Date: / Engineering Mathematics HW1

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1.

$$\frac{dy}{dx}+2xy^2=0 \Rightarrow \frac{dy}{dx}=-2xy^2 \Rightarrow y^{-2}dy=-2x\,dx$$

$$\Rightarrow \int y^{-2}dy=\int -2x\,dx \Rightarrow -\frac{1}{y}=-x^2+C_1 \Rightarrow y=\frac{1}{x^2+C}\ *$$

2.

$$\frac{dy}{dx}=e^{3x}\cdot e^{2y} \Rightarrow \int e^{-2y}dy=\int e^{3x}dx$$

$$\Rightarrow -\frac{1}{2}e^{-2y}=\frac{1}{3}e^{3x}+C_1 \Rightarrow e^{-2y}=\frac{-2}{3}e^{3x}+C_2$$

$$\Rightarrow -2y=\ln\left(\frac{2}{3}e^{3x}+C\right) \Rightarrow y=-\frac{1}{2}\ln\left(\frac{2}{3}e^{3x}+C\right)\ *$$

3、

$$(x^2+y^2)dx+(x^2-xy)dy=0 \qquad \text{let } u=\frac{y}{x}$$

$$y=ux \Rightarrow \frac{dy}{dx}=\frac{du}{dx}x+u$$

$$\xrightarrow{\times\frac{1}{x^2}} \left(1+\left(\frac{y}{x}\right)^2\right)dx+\left(1-\frac{y}{x}\right)dy=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{u^2+1}{u-1}=\frac{du}{dx}x+u \Rightarrow \frac{u^2+1}{u-1}-u=\frac{x}{dx}du$$

$$\Rightarrow \frac{u^2+1}{u-1}-\frac{u^2-u}{u-1}=\frac{u+1}{u-1}=\frac{x}{dx}du \Rightarrow \frac{dx}{x}=\frac{u-1}{u+1}du$$

$$\Rightarrow \int\frac{1}{x}dx=\int\frac{u-1}{u+1}du+C \Rightarrow \ln|x|=u-2\ln|u+1|+C$$

$$=\frac{y}{x}-2\ln\left|\frac{y}{x}+1\right|+C\ *$$

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4.

$$y' = \frac{xy+2y-x-2}{xy-3y+x-3} = \frac{(x+2)(y-1)}{(x-3)(y+1)} = \frac{dy}{dx}$$

$$\Rightarrow \int \frac{x+2}{x-3}\,dx = \int \frac{y+1}{y-1}\,dy \Rightarrow \int 1+\frac{2}{y-1}\,dy = \int 1+\frac{5}{x-3}\,dx$$

$$\Rightarrow y+2\ln|y+1| = x+5\ln|x-3|+C \quad \#$$

5.

$$x^2+1 = x^2(3y^2-1)\frac{dy}{dx} \Rightarrow \int \frac{x^2+1}{x^2}\,dx = \int 3y^2-1\,dy$$

$$\Rightarrow \int (1+x^{-2})\,dx = \int 3y^2-1\,dy \Rightarrow y^3-y = x-\frac{1}{x}+C \quad \#$$

6.

$$y' = \frac{y}{x}+x^3, \quad y(1)=3 \quad \text{let } u=\frac{y}{x}, \quad y'=u'x+u$$

$$u'x+u = u+x^3 \Rightarrow \frac{du}{dx} = x^2 \Rightarrow u = \frac{1}{3}x^3+C_1 = \frac{y}{x} \Rightarrow y = \frac{1}{3}x^4+C_1x$$

$$y(1)=3 \rightarrow C_1 = \frac{8}{3} \Rightarrow y = \frac{1}{3}x^4+\frac{8}{3}x \quad \#$$

7.

$$\int (\cos^2 y)\,dy = x+C$$

$$\Rightarrow \frac{1}{2}\int (1+\cos 2y)\,dy = x+C$$

$$\Rightarrow \frac{1}{2}y+\frac{1}{4}\sin 2y = x+C \quad \#$$

$\cos^2 y - \sin^2 y = \cos 2y$

1) $\cos^2 y + \sin^2 y = 1$

$\cos^2 y = \frac{1}{2}(1+\cos 2y)$

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8.

$$x+y+1+4xy'+4yy'+10y'=0$$

$$\Rightarrow (x+y)+4(x+y)y'+10y'=-1 \qquad \text{let } x+y=u,\ u'=y'+1,\ y'=u'-1$$

$$\Rightarrow u+4uy'+10y'=-1$$

$$\Rightarrow u+1+(4u+10)(u'-1)=0$$

$$\Rightarrow (4u+10)u'=3u+9 \Rightarrow \int \frac{4u+10}{3u+9}du=x+C$$

$$\Rightarrow \int\left(1+\frac{u+1}{3u+9}\right)du=x+C \Rightarrow \frac{4}{3}u-\frac{2}{3}\ln|u+3|=x+C$$

$$\Rightarrow \frac{4}{3}(x+y)-\frac{2}{3}\ln|x+y+3|=x+C \quad \#$$

9.

$$\underbrace{\left(\frac{1}{x}+y\right)}_{M}dx+\underbrace{\left(3y^2+x\right)}_{N}dy=0$$

$M_y(x,y)=1$, $N_x(x,y)=1$ exact.

$$\frac{dv}{dx}=\frac{1}{x}+y \Rightarrow v=\int \frac{1}{x}+y\,dx+k(y)=\ln|x|+xy+k(y)$$

$$\frac{dv}{dy}=x+\frac{d(k(y))}{dy}=N=x+3y^2 \Rightarrow \frac{d(k(y))}{dy}=3y^2 \to k(y)=y^3$$

$$\ln|x|+xy+y^3+C=0 \quad \#$$

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10.

$$\underbrace{(\cos x - x\sin x + y^2)}_{M}\,dx + \underbrace{2xy}_{N}\,dy = 0$$

$$\Rightarrow M_y(x,y) = 2y = N_x(x,y) \quad (\text{exact}.)$$

$$\frac{dv}{dx} = \cos x - x\sin x + y^2,\quad v = \sin x + x\cos x - \sin x + xy^2 + k(y)$$

$$\frac{dv}{dy} = 2xy + \frac{d(k(y))}{dy} = 2xy,\quad ky = c^*$$

$$\sin x + x\cos x - \sin x + xy^2 + c = 0 \quad \#$$