教學　會考一　Oct 15, 2019

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 110 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. $\lim_{x\to c}[f(x)g(x)] = \lim_{x\to c} f(x)\cdot\lim_{x\to c} g(x)$ is true.

2. $\lim_{x\to 2^-}\dfrac{x-3}{x^2-4} = \infty$.

3. If $f(x)=\sqrt{x}$ and $g(x)=\sqrt{3-x}$, then the domain of composite function $g\circ f$ is $[0,9]$.

4. If $\lim_{x\to 0}\dfrac{f(x)}{x} = 1$ then $\lim_{x\to 0} f(x) = 0$.

5. If $\lim_{x\to a} f(x) = 0$, there must be a number c such that $|f(c)| < 10^{-5}$.

6. Since $\sec 0 = 1$ and $\sec\pi = -1$, so by the **Intermediate Value Theroem**, there is a number $x_0 \in (0,\pi)$ such that $\sec x_0 = 0$.

7. The graph of the function $f(x) = \dfrac{\sqrt{x^2+1}}{x+1}$ has two horizontal asymptotes.

8. The graph of $y = \tan 4x$ is a horizontal stretch of the graph of $y = \tan x$ by a factor of 4.

9. The quotient

$$\frac{f(x+h)-f(x-h)}{2h}$$

may have a limit as $h\to 0$ when f has no derivative at x.

10. If $f(x)$ is an even function, then $f'(0) = 0$.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 **5** 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the limit.

$$\lim_{x\to\infty}(\sqrt{x^2+3x}-\sqrt{x^2-2x}).$$

Answer : _____.

2. Find the limit.

$$\lim_{t\to 0}\frac{\tan t\sec 2t}{3t}$$

Answer : _____.

3. Find an equation of the tangent line to the curve

$$y=\frac{\pi\sin x}{x}$$

at $x=\pi$. Answer : _____.

4. At which points the function $f(x)=|x^2-3x|$ is continuous but not differentiable? Answer : _____.

5. How many asymptotes are there in the graph of $f(x)=\dfrac{8}{x^2-16}$?

Answer : _____.

6. Find the limit.

$$\lim_{x\to 0^-} x\left\lfloor\frac{1}{x}\right\rfloor$$

Answer : _____.

7. Let $f(u)=1-\dfrac{1}{u}$ and $u=g(x)=\dfrac{1}{1-x}$. Find the value of $(f\circ g)'$ at $x=-1$.

Answer : _____.

8. Let

$$f(x)=\frac{x^2+x-6}{x^2-4},\quad x\neq 2.$$

Let $F(x)$ be the continuous extension of $f(x)$ to $x=2$. Find $F(2)$.

Answer : _____.

(下頁還有試題)

計算問答證明題　**Please show all your work** (*50* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。　**請詳列計算過程**，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Use implicit differentiation to find the value of dy/dx at the point $(0, 0)$ if $\sin(x + y) = y^2 \cos x$.

2. (*10* points)

a. Find $\dfrac{dy}{dt}$, if $y = \left(1 + \tan^4\left(\dfrac{t}{12}\right)\right)^3$.

b. Find y'' if $y = \tan x$.

3. (*10* points) Let

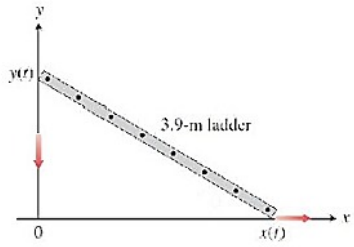
$$f(x) = \begin{cases} x^2 \sin\dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}.$$

Find $f'(0)$.

4. (*10* points) A 3.9-m ladder is leaning against a house when its base starts to slide away (see accompanying figure). By the time the base is 3.6 m from the house, the base is moving at the rate of 1.5 m/s.

a. How fast is the top of the ladder sliding down the wall then?

b. At what rate is the area of the triangle formed by the ladder, wall, and ground changing then?



5. (*10* points) Let $f(x) = \begin{cases} a^2x - 2a, & x \geq 2 \\ 12, & x < 2 \end{cases}$.

a.) For what values of a will $f(x)$ be continuous for all values of x?

Given reasons for your answer.

b.) For what values of a will $f(x)$ be differentiable for all values of x?

Given reasons for your answer.

(試題結束)

微積分聯合教學　會考二　Nov 26, 2019

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 110 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. The definite integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\sqrt{1+x^2}\,dx = 0$.

2. Let f be a continuous function on a closed interval. If f has a local minimum at $x = c$ then $f'(c) = 0$.

3. For any $x, y \in (-\pi, \pi)$, we have $\left|\tan\frac{x}{2} - \tan\frac{y}{2}\right| \geq \frac{|x-y|}{2}$.

4. Let $y = f(x)$ be continuous on the closed interval $[a, b]$, then there exists a number $c \in [a, b]$ such that $\int_a^b f(x)dx = f(c)(b-a)$.

5. If $y = f(x)$, f is increasing and differentiable, and $\Delta x > 0$, then $\Delta y \geq dy$.

6. Suppose $\int_2^{-2} f(x)dx = -4$, $\int_2^5 f(x)dx = 3$, $\int_{-2}^5 g(x)dx = 2$. Then $f(x) \leq g(x)$ on the interval $-2 \leq x \leq 5$.

7. If f is continuous on $[0, 2]$ such that $\int_0^{\frac{1}{2}} f(2x)\,dx = -\frac{1}{2}$ and $\int_1^2 f(x)\,dx = 1$, then $f(c) = 0$ for some point c in [0,2].

8. The function $f(x) = |x^3 - 9x|$ have 5 critical points.

9. If $y = f(x)$ is continuously differentiable and $f(0) = 0$, then

$$\int_0^\alpha \sqrt{1 + [f'(t)]^2}\,dt \geq \sqrt{\alpha^2 + [f(\alpha)]^2}$$

for $\alpha > 0$.

(下頁還有試題)

10. Suppose that the first derivative of $y = f(x)$ is $y' = 6(x+1)(x-2)^2$, there is no inflection points.

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n-1} \frac{\sqrt{n^2-k^2}}{n^2}.$$

Answer : _____.

2. Find the area of the region enclosed by the curves $x + (y-1)^2 = 0$ and $x + 3(y-1)^2 = 1$. Answer : _____.

3. Evaluate $\int \dfrac{\csc\theta}{\csc\theta - \sin\theta}\, d\theta$. Answer : _____.

4. A solid is generated by revolving about the x-axis the region bounded by the graph of the positive continuous function $y = f(x)$, the x-axis, the fixed line $x = a$, and the variable line $x = b$, $b > a$. Its volume, for all b, is $b^2 - ab$. Find $f(x)$. Answer : _____.

5. Find the mass of a thin plate covering the region bounded above by the parabola $y = 4 - x^2$ and below by the x-axis. Assume the density of the plate at the point (x, y) is $\delta = 2x^2$, which is twice the square of the distance from the point to the y-axis. Answer : _____.

6. What values of a and b with $a < b$ maximize the value of $\int_a^b (x - x^2)\, dx$. Answer : _____.

7. Find the length of the curve $x^2 - 4y^3 = 0$ from $x = 0$ to $x = 2$.

Answer : _____.

8. If f is a continuous function, find the value of the following integral

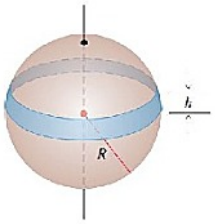
$$I = \int_0^4 \frac{f(4-x)}{f(x) + f(4-x)}\, dx.$$

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (*50* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。 **請詳列計算過程**，否則不予計分。需標明題號但不必抄題。

1. (*10* points) The shaded band shown here is cut from a sphere of radius R by parallel planes h units apart. Show that the surface area of the band is $2\pi Rh$.



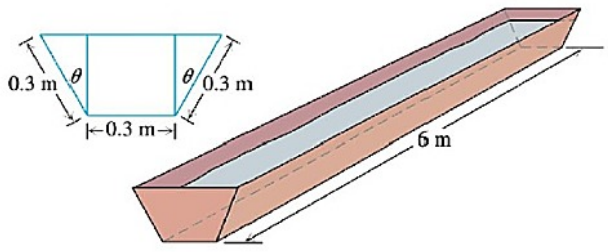
2. (*10* points) Find the linearization of

$$f(x) = 2 - \int_2^{x^2+1} \frac{9}{1+t}dt$$

at $x = 1$.

3. (*10* points) Let $f(x) = \dfrac{2x^2 + x - 1}{x^2 - 1}$. Find **a.** its domain, **b.** critical points, **c.** intervals of increasing/decreasing, **d.** local(relative) maximum/minimum values, **e.** intervals of concavity, **f.** inflection points, **g.** slant(oblique), horizontal and vertical asymptotes. **h.** Then sketch the graph of $f(x)$.

4. (*10* points) The trough (水槽) in the figure is to be made to the dimensions shown. Only the angle θ can be varied. What value of θ will maximize the trough's volume?



5. (*10* points) Find the set of values for k such that $x^3 - 12x - 4 = k$ has more than two (i.e. > 2) distinct real roots.

(試題結束)

微積分聯合教學　會考三　Jan 7, 2020

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 110 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題　**True or False** (*20* points)，請答　**T** (True) 或　**F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. The graph of the function $f(x) = \dfrac{\sqrt{x^2+1}}{x+1}$ has two horizontal asymptotes.

2. The quotient

$$\frac{f(x+h) - f(x-h)}{2h}$$

may have a limit as $h \to 0$ when f has no derivative at x.

3. For any $x, y \in (-\pi, \pi)$, we have $\left|\tan\dfrac{x}{2} - \tan\dfrac{y}{2}\right| \geq \dfrac{|x-y|}{2}$.

4. Let $P(x)/Q(x)$ be a proper rational function where $Q(x)$ factors as a product of distinct linear factors $(x - a_i)$. Then $\displaystyle\int \frac{P(x)}{Q(x)}dx$ may be contain a term involving the arctangent.

5. $\displaystyle\int_1^\infty \frac{dx}{x^p}$ converges for $p < 1$.

6. $\displaystyle\lim_{x\to 1} \frac{x^2+1}{2x+1} = \lim_{x\to 1} \frac{2x}{2} = 1$.

7. If $f(0) = f(1) = 0$ and f'' is continuous on $[0, 1]$ then

$$\int_0^1 f''(x)f(x)dx = -\int_0^1 f'(x)^2 dx.$$

8. If f grows at the same rate as g as $x \to \infty$, and g grows at the same rate as h as $x \to \infty$, then f grows at the same rate as h as $x \to \infty$.

(下頁還有試題)

9. If f is increasing and concave down, then its inverse function is concave down.

10. $\int_1^\infty \frac{1}{x}\sqrt{1+\frac{1}{x}}\,dx$ is convergent.

填充題　**Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let

$$f(x)=\begin{cases} x^2\sin\dfrac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}.$$

Find $f'(0)$. Answer：_____.

2. Evaluate the integral $\int_0^{\pi/4}\tan^3 x dx$. Answer：_____.

3. Find the value $\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$. Answer：_____.

4. Evaluate $\int_0^{\pi}\sin 3x\cos 5x\,dx$. Answer：_____.

公式列表如下，請參考：

a. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$.

b. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$.

5. Let $f(x)=\int_2^x \frac{dt}{\sqrt{1+t^4}}$. Then find $(f^{-1})'(0)$.

Answer：_____.

6. Assuming $N(x)$ is a polynomial with the degree of $N(x)<7$, write the following rational function in the form of partial fraction decomposition.

$$\frac{N(x)}{(x-4)^3(x^2+12)^2}=\frac{A}{x-4}+\cdots$$

Do NOT find the constants in the decomposition.

Answer：_____.

7. Evaluate $\lim_{x\to 0^+}(1+\sin x)^{1/x}$.

Answer：_____.

(下頁還有試題)

8. Which of the following is not true?

a. $x + \ln x = O(x)$ **b.** $x + \ln x = o(x)$

c. $\ln x = O(x)$ **d.** $\ln x = o(x)$

Answer : _____.

計算問答證明題 **Please show all your work** (*50* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。 **請詳列計算過程**，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find the linearization of

$$f(x) = 2 - \int_2^{x^2+1} \frac{9}{1+t} dt$$

at $x = 1$.

2. (*10* points) Use integration, the Direct Comparison Test, or the Limit Comparison Test to test the integrals for convergence.

a. $\displaystyle\int_1^{\infty} \frac{1 - \sin x}{x^2}\, dx$

b. $\displaystyle\int_2^{\infty} \frac{dv}{\sqrt{v-1}}$

3. (*10* points) Evaluate the integrals.

a. $\displaystyle\int_{-2}^{2} \frac{dx}{4 + x^2}$

b. $\displaystyle\int_1^2 x \ln x\, dx$

4. (*10* points) Evaluate the integrals.

a. $\displaystyle\int 5^x dx$

b. $\displaystyle\int_0^2 \frac{\log_2(x+2)}{x+2} dx$

5. (*10* points) Evaluate the integrals.

a. $\displaystyle\int_{-1}^{4} \frac{1}{\sqrt{|x|}}\, dx$

b. $\displaystyle\int_1^2 \frac{1}{x\sqrt{\ln x}}\, dx$

(試題結束)