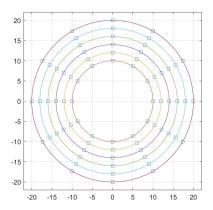
第二次作品: Python 函數繪製的觀念與技巧

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作品目標:利用 Python 函數繪製的觀念與技巧,繪製出一個簡單的圖形,說明程式碼的功能與用途。

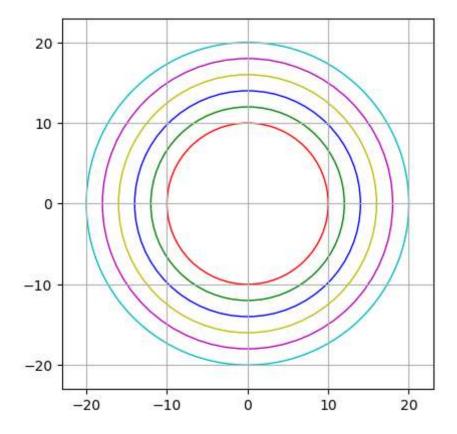
1. 繪製函數



從半徑10到20做6個園

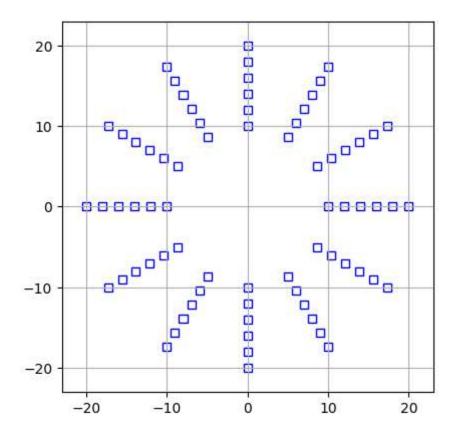
```
import matplotlib.pyplot as plt
In [ ]:
        import numpy as np
        import matplotlib.patches as patches
        # 設定圖片大小
        plt.figure(figsize=(20,2))
        fig, ax = plt.subplots()
        # 圓形
        radius = np.linspace(10, 20, 6)
        color = ['r', 'g', 'b', 'y', 'm', 'c']
        for i in range(6):
            circle = patches.Circle((0, 0), radius=radius[i], fill=False, color=color[i])
            ax.add_patch(circle)
        plt.gca().set aspect('equal', adjustable='box')
        ax.set_xlim(-23, 23)
        ax.set_ylim(-23, 23)
        plt.grid(True)
        plt.show()
```

<Figure size 2000x200 with 0 Axes>

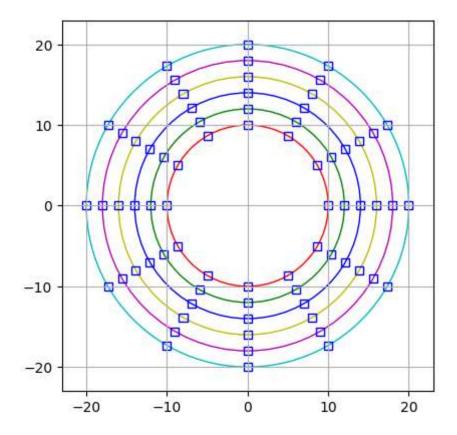


根據座標做正方形

```
In [ ]: import matplotlib.pyplot as plt
                                                                    import numpy as np
                                                                    import matplotlib.patches as patches
                                                                    fig, ax = plt.subplots()
                                                                    radius = np.linspace(10, 20, 6)
                                                                    plt.gca().set aspect('equal', adjustable='box')
                                                                    # 正方形
                                                                    square_x = np.array([1, 3**0.5/2, 1/2, 0, -1/2, -3**0.5/2, -1, -3**0.5/2, -1/2, 0, -1/2, -1/2, -1/2, 0, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1
                                                                    square_y = np.array([0, 1/2, 3**0.5/2, 1, 3**0.5/2, 1/2, 0, -1/2, -3**0.5/2, -1, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -
                                                                    for i in radius:
                                                                                                  for j in range(len(square_x)):
                                                                                                                                 square = patches.Rectangle((square_x[j]*i-0.5, square_y[j]*i-0.5), 1, 1, f:
                                                                                                                                 ax.add patch(square)
                                                                    ax.set_xlim(-23, 23)
                                                                    ax.set_ylim(-23, 23)
                                                                    plt.grid(True)
                                                                    plt.show()
```



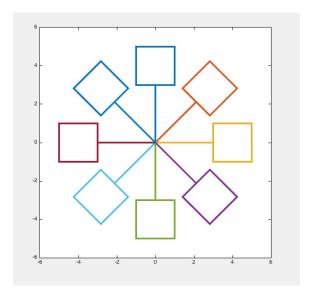
```
import matplotlib.pyplot as plt
In [ ]:
                               import numpy as np
                               import matplotlib.patches as patches
                               fig, ax = plt.subplots()
                               # 圓形
                               n = 6
                               radius = np.linspace(10, 20, n)
                               color = ['r', 'g', 'b', 'y', 'm', 'c']
                               for i in range(n):
                                              circle = patches.Circle((0, 0), radius=radius[i], fill=False, color=color[i])
                                              ax.add_patch(circle)
                               plt.gca().set_aspect('equal', adjustable='box')
                               # 正方形
                               square x = np.array([1, 3**0.5/2, 1/2, 0, -1/2, -3**0.5/2, -1, -3**0.5/2, -1/2, 0,
                               square y = np.array([0, 1/2, 3**0.5/2, 1, 3**0.5/2, 1/2, 0, -1/2, -3**0.5/2, -1, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -1/2, -
                               for i in radius:
                                              for j in range(len(square x)):
                                                             square = patches.Rectangle((square_x[j]*i-0.5, square_y[j]*i-0.5), 1, 1, f;
                                                            ax.add patch(square)
                               ax.set_xlim(-23, 23)
                               ax.set_ylim(-23, 23)
                               plt.grid(True)
                               plt.show()
```



注意事項與討論:

• 本作品的函數繪製·皆以 Python 的 matplotlib.pyplot 模組為主·並以 numpy 模組的 linspace 函數產生 x 值的陣列·以及 array 函數產生 y 值的陣列。

2. 繪製函數



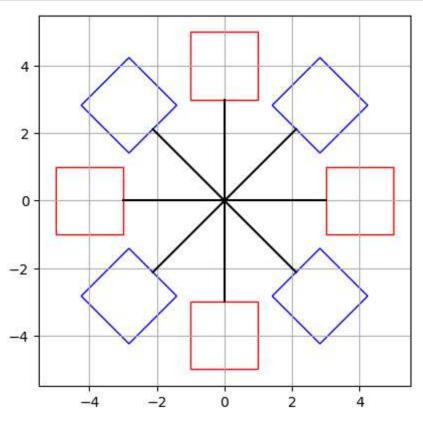
暴力解

```
import matplotlib.pyplot as plt
import numpy as np
import matplotlib.patches as patches

fig, ax = plt.subplots()

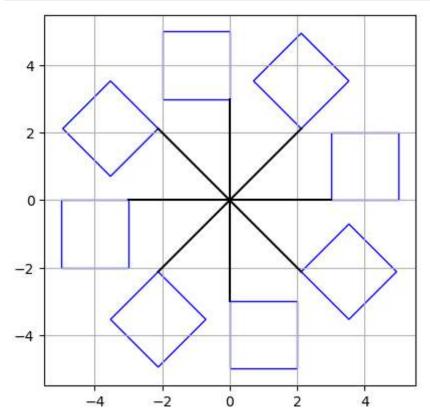
plt.plot([0,0],[3,-3],color='k')
```

```
plt.plot([3,-3],[0,0],color='k')
plt.plot([-3/2**0.5,3/2**0.5],[3/2**0.5,-3/2**0.5],color='k')
plt.plot([-3/2**0.5,3/2**0.5],[-3/2**0.5,3/2**0.5],color='k')
square = patches.Rectangle((-1, 3), 2, 2, fill=False, color='r')
ax.add patch(square)
square = patches.Rectangle((-1, -5), 2, 2, fill=False, color='r')
ax.add patch(square)
square = patches.Rectangle((3, -1), 2, 2, fill=False, color='r')
ax.add patch(square)
square = patches.Rectangle((-5, -1), 2, 2, fill=False, color='r')
ax.add patch(square)
square = patches. Rectangle((4/(2**0.5), 2/(2**0.5)), 2, 2, fill=False, color='b', a
ax.add_patch(square)
square = patches.Rectangle((4/(2**0.5), -6/(2**0.5)), 2, 2, fill=False, color='b',
ax.add patch(square)
square = patches. Rectangle((-4/(2**0.5), 2/(2**0.5)), 2, 2, fill=False, color='b',
ax.add patch(square)
square = patches. Rectangle((-4/(2**0.5), -6/(2**0.5)), 2, 2, fill=False, color='b'
ax.add_patch(square)
plt.gca().set aspect('equal', adjustable='box')
plt.grid(True)
plt.show()
```



讓n變成變數,但正方形沒有對齊

```
import matplotlib.pyplot as plt
In [ ]:
        import numpy as np
        import matplotlib.patches as patches
        fig, ax = plt.subplots()
        n = 8
        angle = 360 / n
        radius = 3
        for i in range(n):
            plt.plot([0, radius*np.cos(np.deg2rad(angle*i))], [0, radius*np.sin(np.deg2rad
            ori_x = radius*np.cos(np.deg2rad(angle*i))
            ori y = radius*np.sin(np.deg2rad(angle*i))
            square = patches.Rectangle((ori_x,ori_y), 2, 2, fill=False, color='b', angle=ar
            ax.add patch(square)
        plt.gca().set_aspect('equal', adjustable='box')
        plt.grid(True)
        plt.show()
```



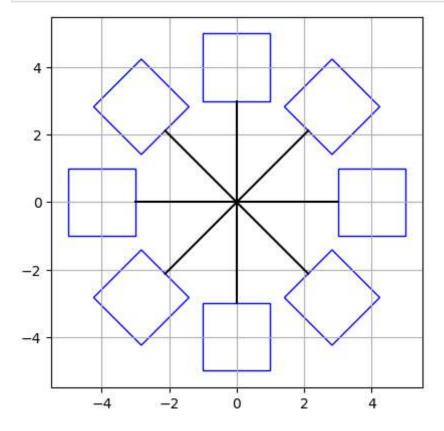
調整正方形的xy座標

```
import matplotlib.pyplot as plt
import numpy as np
import matplotlib.patches as patches

fig, ax = plt.subplots()
n = 8
angle = 360 / n
radius = 3
for i in range(n):
```

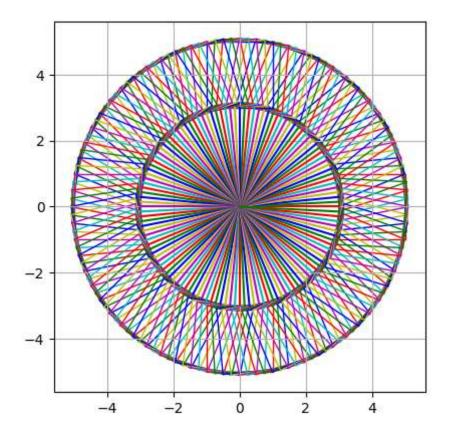
```
plt.plot([0, radius*np.cos(np.deg2rad(angle*i))], [0, radius*np.sin(np.deg2rad
    ori_x = radius*np.cos(np.deg2rad(angle*i))
    ori_y = radius*np.sin(np.deg2rad(angle*i))
    x = radius*np.cos(np.deg2rad(angle*i)) + (2 / 2 * np.sin(np.deg2rad(angle*i)))
    y = radius*np.sin(np.deg2rad(angle*i)) - (2 / 2 * np.cos(np.deg2rad(angle*i)))
    square = patches.Rectangle((x,y), 2, 2, fill=False, color='b', angle=angle*i)
    ax.add_patch(square)

plt.gca().set_aspect('equal', adjustable='box')
    plt.grid(True)
    plt.show()
```



n = 128

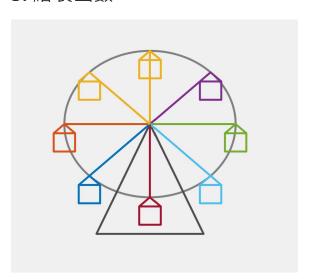
```
In [ ]:
        import matplotlib.pyplot as plt
        import numpy as np
        import matplotlib.patches as patches
        fig, ax = plt.subplots()
        n = 128
        angle = 360 / n
        radius = 3
        color = ['r', 'g', 'b', 'y', 'm', 'c']
        for i in range(n):
            plt.plot([0, radius*np.cos(np.deg2rad(angle*i))], [0, radius*np.sin(np.deg2rad
            ori_x = radius*np.cos(np.deg2rad(angle*i))
            ori_y = radius*np.sin(np.deg2rad(angle*i))
            x = radius*np.cos(np.deg2rad(angle*i)) + (2 / 2 * np.sin(np.deg2rad(angle*i)))
            y = radius*np.sin(np.deg2rad(angle*i)) - (2 / 2 * np.cos(np.deg2rad(angle*i)))
            square = patches.Rectangle((x,y), 2, 2, fill=False, color=color[i%6], angle=angle
            ax.add patch(square)
        plt.gca().set_aspect('equal', adjustable='box')
        plt.grid(True)
        plt.show()
```



注意事項與討論:

• 本作品的函數繪製·皆以 Python 的 matplotlib.pyplot 模組為主·並以 numpy 模組的 linspace 函數產生 x 值的陣列·以及 array 函數產生 y 值的陣列。

3. 繪製函數

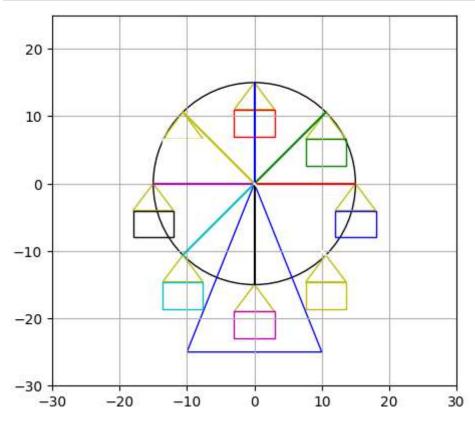


```
import matplotlib.pyplot as plt
import numpy as np
import matplotlib.patches as patches

fig,ax = plt.subplots()

n = 8
angle = 360 / n
radius = 15
```

```
circle = patches.Circle((0, 0), radius=radius, fill=False)
ax.add_patch(circle)
# 繪製三角形
triangle = patches.Polygon([[0, 0], [10, -25], [-10, -25]], fill=False, color='b')
ax.add patch(triangle)
color = ['r', 'g', 'b', 'y', 'm', 'c', 'k', 'w']
for i in range(n):
    plt.plot([0, radius*np.cos(np.deg2rad(angle*i))], [0, radius*np.sin(np.deg2rad
    original x = radius*np.sin(np.deg2rad(angle*i))
    original y = radius*np.cos(np.deg2rad(angle*i))
    triangle = patches.Polygon([[original_x, original_y], [original_x - 3, original_
    ax.add patch(triangle)
    square = patches.Rectangle((original_x - 3, original_y - 8), 6, 4, fill=False,
    ax.add_patch(square)
plt.gca().set_aspect('equal', adjustable='box')
plt.xlim(-30, 30)
plt.ylim(-30, 25)
plt.grid(True)
plt.show()
```



3. 計算卡方右尾面積與自由度對照表

```
In [ ]: from scipy.stats import chi2
import numpy as np
```

```
import pandas as pd

col_F = np.array([0.995, 0.99, 0.975, 0.95, 0.9, 0.1, 0.05, 0.025, 0.01, 0.005])
row_df = np.array([i for i in range(1, 31)])

degrees_of_freedom = pd.DataFrame()

for col_f_value in col_F:
    df = chi2.ppf(col_f_value, row_df)
    degrees_of_freedom.index = row_df

degrees_of_freedom.index = row_df

degrees_of_freedom.columns = np.flip(col_F)

degrees_of_freedom.to_excel('chi2.xlsx')
print(degrees_of_freedom)
```

```
0.025
                                                       0.100
        0.005
                    0.010
                                            0.050
                                                                   0.900
                            5.023886
                                        3.841459
                                                    2.705543
1
     7.879439
                 6.634897
                                                                0.015791
2
    10.596635
                 9.210340
                            7.377759
                                         5.991465
                                                    4.605170
                                                                0.210721
3
    12.838156
                11.344867
                            9.348404
                                         7.814728
                                                    6.251389
                                                                0.584374
4
    14.860259
                13.276704
                           11.143287
                                        9.487729
                                                    7.779440
                                                                1.063623
5
    16.749602
                15.086272
                           12.832502
                                       11.070498
                                                    9.236357
                                                                1.610308
    18.547584
6
                16.811894
                           14.449375
                                       12.591587
                                                   10.644641
                                                                2.204131
7
    20.277740
                18.475307
                           16.012764
                                       14.067140
                                                   12.017037
                                                                2.833107
8
    21.954955
                20.090235
                           17.534546
                                       15.507313
                                                   13.361566
                                                                3.489539
9
                21.665994
                           19.022768
    23.589351
                                       16.918978
                                                   14.683657
                                                                4.168159
10
    25.188180
                23.209251
                           20.483177
                                       18.307038
                                                   15.987179
                                                                4.865182
    26.756849
                24.724970
                           21.920049
                                       19.675138
                                                   17.275009
                                                                5.577785
11
12
    28.299519
               26.216967
                           23.336664
                                       21.026070
                                                   18.549348
                                                                6.303796
13
    29.819471
                27.688250
                           24.735605
                                       22.362032
                                                   19.811929
                                                                7.041505
14
    31.319350
                29.141238
                           26.118948
                                       23.684791
                                                   21.064144
                                                                7.789534
15
    32.801321
                30.577914
                           27.488393
                                       24.995790
                                                   22.307130
                                                                8.546756
                31.999927
                                       26.296228
                                                   23.541829
16
    34.267187
                           28.845351
                                                                9.312236
17
                33.408664
                           30.191009
                                       27.587112
                                                   24.769035
    35.718466
                                                               10.085186
18
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                34.805306
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                                       28.869299
                                                   25.989423
                                                               10.864936
19
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                36.190869
                           32.852327
                                       30.143527
                                                   27.203571
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                                       31.410433
                                                   28.411981
20
    39.996846
                37.566235
                           34.169607
                                                               12.442609
21
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                                       32.670573
                                                   29.615089
                                                               13.239598
22
    42.795655
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                           36.780712
                                       33.924438
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                                                               14.041493
23
    44.181275
               41.638398
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                                       35.172462
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                                                               14.847956
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               42.979820
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                                       36.415029
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    46.927890
               44.314105
                           40.646469
                                       37.652484
25
                                                   34.381587
                                                               16.473408
26
    48.289882
               45.641683
                                       38.885139
                           41.923170
                                                   35.563171
                                                               17.291885
27
    49.644915
                46.962942
                           43.194511
                                       40.113272
                                                   36.741217
                                                               18.113896
28
    50.993376
                48.278236
                           44.460792
                                       41.337138
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29
    52.335618
                49.587884
                           45.722286
                                       42.556968
                                                   39.087470
                                                               19.767744
30
    53.671962
                50.892181
                           46.979242
                                       43.772972
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                                            0.995
        0.950
                                0.990
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                 0.000982
                            0.000157
                                         0.000039
2
     0.102587
                 0.050636
                            0.020101
                                         0.010025
3
     0.351846
                 0.215795
                                         0.071722
                            0.114832
     0.710723
4
                 0.484419
                             0.297109
                                         0.206989
5
     1.145476
                 0.831212
                             0.554298
                                         0.411742
6
     1.635383
                 1.237344
                             0.872090
                                         0.675727
7
                 1.689869
     2.167350
                             1.239042
                                         0.989256
8
     2.732637
                 2.179731
                            1.646497
                                         1.344413
9
     3.325113
                 2.700389
                             2.087901
                                         1.734933
10
                 3.246973
     3.940299
                             2.558212
                                         2.155856
11
     4.574813
                 3.815748
                             3.053484
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12
     5.226029
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                             3.570569
                                         3.073824
13
     5.891864
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                                         9.886234
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```