Calculus Homework Assignment 3

Class: CSIE 3-B

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1. Determine if the alternating series

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^n}{n^2}$$

converges or diverges. It may not satisfy the conditions of the Alternating Series Test.

[§9.6 – 7]

$\lim\limits_{n\to\infty} \frac{2^n}{n^2}$ D.N.E

so $\lim\limits_{n\to\infty} (-1)^{n+1}\frac{2^n}{n^2}$ D.N.E

$\sum\limits_{n=1}^{\infty}(-1)^{n+1}\frac{2^n}{n^2}$ diverges by the n-th term test.

2. The given series below converges absolutely, converges conditionally or diverges? Give reason for your answers.

$$\sum_{n=1}^{\infty}(-1)^n(\sqrt{n+1}-\sqrt{n})$$

[§9.6 – 41]

$(\sqrt{n+1}-\sqrt{n})\left(\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}\right) = \frac{1}{\sqrt{n+1}+\sqrt{n}}$

$\frac{1}{\sqrt{n+1}+\sqrt{n}}$ is decreasing and $\to 0$

$\sum\limits_{n=0}^{\infty}(-1)^n\frac{1}{\sqrt{n+1}+\sqrt{n}}$ converges

but $\lim\limits_{n\to\infty}\frac{\frac{1}{\sqrt{n+1}+\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim\limits_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \frac{1}{2}$

$\frac{1}{\sqrt{n+1}+\sqrt{n}}$ and $\frac{1}{\sqrt{n}}$ both diverge $(p=\frac{1}{2})$

so $\sum\limits_{n=0}^{\infty}(-1)^n(\sqrt{n+1}-\sqrt{n})$ converges conditionally.

3. The given series below converges or diverges? Give reason for your answers.

$$\sum_{n=0}^{\infty}(-1)^n\frac{(n+2)!}{(2n)!}$$

[§9.6 – 61]

$\frac{(n+2)!}{(2n)!}$ is decreasing

and $\lim\limits_{n\to\infty}\frac{(n+2)!}{(2n)!} = 0$

so $\sum\limits_{n=1}^{\infty}(-1)^n\frac{(n+2)!}{(2n)!}$ converges

$\lim\limits_{n\to\infty}\left|\frac{(n+3)!}{(2n+2)!}\cdot\frac{(2n)!}{(n+2)!}\right| = \lim\limits_{n\to\infty}\frac{n+3}{(2n+1)(2n+2)} = 0$

so $\frac{(n+2)!}{(2n)!}$ converges by Ratio Test

so $\sum\limits_{n=0}^{\infty}(-1)^n\frac{(n+2)!}{(2n)!}$ converges absolutely

4. **(a)** Find below series' radius and interval of convergence. For what values of x does the series converges **(b)** absolutely, **(c)** conditionally?

$$\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$$

[§9.7 – 11]

$\lim\limits_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim\limits_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)!}\cdot\frac{n!}{x^n}\right| < 1,\ \forall x$

(a) so the radius is ∞ and $\sum\limits_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$ converges for all x.

(b) $\sum\limits_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$ converges absolute for all x

(c) no x for conditionally converge.

5. Assume that the series $\sum a_n(x-2)^n$ converges for $x=-1$ and diverges for $x=6$. Answer true (T), false (F), or not enough information given (N) for the following statements about the series.

(a) Converges absolutely for $x=1$.
(b) Diverges for $x=-6$.
(c) Diverges for $x=2$.
(d) Converges for $x=0$.
(e) Converges absolutely for $x=5$.
(f) Diverges for $x=4.9$.
(g) Diverges for $x=5.1$.
(h) Converges absolutely for $x=4$.

[§9.7 – 63]

6. Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by $f(x)=\sin x$ at $a=\dfrac{\pi}{4}$.

[§9.8 – 7]

$f(x)=\sin x,\ f'(x)=\cos x,\ f''(x)=-\sin x,\ f'''(x)=-\cos x$

$f(a)=\frac{\sqrt{2}}{2},\ f'(a)=\frac{\sqrt{2}}{2},\ f''(a)=-\frac{\sqrt{2}}{2},\ f'''(a)=-\frac{\sqrt{2}}{2}$

$P_0(x)=\frac{\sqrt{2}}{2}$

$P_1(x)=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)$

$P_2(x)=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)+\frac{\sqrt{2}}{2}\cdot\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2$

$P_3(x)=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)+\frac{\sqrt{2}}{4}\left(x-\frac{\pi}{4}\right)^2+\frac{\sqrt{2}}{12}\left(x-\frac{\pi}{4}\right)^3$

7. Find the first three nonzero terms of the Maclaurin series for each function and the values of x for which the series converges absolutely.

$$f(x)=\cos x-\left(\frac{2}{(1-x)}\right)$$

[§9.8 – 35]

$\cos(x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}=1-\frac{1}{2}x^2+\frac{1}{24}x^4-\cdots$

$\frac{2}{1-x}=2\sum_{n=0}^{\infty}x^n=2+2x+2x^2+\cdots$

$\cos x-\left(\frac{2}{1-x}\right)=-1-2x-\frac{5}{2}x^2+\cdots$

$\cos x$ converges on $(-\infty,\infty)$

$\frac{2}{1-x}$ converges on $(-1,1)$

so $\cos x-\left(\frac{2}{1-x}\right)$ converges absolutely on $(-1,1)$

8. The Taylor polynomial of order 2 generated by a twice-differentiable function $f(x)$ at $x=a$ is called the *quadratic approximation* of f at $x=a$. Given

$$f(x)=\ln(\cos x).$$

(a) Find the Taylor polynomial of order 1 of f at $x=0$.
(b) Find the quadratic approximation of f at $x=0$.

[§9.8 – 45]

$f(x)=\ln(\cos x)$, $f(0)=0$

$f'(x)=\frac{-\sin x}{\cos x}=-\tan x$, $f'(0)=0$

$f''(x)=-\sec^2 x$, $f''(0)=-1$

$L(x)=0,\quad Q(x)=-\frac{x^2}{2}$

(The End)