Calculus Homework Assignment 4

5. Replace the polar equation $r^2 = -4r\cos\theta$ with equivalent Cartesian equation. Then describe or identify the graph. [§10.3 #45]

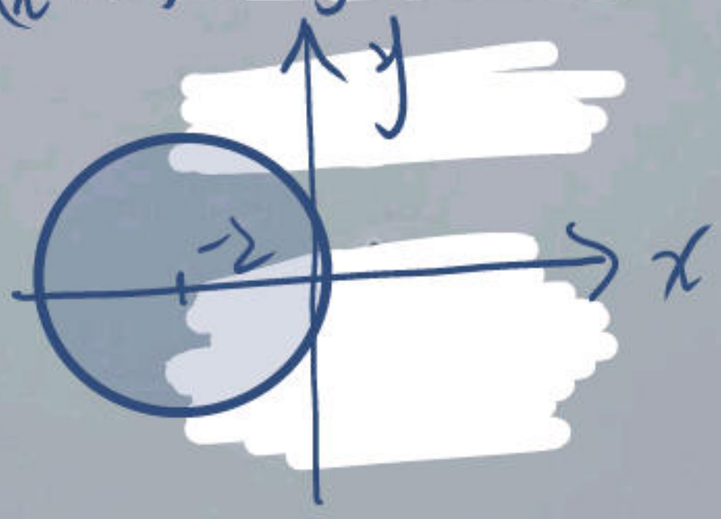
$x = r\cos\theta$, $y = r\sin\theta$

$x^2 + y^2 = r^2(\cos^2\theta + \sin^2\theta) = r^2$

$x^2 + y^2 = -4x$

$x^2 + 4x + y^2 = 0$

$(x+2)^2 + y^2 = 2^2$



6. Replace the Cartesian equation $(x-3)^2 + (y+1)^2 = 4$ with equivalent polar equation. [§10.3 #65]

$x = r\cos\theta$, $y = r\sin\theta$

$(x-3)^2 + (y+1)^2 = 4$

$x^2 - 6x + 9 + y^2 + 2y + 1 = 4$

$r^2 - 6r\cos\theta + 2r\sin\theta = -6$ *

7. Equation (1) in Section 10.2 gives the formula for the derivative y' of a polar curve $r = f(\theta)$. The second derivative is $\dfrac{d^2y}{dx^2} = \dfrac{dy'/d\theta}{dx/d\theta}$ (see Equation (2) in Section 10.2). Find the slope and concavity of the curve $r = \sin\theta$ at the given points $\theta = \pi/6, \pi/3$. [§10.4 #21]

$x = r\cos\theta = \sin\theta\cos\theta$

$y = r\sin\theta = \sin^2\theta$

$\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

$\theta = \frac{\pi}{6}$, slope $= \frac{\sqrt{3}}{4}$

$\theta = \frac{\pi}{3}$, slope $= -\frac{\sqrt{3}}{4}$

$\dfrac{d^2y}{dx^2} = \dfrac{2(\cos^2\theta - \sin^2\theta)}{-2\sin\theta\cos\theta - 2\cos\theta\sin\theta}$

$\theta = \frac{\pi}{6} \to \frac{d^2y}{dx^2} = \frac{-\sqrt{3}}{3}$ ↘

$\theta = \frac{\pi}{3} \to \frac{d^2y}{dx^2} = \frac{\sqrt{3}}{3}$ ↗

8. Which of the following has the same graph as $r = 1 - \cos\theta$? **a.** $r = -1 - \cos\theta$ **b.** $r = 1 + \cos\theta$ Confirm your answer with algebra. [§10.4 #33]

$r = 1 - \cos\theta$

$r^2 = r - r\cos\theta$

$x^2 + y^2 = \sqrt{x^2+y^2} - x$

$(x^2 + y^2 + x)^2 = x^2 + y^2$

$x^2 + y^2 + x = -\sqrt{x^2+y^2}$

$x^2 + y^2 = -\sqrt{x^2+y^2} - x$

$r^2 = -r - r\cos\theta$

$r = -1 - \cos\theta$ A: a

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