

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $f(x) = g(x)$ when $x \neq 0$, then $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} g(x)$, provided the limits exist.
2. Suppose $f(x)$ is a function on the closed interval $[1, 3]$ and $f(1) \cdot f(3) < 0$, then there exists a number c in $(1, 3)$ such that $f(c) = 0$.
3. $f(x) = x^{1/3}$ is differentiable for all $x \in (-\infty, \infty)$.
4. If $f(x) = \frac{1}{x}$. Then the domain of $f \circ f$ is $(-\infty, \infty)$.
5. $f(x) = \begin{cases} x \sin(1/x) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ is not differentiable at $x = 0$.
6. The function $f(x) = \sin|x|$ is not differentiable at $x = 0$.
7. A vertical line can intersect the graph of a function of x at most once.
8. Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{-x}$. Then the domain of f and the range of g are the same.
9. The statement $\lim_{x \rightarrow c} f(x) = L$ means that for some $\epsilon > 0$ there exists $\delta > 0$ such that if $0 < |x - c| < \delta$ then $|f(x) - L| < \epsilon$.
10. The derivative of an even function is even, if it exists.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the limit $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}$. Answer : _____.
2. If f is continuous at 5, $f(5) = 2$ and $f(4) = -3$, then find $\lim_{x \rightarrow 1} f(3x^2 + 2)$.
Answer : _____.
3. Find $\lim_{x \rightarrow 0} \left[\left(\cos \frac{1}{x} \right) (\sin x) \right]$. Answer : _____.
4. Find the domain of the function $f(x) = \frac{1}{|x^2 - 1|} + \sqrt{x}$.
Answer : _____.
5. If $f(x) = \begin{cases} \frac{1 - \cos x}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$, find $f'(0)$. Answer : _____.
6. Find the equation of the line that is tangent to the graph of $f(x) = 2\sqrt{x}$ and parallel to $x - y - 6 = 0$. Answer : _____.
7. Let $f(x) = \begin{cases} \frac{x^2 - a^2}{x - a} & , x \neq a \\ 8 & , x = a \end{cases}$. Find a if f is continuous on the entire real number line. Answer : _____.
8. Given $xy - 1 = 2x + y^2$, find the value of $\frac{d^2y}{dx^2}$ at the point $\left(\frac{-1}{2}, 0 \right)$.
Answer : _____.

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Find the derivative of the function

$$f(x) = \sqrt{1 - 2x}$$

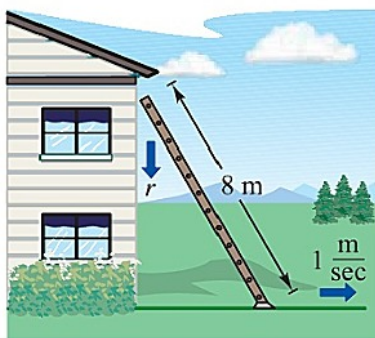
by using the definition of the derivative. State the domain of the function and the domain of its derivative.

(下頁還有試題)

2. (10 points) A ladder 8 meters long is leaning against the wall of a house (see figure).

The base of the ladder is pulled away from the wall at a rate of 1 meter per second.

- How fast is the top of the ladder moving down the wall when its base is 2 m from the wall?
- Consider the triangle formed by the side of the house, the ladder, and the ground. Find the rate at which the area of the triangle is changing when the base of the ladder is 2 m from the wall.



3. (10 points) Find the vertical asymptotes (if any) of the graph of the function.

$$h(x) = \frac{x^2 - 9}{x^3 + 3x^2 - x - 3}$$

4. (10 points) Find the derivative of the function $y = (\sin(\tan 2x))^3$.

5. (10 points) Use the $\epsilon - \delta$ definition of limit to prove that

$$\lim_{x \rightarrow 1} \sqrt{x+3} = 2.$$

6. (10 points) Find the slope of the tangent line to the graph of $\sin y = xy$ at the point $(0, \pi)$.

(試題結束)

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. A function $y = f(x)$ is integrable on the closed interval $[-1, 1]$, then $y = f(x)$ is continuous on $[-1, 1]$.
2. Let $y = g(x)$ be continuous on the closed interval $[3, 5]$. Then the range of g is a set of a single real number or a closed interval.
3. Let $y = f(x)$ be integrable on the closed interval $[a, b]$. Then there exists a number c in $[a, b]$ such that $\int_a^b f(x) dx = f(c)(b - a)$.
4. A function that is neither increasing nor decreasing can not be one-to-one and hence does not have an inverse.
5. If $y = f(x)$, f is increasing and differentiable, and $\Delta x > 0$, then $\Delta y \geq dy$.
6. $\int_a^b f(x)g(x) dx = \left[\int_a^b f(x) dx \right] \left[\int_a^b g(x) dx \right]$.
7. The integral $\int_0^2 (2x - 1)dx$ is the area of the region bounded by the graph of $f(x) = 2x - 1$ and the x -axis for $0 \leq x \leq 2$.
8. $|\cos x - \cos y| \leq |x - y|$ for all x and y .
9. If $(c, f(c))$ is a point of inflection of the graph of f , then either $f''(c) = 0$ or f'' does not exist at $x = c$.

(下頁還有試題)

10. Let $f(x)$ be a continuous function on an open interval I containing a and

$$F(x) = \int_a^x f(t) dt \text{ for } x \in I. \text{ Then } F(x) \text{ is differentiable on } I.$$

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the average value of $f(x) = \tan x$ on the interval $\left[0, \frac{\pi}{4}\right]$.

Answer : _____.

2. Let

$$F(x) = \int_{\pi}^{\ln x} \cos(e^t) dt.$$

Find $F'(\pi)$. Answer : _____.

3. Find $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{3i}{n}\right) \left(\frac{3}{n}\right)$.

Answer : _____.

4. Find $\int \frac{3^{2x}}{1 + 3^{2x}} dx$.

Answer : _____.

5. Find the limit

$$\lim_{x \rightarrow \infty} (4x - \sqrt{16x^2 + x}).$$

Answer : _____.

6. Find the definite integral

$$\int_1^4 (3 - |x - 3|) dx.$$

Answer : _____.

7. Find an equation of the tangent line to the graph of $y = 3x \arcsin x$ at the point

$\left(\frac{1}{2}, \frac{\pi}{4}\right)$. Answer : _____.

8. Find the absolute maximum for $f(x) = 3x^4 - 4x^3$ on the interval $[-1, 2]$.

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Find the derivative of the function $f(x) = x^{e^x}$ on the interval $(0, \infty)$.
2. (10 points) Find the area of the region bounded by the graph of $y = e^x \cos(e^x)$, the x -axis, and the vertical lines $x = -1$ and $x = 0$.
3. (10 points) Let f be continuous on the closed interval $[a, b]$, and differentiable on (a, b) . If $f'(x) > 0$ on (a, b) , then f is increasing on $[a, b]$.
4. (10 points) Let $f(x) = 3x^{2/3} - 2x$. Find **a.** its domain, **b.** critical numbers, **c.** intervals of increasing/decreasing, **d.** relative maximum/minimum values, **e.** intervals of concavity, **f.** inflection points, **g.** asymptotes. **h.** Then sketch the graph of $f(x)$.

5. (10 points)

a. Show that

$$f(x) = \int_0^{1/x} \frac{1}{t^2 + 1} dt + \int_0^x \frac{1}{t^2 + 1} dt$$

is constant for $x > 0$.

b. Show that

$$f(x) = \int_2^x \sqrt{1+t^2} dt$$

is one-to-one and find $(f^{-1})'(0)$.

6. (10 points) Four feet of wire is to be used to form a square and a circle. How much of the wire should be used for the square and how much should be used for the circle to enclose the maximum total area?

(試題結束)

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. Suppose $f(x)$ is a function on the closed interval $[1, 3]$ and $f(1) \cdot f(3) < 0$, then there exists a number c in $(1, 3)$ such that $f(c) = 0$.

2. A function $y = f(x)$ is integrable on the closed interval $[-1, 1]$, then $y = f(x)$ is continuous on $[-1, 1]$.

3. The integral $\int_0^2 (2x - 1)dx$ is the area of the region bounded by the graph of $f(x) = 2x - 1$ and the x -axis for $0 \leq x \leq 2$.

4. $\lim_{x \rightarrow 0^+} \frac{1 - \cos x}{x + x^3} = \lim_{x \rightarrow 0^+} \frac{\sin x}{1 + 3x^2} = \lim_{x \rightarrow 0^+} \frac{\cos x}{6x} = \infty$.

5. Since $\lim_{b \rightarrow \infty} \int_{-b}^b \frac{2x}{x^2 + 1} dx = 0$, we have

$$\int_{-\infty}^{\infty} \frac{2x}{x^2 + 1} dx = \lim_{b \rightarrow \infty} \int_{-b}^b \frac{2x}{x^2 + 1} dx = 0.$$

6. $\frac{8x^3 + 13x}{(x^2 + 2)^2}$ can be put in the form $\frac{Ax + B}{x^2 + 2} + \frac{Cx + D}{(x^2 + 2)^2}$.

7. If $f'(x)$ is continuous on $[0, 1]$ and $f(0) = 0$, then

$$\int_0^\alpha \sqrt{1 + [f'(x)]^2} dx \geq \sqrt{\alpha^2 + [f(\alpha)]^2}$$

for $0 \leq \alpha \leq 1$.

(下頁還有試題)

8. $\int_1^{\infty} \frac{1}{x\sqrt{\frac{1}{2}}} dx$ converges.
9. If $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 1$, then $\lim_{x \rightarrow \infty} [f(x) - g(x)] = 0$.
10. $\int_{-1}^1 \frac{1}{x^2} dx = -2$.

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let $f(x) = \int_2^x \sqrt{1+t^2} dt$. Find $(f^{-1})'(0)$.

Answer : _____.

2. Find $\int_0^{\pi/2} \cos x \cos 2x dx$.

Answer : _____.

3. Evaluate $\lim_{x \rightarrow 0^+} (\sin x)^x$.

Answer : _____.

4. Find the volumes of the solid generated by revolving the region bounded by the graphs of the equations $y = \sqrt{x}$, $y = 0$, $x = 4$ about the line $x = 4$.

Answer : _____.

5. Find $\int_{-1}^{1/2} \frac{1}{\sqrt{8-2x-x^2}} dx$.

Answer : _____.

6. Find the area of the region bounded by the graphs of $x = 3 - y^2$ and $x = y + 1$.

Answer : _____.

7. Evaluate $\int_{\sqrt{3}}^2 \frac{\sqrt{x^2-3}}{x} dx$.

Answer : _____.

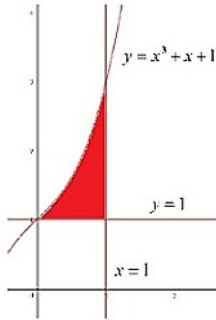
8. Evaluate $\int_1^{\infty} (1-x)e^{-x} dx$.

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (10 points) Given $xy - 1 = 2x + y^2$, find the value of $\frac{d^2y}{dx^2}$ at the point $\left(\frac{-1}{2}, 0\right)$.
2. (10 points) Evaluate $\int \frac{\sqrt{x}}{x-1} dx$.
3. (10 points) Evaluate $\int_0^\infty \frac{dx}{2\sqrt{x}(x+1)}$.
4. (10 points) Find the integral.
 - a. $\int_0^{\frac{\pi}{4}} \sec^5 x \tan^3 x dx$.
 - b. $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^4 x dx$.
5. (10 points) Let $f(x) = \frac{x^3}{12} + \frac{1}{x}$, $1 \leq x \leq 2$.
 - a. Find the length of the graph of f .
 - b. Find the area of the surface generated by revolving the curve $y = f(x)$ about the x -axis.
6. (10 points) Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3 + x + 1$, $y = 1$, and $x = 1$ about the line $x = 2$.



(試題結束)