Calculus Homework Assigment 1

Class 班: CSIE 1-B

Student Number 學號: 110502567

Name 姓名: 蔡淵丞

1. Find the domain of $y = \dfrac{x+3}{4-\sqrt{x^2-9}}$. [§1.1 #21]

$x^2-9 \geq 0 \Rightarrow x \leq -3$, or $x \geq 3$

$\sqrt{x^2-9} \neq 4 \Rightarrow x \neq \pm 5$

The domain of $y = \dfrac{x+3}{4-\sqrt{x^2-9}}$ is $(-\infty, -5), (-5, -3], [3, 5), (5, +\infty)$

2. Say whether the function is even, odd, or neither. Give reasons for your answer.

a. $\sin 2x$

b. $\cos 3x$ [§1.1 #59,61]

a. $f(x) = \sin 2x$

$f(-x) = \sin -2x = -\sin 2x = -f(x)$

$y = \sin 2x$ is an odd function

b. $f(x) = \cos 3x$

$f(-x) = \cos -3x = \cos 3x = f(x)$

$y = \cos 3x$ is an even function

3. $f(x) = \sqrt{x+1}, \quad g(x) = \dfrac{1}{x}$

a. Write formulas for $f \circ g$ and $g \circ f$.

b. Find the domains and ranges of $f \circ g$ and $g \circ f$.

[§1.2 #17]

a. $f \circ g(x) = f(g(x)) = f(\frac{1}{x})$

$= \sqrt{\frac{1}{x} + 1}$

$g \circ f(x) = g(f(x)) = g(\sqrt{x+1})$

$= \dfrac{1}{\sqrt{x+1}}$

b. The domain of $f \circ g(x) = \sqrt{\frac{1}{x} + 1}$ is $[-1, 0)$

The domain of $g \circ f(x) = \dfrac{1}{\sqrt{x+1}}$ is $(-1, +\infty)$

4. Let $f(x) = \dfrac{x}{x-2}$. Find a function $y = g(x)$ so that $f \circ g(x) = x$. [§1.2 #19]

$f \circ g(x) = \dfrac{g(x)}{g(x)-2} = x$

$\Rightarrow x \cdot g(x) - 2x = g(x)$

$(x-1)\, g(x) = 2x$

$g(x) = \dfrac{2x}{x-1}, \quad x \neq 1$

(Turn over please 請翻頁)

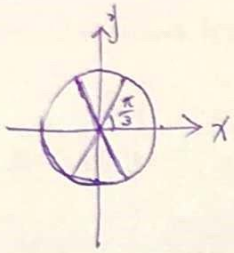
Calculus Homework Assignment 1

5. Solve for the angle θ, where $0 \le \theta \le 2\pi$.

$$\sin^2\theta = \frac{3}{4}$$

[§1.3 #51]

$\sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2}$



$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

6. Solve for the angle θ, where $0 \le \theta \le 2\pi$.

$$\sin 2\theta - \cos\theta = 0$$

[§1.3 #53]

$\sin 2\theta - \cos\theta = 2\sin\theta\cos\theta - \cos\theta = 0$

$= \cos\theta(2\sin\theta - 1) = 0$

if $2\sin\theta - 1 = 0 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

if $\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

7. Find (a) the slope of the curve at the given point P, and (b)an equation of the tangent line at P.

$$y = x^2 - 2x - 3, \quad P(2, -3)$$

[§2.1 #11]

Let $Q(2+h, (2+h)^2 - 2(2+h) - 3) \in y = x^2 - 2x - 3$

$h^2 + 4h + 4 - 2h - 4 - 3 = h^2 + 2h - 3$

$Q(2+h, h^2 + 2h - 3)$

$P(2, -3)$

The slope of the secant $\overline{PQ}$ is $\frac{h^2+2h-3-(-3)}{2+h-2} = \frac{h^2+2h}{h} = h+2$

As Q approaches P along the curve, h approaches zero and the secant slope $h+2$ approaches 2. (a)

equation: $y+3 = 2x-4 \Rightarrow y = 2x-7$ (b) (The end 結束)

8. Find (a) the slope of the curve at the given point P, and (b)an equation of the tangent line at P.

$$y = \sqrt{x}, \quad P(4, 2)$$

[§2.1 #17]

Let L be the tangent line at P and $Q(4+h, \sqrt{4+h}) \in y = \sqrt{x}$

The slope of the secant $\overline{PQ}$ is $\frac{\sqrt{4+h}-2}{h} = \frac{h}{h(\sqrt{4+h}+2)} = \frac{1}{\sqrt{4+h}+2}$

As Q approaches P along the curve, h approaches zero and the secant slope $\frac{1}{\sqrt{4+h}+2}$ approaches $\frac{1}{4}$ (a)

$L: y - 2 = \frac{1}{4}(x-4)$

$\Rightarrow y = \frac{1}{4}x + 1$ (b)