GEORG-AUGUST-UNIVERSITÄT
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Internship Report

Introduction to Modelling Opinion Dynamics

prepared by

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1. Introduction

The purpose of this internship report is to prepare myself for the upcoming bachelor thesis „*Modelling Opinion Dynamics in Social Networks with different Topologies*“. It should give an overview about the basics of the related topics, the mathematical and computational methods including some literature research. At the end there will be a summary of all important concepts plus an outlook for the goal of the thesis. Therefore the report can be seen as the foundation of it.

To begin with I will explain some basic theory of network science, especially the commonly used terms and principles. After that I will move on to talk about opinion dynamics in social networks and how to model those with different models and assumptions. In the context of three different model classes I will recreate some exemplary simulations of the time development of all opinions in the network and qualitatively explain the mechanisms and known results. In the third chapter I will introduce the model of social differentiation which I used for my research questions and added a few modifications. To get a feeling for the model properties I will reproduce the most important past results as well. There I take a deeper look into the impact of the initial conditions regarding to the development of the system and its final state. For this purpose some useful and meaningful metrics need to be set in a way that makes the results comparable and helps to fully understand not only how the system behaves but also why it does according to the model. Particularly for further research it is necessary to get those insights and know exactly what parameter values and their combinations lead to. The final thesis will include these quantitative measurements but for this report I will rather stick to qualitative observations due to page limitation.

As already said the last part of this report will include a summary of the gained knowledge and the most important results. On top of that a short discussion about the advantages, applicability and limitations of the model will follow before finishing with a brief outlook for other research questions that will be investigated in my bachelor thesis.

At `https://github.com/VincentBrockers/internship_modelling_od` all code and implementations for recreating the simulations and plots are available to look at.

2. Basics

2.1. Network Science

A network is an interconnected or interrelated chain, group or system. In general, its entities are called *nodes* who are connected by *links* [1, p. 1]. In network science many different types of complex networks are studied, for example computer networks, transportation networks, biological networks or social networks, where this report will focus on. A network can be *directed* or *undirected* meaning that links either represent a certain direction between nodes or just a bi-directional connection. On top of that links can be *weighted* or *unweighted* in a sense that an associated weight represents a quantity, e.g. the influence of one person on another [1, p. 15]. If wanted and useful these features can nicely be visualized in a graph. Another thing that should be mentioned is the case when two different types of nodes occur, for example in social media the users and the posts they interact with. This is called a *bipartite* network [1, p. 17]. After defining those network properties one can start to analyze it further. There are a lot of metrics that can be used like the *average degree*, *average shortest path length*, *clustering coefficient*, *connectedness* or *node centrality* but since these are not really needed in my analysis I will skip a deeper explanation here. Another way to do it, is by e.g. plotting the time development of the system, looking at the outcome for different initial conditions and comparing those developments with other useful metrics for social networks like the distribution of initial vs. final opinion, the amount of peaks or the standard deviation of the final opinion distribution and so on [2, p. 3].

2.2. Opinion Dynamics in Social Networks

For my research I am particularly interested in opinion dynamics, meaning how opinions of people in a social network spread, influence each other and thus develop over time. Describing and analyzing this process is very complex since each individual behaves differently in the way of perceiving and dealing with other opinions plus the interaction between people is strongly dependent on the network topology [3, p. 2]. With a huge variety of methods, models and assumptions a lot of insights were gained in this field since 1970 and still there is much to discover [4, p. 23]. In order to do so, one has to chose what small part of a big process can be described by a mathematical model and for what cases the assumptions hold to make predictions for the real world.

Models of opinion dynamics represent an opinion as a number or a set of numbers. Usually those models are divided into two categories, whether they use discrete or continuous opinions, which can be separated into other subcategories. In reality, unless we look at binary positions like buy/sell or Android/iPhone, opinions are better represented by a continuous spectrum for example from very progressive (-1) to very conservative ($+1$) [1, p. 200]. In the modelling process, each node of the network randomly (with respect to a wanted probability distribution) gets a number assigned from this spectrum. Then the opinions change over and over again as they get updated by a certain rule that depends on whether the interacting nodes are connected and what their opinion distance is (see section 2.3). Under the right conditions, the system eventually will reach a stationary state. Typically these states are *consensus*, *polarization* and *fragmentation*, who are characterized by their uni-, bi-, or multimodal final opinion distribution [1, p. 203].

2.2.1. Concepts from Psychology and Sociology

All of the described mechanisms are based one some important principles of psychology and sociology. To make everything reasonable one should not forget what the underlying forces for the given opinion dynamics are. Mainly the interaction and behaviour between people in social networks can be described by the following concepts [4].

- *social influence*: individuals change their behavior to meet the demands of a social environment (neighbors becoming more similar to neighbors) [5] [1, p. 205].
- *homophily*: tendency of individuals to associate and bond with similar others (similar nodes becoming neighbors) [6] [1, p. 205].
- *confirmation bias*: tendency to search for, interpret, favor, and recall information in a way that confirms or supports one's prior beliefs or values [7].
- *emotional contagion*: tendency to automatically mimic and synchronize behaviour and beliefs of another person, and consequently, to converge emotionally. [8].
- *xenophobia*: the larger the dissimilarity between two interacting individuals, the more they evaluate each other negatively, triggering repulsive behaviour [9].

2.3. Models of Social Influence

Due to the huge amount of opinion modelling studies in literature it is not easy to classify and categorize every contribution. One approach of dealing with this was done by Andreas Flache et. al. [4] which categorization in three classes of models I will refer to. Here the models were grouped together if their assumptions about social influence were implemented in a similar way. From a mathematical point of view the opinion update rule of those models is generally structured like the following form (2.1). The new opinion $o_{i,t+1}$ of an agent (node) i at the time step $t+1$ is calculated by

$$o_{i,t+1} = o_{i,t} + f_w(o_{i,t}, o_{j,t}) \cdot (o_{j,t} - o_{i,t}) \tag{2.1}$$

with $o_{i,t}$ the opinion of agent i at time step t, $o_{j,t}$ the opinion of agent j at time step t and the so called weight function f_w that scales the opinion change depending on both agents current opinions. This formalization is used for randomly picking two agents i, j and updating the first picked agents opinion if they are connected. Another way to update an agents opinion is by assuming the whole state of the network in fact every other opinion has an influence. This would be implemented by taking the sum of the (normalized by weights) second term in (2.1) over all agents j [4, p. 7]. It should be mentioned that the definition of f_w may vary by some small adaptions, still the mechanisms that arise in each model class are similar in the core.

2.3.1. Assimilative Influence

The first model category to look at is the one for models with assimilative influence. Its core assumptions are that individuals who are connected by a structural relationship always influence each other towards reducing opinion differences. This results in the weight function being defined as

$$f_w(o_{i,t}, o_{j,t}) = \mu \tag{2.2}$$

for all opinion distances with the parameter μ ($0 < \mu \leq 1$) that scales the strength of opinion convergence. Because of this property in the long run, if all agents in the network stay connected, the system inevitably will reach consensus [4, p. 7].

In the following figures 2.1, 2.2, 2.3 a few recreated exemplary simulations are shown to qualitatively understand the impact of the initial opinion distribution (mean value 0.5, varying standard deviation σ), the network size N and the parameter μ.

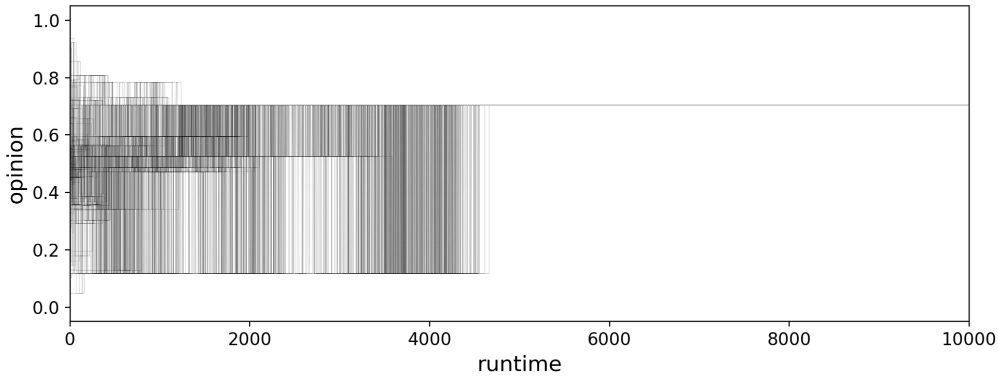

Figure 2.1.: Opinion timeline for $N = 100$ agents, initial opinion normal distribution with standard deviation $\sigma \approx 0.2$ and convergence parameter $\mu = 1$.

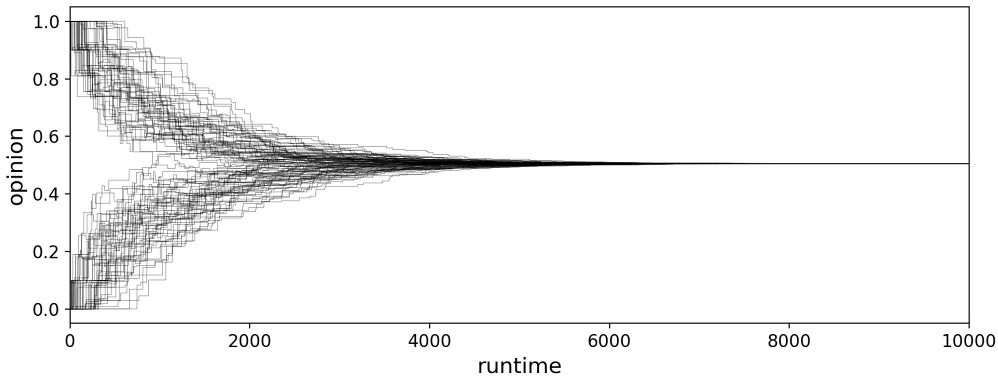

Figure 2.2.: Opinion timeline for $N = 100$ agents, initial opinion distribution with maximum polarization ($\sigma = 0.5$) and convergence parameter $\mu = 0.1$.

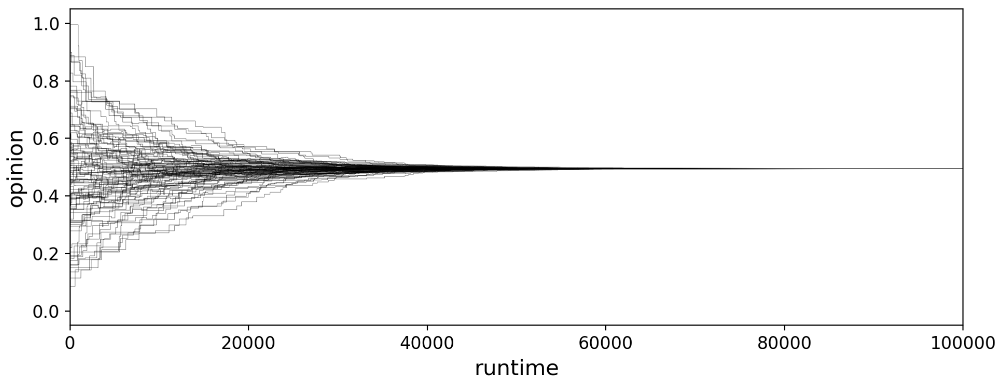

Figure 2.3.: Opinion timeline for $N = 1000$ agents, initial opinion normal distribution with standard deviation $\sigma \approx 0.2$ and convergence parameter $\mu = 0.1$.

In the first figure 2.1 one can see that the opinion dynamics for $\mu = 1$ result in pretty drastic opinion changes with big jumps. If we look back at equation (2.1) with our given weight function this makes sense since the updated opinion $o_{i,t+1}$ from agent i is just set equal with the old opinion $o_{j,t}$ from agent j. After some iteration steps the formation of subgroups occur, meaning that groups of multiple agents adapted the same opinion. In the end there are only two subgroups left which always converge to one group of the opinion from one of the subgroups before, most likely the one with more agents. Another side effect of the big opinion changes is that the final consensus opinion differs significantly from the mean value 0.5.

In contrast to that the second figure 2.2 shows that $\mu = 0.1$ instead leads to much smoother but a bit slower opinion dynamics due to smaller opinion changes. Furthermore it gets clear that for this model even maximum initial opinion polarization results in consensus since the weight function always causes attraction between agents.

The last figure 2.3 illustrates that on one hand the system behaves pretty identical for a different initial distribution, the smaller standard deviation just results in faster convergence for the same μ. On the other hand the runtime scales linearly with the network size N as the behaviour for $N = 1000$ is the same as for $N = 100$ but with a runtime ten times longer. To enhance these statements one can compare this plot with the figures A.1, A.2 in the appendix as well. In both this and the former case the final opinion barely distinguishes from the mean value.

2.3.2. Similarity Biased Influence

Models with similarity biased influence assume that only sufficiently similar individuals (depending on additional psychological mechanisms) can influence each other towards reducing opinion differences. Basically this leads to the former weight function but with limiting the opinion convergence as proposed in the new definition

$$f_w(o_{i,t}, o_{j,t}) = \begin{cases} \mu \, , & \text{if } |o_{i,t} - o_{j,t}| \leq \varepsilon \\ 0 \, , & \text{otherwise} \end{cases} \tag{2.3}$$

with the tolerance parameter ε that sets the threshold where the influence of another agent is cut off. Depending on the initial opinion distribution and the choice of ε the system can reach various final states. It is possible to reach either consensus, fragmentation or bi-polarization [4, p. 10]. Consensus can be achieved by setting the tolerance parameter big enough so the opinions of all agents will converge. Setting a smaller parameter will cause the appearance of opinion clusters because after some initial convergence time the agents form clusters in which they are stuck due to being outside the possible interaction range with agents from other clusters. Bi-polarization though is only reached via placing stubborn extremists (agents that have a fixed opinion) at the borders of the opinion space so they will drag other agents towards their extremes, if the tolerance parameter is not set too small. Furthermore similarity biased influence models are so called bounded confidence models meaning the opinions of all agents never leave the initial opinion range [10].

As done for the former model class I will take a briefly look into the time development of the system for different initial conditions. The following figures 2.4, 2.5, 2.6 display three exemplary simulation for $N = 100$ agents, $\mu = 0.1$ but varying ε and σ.

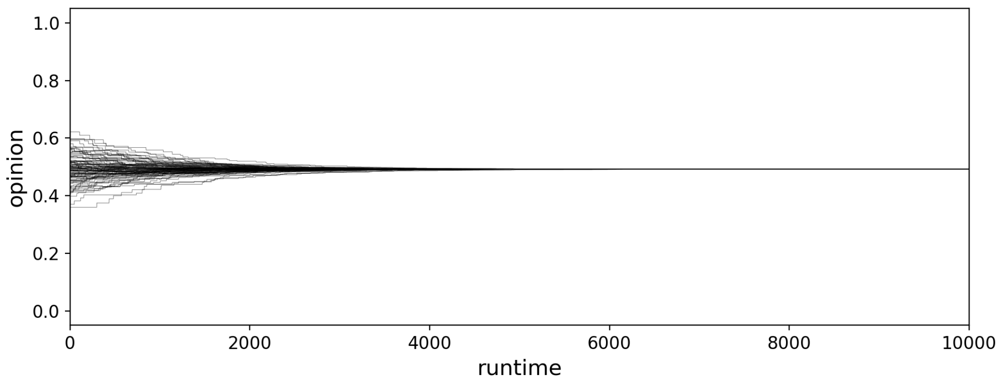

Figure 2.4.: Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma \approx 0.05$, tolerance parameter $\varepsilon = 0.15$ and convergence parameter $\mu = 0.1$.

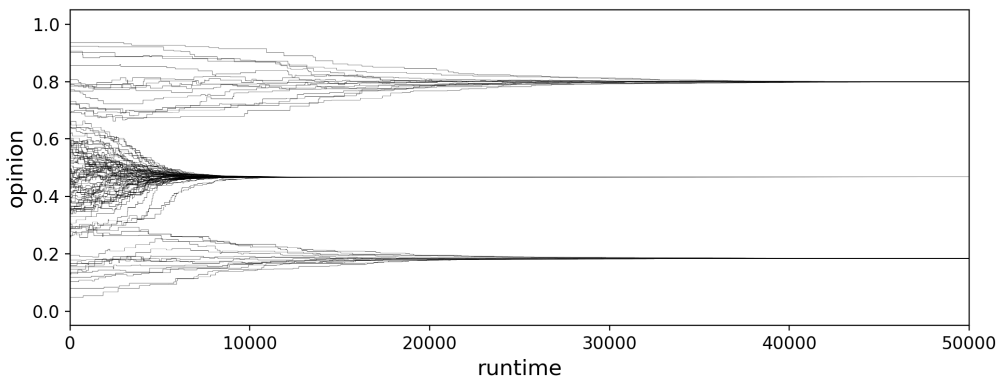

Figure 2.5.: Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma \approx 0.2$, tolerance parameter $\varepsilon = 0.15$ and convergence parameter $\mu = 0.1$.

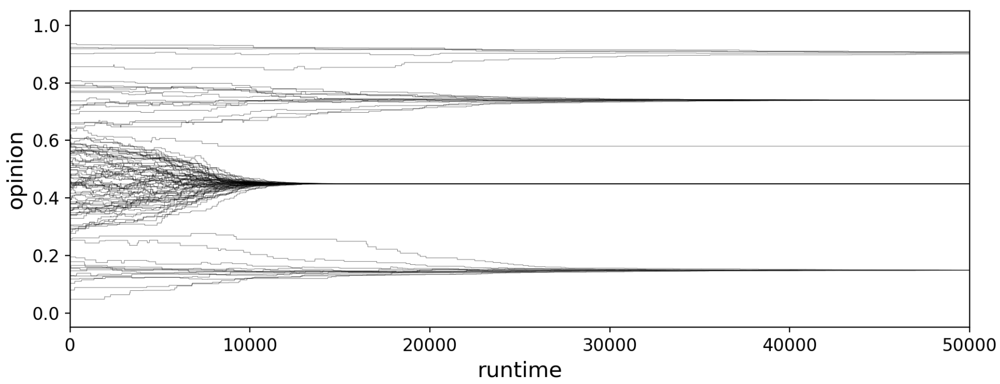

Figure 2.6.: Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma \approx 0.2$, tolerance parameter $\varepsilon = 0.1$ and convergence parameter $\mu = 0.1$.

The first figure 2.4 shows that consensus can be achieved by setting the tolerance parameter ε at a value where in combination with the initial opinion distribution the majority of absolute opinion distances between agents are below ϵ so the weight function basically is identical as in the former model class.

In the next figure 2.5 one can see what happens if the standard deviation is big enough to have sufficient agents with absolute opinion distances above ε resulting in the formation of opinion clusters, a final state of fragmentation. This happens as distant agents do not interact at all ($f_w = 0$) but rather converge with close agents to one group of one opinion that is too distant from other opinions to change as well.

Lastly, figure 2.6 emphasizes the impact of ε regarding the amount of opinion clusters. For smaller ε it is, with the same argumentation as before, possible for the system to form more groups of agents the same opinion in comparison to a system with equal σ. As seen in the timeline it is even possible for one agent to form a group alone since neither the agents above or below can attract him as every absolute opinion distance is greater than ε. Another mechanism which can be observed for systems resulting in fragmented opinion states is that the opinion dynamics are slower due to the fact that many of the interactions between agents are not opinion changes, in fact only those who have sufficiently close opinions.

In the appendix figures A.3, A.4, A.5 again show that the speed of the opinion dynamics for same initial conditions is dependent on μ, as a value twice as big reduces the runtime for the same outcome by the half. Also the systems behaviour scales linearly with the amount of agents N as seen in the previous model class.

2.3.3. Repulsive Influence

Lastly, models with repulsive influence assume that if individuals are too dissimilar (depending on additional psychological mechanisms) they can also influence each other towards increasing opinion differences. The weight function for models of this type can be defined as

$$f_w(o_{i,t}, o_{j,t}) = \mu \left(1 - \frac{1}{\varepsilon} \left|o_{i,t} - o_{j,t}\right|\right) \; . \tag{2.4}$$

In this form the tolerance parameter ε sets the threshold whether the opinion of the agent gets reinforced for absolute opinion distances below or rejected for a distance above it. Furthermore not only the sign of μ will be decided, the further the absolute opinion distance varies from the threshold, the bigger the influence in the resulting direction. Due to this characteristic models with repulsive influence behave in a very flexible way. It is possible to realize consensus, fragmentation and bi-polarization even without placing stubborn extremists and for many initially moderate agents just as a result of the repulsive behaviour. Without truncating the opinions at some value or cutting agents connections it would be possible for the opinions to leave the initial range. Thus such models are called unbound confidence models as well [10].

The following figures 2.7, 2.8, 2.9 again show some exemplary simulations for $N = 100$ agents, $\mu = 0.1$ but varying tolerance parameter ε and standard derivation σ. To keep the initial opinion range I just used a simple truncating rule. Whenever an agent would cross the upper or lower border according to the opinion updating rule, the opinion is just set to the borders value 0 or 1.

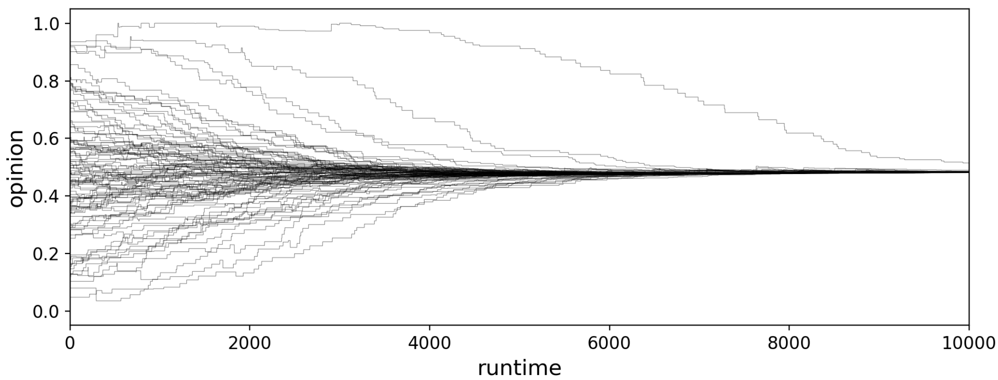

Figure 2.7.: Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma \approx 0.2$, tolerance parameter $\varepsilon = 0.55$ and convergence parameter $\mu = 0.1$.

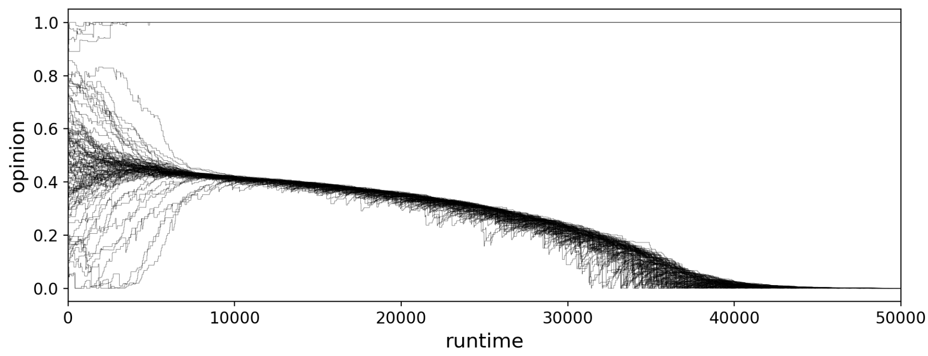

Figure 2.8.: Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma \approx 0.2$, tolerance parameter $\varepsilon = 0.48$ and convergence parameter $\mu = 0.1$.

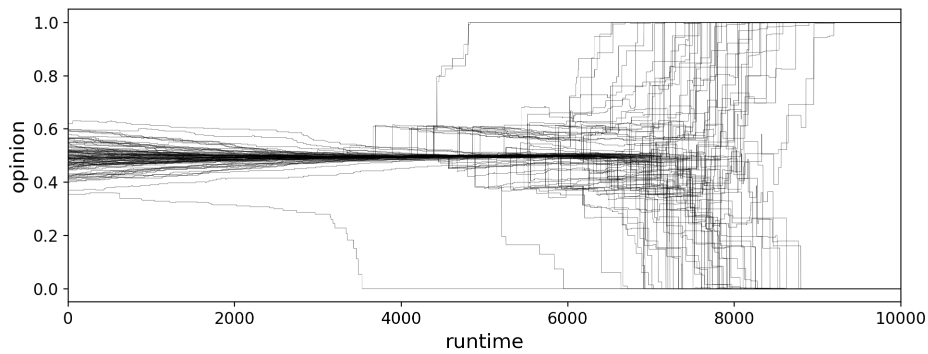

Figure 2.9.: Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma \approx 0.05$, tolerance parameter $\varepsilon = 0.15$ and convergence parameter $\mu = 0.1$.

As for the two previous model classes it is possible to reach consensus , as seen in figure 2.7, by setting the tolerance parameter ε big enough to ensure as many attractions between agents as possible for the given initial opinion distribution. Even though repulsive opinion changes can happen, if no agent has a greater absolute opinion distance than ε regarding the convergence point, consensus is unavoidable.

In the next figure 2.8 one can see what happens, if this would be the case. A slightly smaller choice of ε causes that even a few agents who were pushed to the upper border in the beginning are always being rejected by the majority which as a consequence of that slowly move to the other border until a state of bi-polarization is reached.

In figure 2.9 one can see another phenomenon if both the initial standard deviation σ and ε is pretty small. At first it seems like consensus will be achieved but after a while the one agent that went to one border pushes a few other agents to the opposite border such that in the end all agents are rejected until a final polarized state.

In the appendix in figures A.6, A.7, A.8 additionally show again how μ scales the speed of the dynamics, how a final polarized state can be reached immediately and how extending the opinion range leads to faster dynamics as well due to bigger weight function values.

3. The Model of Social Differentiation

Since only models with repulsive influence can reach bi-polarization from various initial conditions, which is especially seen in social media [11, p. 29], I also chose one to work with, namely the model of social differentiation [12, p. 76]. The core assumptions are motivated again by the concepts explained in section 2.2.1.

3.1. Model Properties

As seen before, each member of a population is represented by an agent i that holds the opinion $o_{i,t} \in [0, 1]$. During every time step t of the simulation the program randomly picks one of the N agents and updates its current opinion as

$$o_{i,t+1} = o_{i,t} + \frac{\sum_{j=1}^{N} f_w(o_{i,t}, o_{j,t}) \cdot (o_{j,t} - o_{i,t})}{\sum_{j=1}^{N} f_w(o_{i,t}, o_{j,t})} + \xi_{i,t} \tag{3.1}$$

with the weight function

$$f_w(o_{i,t}, o_{j,t}) = \begin{cases} \left(1 - \frac{1}{\varepsilon} \cdot |o_{i,t} - o_{j,t}|\right)^a , & \text{if } |o_{i,t} - o_{j,t}| \leq \varepsilon \\ -\left(\frac{1}{\varepsilon} \cdot |o_{i,t} - o_{j,t}| - 1\right)^a , & \text{if } |o_{i,t} - o_{j,t}| > \varepsilon \end{cases} . \tag{3.2}$$

and the parameter

$$\xi_{i,t} = \mathcal{N}\left(0,\ \frac{s}{N} \cdot \sum_{j=1}^{N} e^{-|o_{i,t} - o_{j,t}|}\right) \tag{3.3}$$

which implemented to add a bit of noise and drive the agent away from his current opinion as a mechanism of “striving for uniqueness” [12, p. 77]. The factor is determined by drawing a value from a normal distribution with mean 0 and the standard deviation $s/N \cdot \sum_{j=1}^{N} e^{-|o_{i,t} - o_{j,t}|}$ that gets bigger the more agents have a low opinion distance regarding the currently viewed agent. The factor s/N is for scaling and normalizing purposes. In the weight function the tolerance parameter ε occurs again to set the critical opinion distance at which the sign switches plus the exponent $a > 0$ appears to vary the shape of the weight function [12, p. 76].

3.2. Reproduction of Past Results

In this section I want to present my reproduction of the most important past results for the described model. At first I will take a look at two cases in figures 3.1, 3.2 without "striving for uniqueness", so $s = \xi_{i,t} = 0$ for all i, t, for a fully connected network with $N = 100$, $\varepsilon = 0.5$ but varying initial standard deviation σ for a normal distribution with mean value 0.5. The exponent of the weight function was set to $a = 100$. After that figures 3.3, 3.4, 3.5 illustrates three cases where the previous conditions are only changed in $s = 0.025$ and $\varepsilon = 1$ so with "striving for uniqueness" but only for attractive opinion changes. As done before, the opinions are simply truncated at the according border.

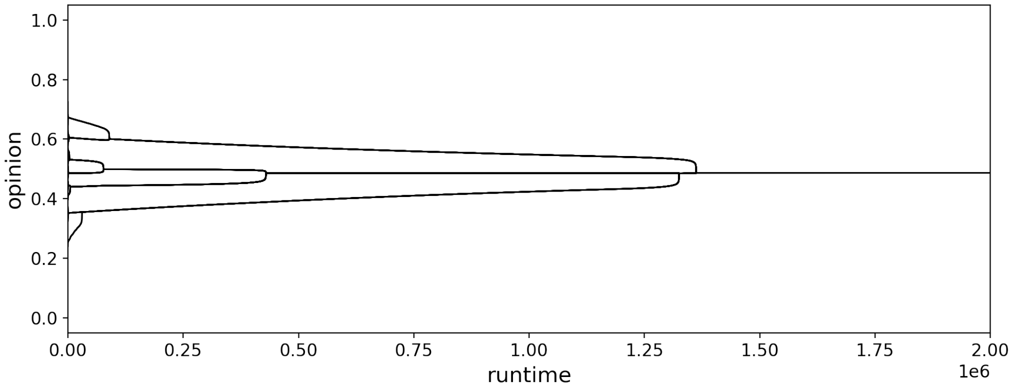

Figure 3.1.: Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma \approx 0.1$, tolerance parameter $\varepsilon = 0.5$ and $s = 0, a = 100$.

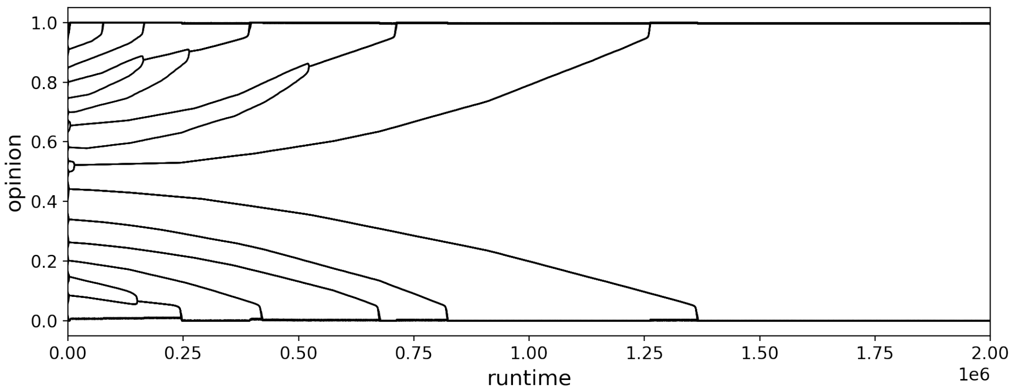

Figure 3.2.: Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma \approx 0.3$, tolerance parameter $\varepsilon = 0.5$ and $s = 0, a = 100$.

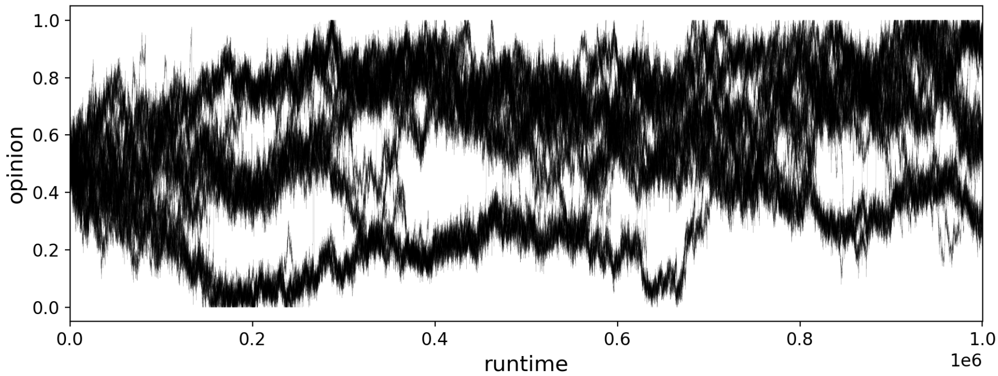

Figure 3.3.: Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma = 0$, tolerance parameter $\varepsilon = 1$ and $s = 0.025, a = 100$.

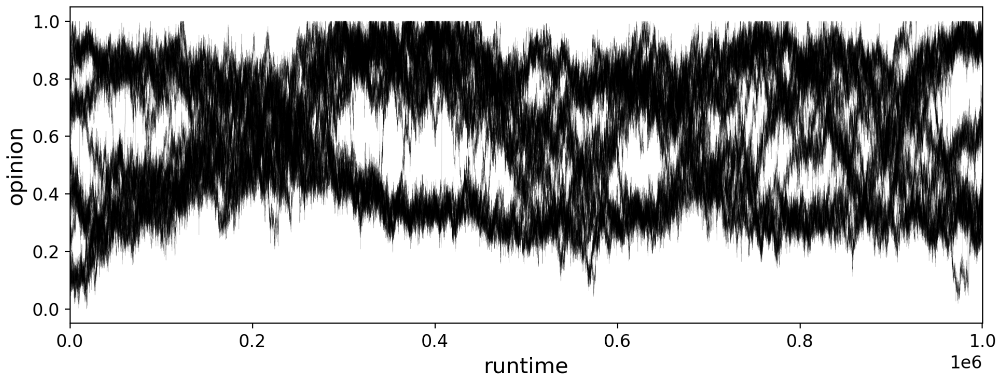

Figure 3.4.: Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma \approx 0.3$, tolerance parameter $\varepsilon = 1$ and $s = 0.025, a = 100$.

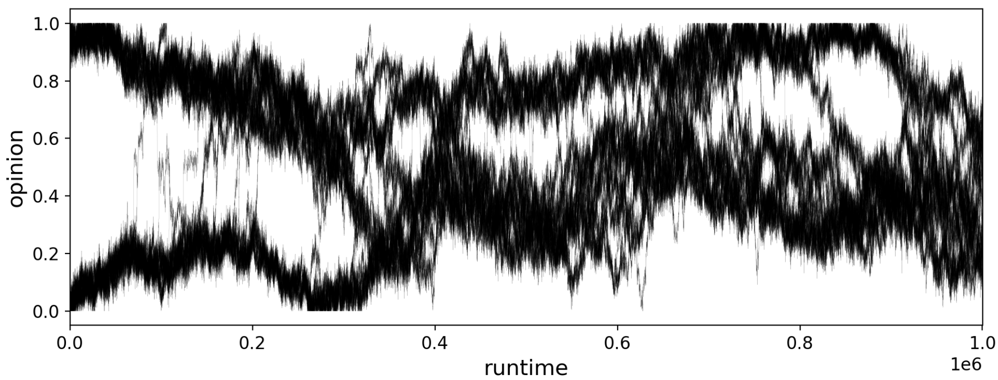

Figure 3.5.: Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma = 0.5$, tolerance parameter $\varepsilon = 1$ and $s = 0.025, a = 100$.

The first two figures 3.1, 3.2 for $s = 0, \varepsilon = 0.5$ show a known behaviour at first glance. For a small initial standard deviation even with allowed repulsive opinion changes consensus can be reached in the end. If in contrast to that the standard deviation is set higher, the majority of interactions are rejections so a bi-polarized state will be reached. The new phenomenon observed is the formation of branches consisting of very distinct subgroups which merge to new or existing branches that converge to the final state. The reason for that is the value $a = 100$ as discussed later. Another interesting observation is that in the case of bi-polarization the final branches are not exactly converging towards the maximal/minimal opinion. This makes sense since even opinions initially sitting at the border will be dragged towards the center sometimes. In the end the two final branches have an absolute opinion distance above $\varepsilon = 0.5$ which makes the system stable.

Adding the mechanism of “striving for uniqueness” results in a completely new behaviour as seen in figures 3.3, 3.4, 3.5. During all times visible fluctuations within a certain range $\xi > 0$ occur on top of the initial opinion changes. Besides that, it is also possible to declare the formation of subgroups within branches that eventually wander into others but are never stable as a consequence of fluctuations. It seems like there are time intervals in which the current state of the system is especially chaotic or ordered in a sense of less or more distinguishable branches with bigger spaces between them.

3.3. Adaptions for Application on Social Media

For the applications of the model on social media a few modifications of the weight function f_w were implemented which I found to be useful since very distant opinions do not necessarily result in stronger repulsion. With them the new definition is given by

$$f_w(o_{i,t}, o_{j,t}) = \begin{cases} \mu \left(1 - \frac{1}{\varepsilon} |o_{i,t} - o_{j,t}|\right)^a \ , & \text{if } |o_{i,t} - o_{j,t}| \leq \varepsilon \\ -\frac{\mu}{r} \left(\frac{1}{\varepsilon} |o_{i,t} - o_{j,t}| - 1\right)^a \ , & \text{if } \varepsilon < |o_{i,t} - o_{j,t}| \leq \Omega \\ 0 \ , & \text{otherwise} \end{cases} \tag{3.4}$$

including a new parameter $r > 0$ to scale the repulsive term if wanted and the interaction width $\Omega > \varepsilon$ to ensure that there is no possibility of opinions drifting towards infinity but rather a certain point where an extreme opinion stays and can not be interacted with. I also introduced the convergence parameter μ again to ensure smoother opinion changes. The results for these adaptions will be presented in my final thesis as it would exceed the limit of this report otherwise.

4. Summary and Discussion

In the beginning of this report I defined the term network advancing to just social networks and the possibilities of how to model opinion dynamics in those. There I pointed out the usage of models with a continuous opinion spectrum and their typical final states consensus, fragmentation or bi-polarization. In the context of three different model classes (with assimilative, similarity bias, repulsive influence) I illustrated those outcomes and talked about the mechanisms leading to those.

One important thing to mention here is that I limit myself to one dimensional models as they are very intuitive to understand, still produce various outcomes with complex dynamics and have a faster computational time which I will benefit from when putting one model into other network topologies that simulate the processes in social media. On top of that I just vaguely listed up the psychological and sociological principles for the underlying forces of the dynamics. To really explain and justify their application in the context of the models one has to extend the argumentation which I will give in the final thesis.

Nonetheless I learned some very important lessons from comparing the three model classes. Their network size N and the convergence parameter μ scales linearly to the runtime, while the tolerance ε is the parameter mainly determining the outcome. There may be cases where μ is also relevant for the outcome due to the scaling of opinion changes which I have to investigate further. Also the statements with respect to the runtime must be verified again by more simulations measuring the convergence regarding some metric like a certain threshold around the mean value. Lastly it gets clear that models with repulsive influence should be used in context of social media since we especially observe very polarized communities and platforms there and such models deliver the most flexible dynamics and possibilities for adaptions as seen in the model of social differentiation.

This model shows how the formation of very distinct subgroups can arise. A look at the exponent $a = 100$ explains this, as the weight function has a really steep curve such that only very close and very distant agents will change their opinion significantly. When the standard deviation is small enough only the first effect is relevant and the formed subgroups converge to consensus. A bigger standard deviation makes the second effect relevant as well such the subgroups converge to either one border and a bi-polarized state is reached.

Adding the mechanism “striving for uniqueness” was done as an approach to model the fact that when humans in a bigger group perceive that they are very similar to others, they tend to change their opinion towards becoming more unique and individual. If this assumption holds as well for users in social media has to be determined in the future. Still I learned much from doing simulations with such varying noise on every opinion change. It turns out that fluctuations reduce the sensitivity of the system to the initial opinion distribution. For both extreme cases, initial consensus and polarization, the dynamics transition into a similar behaviour of forming and dissolving subgroups within a certain “tube” of opinions that fluctuates around itself repeatedly as the system can never reach a stable state.

Furthermore it occurred to myself that the model initially was implemented with setting values of the weight function to -10^{-5} or 10^{-5} if it would return values closer to zero otherwise to ensure weak interaction is not falsely seen as no interaction by the program due to floating point inaccuracies. The problem with that is the creation of a new invisible hyperparameter that could change the dynamics which in fact can be observed as seen in figure A.9 as I left out this limitation and ran the simulation for the same initial conditions. The system shows a behaviour that can not be explained by statistical fluctuations only as there are only two branches left that do not have a flux of agents between them. Another little modification I noticed is that the quotient in the opinion update rule needs to be limited for the case of initial polarization as it can get really big for opinions only beeing set to 0 or 1. Without such limitations one group will quickly move to the other border as figure A.10 shows. So for recreating 3.5 I only allowed $|f_w| \leq 0.075$. In general this showed me to be more careful of artificially changing the dynamics with small code adaptions and invisible hyperparameters and also to differentiate between statistical fluctuations or those hidden mechanisms.

5. Outlook

In the upcoming bachelor thesis I will continue my work on analyzing opinion dynamics with the adapted model of social differentiation. My goal is to produce more quantitative results with introducing new metrics and doing much more simulations to reduce statistical fluctuations and gain a wider knowledge about the parameter space. On top of that I want to find out what happens if I put the model into networks with different topologies such as seen in various social media platforms.

A. Appendix

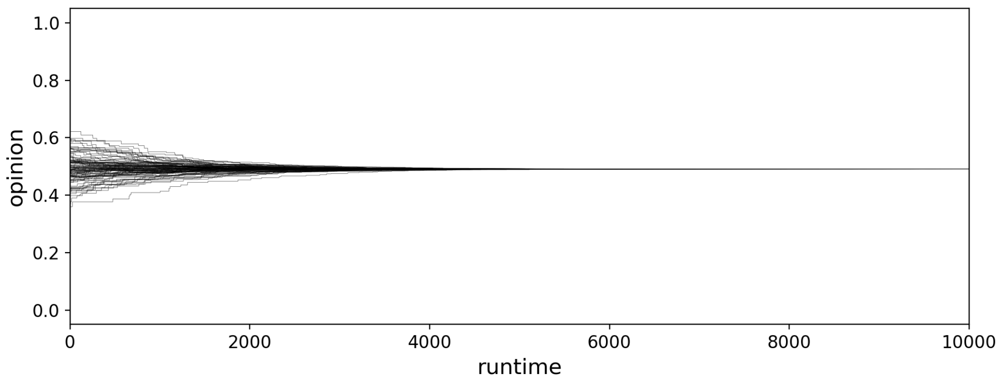

Figure A.1.: (Model with assimilative influence) Opinion timeline for $N = 100$ agents, initial opinion normal distribution with standard deviation $\sigma \approx 0.05$ and convergence parameter $\mu = 0.1$.

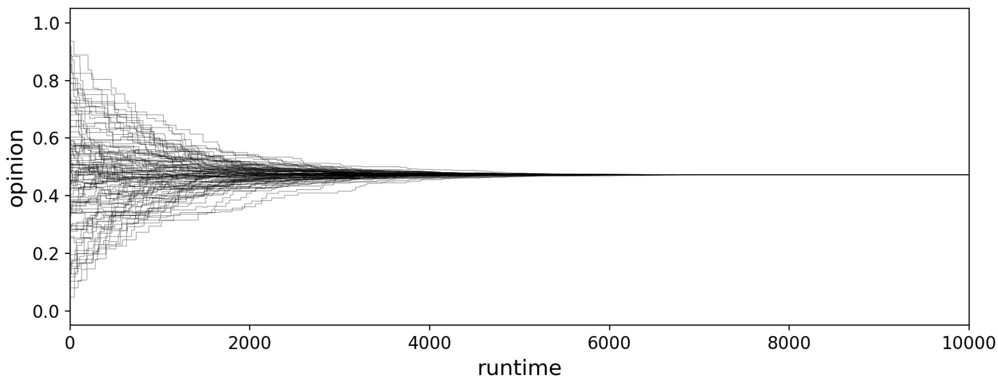

Figure A.2.: (Model with assimilative influence) Opinion timeline for $N = 100$ agents, initial opinion normal distribution with standard deviation $\sigma \approx 0.2$ and convergence parameter $\mu = 0.1$.

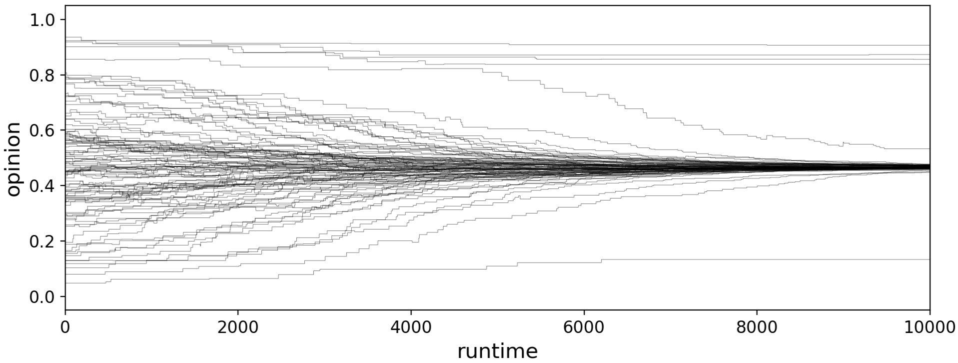

Figure A.3.: (Model with similarity biased influence) Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma \approx 0.2$, tolerance parameter $\varepsilon = 0.3$ and convergence parameter $\mu = 0.05$.

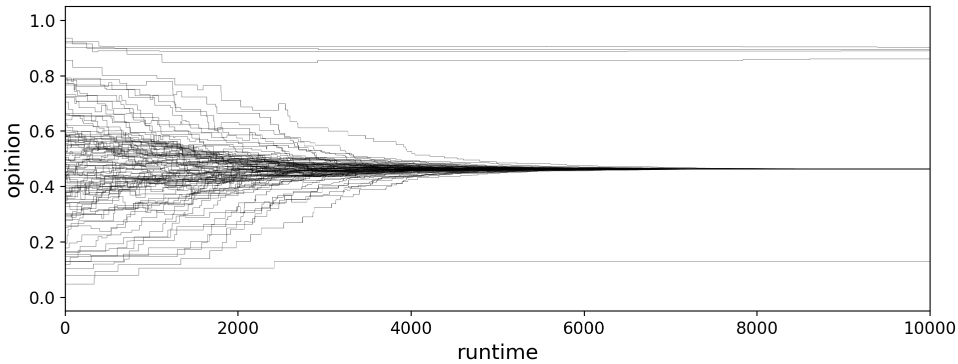

Figure A.4.: (Model with similarity biased influence) Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma \approx 0.2$, tolerance parameter $\varepsilon = 0.3$ and convergence parameter $\mu = 0.1$.

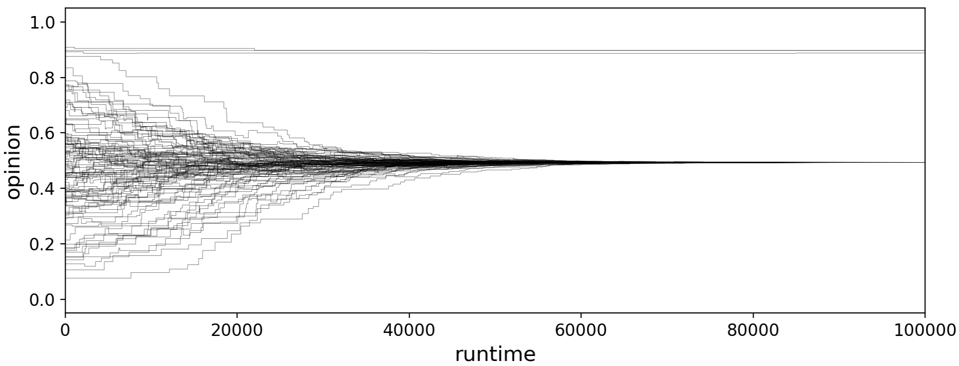

Figure A.5.: (Model with similarity biased influence) Opinion timeline for $N = 1000$ agents, initial standard deviation $\sigma \approx 0.2$, tolerance parameter $\varepsilon = 0.3$ and convergence parameter $\mu = 0.1$.

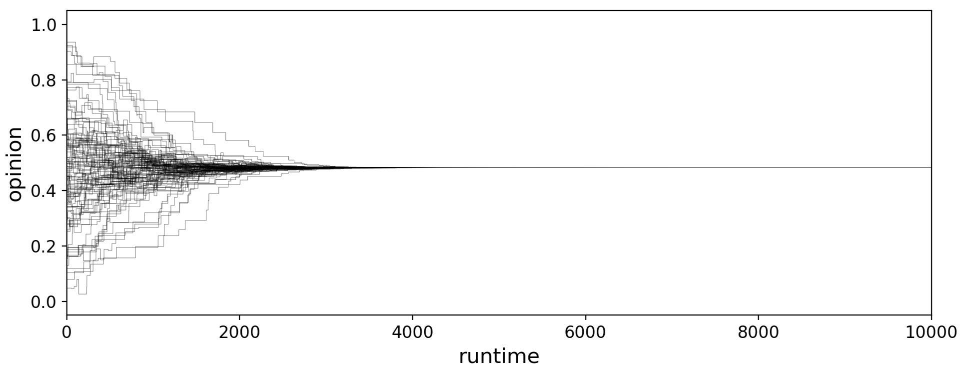

Figure A.6.: (Model with repulsive influence) Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma \approx 0.2$, tolerance parameter $\varepsilon = 0.55$ and convergence parameter $\mu = 0.3$.

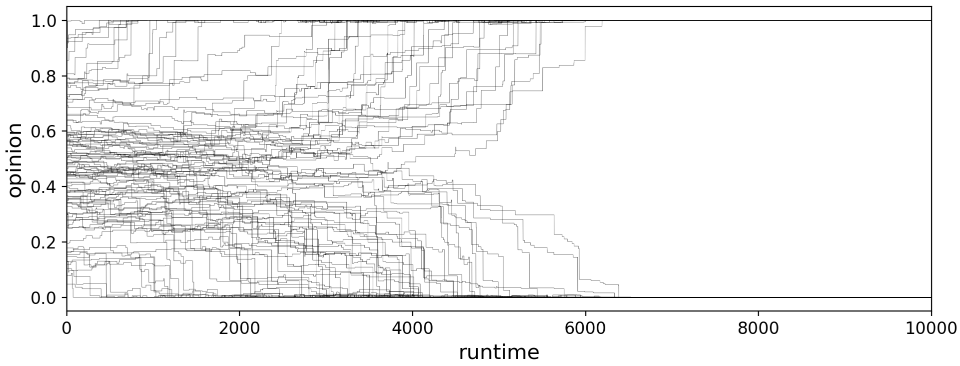

Figure A.7.: (Model with repulsive influence) Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma \approx 0.2$, tolerance parameter $\varepsilon = 0.35$ and convergence parameter $\mu = 0.1$.

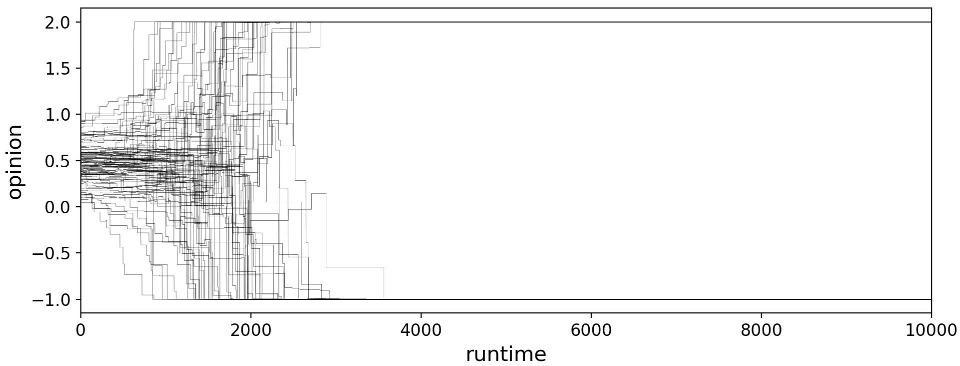

Figure A.8.: (Model with repulsive influence) Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma \approx 0.2$, tolerance parameter $\varepsilon = 0.35$ and convergence parameter $\mu = 0.1$. Bigger opinion range, $o_{i,t} \in [-1, 2]$.

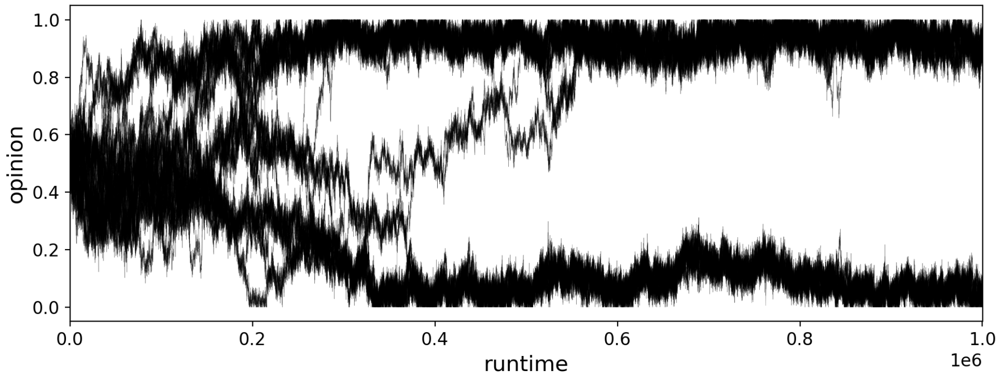

Figure A.9.: (Model of social differentiation, allowing $|f_w| < 10^{-5}$) Opinion timeline for $N = 100$ agents, initial standard deviation $\sigma = 0$, tolerance parameter $\varepsilon = 1$ and $s = 0.025, a = 100$.

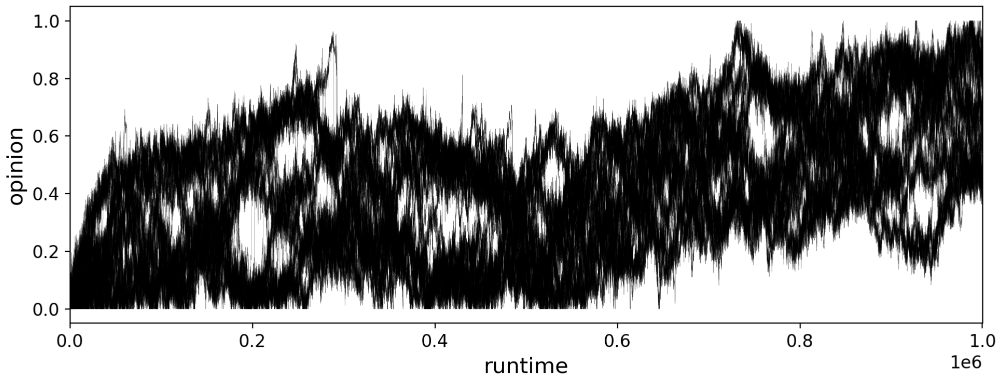

Figure A.10.: (Model of social differentiation, allowing $|f_w| > 0.075$) Opinion timeline for $N = 100$ agents, initial polarization ($\sigma = 0.5$), tolerance parameter $\varepsilon = 1$ and $s = 0.025, a = 100$.

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