Calculus Homework Assigment 4

Class 班: CSIE 1-B

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1. Identify the particle's path by finding a Cartesian equation for it. Graph the Cartesian equation. (The graphs will vary with the equation used.) Indicate the portion of the graph traced by the particle and the direction of motion.

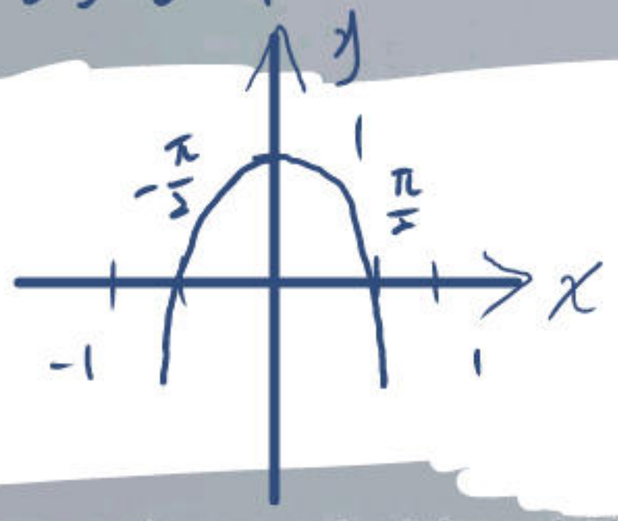
$$x = \sin t, \quad y = \cos 2t, \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$$

[§10.1 #9]

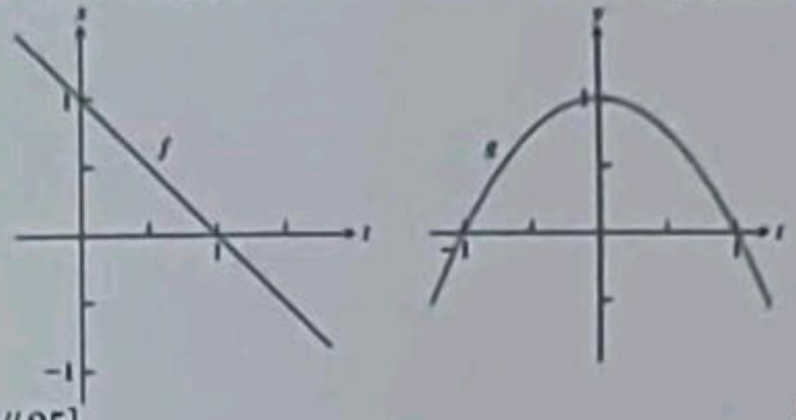
$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$y = \cos 2t = 1 - 2\sin^2 t = 1 - 2x^2$



2. Use the given graphs of $x = (t)$ and $y = g(t)$ to sketch the corresponding parametric curve in the xy-plane.



[§10.1 #25]

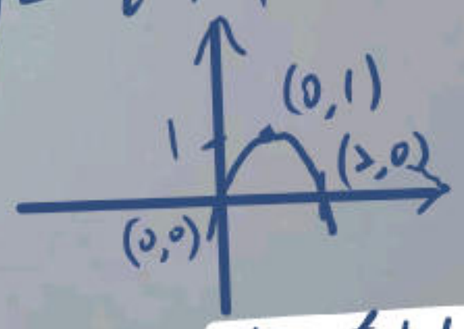
$x = -t + 1$

$y = -t^2 + 1$

$y = -(1-x)^2 + 1$

$y = -x^2 + 2x$

$x^2 - 2x + y = 0$



$x = -t + 1, \ y = -t^2 + 1, \ -\infty < t < \infty$

3. Find the area under one arch of the cycloid

$$x = a(t - \sin t), \quad y = a(1 - \cos t)$$

[§10.2 #21]

$x = a(t - \sin t)$

$dx = a(1 - \cos t)\,dt$

$0 = a(t - \sin t)$

$\Rightarrow t = 0$

$2a\pi = a(t - \sin t)$

$t = 2\pi$

$\int_0^{2a\pi} y\,dx$

$\int_0^{2\pi} a(1-\cos t)\,a(1-\cos t)\,dt$

$= a^2 \int_0^{2\pi} 1 - 2\cos t + \frac{1+\cos 2t}{2}\,dt$

$= a^2 \int_0^{2\pi} \frac{3}{2} - 2\cos t + \frac{1}{2}\cos 2t\,dt$

$= a^2 \left[\frac{3}{2}t - 2\sin t + \frac{1}{4}\sin 2t\,dt\right]_0^{2\pi}$

$= 3a^2\pi$ //

4. Find the length of the curve

$$x = t^2/2, \quad y = (2t+1)^{3/2}/3, \quad 0 \le t \le 4.$$

[§10.2 #27]

$\frac{dx}{dt} = t$

$\frac{dy}{dt} = \sqrt{2t+1}$

$\int_0^4 \sqrt{t^2 + (\sqrt{2t+1})^2}\,dt$

$= \left[\frac{t^2}{2} + t\right]_0^4$

$= 8 + 4 = 12$ //

(Turn over please 請翻頁)