Engineering Mathematics HW1

(CH1.1 Basic Concepts. Modeling

~CH1.4 Exact ODEs. Integrating Factors)

Deadline:2025/3/23 23:59

1. (a) Verify that y is a solution of the ODE. (b) Determine from y the particular solution of the IVP.

$$yy' = 4x, \quad y^2 - 4x^2 = c \ (y > 0), \quad y(1) = 4$$

$\frac{d}{dx} y^2 - 4x^2 = \frac{d}{dx} c \Rightarrow 2yy' - 8x = 0 \Rightarrow yy' = 4x$ so $y^2 - 4x^2 = c$ is a sol. $\quad 4^2 - 4(1)^2 = 12 \Rightarrow$ particular sol : $\underline{y^2 - 4x^2 = 12}$ (at $y(1)=4$)

2. Find a general solution. Show the steps of derivation. Check your answer by substitution.

check : $-\frac{x}{y} = x + C$

$\frac{-y + xy'}{y^2} = 1 \Rightarrow xy' = y^2 + y$ #

$$xy' = y^2 + y \quad (Set\ \frac{y}{x} = u)$$

$y' = \frac{y}{x} y + \frac{y}{x}$,

$y' = uy + u = u(y+1)$

$y = xu$, $y' = u + xu'$

$u + xu' = u^2 x + u$

$xu' = u^2 x$

$u' = u^2$

$\frac{u'}{u^2} = 1$

$-u^{-1} = x + C$

$\underline{-\frac{x}{y} = x + C}$ #

3. Solve the IVP. Show the steps of derivation, beginning with the general solution.

$$y' = -4x/y, \qquad y(2) = 3$$

$yy' = -4x \Rightarrow \frac{1}{2} y^2 = -2x^2 + C \qquad C - 8 = \frac{9}{2} \Rightarrow C = \frac{25}{2} \qquad \underline{\underline{y^2 = -4x^2 + 25}}$

4. Test for exactness. If exact, solve. If not, use an integrating factor as given or obtained by inspection or by the theorems in the text. Also, if an initial condition is given, find the corresponding particular solution.

(a) $\underbrace{sin(x)\, cos(y)}_{M}\, dx + \underbrace{cos(x)\, sin(y)}_{N}\, dy = 0$

(b) $(x^2 + y^2)\, dx - 2xy\, dy = 0$

(a) $\frac{\partial}{\partial y} \sin(x)\cos(y) = -\sin(x)\sin(y)$, $\frac{\partial}{\partial x} \cos(x)\sin(y) = -\sin(x)\sin(y)$ ... exact!

$\Phi = \int \sin(x)\cos(y)\, dx = -\cos(x)\cos(y) + C(y) \qquad \frac{\partial}{\partial y}\Phi = \cos(x)\sin(y)$, so $C(y) = C$

Sol: $\underline{\underline{\cos(x)\cos(y) = C}}$

(b) $\frac{\partial}{\partial y} M = 2y$, $\frac{\partial}{\partial x} N = -2y \Rightarrow$ not exact!

integrating factor $u = \frac{1}{x^2}$

$\underbrace{(1 + \frac{y^2}{x^2})}_{M'} dx - \underbrace{2\frac{y}{x}}_{N'} dy = 0$

$\frac{\partial M'}{\partial y} = \frac{2y}{x^2}$, $\frac{\partial N'}{\partial x} = \frac{2y}{x^2}$

$\Phi = \int M' dx = x - \frac{y^2}{x} + C(y)$

$\frac{\partial \Phi}{\partial y} = -\frac{2y}{x} + C'(y) = N'$, so $C(y) = C$

Sol: $\underline{\underline{x - \frac{y^2}{x} = C}}$