Engineering Mathematics HW1

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(1) $y' = e^{3x+2y}$

sol: $\frac{dy}{dx} = e^{3x} \cdot e^{2y} \Rightarrow e^{-2y}dy = e^{3x}dx$

$\Rightarrow \int e^{-2y}dy = \int e^{3x}dx \Rightarrow -\frac{1}{2}e^{-2y} = \frac{1}{3}e^{3x} + c$

$\Rightarrow \ln\left(\frac{-2}{3}e^{3x} + c\right) = -2y \Rightarrow y = -2\ln\left(\frac{-2}{3}e^{3x} + c\right)$ #

(2) $y' = (1+x)e^{x+y}$

sol: $e^{-y}dy = (1+x)e^{x}dx \Rightarrow \int e^{-y}dy = \int (1+x)e^{x}dx$

$\Rightarrow -e^{-y} = xe^{x} + c \Rightarrow y = -\ln(-xe^{x} + c)$ #

(3) $\frac{1}{x} + y + (3y^2 + x)y' = 0$

sol: $y' = \frac{-\frac{1}{x} - y}{3y^2 + x}$

(4) $xy' + y = y^2$

sol: $\frac{x}{dx} = \frac{y^2 - y}{dy} \Rightarrow \int \frac{1}{x}dx = \int \frac{1}{y^2 - y}dy$

$\boxed{\frac{1}{y^2 - y} = \frac{A}{y} + \frac{B}{y-1} \Rightarrow \begin{cases} A = -1 \\ B = 1 \end{cases}}$

$\Rightarrow \ln|x| + c = \ln\left|\frac{y-1}{y}\right|$

$\Rightarrow \frac{y-1}{y} = e^{c} \cdot |x| = c \cdot |x|$

$\Rightarrow \frac{y - 1 - yc|x|}{y} = 0 \Rightarrow \frac{y(1 - c|x|) - 1}{y} = 0 \Rightarrow y = \frac{1}{1 - c|x|}$ #

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(5) $\sin x \cos y \, dx + \cos x \sin y \, dy = 0$

sol: $\int \frac{\sin y}{\cos y} dy = -\int \frac{\sin x}{\cos x} \Rightarrow \int \tan y \, dy = -\int \tan x \, dx$

$\Rightarrow \ln|\cos y| = -\ln|\cos x| + C_1$

$\Rightarrow |\cos y| = C \cdot |\cos x|$

$\Rightarrow y = \cos^{-1}(\cos x \cdot C)$ ※

$$\boxed{\begin{aligned} &\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = \ln|\cos x| + C \\ &u = \cos x \\ &du = \sin x \, dx \quad = \int \frac{1}{u} du \end{aligned}}$$

(6) $y' + 2xy^2 = 0$

sol: $\int y^{-2} dy = \int -2x \, dx \Rightarrow -y^{-1} = -x^2 + C_1 \Rightarrow y = \frac{1}{x^2 + C}$ ※

(7) $y' = y^2 - 4$

sol: $\int \frac{1}{y^2-4} dy = \int dx \Rightarrow \frac{1}{4} \int \left(\frac{1}{y-2} - \frac{1}{y+2}\right) dy = x + C_1 \Rightarrow \frac{1}{4} \ln\left|\frac{y-2}{y+2}\right| = x + C_1$

$$\boxed{\frac{1}{y^2-4} = \frac{A}{y-2} + \frac{B}{y+2} \Rightarrow \begin{aligned} A &= \tfrac{1}{4} \\ B &= -\tfrac{1}{4} \end{aligned}}$$

$\Rightarrow \ln\left|\frac{y-2}{y+2}\right| = 4x + C_2 \Rightarrow \frac{y-2}{y+2} = C_3 \cdot e^{4x}$

$$\frac{y-2-(y+2)ce^{4x}}{y+2} = 0$$

(8) $xy' = x + y$, $y(1) = 3$

sol: $y' = 1 + \frac{y}{x}$, Let $u = \frac{y}{x}$, $y' = u'x + u$

$u'x + u = 1 + u \Rightarrow u'x = 1 \Rightarrow \int du = \int \frac{1}{x} dx \Rightarrow u = \ln|x| + C_1$

$y = x\ln|x| + C$, $C = 3 \Rightarrow y = x\ln|x| + 3$ ※

(9) $xy' = y + 3x^4\cos^2\left(\frac{y}{x}\right)$, $y(1) = 0$

sol: $y' = \frac{y}{x} + 3x^3\cos^2\left(\frac{y}{x}\right)$, Let $u = \frac{y}{x}$, $y' = u'x + u$

$u'x + u = u + 3x^3\cos^2 u \Rightarrow u' = 3x\cos^2 u \Rightarrow \int \cos^{-2} u \, du = \int 3x \, dx$

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(10) $2xy^3-3y-(3x+\alpha x^2y^2-2\alpha y)y'=0$

sol: $\underset{M}{(2xy^3-3y)}dx+\underset{N}{(-3x-\alpha x^2y^2+2\alpha y)}dy=0$

$\frac{\partial M}{\partial y}=6xy^2-3$, $\frac{\partial N}{\partial x}=-3-2\alpha xy^2 \Longrightarrow \alpha=\boxed{-3}$

$u=\int 2xy^3-3y\,dx+k(y)=x^2y^3-3xy+k(y)$

$\Rightarrow 3x^2y^2-3x+\frac{dk}{dy}=N=-3x+3x^2y^2-6y$

$\Rightarrow \frac{dk}{dy}=-6y \Rightarrow k(y)=-3y^2$

$\Rightarrow u=x^2y^3-3xy-3y^2=C$ #