Optimization Methods: PhD Course Introduction

A Introduction

Optimization is at the heart of modern science, engineering, and data-driven decision making. This course provides a rigorous and comprehensive introduction to the theory and practice of optimization methods, with a focus on both classical and advanced techniques relevant for PhD-level research and applications.

A.1 Course Objectives

- Understand the mathematical foundations of linear, integer, and combinatorial optimization.
- Master classical algorithms such as the Simplex Method, Branch-and-Bound, and Cutting Planes.
- Explore advanced topics including duality, decomposition, column generation, and Branch-and-Price.
- Develop the ability to model, analyze, and solve real-world optimization problems.
- Gain insight into current research directions and open problems in optimization.

A.2 Course Structure

The course is organized into two main phases:

1. **Classic Methods:** Linear programming, Simplex, Integer programming, Branch-and-Bound, Cutting Planes.
2. **Advanced Methods:** Duality, decomposition, column generation, Branch-and-Price, and large-scale optimization.

Each topic is presented with formal theory, mathematical proofs, illustrative examples, and practical exercises.

A.3 Prerequisites

A solid background in linear algebra, basic mathematical analysis, and some familiarity with algorithms is recommended. Familiarity with C++ programming is also helpful, as computational examples and assignments use C++.

A.4 Evaluation Criteria

The final grade for the course is determined as follows:

Phase	**Description**	**Weight**
First Phase	Activity 1: Implementation and Report using a commercial solver	15%
	Evidence 1: Report on the implementation of a cutting planes generation method	20%
Second Phase	Activity 2: Duality exercises	15%
	Evidence 2: Implementation and Report of a column generation method	20%
Final Project	Implementation and Report of a Branch-and-Price method	30%

B References

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[2] L. A. Wolsey, *Integer Programming*, Wiley-Interscience, 1998.

[3] G. L. Nemhauser and L. A. Wolsey, *Integer and Combinatorial Optimization*, Wiley-Interscience, 1988.

[4] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[5] A. Schrijver, *Theory of Linear and Integer Programming*, Wiley, 1998.

[6] M. Conforti, G. Cornuéjols, and G. Zambelli, *Integer Programming*, Springer, 2014.

[7] R. J. Vanderbei, *Linear Programming: Foundations and Extensions*, Springer, 2020.

[8] R. K. Martin, *Large Scale Linear and Integer Optimization: A Unified Approach*, Springer, 1999.