Calculus Homework Assigment 3

Class 班: CSIE 1-B

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1. Find the series' radius and interval of convergence. For what values of x does the series converge absolutely, conditionally?

$$\sum_{n=1}^{\infty} \frac{1+2+3+\ldots+n}{1^2+2^2+3^2+\ldots+n^2} x^n$$

[§9.7 #35]

$$R = \frac{a_n}{a_{n+1}} = \lim_{n\to\infty} \frac{\left(\frac{n(n+1)}{(n+1)(n+2)(2n+3)}\right)}{\left(\frac{(n+1)(n+2)}{(n+1)(n+2)(2n+3)}\right)} = 1$$

$$\begin{cases} x = 1 : \sum_{n=1}^{\infty} \frac{1+2+\cdots+n}{1^2+2^2+\cdots+n^2} = \text{div.} \\ x = -1 : \sum_{n=1}^{\infty} \frac{1+2+\cdots+n}{1^2+2^2+\cdots+n^2} = (-1)^n \text{ conv.} \end{cases}$$

so $-1 < x < 1$ abs conv.
ratio $= 1$ interval $[-1, 1]$

2. Use Theorem 20 to find the following series' interval of convergence and, within this interval, the sum of the series as a function of x.

$$\sum_{n=0}^{\infty} \left(\frac{x^2+1}{3}\right)^n$$

[§9.7 #47]

$$R = \frac{a_n}{a_{n+1}} = \lim_{n\to\infty} \frac{\left(\frac{x^2+1}{3}\right)^n}{\left(\frac{x^2+1}{3}\right)^{n+1}} = 1$$

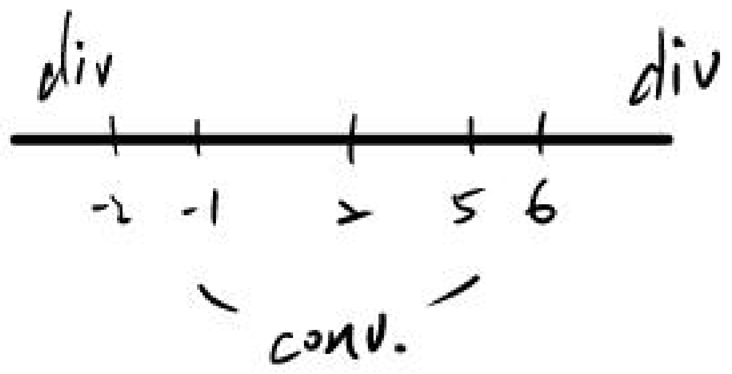
for $\left|\frac{x^2+1}{3}\right| < 1$, $x^2 < 2 \Rightarrow -\sqrt{2} < x < 2$

$$\frac{1}{1-\left(\frac{x^2+1}{3}\right)} = \frac{3}{4-x^2}$$

3. Assume that the series $\sum a_n(x-2)^n$ converges for $x = -1$ and diverges for $x = 6$. Answer true (T), false (F), or not enough information given (N) for the following statements about the series.

T **a.** Converges absolutely for x = 1
T **b.** Diverges for x = -6
F **c.** Diverges for x = 2
T **d.** Converges for x = 0
N **e.** Converges absolutely for x = 5
F **f.** Diverges for x = 4.9
N **g.** Diverges for x = 5.1
T **h.** Converges absolutely for x = 4

[§9.7 #63]



4. Find the Taylor polynomials of orders 0,1,2, and 3 generated by $f(x) = \ln x$ at $a = 1$. [§9.8 #3]

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(1)}{n!}(x-1)^n$$

$$P_0(x) = f(1) = 0$$

$$P_1(x) = \frac{1}{1}(x-1) = x-1$$

$$P_2(x) = x-1-\frac{(x-1)^2}{2!} = -\frac{x^2}{2}+2x-\frac{3}{2}$$

$$P_3(x) = -\frac{x^2}{2}+2x-\frac{3}{2}+\frac{2!(x-1)^3}{3!} = \frac{x^3}{3}-\frac{3x^2}{2}+3x-\frac{11}{6}$$

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5. Find the Maclaurin series for the function.

$$x \sin x$$

[§9.8 #23]

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$x \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+2}$$

6. Find the first three nonzero terms of the Maclaurin series for each function and the values of x for which the series converges absolutely.

$$f(x) = x^4 e^{x^2}$$

[§9.8 #39]

$$e^{x^2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$$

$$x^4 e^{x^2} = \sum_{n=0}^{\infty} \frac{x^{2n+4}}{n!} = x^4 + x^6 + \frac{x^8}{2}$$

$$R = \lim_{n \to \infty} \frac{|a_n|}{|a_{n+1}|} = \lim_{n \to \infty} \frac{\left(\frac{1}{n!}\right)}{\left(\frac{1}{(n+1)!}\right)} = \lim_{n \to \infty} (n+1) = \infty$$

7. Use substitution to find the Taylor series at $x = 0$ of the following function.

$$\ln(3 + 6x)$$

[§9.9 #11]

let $f(x) = \ln(x+3) = \ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n-1)!}{n!\, 3^n}$

$$= \ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3^n \cdot n} x^n$$

$$f(6x) = \ln(6x+3) = \ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3^n \cdot n} (6x)^n$$

$$= \ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot 2^n}{n} x^n$$ *

8. Find the first four nonzero terms in the Maclaurin series for the function.

$$\cos(e^x - 1)$$

[§9.9 #37]

$\cos(e^x - 1) = 1 \quad \frac{x^0}{(0!)}$

$-\sin(e^x-1)e^x = 0 \quad \frac{x^2}{(2!)}$

$-\cos(e^x-1)e^{2x} - \sin(e^x-1)e^x = -1$

$\sin(e^x-1)e^{3x} - 2\cos(e^x-1)e^{2x} - \cos(e^x-1)e^{2x}$

$-\sin(e^x-1)e^x = -3 \left(\frac{x^3}{7!}\right)$

$\cos(e^x-1)e^{4x} + 3\sin(e^x-1)e^{3x} + 3\sin(e^x-1)e^{3x}$

$+6\cos(e^x-1)e^{2x} - \cos(e^x-1)e^{2x} - \sin(e^x-1)e^x$

$= -6 \left(\frac{x^4}{4!}\right)$

A: $\frac{x^0}{0!}, \frac{x^2}{2!}, \frac{x^3}{3!}, \frac{x^4}{4!}$

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