

Calculus Homework Assignment 6

Class 班: CSIE 1-B

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1. Which formula is not equivalent to the other two?

a. $\sum_{k=2}^4 \frac{(-1)^{k-1}}{k-1}$ b. $\sum_{k=0}^2 \frac{(-1)^k}{k+1}$ c. $\sum_{k=-1}^1 \frac{(-1)^k}{k+2}$

[§5.2 #9]

a. $\sum_{k=2}^4 \frac{(-1)^{k-1}}{k-1} = -1 + \frac{1}{2} - \frac{1}{3}$

b. $\sum_{k=0}^2 \frac{(-1)^k}{k+1} = 1 - \frac{1}{2} + \frac{1}{3}$

c. $\sum_{k=-1}^1 \frac{(-1)^k}{k+2} = -1 + \frac{1}{2} - \frac{1}{3}$

b.

2. Evaluate the sum.

$$\sum_{k=7}^{30} (\sqrt{k-4} - \sqrt{k-3})$$

[§5.2 #35]

$$\sqrt{3} - \sqrt{4} + \sqrt{4} - \sqrt{5} + \dots + \sqrt{29} - \sqrt{30} + \sqrt{30} - \sqrt{31}$$

$$= \sqrt{3} - \sqrt{31} = -2\sqrt{3}$$

$$\underline{\underline{-2\sqrt{3}}}$$

3. Suppose that f is integrable and that $\int_0^2 f(x) dx = 3$ and $\int_0^4 f(x) dx = 7$. Find

a. $\int_3^4 f(x) dx$ 4

b. $\int_4^3 f(t) dt$ -4

[§5.3 #13]

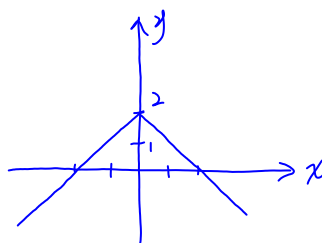
a. $\int_3^4 f(x) dx = \int_0^4 f(x) dx - \int_0^3 f(x) dx = 7 - 3 = 4$

b. $\int_4^3 f(t) dt = -\int_3^4 f(x) dx = -4$

4. Graph the integrals and use unknown area formulas to evaluate the integrals.

$$\int_{-1}^1 (2 - |x|) dx$$

[§5.3 #21]



$$\int_{-1}^1 (2 - |x|) dx = \int_{-1}^0 (2 + x) dx + \int_0^1 (2 - x) dx$$

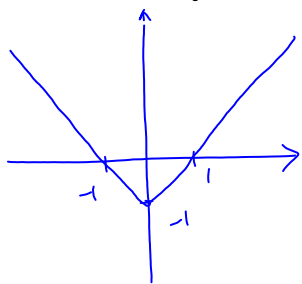
$$= \frac{3}{2} + \frac{3}{2} = 3$$

3

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5. Graph the function $g(x) = |x| - 1$ and find its average value over the interval $[-1, 1]$.

[§5.3 #61]



$$\begin{aligned} & \int_{-1}^1 (|x| - 1) dx \\ &= \int_{-1}^1 |x| dx - \int_{-1}^1 1 dx \\ &= 1 - 2 = -1 \end{aligned}$$

$$\frac{-1}{1 - (-1)} = 2$$

$$\underline{\underline{2}}$$

6. Find $\frac{dy}{dx}$

$$y = \int_0^{\sqrt{x}} \cos t \, dt \quad [\text{§5.4 \#33}]$$

Let $u = \sqrt{x}$

$$\frac{dy}{dx} = \frac{d}{du} \int_0^u \cos t \, dt \cdot \frac{du}{dx}$$

$$= \cos u \cdot \frac{du}{dx}$$

$$= \frac{1}{2\sqrt{x}} \cdot \cos \sqrt{x}$$

$$\underline{\underline{\frac{1}{2\sqrt{x}} \cdot \cos \sqrt{x}}}$$

7. Find $\frac{dy}{dx}$

$$y = \int_0^{x^4} \sqrt{u} \, du \quad [\text{§5.4 \#35}]$$

Let $t = x^4$

$$\frac{dy}{dx} = \frac{d}{dt} \int_0^t \sqrt{u} \, du \cdot \frac{dt}{dx}$$

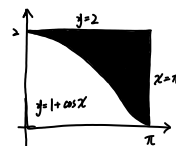
$$= \sqrt{t} \cdot \frac{dt}{dx} = 4x^3 \sqrt{x^4} = 4x^5$$

$$\underline{\underline{4x^5}}$$

8. Find the area of the shaded region.

$$2\pi - \int_0^\pi (1 + \cos x) dx = 2\pi - [x + \sin x]_0^\pi$$

$$= 2\pi - \pi = \pi$$



[§5.4 #51]

$$\underline{\underline{\pi}}$$