Transportation Problem

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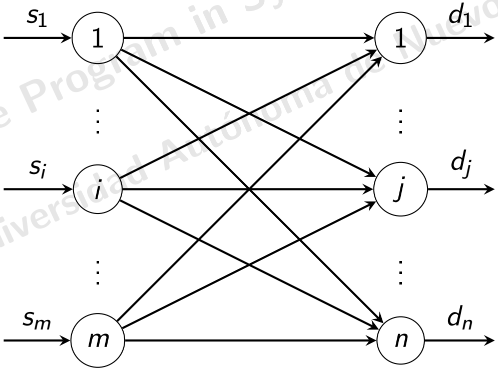
Content

Agenda

1. Optimization Model
2. Balanced Transportation Problem
3. Solution Approaches

Definition

The transportation problem is a special type of LP problem where the objective is to minimize the cost of transportation of a product from a number of origins (suppliers) to a number of destinations (customers). Each origin has a number of products that it can supply, and each destination has a demand of products.



General Model

Decision

Number of products to transport from each origin to each destination.

Objective

Minimize the total transportation cost.

Constraints

- The origins' supply capacity cannot be exceeded.
- The destinations' demand has to be satisfied.

General Model

Decision Variables

- x_{ij}: the number of products sent from origin i to destination j.

Objective

- $z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$, where c_{ij} is the cost of transporting one unit of product from origin i to destination j.

Constraints

- Supply Capacity of Origin i: $\sum_{j=1}^{n} x_{ij} \leq s_i$,
- Demand of destination j: $\sum_{i=1}^{m} x_{ij} \geq d_j$.

General Model

The Mathematical Model:

$$
\begin{aligned}
Min \quad z = & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} && && \\
s.t. \quad & \sum_{j=1}^{n} x_{ij} && \leq s_i, && \forall i \in \{1, ..., m\}, \\
& \sum_{i=1}^{m} x_{ij} && \geq d_j, && \forall j \in \{1, ..., n\}, \\
& x_{ij} && \geq 0, && \forall i \in \{1, ..., m\}, \ \forall j \in \{1, ..., n\}.
\end{aligned}
$$

Agenda

1. Optimization Model
2. Balanced Transportation Problem
3. Solution Approaches

Definition

Definition of a Balanced Transportation Problem

A transportation problem is said **balanced** when the total number of products that the origins can supply is equal to the total demand of the destinations (i.e. $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$).

Mathematical Model

Mathematical Model of a Balanced Transportation Problem

$$
\begin{aligned}
Min \quad z = \ & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} & & \\
s.t. \quad & \sum_{j=1}^{n} x_{ij} & = \ & s_i, \quad \forall i \in \{1, ..., m\}, \\
& \sum_{i=1}^{m} x_{ij} & = \ & d_j, \quad \forall j \in \{1, ..., n\}, \\
& x_{ij} & \geq \ & 0, \quad \forall i \in \{1, ..., m\}, \ \forall j \in \{1, ..., n\}.
\end{aligned}
$$

The PowerCo Problem

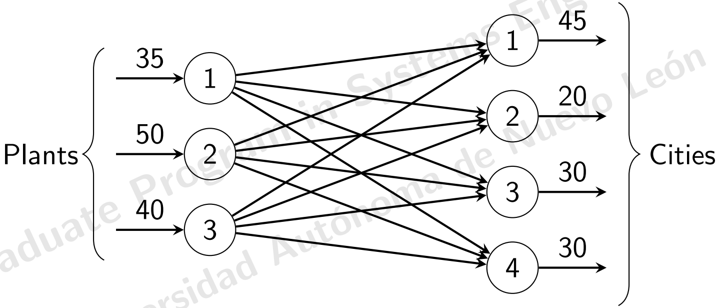
PowerCo has three electric power plants that supply the power needs of four cities. Each power plant can supply the following numbers of kilowatt-hours (kwh) of electricity: plant 1, 35 million; plant 2, 50 million; plant 3, 40 million. The peak power demands in these cities which occur at the same time (2 p.m.), are as follows (in kwh): city 1, 45 million; city 2, 20 million; city 3, 30 million; city 4, 30 million.

The costs of sending 1 million kwh of electricity from plant to city depend on the distance the electricity must travel. Formulate an LP to minimize the cost of meeting each city's peak power demand.

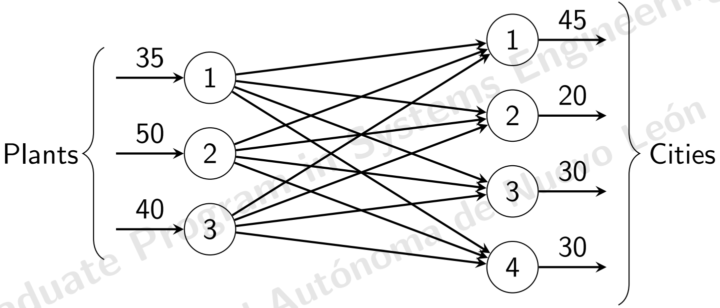
	TO				
FROM	City 1	City 2	City 3	City 4	SUPPLY*
Plant 1	\$8	\$6	\$10	\$9	35
Plant 2	\$9	\$12	\$13	\$7	50
Plant 3	\$14	\$9	\$16	\$5	40
DEMAND*	45	20	30	30	

*Million kwh

The PowerCo Problem



The PowerCo Problem



125 = 125

BALANCED TP

The PowerCo Problem

Decision Variables

- x_{ij}: the number of Mkwh of electricity sent from Plant i to City j.

Objective

- $$\begin{aligned} z = \quad & 8x_{11}+ \quad 6x_{12}+ \quad 10x_{13}+ \quad 9x_{14}+ \\ & 9x_{21}+ \quad 12x_{22}+ \quad 13x_{23}+ \quad 7x_{24}+ \\ & 14x_{31}+ \quad 9x_{32}+ \quad 16x_{33}+ \quad 5x_{34}. \end{aligned}$$

Constraints

- Supply of Plants.
- Demand of Cities.

The PowerCo Problem

Decision Variables

Objective

Constraints

- Supply of Plant 1: $x_{11} + \quad x_{12} + \quad x_{13} + \quad x_{14} = \quad 35$
  Supply of Plant 2: $x_{21} + \quad x_{22} + \quad x_{23} + \quad x_{24} = \quad 50$
  Supply of Plant 3: $x_{31} + \quad x_{32} + \quad x_{33} + \quad x_{34} = \quad 40$
- Demand of City 1: $x_{11} + \quad x_{21} + \quad x_{31} = \quad 45$
  Demand of City 2: $x_{12} + \quad x_{22} + \quad x_{32} = \quad 20$
  Demand of City 3: $x_{13} + \quad x_{23} + \quad x_{33} = \quad 30$
  Demand of City 4: $x_{14} + \quad x_{24} + \quad x_{34} = \quad 30$

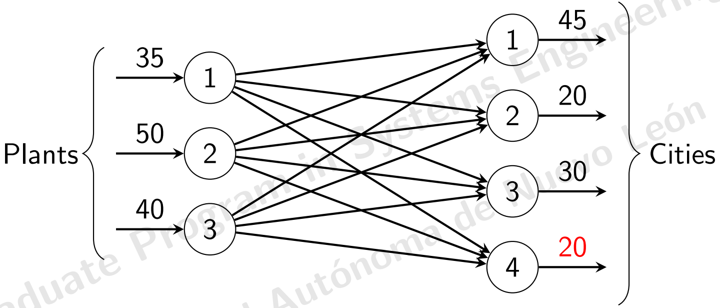
The PowerCo Problem

The Model

$$\begin{array}{rrrrrrl}
Min\ z = & 8x_{11}+ & 6x_{12}+ & 10x_{13}+ & 9x_{14}+ & & \\
 & 9x_{21}+ & 12x_{22}+ & 13x_{23}+ & 7x_{24}+ & & \\
 & 14x_{31}+ & 9x_{32}+ & 16x_{33}+ & 5x_{34} & & \\
s.t. & x_{11}+ & x_{12}+ & x_{13}+ & x_{14} & = & 35 \\
 & x_{21}+ & x_{22}+ & x_{23}+ & x_{24} & = & 50 \\
 & x_{31}+ & x_{32}+ & x_{33}+ & x_{34} & = & 40 \\
 & x_{11}+ & x_{21}+ & x_{31} & & = & 45 \\
 & x_{12}+ & x_{22}+ & x_{32} & & = & 20 \\
 & x_{13}+ & x_{23}+ & x_{33} & & = & 30 \\
 & x_{14}+ & x_{24}+ & x_{34} & & = & 30 \\
\end{array}$$

$$x_{ij} \geq 0,\ \forall i \in \{1,2,3\},\ \forall j \in \{1,2,3,4\}$$

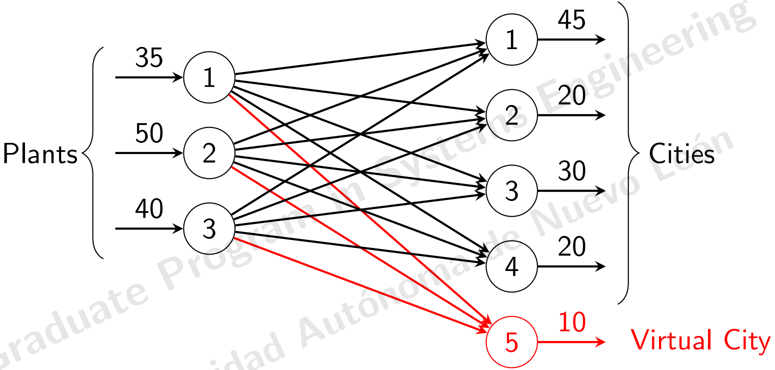
Supply > Demand



125 > 115

UNBALANCED TP

Supply > Demand



125 = 125

BALANCED TP

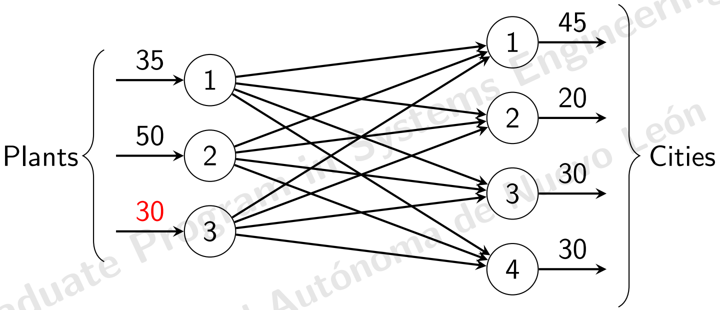
Supply > Demand

The Model

$$
\begin{array}{llllllll}
Min\ z = & 8x_{11}+ & 6x_{12}+ & 10x_{13}+ & 9x_{14}+ & & & \\
 & 9x_{21}+ & 12x_{22}+ & 13x_{23}+ & 7x_{24}+ & & & \\
 & 14x_{31}+ & 9x_{32}+ & 16x_{33}+ & 5x_{34} & & & \\
s.t. & x_{11}+ & x_{12}+ & x_{13}+ & x_{14}+ & \color{red}{x_{15}} & = & 35 \\
 & x_{21}+ & x_{22}+ & x_{23}+ & x_{24}+ & \color{red}{x_{25}} & = & 50 \\
 & x_{31}+ & x_{32}+ & x_{33}+ & x_{34}+ & \color{red}{x_{35}} & = & 40 \\
 & x_{11}+ & x_{21}+ & x_{31} & & & = & 45 \\
 & x_{12}+ & x_{22}+ & x_{32} & & & = & 20 \\
 & x_{13}+ & x_{23}+ & x_{33} & & & = & 30 \\
 & x_{14}+ & x_{24}+ & x_{34} & & & = & 20 \\
 & \color{red}{x_{15}+} & \color{red}{x_{25}+} & \color{red}{x_{35}} & & & \color{red}{=} & \color{red}{10}
\end{array}
$$

$$x_{ij} \geq 0,\ \forall i \in \{1,2,3\},\ \forall j \in \{1,2,3,4,\color{red}{5}\}$$

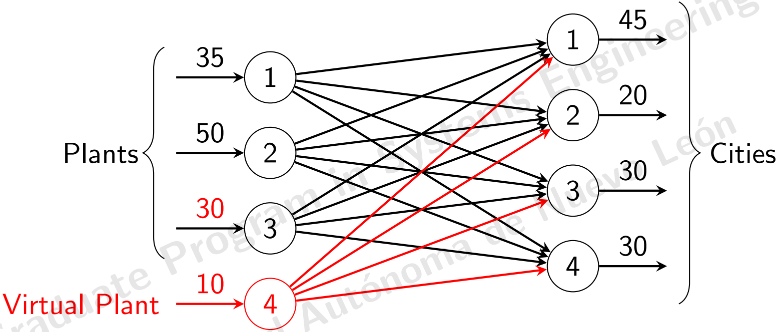
Supply < Demand



115 < 125

UNBALANCED TP

Supply $<$ Demand



125 = 125

BALANCED TP

Supply < Demand

The Model

$$\begin{array}{rrrrrcl}
Min\ z = & 8x_{11}+ & 6x_{12}+ & 10x_{13}+ & 9x_{14}+ & & \\
 & 9x_{21}+ & 12x_{22}+ & 13x_{23}+ & 7x_{24}+ & & \\
 & 14x_{31}+ & 9x_{32}+ & 16x_{33}+ & 5x_{34}+ & & \\
 & 20x_{41}+ & 40x_{42}+ & 30x_{43}+ & 10x_{44} & & \\
s.t. & x_{11}+ & x_{12}+ & x_{13}+ & x_{14} & = & 35 \\
 & x_{21}+ & x_{22}+ & x_{23}+ & x_{24} & = & 50 \\
 & x_{31}+ & x_{32}+ & x_{33}+ & x_{34} & = & 30 \\
 & x_{41}+ & x_{42}+ & x_{43}+ & x_{44} & = & 10 \\
 & x_{11}+ & x_{21}+ & x_{31}+ & x_{41} & = & 45 \\
 & x_{12}+ & x_{22}+ & x_{32}+ & x_{42} & = & 20 \\
 & x_{13}+ & x_{23}+ & x_{33}+ & x_{43} & = & 30 \\
 & x_{14}+ & x_{24}+ & x_{34}+ & x_{44} & = & 30 \\
 & \multicolumn{6}{l}{x_{ij} \geq 0,\ \forall i \in \{1,2,3,4\},\ \forall j \in \{1,2,3,4\}}
\end{array}$$

Supply $<$ Demand

The Model

$$
\begin{array}{lrrrrll}
Min\ z = & 8x_{11}+ & 6x_{12}+ & 10x_{13}+ & 9x_{14}+ & & \\
 & 9x_{21}+ & 12x_{22}+ & 13x_{23}+ & 7x_{24}+ & & \\
 & 14x_{31}+ & 9x_{32}+ & 16x_{33}+ & 5x_{34}+ & & \\
 & 20x_{41}+ & 40x_{42}+ & 30x_{43}+ & 10x_{44} & & \rightarrow \textit{Penalization Cost} \\
s.t. & x_{11}+ & x_{12}+ & x_{13}+ & x_{14} = & 35 & \\
 & x_{21}+ & x_{22}+ & x_{23}+ & x_{24} = & 50 & \\
 & x_{31}+ & x_{32}+ & x_{33}+ & x_{34} = & 30 & \\
 & x_{41}+ & x_{42}+ & x_{43}+ & x_{44}= & 10 & \\
 & x_{11}+ & x_{21}+ & x_{31}+ & x_{41} = & 45 & \\
 & x_{12}+ & x_{22}+ & x_{32}+ & x_{42} = & 20 & \\
 & x_{13}+ & x_{23}+ & x_{33}+ & x_{43} = & 30 & \\
 & x_{14}+ & x_{24}+ & x_{34}+ & x_{44} = & 30 & \\
\end{array}
$$

$$x_{ij} \geq 0,\ \forall i \in \{1,2,3,4\},\ \forall j \in \{1,2,3,4\}$$

Agenda

1. Optimization Model
2. Balanced Transportation Problem
3. Solution Approaches

Northwest Corner Method

Cities

Plants	1	2	3	4	
1	8	6	10	9	35
2	9	12	13	7	50
3	14	9	16	5	40
	45	20	30	30	

$z = 0$

Northwest Corner Method

Cities

Plants	1	2	3	4	
1	8 35	6	10	9	~~35~~ 0
2	9	12	13	7	50
3	14	9	16	5	40
	45	20	30	30	

$z = 0$

Northwest Corner Method

Cities

Plants

	1	2	3	4	
1	8 35	▨ 6	▨ 10	▨ 9	~~35~~ 0
2	9	12	13	7	50
3	14	9	16	5	40
	~~45~~ 10	20	30	30	

$z = 8 \times 35$

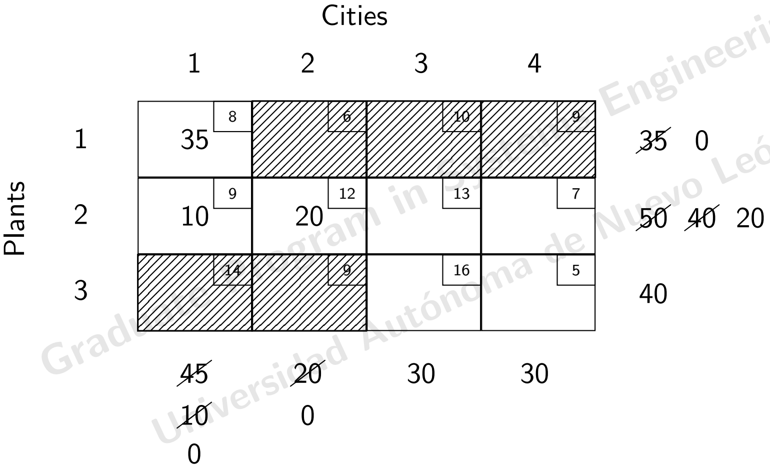
Northwest Corner Method

Cities

Plants	1	2	3	4	
1	8 35	▨ 6	▨ 10	▨ 9	~~35~~ 0
2	9 10	12	13	7	~~50~~ 40
3	▨ 14	9	16	5	40
	~~45~~ ~~10~~ 0	20	30	30	

$$z = 8 \times 35 + 9 \times 10$$

Northwest Corner Method



Cities

Plants

	1	2	3	4	
1	8 35	▨ 6	▨ 10	▨ 9	~~35~~ 0
2	9 10	12 20	13	7	~~50~~ ~~40~~ 20
3	▨ 14	▨ 9	16	5	40
	~~45~~ ~~10~~ 0	~~20~~ 0	30	30	

$$z = 8 \times 35 + 9 \times 10 + 12 \times 20$$

Northwest Corner Method

Cities

		1	2	3	4	
Plants	1	8 35	▨ 6	▨ 10	▨ 9	~~35~~ 0
	2	9 10	12 20	13 20	▨ 7	~~50~~ ~~40~~ ~~20~~ 0
	3	▨ 14	▨ 9	16	5	40
		~~45~~ ~~10~~ 0	~~20~~ 0	~~30~~ 10	30	

$$z = 8 \times 35 + 9 \times 10 + 12 \times 20 + 13 \times 20$$

Northwest Corner Method

Cities

Plants

		1	2	3	4	
Plants	1	8 35	▨ 6	▨ 10	▨ 9	~~35~~ 0
	2	9 10	12 20	13 20	▨ 7	~~50~~ ~~40~~ ~~20~~ 0
	3	▨ 14	▨ 9	16 10	5	~~40~~ 30
		~~45~~ ~~10~~ 0	~~20~~ 0	~~30~~ ~~10~~ 0	30	

$$z = 8 \times 35 + 9 \times 10 + 12 \times 20 + 13 \times 20 + 16 \times 10$$

Northwest Corner Method

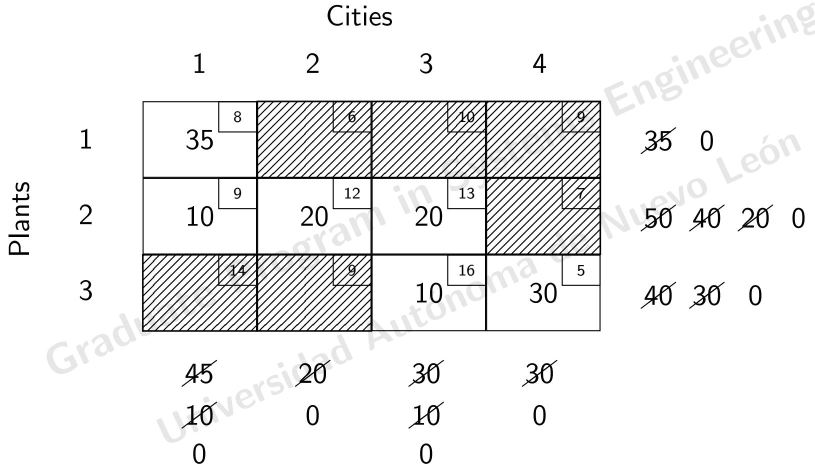
Cities

Plants

	1	2	3	4	
1	8 35	▨ 6	▨ 10	▨ 9	~~35~~ 0
2	9 10	12 20	13 20	▨ 7	~~50~~ ~~40~~ ~~20~~ 0
3	▨ 14	▨ 9	16 10	5 30	~~40~~ ~~30~~ 0
	~~45~~ ~~10~~ 0	~~20~~ 0	~~30~~ ~~10~~ 0	~~30~~ 0	

$$z = 8 \times 35 + 9 \times 10 + 12 \times 20 + 13 \times 20 + 16 \times 10 + 5 \times 30$$

Northwest Corner Method



		Cities 1	2	3	4	
Plants	1	8 35	▨ 6	▨ 10	▨ 9	~~35~~ 0
	2	9 10	12 20	13 20	▨ 7	~~50~~ ~~40~~ ~~20~~ 0
	3	▨ 14	▨ 9	16 10	5 30	~~40~~ ~~30~~ 0
		~~45~~ ~~10~~ 0	~~20~~ 0	~~30~~ ~~10~~ 0	~~30~~ 0	

$z = 1180$

Minimum Cost Method

Cities

Plants

Plants	1	2	3	4	
1	8	6	10	9	35
2	9	12	13	7	50
3	14	9	16	5	40
	45	20	30	30	

$z = 0$

Minimum Cost Method

Cities

Plants

	1	2	3	4	
1	8	6	10	9 ▨	35
2	9	12	13	7 ▨	50
3	14	9	16	5 30	~~40~~ 10
	45	20	30	~~30~~ 0	

$z = 5 \times 30$

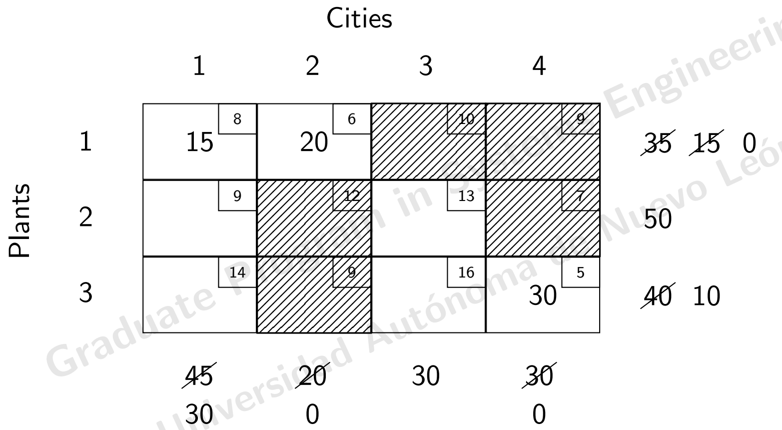
Minimum Cost Method

Cities

Plants	1	2	3	4	
1	8	6 20	10	▨ 9	~~35~~ 15
2	9	▨ 12	13	▨ 7	50
3	14	▨ 9	16	5 30	~~40~~ 10
	45	~~20~~ 0	30	~~30~~ 0	

$$z = 5 \times 30 + 6 \times 20$$

Minimum Cost Method



Cities

Plants	1	2	3	4	
1	8 15	6 20	▨ 10	▨ 9	~~35~~ ~~15~~ 0
2	9	▨ 12	13	▨ 7	50
3	14	▨ 9	16	5 30	~~40~~ 10
	~~45~~ 30	~~20~~ 0	30	~~30~~ 0	

$$z = 5 \times 30 + 6 \times 20 + 8 \times 15$$

Minimum Cost Method

Cities

		1	2	3	4	
Plants	1	$\boxed{8}$ 15	$\boxed{6}$ 20	▨ $\boxed{10}$	▨ $\boxed{9}$	~~35~~ ~~15~~ 0
	2	$\boxed{9}$ 30	▨ $\boxed{12}$	$\boxed{13}$	▨ $\boxed{7}$	~~50~~ 20
	3	▨ $\boxed{14}$	▨ $\boxed{9}$	$\boxed{16}$	$\boxed{5}$ 30	~~40~~ 10
		~~45~~ ~~30~~ 0	~~20~~ 0	30	~~30~~ 0	

$$z = 5 \times 30 + 6 \times 20 + 8 \times 15 + 9 \times 30$$

Minimum Cost Method

Cities

Plants	1	2	3	4	
1	8 15	6 20	10 ▨	9 ▨	~~35~~ ~~15~~ 0
2	9 30	12 ▨	13 20	7 ▨	~~50~~ ~~20~~ 0
3	14 ▨	9 ▨	16	5 30	~~40~~ 10
	~~45~~ ~~30~~ 0	~~20~~ 0	~~30~~ 10	~~30~~ 0	

$$z = 5 \times 30 + 6 \times 20 + 8 \times 15 + 9 \times 30 + 13 \times 20$$

Minimum Cost Method

Cities

Plants	1	2	3	4	
1	8 15	6 20	▨ 10	▨ 9	~~35~~ ~~15~~ 0
2	9 30	▨ 12	13 20	▨ 7	~~50~~ ~~20~~ 0
3	▨ 14	▨ 9	16 10	5 30	~~40~~ ~~10~~ 0
	~~45~~ ~~30~~ 0	~~20~~ 0	~~30~~ ~~10~~ 0	~~30~~ 0	

$$z = 5 \times 30 + 6 \times 20 + 8 \times 15 + 9 \times 30 + 13 \times 20 + 16 \times 10$$

Minimum Cost Method

Cities

Plants	1	2	3	4	
1	8 15	6 20	▨ 10	▨ 9	~~35~~ ~~15~~ 0
2	9 30	▨ 12	13 20	▨ 7	~~50~~ ~~20~~ 0
3	▨ 14	▨ 9	16 10	5 30	~~40~~ ~~10~~ 0
	~~45~~ ~~30~~ 0	~~20~~ 0	~~30~~ ~~10~~ 0	~~30~~ 0	

$z = 1080$

Vogel Approximation Method

Cities

Plants

	1	2	3	4	
1	8	6	10	9	35
2	9	12	13	7	50
3	14	9	16	5	40
	45	20	30	30	

$z = 0$

Vogel Approximation Method

Cities

Plants

	1	2	3	4		
1	8	6	10	9	35	8-6=2
2	9	12	13	7	50	9-7=2
3	14	9	16	5	40	9-5=4
	45	20	30	30		
	9-8=1	9-6=3	13-10=3	7-5=2		

$z = 0$

Vogel Approximation Method

Cities

Plants

	1	2	3	4		
1	8	6	10	▨ 9	35	8-6=2
2	9	12	13	▨ 7	50	9-7=2
3	14	9	16	5 30	~~40~~ 10	9-5=4
	45	20	30	~~30~~ 0		
	9-8=1	9-6=3	13-10=3	7-5=2		

$$z = 5 \times 30$$

Vogel Approximation Method

Cities

Plants	1	2	3	4		
1	8	6	10	▨ 9	35	8-6=2
2	9	12	13	▨ 7	50	12-9=3
3	14	9	16	5 30	~~40~~ 10	14-9=5
	45	20	30	~~30~~ 0		
	9-8=1	9-6=3	13-10=3	-		

$z = 5 \times 30$

Vogel Approximation Method

Cities

Plants

	1	2	3	4		
1	8	6	10	▨ 9	35	8-6=2
2	9	12	13	▨ 7	50	12-9=3
3	▨ 14	9 10	▨ 16	5 30	~~40~~ ~~10~~ 0	14-9=5
	45	~~20~~ 10	30	~~30~~ 0		
	9-8=1	9-6=3	13-10=3	-		

$z = 5 \times 30 + 9 \times 10$

Vogel Approximation Method

Cities

Plants	1	2	3	4		
1	8	6	10	▨ 9	35	8-6=2
2	9	12	13	▨ 7	50	12-9=3
3	▨ 14	9 10	▨ 16	5 30	~~40~~ ~~10~~ 0	-
	45	~~20~~ 10	30	~~30~~ 0		
	9-8=1	12-6=6	13-10=3	-		

$z = 5 \times 30 + 9 \times 10$

Vogel Approximation Method

Cities

Plants

	1	2	3	4		
1	8	6 10	10	▨ 9	~~35~~ 25	8-6=2
2	9	▨ 12	13	▨ 7	50	12-9=3
3	▨ 14	9 10	▨ 16	5 30	~~40~~ ~~10~~ 0	-
	45	~~20~~ ~~10~~ 0	30	~~30~~ 0		
	9-8=1	12-6=6	13-10=3	-		

$$z = 5 \times 30 + 9 \times 10 + 6 \times 10$$

Vogel Approximation Method

Cities

Plants	1	2	3	4		
1	8	6 10	10	▨ 9	~~35~~ 25	10-8=2
2	9	▨ 12	13	▨ 7	50	13-9=4
3	▨ 14	9 10	▨ 16	5 30	~~40~~ ~~10~~ 0	-
	45	~~20~~ ~~10~~ 0	30	~~30~~ 0		
	9-8=1	-	13-10=3	-		

$$z = 5 \times 30 + 9 \times 10 + 6 \times 10$$

Vogel Approximation Method

Cities

Plants	1	2	3	4		
1	▨ 8	6 10	10	▨ 9	~~35~~ 25	10-8=2
2	9 45	▨ 12	13	▨ 7	~~50~~ 5	13-9=4
3	▨ 14	9 10	▨ 16	5 30	~~40~~ ~~10~~ 0	-
	~~45~~ 0	~~20~~ ~~10~~ 0	30	~~30~~ 0		
	9-8=1	-	13-10=3	-		

$$z = 5 \times 30 + 9 \times 10 + 6 \times 10 + 9 \times 45$$

Vogel Approximation Method

Cities

		1	2	3	4		
Plants	1	▨ 8	6 10	10	▨ 9	~~35~~ 25	-
	2	9 45	▨ 12	13	▨ 7	~~50~~ 5	-
	3	▨ 14	9 10	▨ 16	5 30	~~40~~ ~~10~~ 0	-
		~~45~~ 0	~~20~~ ~~10~~ 0	30	~~30~~ 0		
		-	-	13-10=3	-		

$$z = 5 \times 30 + 9 \times 10 + 6 \times 10 + 9 \times 45$$

Vogel Approximation Method

Cities

Plants	1	2	3	4		
1	▨ 8	6 10	10 25	▨ 9	~~35~~ ~~25~~ 0	-
2	9 45	▨ 12	13	▨ 7	~~50~~ 5	-
3	▨ 14	9 10	▨ 16	5 30	~~40~~ ~~10~~ 0	-
	~~45~~	~~20~~	~~30~~	~~30~~		
	0	~~10~~	5	0		
		0				
	-	-	13-10=3	-		

$$z = 5 \times 30 + 9 \times 10 + 6 \times 10 + 9 \times 45 + 10 \times 25$$

Vogel Approximation Method

Cities

Plants

	1	2	3	4	
1	▨ 8	6 10	10 25	▨ 9	~~35~~ ~~25~~ 0
2	9 45	▨ 12	13	▨ 7	~~50~~ 5
3	▨ 14	9 10	▨ 16	5 30	~~40~~ ~~10~~ 0
	~~45~~ 0	~~20~~ ~~10~~ 0	~~30~~ 5	~~30~~ 0	

$$z = 5 \times 30 + 9 \times 10 + 6 \times 10 + 9 \times 45 + 10 \times 25$$

Vogel Approximation Method

Cities

Plants

	1	2	3	4			
1	▨ 8	6 10	10 25	▨ 9	~~35~~	~~25~~	0
2	9 45	▨ 12	13 5	▨ 7	~~50~~	~~5~~	0
3	▨ 14	9 10	▨ 16	5 30	~~40~~	~~10~~	0
	~~45~~	~~20~~	~~30~~	~~30~~			
	0	~~10~~	~~5~~	0			
		0	0				

$$z = 5 \times 30 + 9 \times 10 + 6 \times 10 + 9 \times 45 + 10 \times 25 + 13 \times 5$$

Vogel Approximation Method

Cities

Plants	1	2	3	4			
1	▨ 8	6 10	10 25	▨ 9	~~35~~	~~25~~	0
2	9 45	▨ 12	13 5	▨ 7	~~50~~	~~5~~	0
3	▨ 14	9 10	▨ 16	5 30	~~40~~	~~10~~	0
	~~45~~	~~20~~	~~30~~	~~30~~			
	0	~~10~~	~~5~~	0			
		0	0				

$z = 1020$