Calculus Homework Assigment 3

Class 班: CSIE 1-B

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1. Find the limits.

a. $\displaystyle\lim_{t\to-\infty}\frac{2-t+\sin t}{t+\cos t}$

b. $\displaystyle\lim_{x\to-\infty}\frac{\sqrt[3]{x}-\sqrt[5]{x}}{\sqrt[3]{x}+\sqrt[5]{x}}$

[§2.6 #11,29]

a. $\displaystyle\lim_{t\to-\infty}\frac{2-t+\sin t}{t+\cos t} \overset{x=-t}{=} \lim_{x\to\infty}\frac{2+x-\sin x}{-x+\cos x}$

$\displaystyle = \lim_{x\to\infty}\frac{1+\frac{2}{x}-\frac{\sin x}{x}}{-1+\frac{\cos x}{x}} = \frac{1+0+0}{-1+0} = -1$

b. $\displaystyle\lim_{x\to-\infty}\frac{x^{\frac{1}{3}}-x^{\frac{1}{5}}}{x^{\frac{1}{3}}+x^{\frac{1}{5}}} = \lim_{x\to-\infty}\frac{x^5-x^3}{x^5+x^3}$

$\displaystyle = \lim_{x\to-\infty}\frac{1-\frac{1}{x^2}}{1+\frac{1}{x^2}} = 1$

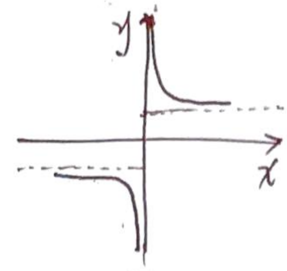
2. Determine the domain and range of the function. Use various limits to find the asymptotes and the range.

$$y=\frac{\sqrt{x^2+4}}{x}$$

[§2.6 #71]

Domain : $(-\infty,0)\cup(0,\infty)$

Range : $(-\infty,-1)\cup(1,\infty)$



$\displaystyle\lim_{x\to+\infty}\frac{\sqrt{x^2+4}}{x} = \lim_{x\to\infty}\sqrt{1+\frac{4}{x^2}} = 1$

$\displaystyle\lim_{x\to-\infty}\frac{\sqrt{x^2+4}}{x} \overset{x=-t}{=} \lim_{t\to\infty}\frac{\sqrt{t^2+4}}{-t} = -1$

$\displaystyle\lim_{x\to0^+}\frac{\sqrt{x^2+4}}{x} = +\infty,\quad \lim_{x\to0^-}\frac{\sqrt{x^2+4}}{x} = -\infty$

$y=\frac{\sqrt{x^2+4}}{x}$ has 3 asymptotes : $\begin{cases} y=1 \text{ (hori.)} \\ y=-1 \text{ (hori.)} \\ x=0 \text{ (vert.)}\end{cases}$

3. Find the slope of the function's graph at the given point. Then find an equation for the line tangent to the graph there.

$$g(x)=\frac{x}{x-2},\quad (3,3)$$

[§3.1 #13]

$\displaystyle\lim_{h\to0}\frac{g(3+h)-g(3)}{h} = \lim_{h\to0}\frac{\left(\frac{h+3}{h+1}\right)-3}{h}$

$\displaystyle = \lim_{h\to0}\frac{-2h}{h(h+1)} = -2$ (slope).

$y-3=-2(x-3)$

$\Rightarrow$ $y=-2x+9$ (equation)

4. Does the graph of

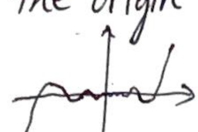
$$f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right), & x\neq0 \\ 0, & x=0\end{cases}$$

have a tangent at the origin? Give reasons for your answer.

[§3.1 #35]

$\displaystyle\lim_{h\to0}\frac{f(h)-f(0)}{h} = \lim_{h\to0} h\cdot\sin\frac{1}{h} = 0$

Since $f(x)$ is differentiable at the origin, it has a tangent at the origin ($y=0$).



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5. Using the definition, calculate the derivatives of the function. Then find the values of the derivatives as specified.

$$g(t) = \frac{1}{t^2};\quad g'(-1),\ g'(2),\ g'(\sqrt{3})$$

[§3.2 #3]

$g(t) = t^{-2}$

$g'(t) = -2t^{-3} = \frac{-2}{t^3}$

$g'(-1) = \frac{-2}{(-1)^3} = \underline{\underline{2}}$

$g'(2) = \frac{-2}{(2)^3} = \underline{\underline{-\frac{1}{4}}}$

$g'(\sqrt{3}) = \frac{-2}{3\sqrt{3}} = \underline{\underline{\frac{-2\sqrt{3}}{9}}}$

6. Determine if the piecewise-defined function is differentiable at the origin.

$$f(x) = \begin{cases} 2x + \tan x, & x \geq 0 \\ x^2, & x < 0 \end{cases}$$

[§3.2 #43]

$\lim_{h\to 0^+} \frac{f(h)-f(0)}{h} = \lim_{h\to 0^+} \frac{2h+\tan h}{h} = \lim_{h\to 0^+} \left(2+\frac{\tan h}{h}\right) = 3$

$\lim_{h\to 0^-} \frac{f(h)-f(0)}{h} = \lim_{h\to 0^-} x^2 = 0$

$\lim_{h\to 0} \frac{f(h)-f(0)}{h}$ D.N.E.

so $f(x)$ is undifferentiable at the origin.

7. Find the derivatives of

$$v = (1-t)(1+t^2)^{-1}.$$

[§3.3 #21]

$v = \frac{1-t}{1+t^2}$

$\frac{dv}{dt} = \frac{-1\cdot(1+t^2)-(1-t)\cdot(2t)}{1+2t^2+t^4}$

$= \frac{-1-t^2-2t+2t^2}{t^4+2t^2+1} = \underline{\underline{\frac{t^2-2t-1}{t^4+2t^2+1}}}$

8. Assume that functions f and g are differentiable with $f(1) = 2$, $f'(1) = -3$, $g(1) = 4$, and $g'(1) = -2$. Find the equation of the line tangent to the graph of $F(x) = f(x)g(x)$ at $x = 1$. [§3.3 #51]

1° $F(1) = f(1)\cdot g(1) = 2\times 4 = 8$

2° $F'(1) = f'(1)g(1) + f(1)g'(1) = (-3)\times 4 + 2\times(-2) = -16$

3° $y - 8 = -16(x-1)$

$\Rightarrow \underline{y = -16x + 24}$

(The end 結束)