

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. The function $f(x) = x|x|$ is not differentiable at $x = 0$.
2. If $\lim_{x \rightarrow 0} |f(x)| = 1$ then $\lim_{x \rightarrow 0} f(x) = 1$.
3. If $f(x) = x^2$ and $g(x) = \sqrt{x}$. Then $(f \circ g)(x) = x$ and hence the domain of $f \circ g$ is $(-\infty, \infty)$.
4. If f is a continuous function on $[a, b]$ and $f(a) < k < f(b)$, then there exists only one number $c \in [a, b]$ such that $f(c) = k$.
5. There exists a continuous function f such that f is not differentiable at $x = 3$.
6. If f is even then f' is odd, and if f is odd then f' is even.

7. The graph of

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

has a tangent at the origin.

8. The function $f(x) = \frac{1 - \cos x}{x}$ has a continuous extension to $x = 0$.
9. The line $x = 0$ is a vertical asymptote of the graph of the function $f(x) = \frac{\sin x}{x}$.
10. The graph of the function $f(x) = \frac{x+1}{|x|+1}$ has two horizontal asymptotes.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let $f(u) = \left(\frac{u-1}{u+1}\right)^2$ and $u = g(x) = \frac{1}{x^2} - 1$. Find the value of $(f \circ g)'$ at $x = 1$. Answer : _____.

2. Let $f(x) = \frac{1}{\sqrt{x-1}}$. Find the domain of the composite function $f \circ f$.
Answer : _____.

3. Find $\lim_{\theta \rightarrow \pi/3} \frac{\sin \theta - \frac{\sqrt{3}}{2}}{\theta - \frac{\pi}{3}}$. Answer : _____.

4. Find the linearization of $f(x) = \sqrt{1+x}$ at $x = 3$.
Answer : _____.

5. Evaluate $\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 8x}$. Answer : _____.

6. Let $y = \sec x$. Find the value of $\frac{d^2y}{dx^2}$ at the point $x = \frac{\pi}{4}$.
Answer : _____.

7. Find the limit.

$$\lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 25} - \sqrt{x^2 - 1} \right).$$

Answer : _____.

8. Find the limit.

$$\lim_{x \rightarrow \frac{\pi}{2}^-} (\sec x - \tan x).$$

Answer : _____.

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1. (10 points) Using the ϵ - δ definition of limit to prove that $\lim_{x \rightarrow 2} f(x) = 4$ if

$$f(x) = \begin{cases} x^2, & \text{if } x \neq 2 \\ 1, & \text{if } x = 2. \end{cases}$$

(下頁還有試題)

2. (10 points) Find all asymptotes (horizontal, oblique and vertical) of the graph of

$$f(x) = \frac{x^2 - 3}{2x - 4}.$$

3. (10 points) Find dy/dx of the curve

$$x \sin 2y = y \cos 2x$$

by using implicit differentiation and find the line that is tangent to the curve at the point $(\pi/4, \pi/2)$.

4. (10 points) For all values of the constants a and b for which the function

$$f(x) = \begin{cases} \frac{2\sin^2 x}{x}, & \text{if } x > 0 \\ ax + b \cos x, & \text{if } x \leq 0 \end{cases}$$

is

- a. continuous at $x = 0$.
- b. differentiable at $x = 0$.

5. (10 points) Find dy/dx .

- a. $y = \sqrt{1 + \cos(x^2)}$
- b. $y = \frac{(x+1)(x+2)}{(x-1)(x-2)}$

6. (10 points) A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 km north of the intersection and the car is 0.8 km to the east, the police determine with radar that the distance between them and the car is increasing at 30 km/h. If the cruiser is moving at 100 km/h at the instant of measurement, what is the speed of the car?

(試題結束)

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(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. Let f be a differentiable function defined on the open interval (a, b) . Suppose that $|f(x) - f(y)| \leq |x - y|$ for some distinct $x, y \in (a, b)$, then there exists a number $c \in (a, b)$ such that $|f'(c)| \leq 1$.
2. If $f'(x)$ is continuous on $[0, 1]$ and $f(0) = 0$, then $\int_0^\alpha \sqrt{1 + [f'(x)]^2} dx < \sqrt{\alpha^2 + [f(\alpha)]^2}$ for $0 \leq \alpha \leq 1$.
3. The function $f(x) = |x^3 - 9x|$ have 2 critical points.
4. Suppose that f has a positive derivative for all values of x and that $f(1) = 0$. Let $g(x) = \int_0^x f(t) dt$. Then g has a local minimum at $x = 1$.
5. Let $y = f(x)$ be integrable on the closed interval $[a, b]$, then there exists a number $c \in [a, b]$ such that $\int_a^b f(x) dx = f(c)(b - a)$.
6. Let $f(x) = \int_0^{1/x} \frac{1}{t^2 + 1} dt + \int_0^x \frac{1}{t^2 + 1} dt$. Then $f(x)$ is a constant for $x > 0$.
7. The definite integral $\int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \frac{x^2 \sin x}{1 + x^6} dx = 0$.
8. If $f(x)$ is continuous on $(-\infty, \infty)$, then

$$\int_0^x \left(\int_0^u f(t) dt \right) du = \int_0^x f(u) (x - u) du.$$

(下頁還有試題)

9. Suppose that f has a positive derivative for all values of x and that $f(1) = 0$.

Let $g(x) = \int_0^x f(t) dt$. Then the graph of g has an inflection point at $x = 1$.

10. If $f'(c) = 0$, then f has a local maximum or minimum at c .

填充題 **Short answer questions** (40 points), 每題 5 分。

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1. Find the length of the curve $y = x^{\frac{3}{2}}$ from $x = 1$ to $x = 2$.

Answer : _____.

2. Find the area of the region enclosed by the curves $x + y^2 = 1$ and $2x + y^2 = 0$.

Answer : _____.

3. Find $\frac{d}{d\theta} \int_0^{\tan \theta} \frac{1}{1-y^2} dy$. Answer : _____.

4. Suppose the function $f(x) = ax^3 + bx^2 + cx + d$ has a local maximum 1 at $x = 1$, and $(0, 0)$ is an inflection point of f . Find the value of c .

Answer : _____.

5. Find the area of the surface generated by revolving the line segment

$$ax + by = ab, \quad 0 \leq x \leq b,$$

about the x -axis. Answer : _____.

6. Use the integral to evaluate

$$\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \cdots + \frac{1}{\sqrt{n}\sqrt{n+n}} \right).$$

Answer : _____.

7. Suppose that $F(x)$ is an antiderivative of $f(x) = \frac{\sin x}{x}$, $x > 0$. Express

$$\int_1^3 \frac{\sin 2x}{x} dx$$

in terms of F . Answer : _____.

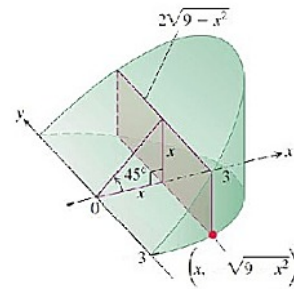
8. Find the average value of $f(x) = x - 2|x|$ on $[-1, 2]$.

Answer : _____.

(下頁還有試題)

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- (10 points) Let $f(x) = \frac{x^2 - 3}{x - 2}, x \neq 2$. Find **a.** critical numbers, **b.** intervals of increasing/decreasing, **c.** local(relative) maximum/minimum values, **d.** intervals of concavity, **e.** inflection points, **f.** slant(oblique), horizontal and vertical asymptotes. **g.** Then sketch the graph of $f(x)$.
- (10 points) A rectangular page is to contain 36 square inches of print. The margins on each side are $1\frac{1}{2}$ inches. Find the dimensions of the page such that the least amount of paper is used.
- (10 points) Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$, the x -axis, and the line $x = 4$ about the given lines.
 - the x -axis
 - the y -axis
- (10 points) Evaluate the integrals.
 - $\int \frac{\sin(2x + 1)}{\cos^2(2x + 1)} dx$
 - $\int \sin^3 x \cos^2 x dx$
- (10 points) A curved wedge is cut from a circular cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at a 45° angle at the center of the cylinder. Find the volume of the wedge.



- (10 points) Show that the function

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

has no Riemann integral over $[0, 1]$.

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1. The function $f(x) = \frac{1 - \cos x}{x}$ has a continuous extension to $x = 0$.
2. The graph of the function $f(x) = \frac{x+1}{|x|+1}$ has two horizontal asymptotes.
3. Let $y = f(x)$ be integrable on the closed interval $[a, b]$, then there exists a number $c \in [a, b]$ such that $\int_a^b f(x) dx = f(c)(b-a)$.
4. $\int_{-\infty}^{\infty} f(x) dx$ may not equal $\lim_{b \rightarrow \infty} \int_{-b}^b f(x) dx$.
5. Functions that are neither increasing nor decreasing may still be one-to-one and have an inverse.
6. If $x > 0$, then $\frac{d}{dx} x^x = x^x \ln x$.
7. $\tan^{-1} \left(\tan \frac{2\pi}{3} \right) = \frac{2\pi}{3}$.
8. $\int_0^3 \frac{dx}{(x-2)^2} = \left[\frac{-1}{x-2} \right]_0^3 = -\frac{3}{2}$.
9. $e^x + x^4 = O(e^x)$ and $e^x = O(e^x + x^4)$.
10. $\frac{x^3+1}{x^2(x^2+4)}$ can be put in the form $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4}$ for some rational numbers A, B, C, D .

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1. For all values of the constants a and b for which the function

$$f(x) = \begin{cases} \frac{2\sin^2 x}{x}, & \text{if } x > 0 \\ ax + b \cos x, & \text{if } x \leq 0 \end{cases}$$

is continuous and differentiable at $x = 0$.

Answer : _____.

2. Find an equation of the tangent line to the graph of $y = \sin^{-1} x$ at the point

$\left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$. Answer : _____.

3. Find $f'(0)$ for

$$f(x) = \begin{cases} e^{-1/x^2}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$$

Answer : _____.

4. Order the following functions from slowest growing to fastest growing as $x \rightarrow \infty$.

a. e^x b. x^x c. $(\ln x)^x$ d. $e^{\frac{x}{3}}$.

Answer : _____.

5. Let $f(x) = x^3 - 3x^2 - 1$, $x \geq 2$. Find the value of $\frac{df^{-1}}{dx}$ at the point $x = -1 = f(3)$. Answer : _____.

6. Evaluate $\int_{-\pi/4}^{\pi} \cos x \cos 3x \, dx$.

Answer : _____.

公式列表如下, 請參考 :

a. $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$.

b. $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$.

7. Evaluate $\int_1^4 \frac{dx}{(1 + \sqrt{x})^3}$.

Answer : _____.

8. Evaluate $\int_1^e x(\ln x)^2 \, dx$.

Answer : _____.

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1. (10 points) Use the integral to evaluate

$$\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \cdots + \frac{1}{\sqrt{n}\sqrt{n+n}} \right).$$

2. (10 points) Evaluate the integrals.

a. $\int \tan^4 x \, dx$

b. $\int \frac{x^2 \, dx}{\sqrt{9-x^2}}$

3. (10 points) Evaluate

$$\int \frac{-2x+4}{(x^2+1)(x-1)^2} \, dx.$$

4. (10 points) Find the limits.

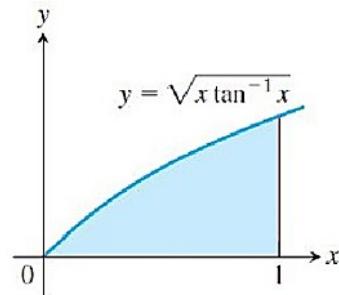
a. $\lim_{x \rightarrow \infty} \frac{\ln(x+1)}{\log_2 x}$

b. $\lim_{x \rightarrow 0^+} x^x$

5. (10 points) Investigate the convergence of

$$\int_1^{\infty} \frac{1-e^{-x}}{x} \, dx.$$

6. (10 points) Consider the region bounded by the graphs of $y = \sqrt{x \tan^{-1} x}$ and $y = 0$ for $0 \leq x \leq 1$. Find the volume of the solid formed by revolving this region about the x -axis (see accompanying figure).



(試題結束)