微積分聯合教學　　會考一　　Oct 20, 2020

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 110 分。為避免產生爭議，不建議使用鉛筆作答，假設使用鉛筆，除分數加總錯誤外，均不受理成績更改。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False) 。每題 2 分。(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $\lim_{x\to c} f(x) \neq 0$ then $\lim_{x\to c} |f(x)| \neq 0$.

2. Suppose $f(x)$ is a function defined on the closed interval $[1,3]$ and $f(1)\cdot f(3) < 0$, then there exists a number c in $(1,3)$ such that $f(c) = 0$.

3. If f and g both are odd functions and the range of g lies in the domain of f, then $f(g(x))$ is an odd function of x.

4. The statement $\lim_{x\to c} f(x) = L$ means that for some $\epsilon > 0$ there exists $\delta > 0$ such that if $0 < |x-c| < \delta$ then $|f(x) - L| < \epsilon$.

5. The statement $\lim_{x\to c}(f\circ g)(x) = \lim_{x\to c}(f(g(x)) = f(\lim_{x\to c} g(x))$ is true.

6. If $f'(a)$ does not exist, then it is still possible that f is continuous at a.

7. $\lim_{x\to(-2)^+} \dfrac{8}{x^2-4} = \infty$.

8. $\dfrac{d^{99}}{dx^{99}}\cos x = \cos\left(x + \dfrac{\pi}{2}\right)$.

9. If $f(x)$ is an even function of x, then $f'(x)$ is an odd function of x.

10. If $y = \sec x$, then $\dfrac{d^2y}{dx^2} = \sec^3 x + \sec x \tan x$.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 5 分。(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find $\lim_{x\to-\infty}(2x+\sqrt{4x^2+3x-2})$.

Answer: _____.

2. Find the linearization $L(x)$ of

$$f(x)=\cos(x^2+x)+\frac{1}{2x+1}$$

at $x=0$. Answer:_____.

3. Suppose A, B are two real numbers such that $\lim_{x\to0^+} f(x)=A$ and $\lim_{x\to0^-} f(x)=B$. Find

$$\lim_{x\to0^-} f(x^3-x).$$

Answer:_____.

4. Find $\lim_{\theta\to\pi/4}\dfrac{\tan\theta-1}{\theta-\pi/4}$.

Answer:_____.

5. Let $y=x^3-3\sqrt{x}$. Find dy.

Answer:_____.

6. Find the natural domain of the function $f(x)=\sin(\sqrt{1-x})-\dfrac{1}{\sqrt{x}}$.

Answer:_____.

7. Find $\lim_{x\to0}\dfrac{x-x\cos x}{\sin^2 3x}$.

Answer:_____.

8. Find the values of a and b so that the function

$$f(x)=\begin{cases} ax^2-b, & \text{if } x>-1 \\ bx^3-2, & \text{if } x\le-1 \end{cases}$$

is differentiable for all x-values. Answer:_____.

(下頁還有試題)

計算問答證明題 **Show all your work** (*50* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。**詳列計算過程**，否則不予計分。需標明題號但不必抄題。

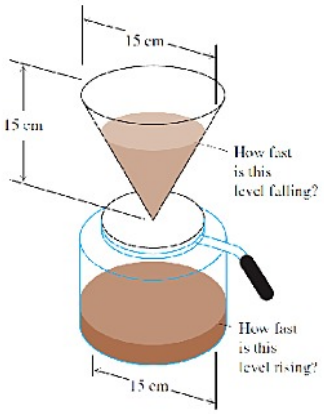
1. (*10* points) Find dy/dx of the curve

$$x^2 \cos^2(2y) - \sin(2y) = 0$$

by using implicit differentiation and find the equation of the line that is tangent to the curve at the point $\left(0, \frac{\pi}{2}\right)$.

2. (*10* points) Let $f(x)$ be a function satisfying $|f(x)| \leq x^2$ for $-1 \leq x \leq 1$. Show that f is differentiable at $x = 0$ and find $f'(0)$.

3. (*10* points) Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of 160 cm^3/min. (The volume of the circular cone with base radius r and height h is $V = \frac{1}{3}\pi h r^2$.)



a. How fast is the level in the pot rising when the coffee in the cone is 12 cm deep?

b. How fast is the level in the cone falling then?

4. (*10* points) Find all horizontal and vertical asymptotes of the graph of

$$f(x) = \frac{|x|^3 + 1}{x^3 + 2}.$$

5. (*10* points) Find dy/dx.

a. $y = (\cos^2 3x)\tan(x^{-2})$

b. $y = \dfrac{(x+1)(x+2)}{(x-1)(x-2)}$

(試題結束)

微積分聯合教學　　會考二　　Dec 01, 2020

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是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False) 。每題 2 分。(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. The function $f(x) = |x^3 - 9x|$ have 5 critical points.

2. $\int \sqrt{2x+1}\,dx = \sqrt{x^2+x} + C.$

3. If f is odd, continuous and $\int_0^1 f(x)\,dx = 3$, then $\int_{-1}^0 f(x)\,dx = -3$.

4. Suppose that $f(0) = 2$ and $f'(x) = 0$ for all $x \in \mathbb{R}$. Then $f(-1) = 2$.

5. Let $f(0) = -3$ and $f'(x) \le 5$ for all $x \in \mathbb{R}$. Then $f(2)$ can be 8.

6. $\lim_{n\to\infty} \dfrac{\sqrt{1}+\sqrt{2}+\sqrt{3}+\cdots+\sqrt{n}}{n\sqrt{n}} = \dfrac{2}{3}.$

7. If $\int_0^1 |f(x)|\,dx$ exists, then $\int_0^1 f(x)\,dx$ exists.

8. If f is continuous on (a, b), then f must attain a minimum and a maximum.

9. A continuous function is an integrable function.

10. If $f''(x_0) = 0$, then x_0 is a point of inflection of f.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 5 分。(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find $\dfrac{dy}{dx}$, if

$$y = \int_0^{\sin x} \frac{dt}{\sqrt{1-t^2}}, \quad |x| < \frac{\pi}{2}.$$

Answer: _____.

2. Calculate the smallest distance from the parabola $y^2 = 2x$ to the point $(1, 4)$.

Answer:_____.

3. Evaluate the integral

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\, dt.$$

Answer:_____.

4. Find the area of the region enclosed by the graphs of $y = x$, $y = \dfrac{x^2}{4}$ and $y = 1$.

Answer:_____.

5. Use the integral to evaluate

$$\lim_{n\to\infty}\left[\frac{1^2}{n^3} + \frac{2^2}{n^3} + \frac{3^2}{n^3} + \cdots + \frac{(n-1)^2}{n^3}\right].$$

Answer:_____.

6. Find the average value of

$$f(x) = \sqrt{16 - x^2}$$

on $[-4, 0]$. Answer:_____.

7. Suppose f is a continuous function having

$$\int_1^x f(y)dy = x^3 - 2x^2 + 1.$$

Find $f(x)$. Answer:_____.

8. Find the length of the curve

$$y = (1/3)(x^2 + 2)^{3/2}$$

from $x = 0$ to $x = 3$. Answer:_____.

(下頁還有試題)

計算問答證明題 **Show all your work** (*50* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。**詳列計算過程**，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Graph the function $f(x) = x^4 - 2x^2$.

a. Find the intervals on which f is increasing and the intervals on which f is decreasing. (3 points)

b. Find where f is concave up and where f is concave down. (3 points)

c. Plot some specific points, such as the local maximum and minimum points, inflection points. Then sketch the curve. (4 points)

2. (*10* points)

a. Find the volume of the solid generated by revolving the region bounded by

$$y = \sec x, \quad y = \sqrt{2}, \quad -\frac{\pi}{4} \le x \le \frac{\pi}{4}$$

about the x-axis. (5 points)

b. Find the volume of the solid generated by revolving the region bounded by

$$y = -x(x-2), \quad y = 0$$

about the line $x = -1$. (5 points)

3. (*10* points) Find the center of mass of a thin plate covering the region bounded above by the parabola $y = 9 - x^2$ and below by the x-axis, if the density of the plate at the point (x, y) is $\delta(x) = x^2$.

4. (*10* points) Evaluate the following integrals

a. $\int \csc(x)\cot(x)dx$. (5 points)

b. $\int_0^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3-x}}\, dx$. (5 points)

5. (*10* points) If $f(x) = x^3 + 3x + \cos x$, prove that f has at most one fixed point. (A point x_0 is called a fixed point of f if $f(x_0) = x_0$.)

(試題結束)

微積分聯合教學　　會考三　　Jan 12, 2021

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1. Suppose $f(x)$ is a function defined on the closed interval $[1,3]$ and $f(1)\cdot f(3)<0$, then there exists a number c in $(1,3)$ such that $f(c)=0$.

2. The statement $\lim\limits_{x\to c}(f\circ g)(x)=\lim\limits_{x\to c}(f(g(x))=f(\lim\limits_{x\to c}g(x))$ is true.

3. $\displaystyle\int_1^2 \frac{dx}{x(\ln x)^p}$ converges for $p<1$.

4. $\displaystyle\int_1^\infty \frac{\sin^2 x}{x^2}\,dx$ converges.

5. If $f,g:(-1,1)\to\mathbb{R}$ with $f,g>0$ on $(-1,1)$, $\lim\limits_{x\to 0}f(x)=\lim\limits_{x\to 0}g(x)=0$ then $\lim\limits_{x\to 0}f(x)^{g(x)}=1$. That is "$0^0=1$".

6. $\displaystyle\int_0^3 (x-1)^{-3}\,dx \;=\left[\frac{-1}{2(x-1)^2}\right]_0^3=\frac{3}{8}.$

7. Because $\cos\left(-\dfrac{\pi}{3}\right)=\dfrac{1}{2}$, it follows that $\cos^{-1}\left(\dfrac{1}{2}\right)=-\dfrac{\pi}{3}$.

8. If f is decreasing, concave up and $f'\neq 0$, then its inverse function is concave down.

9. The function $f(x)=x^{1/(x-1)}$ has a continuous extension to $x=1$.

10. $\displaystyle\int_0^\infty \frac{1}{x^p}dx$ diverges for all $p\in\mathbb{R}$.

(下頁還有試題)

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1. Find the values of a and b so that the function

$$f(x) = \begin{cases} ax^2 - b, & \text{if } x > -1 \\ bx^3 - 2, & \text{if } x \le -1 \end{cases}$$

is differentiable for all x-values. Answer: _____.

2. Evaluate the limit:

$$\lim_{x \to 0^+} \frac{(\tan^{-1}\sqrt{x})^2}{x\sqrt{x+1}}.$$

Answer:_____.

3. Evaluate the integral.

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec^4 x}{\tan x}\, dx$$

Answer:_____.

4. Let $f(x) = x^3 + 27\,x + 31$. Find $(f^{-1})'(31)$.

Answer:_____.

5. Evaluate the integral.

$$\int_0^1 2^{-\theta}\, d\theta$$

Answer:_____.

6. Evaluate the integral.

$$\int \cos^3\theta \sin 2\theta\, d\theta$$

Answer:_____.

7. Let

$$f(x) = \begin{cases} e^{-1/x^2}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$$

Find $f'(0)$. Answer:_____.

8. Order the following functions from slowest growing to fastest growing as $x \to \infty$.

a. e^{x^2}, **b.** x^2, **c.** $\ln(1+x^4)$, **d.** x^x.

Answer:_____.

(下頁還有試題)

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1. (*10* points) If $f(x) = x^3 + 3x + \cos x$, prove that f has at most one fixed point. (A point x_0 is called a fixed point of f if $f(x_0) = x_0$.)

2. (*10* points) Evaluate the integrals.

a. $\displaystyle\int_0^{\pi/4} \sec^3 x dx$

b. $\displaystyle\int_0^{\ln 4} \frac{e^t}{\sqrt{e^{2t}+9}}\, dt$

3. (*10* points) Determine if

$$\int_0^1 \frac{dt}{t - \sin t}$$

converges or not.

4. (*10* points) Find the limits.

a. $\displaystyle\lim_{x\to 0^+} x^x$

b. $\displaystyle\lim_{x\to\infty} \left(x - \sqrt{x^2+x}\right)$

5. (*10* points) Evaluate

$$\int \frac{5x^3 - 6x^2 + 13x - 28}{(x-1)^2(x^2+2x+5)} dx.$$

(試題結束)