

Sensitivity Analysis

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Graduate Program in Systems Engineering
Universidad Autónoma de Nuevo León

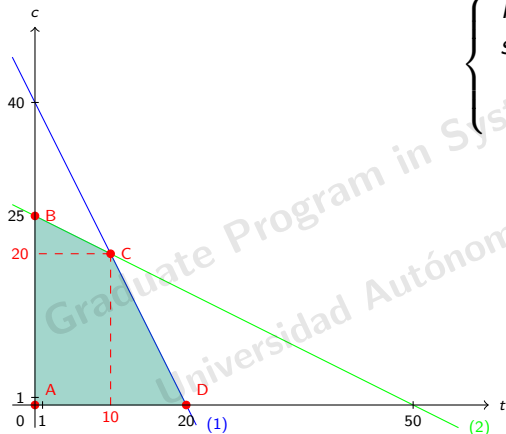
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The Carpenter's Problem



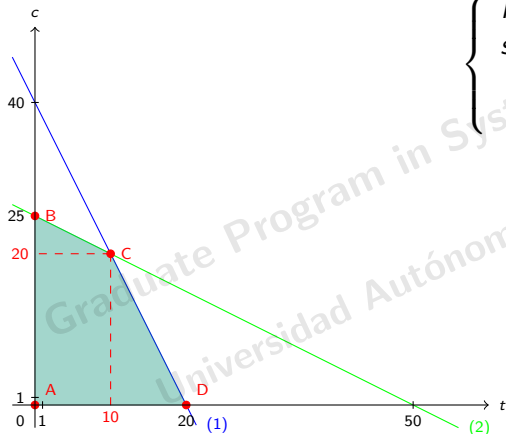
$$\begin{cases} \text{Max} & 500t + 300c \\ \text{s.t.} & 2t + c \leq 40 \\ & t + 2c \leq 50 \\ & t \geq 0, c \geq 0 \end{cases}$$

	t	c	z
A	0	0	0
B	0	25	7500
C	10	20	11000
D	20	0	10000

Change in objective (500 \rightarrow 200)

$$\left\{ \begin{array}{ll} \text{Max} & 200t + 300c \\ \text{s.t.} & 2t + c \leq 40 \\ & t + 2c \leq 50 \\ & t \geq 0, c \geq 0 \end{array} \right.$$

Change in objective (500 \rightarrow 200)



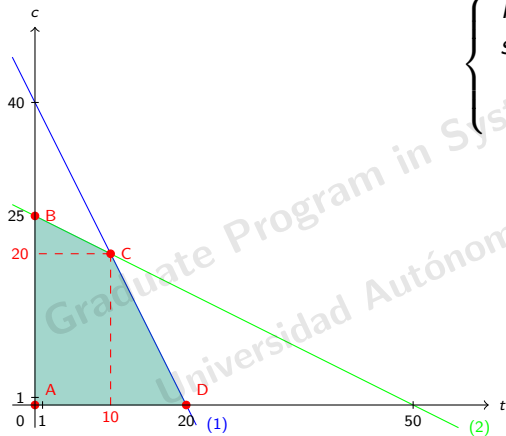
$$\begin{cases} \text{Max} & 200t + 300c \\ \text{s.t.} & 2t + c \leq 40 \\ & t + 2c \leq 50 \\ & t \geq 0, c \geq 0 \end{cases}$$

	t	c	z
A	0	0	0
B	0	25	7500
C	10	20	8000
D	20	0	4000

Change in objective (500 \rightarrow 100)

$$\left\{ \begin{array}{ll} \text{Max} & 100t + 300c \\ \text{s.t.} & 2t + c \leq 40 \\ & t + 2c \leq 50 \\ & t \geq 0, c \geq 0 \end{array} \right.$$

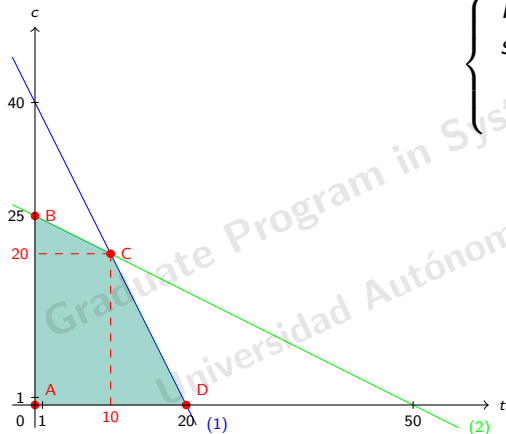
Change in objective (500 \rightarrow 100)



$$\begin{cases} \text{Max} & 100t + 300c \\ \text{s.t.} & 2t + c \leq 40 \\ & t + 2c \leq 50 \\ & t \geq 0, c \geq 0 \end{cases}$$

	t	c	z
A	0	0	0
B	0	25	7500
C	10	20	7000
D	20	0	2000

The Carpenter's Problem

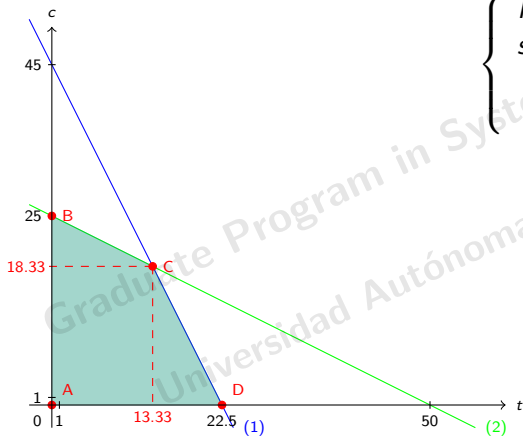


$$\begin{cases} \text{Max} & 500t + 300c \\ \text{s.t.} & 2t + c \leq 40 \\ & t + 2c \leq 50 \\ & t \geq 0, c \geq 0 \end{cases}$$

	t	c	z
A	0	0	0
B	0	25	10000
C	10	20	13000
D	20	0	10000

Change in rhs (40 \rightarrow 45)

$$\left\{ \begin{array}{lll} \text{Max} & 500t + 300c & \\ \text{s.t.} & 2t + c & \leq 45 \\ & t + 2c & \leq 50 \\ & t \geq 0, c \geq 0 & \end{array} \right.$$

Change in rhs (40 \rightarrow 45)

$$\begin{cases} \text{Max} & 500t + 300c \\ \text{s.t.} & 2t + c \leq 45 \\ & t + 2c \leq 50 \\ & t \geq 0, c \geq 0 \end{cases}$$

	t	c	z
A	0	0	0
B	0	25	10000
C	13.33	18.33	12166.33
D	22.5	0	11250

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1 Introduction

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Original problem

$$\begin{aligned}
 \text{Max } z &= 500t + 300c \\
 \text{s.t.} \quad &2t + c \leq 40 \\
 &t + 2c \leq 50 \\
 &t \geq 0, c \geq 0
 \end{aligned}$$

Standard form

$$\begin{aligned}
 \text{Max } z &= 500t + 300c \\
 \text{s.t.} \quad &2t + c + s_1 = 40 \\
 &t + 2c + s_2 = 50 \\
 &t \geq 0, c \geq 0, s_1 \geq 0, s_2 \geq 0
 \end{aligned}$$

Standard form

$$\begin{aligned}
 \text{Max } z &= 500t + 300c \\
 \text{s.t.} \quad &2t + c + s_1 = 40 \\
 &t + 2c + s_2 = 50 \\
 &t \geq 0, c \geq 0, s_1 \geq 0, s_2 \geq 0
 \end{aligned}$$

Initial tableau

B.V.	t	c	s ₁	s ₂	rhs
s ₁	2	1	1	0	40
s ₂	1	2	0	1	50
z	-500	-300	0	0	0

Initial tableau

B.V.	t	c	s ₁	s ₂	rhs
s ₁	2	1	1	0	40
s ₂	1	2	0	1	50
z	-500	-300	0	0	0

Final tableau

B.V.	t	c	s ₁	s ₂	rhs
t	1	0	2/3	-1/3	10
c	0	1	-1/3	2/3	20
z	0	0	700/3	100/3	11,000

Initial tableau

B.V.	t	c	s ₁	s ₂	z	rhs
s ₁	2	1	1	0	0	40
s ₂	1	2	0	1	0	50
z	-500	-300	0	0	1	0

Final tableau

B.V.	t	c	s ₁	s ₂	z	rhs
t	1	0	2/3	-1/3	0	10
c	0	1	-1/3	2/3	0	20
z	0	0	700/3	100/3	1	11,000

Initial tableau

B.V.	t	c	s ₁	s ₂	z	rhs
s ₁	2	1	1	0	0	40
s ₂	1	2	0	1	0	50
z	-500	-300	0	0	1	0

Final tableau

B.V.	t	c	s ₁	s ₂	z	rhs
t	1	0	2/3	-1/3	0	10
c	0	1	-1/3	2/3	0	20
z	0	0	700/3	100/3	1	11,000

Final tableau

B.V.	t	c	s ₁	s ₂	z	rhs
t	1	0	2/3	-1/3	0	10
c	0	1	-1/3	2/3	0	20
z	0	0	700/3	100/3	1	11,000

$$B = \begin{pmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 700/3 & 100/3 & 1 \end{pmatrix}$$

Initial tableau: Change in objective (500 \rightarrow 200)

B.V.	t	c	s ₁	s ₂	z	rhs
s ₁	2	1	1	0	0	40
s ₂	1	2	0	1	0	50
z	-200	-300	0	0	1	0

$$\begin{pmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 700/3 & 100/3 & 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ -200 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 300 \end{pmatrix}$$

Initial tableau: Change in objective (500 \rightarrow 200)

B.V.	t	c	s ₁	s ₂	z	rhs
s ₁	2	1	1	0	0	40
s ₂	1	2	0	1	0	50
z	-200	-300	0	0	1	0

$$\begin{pmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 700/3 & 100/3 & 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ -200 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 300 \end{pmatrix}$$

Final tableau

B.V.	t	c	s ₁	s ₂	z	rhs
t	1	0	2/3	-1/3	0	10
c	0	1	-1/3	2/3	0	20
z	300	0	700/3	100/3	1	11,000

Initial tableau: Change in objective (500 \rightarrow 200)

B.V.	t	c	s ₁	s ₂	z	rhs
s ₁	2	1	1	0	0	40
s ₂	1	2	0	1	0	50
z	-200	-300	0	0	1	0

$$\begin{pmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 700/3 & 100/3 & 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ -200 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 300 \end{pmatrix}$$

Final tableau: Optimal!!

B.V.	t	c	s ₁	s ₂	z	rhs
t	1	0	2/3	-1/3	0	10
c	0	1	-1/3	2/3	0	20
z	0	0	100/3	400/3	1	8,000

Initial tableau: Change in objective (500 \rightarrow 100)

B.V.	t	c	s ₁	s ₂	z	rhs
s ₁	2	1	1	0	0	40
s ₂	1	2	0	1	0	50
z	-100	-300	0	0	1	0

$$\begin{pmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 700/3 & 100/3 & 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ -100 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 400 \end{pmatrix}$$

Initial tableau: Change in objective (500 \rightarrow 100)

B.V.	t	c	s ₁	s ₂	z	rhs
s ₁	2	1	1	0	0	40
s ₂	1	2	0	1	0	50
z	-100	-300	0	0	1	0

$$\begin{pmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 700/3 & 100/3 & 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ -100 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 400 \end{pmatrix}$$

Final tableau

B.V.	t	c	s ₁	s ₂	z	rhs
t	1	0	2/3	-1/3	0	10
c	0	1	-1/3	2/3	0	20
z	400	0	700/3	100/3	1	11,000

Initial tableau: Change in objective (500 \rightarrow 100)

B.V.	t	c	s ₁	s ₂	z	rhs
s ₁	2	1	1	0	0	40
s ₂	1	2	0	1	0	50
z	-100	-300	0	0	1	0

$$\begin{pmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 700/3 & 100/3 & 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ -100 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 400 \end{pmatrix}$$

Final tableau: Not Optimal!!

B.V.	t	c	s ₁	s ₂	z	rhs
t	1	0	2/3	-1/3	0	10
c	0	1	-1/3	2/3	0	20
z	0	0	-100/3	500/3	1	7,000

Initial tableau: Change in objective (500 \rightarrow 100)

B.V.	t	c	s ₁	s ₂	z	rhs
s ₁	2	1	1	0	0	40
s ₂	1	2	0	1	0	50
z	-100	-300	0	0	1	0

$$\begin{pmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 700/3 & 100/3 & 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ -100 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 400 \end{pmatrix}$$

Final tableau: Optimal!!

B.V.	t	c	s ₁	s ₂	z	rhs
s ₁	3/2	0	1	-1/2	0	15
c	1/2	1	0	1/2	0	25
z	50	0	0	450/3	1	7,500

Initial tableau: Change in rhs (40 \rightarrow 45)

B.V.	t	c	s ₁	s ₂	z	rhs
s ₁	2	1	1	0	0	45
s ₂	1	2	0	1	0	50
z	-500	-300	0	0	1	0

$$\begin{pmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 700/3 & 100/3 & 1 \end{pmatrix} \times \begin{pmatrix} 45 \\ 50 \\ 0 \end{pmatrix} = \begin{pmatrix} 40/3 \\ 55/3 \\ 36,500/3 \end{pmatrix}$$

Initial tableau: Change in rhs (40 \rightarrow 45)

B.V.	t	c	s ₁	s ₂	z	rhs
s ₁	2	1	1	0	0	45
s ₂	1	2	0	1	0	50
z	-500	-300	0	0	1	0

$$\begin{pmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 700/3 & 100/3 & 1 \end{pmatrix} \times \begin{pmatrix} 45 \\ 50 \\ 0 \end{pmatrix} = \begin{pmatrix} 40/3 \\ 55/3 \\ 36,500/3 \end{pmatrix}$$

Final tableau: Optimal!!

B.V.	t	c	s ₁	s ₂	z	rhs
t	1	0	2/3	-1/3	0	40/3
c	0	1	-1/3	2/3	0	55/3
z	0	0	700/3	100/3	1	36,500/3