2C-L2 Convolution in frequency domain

1. Intro
    a. Previously
        i. the idea of decomposing the images based on frequency(Fre)
            1. decomposing via base set, Fourier Set (Sin & Cos)
        ii. Fourier Transform
            1. basic
                a. f(x) => F(ω)
                b. a complex number(real(cos) + imaginary(sin)), encode the A and phi inside it.
            2. meaning
                a. Represent the signal as an infinite weighted sum of an infinite number of sinusoids:

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi ux} dx$$

                b. go from spatial domain to the Fre domain
            1. 2D - with two parameters

- The two dimensional version: .

$$F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-i2\pi(ux+vy)} dx\, dy \frac{1}{2}$$

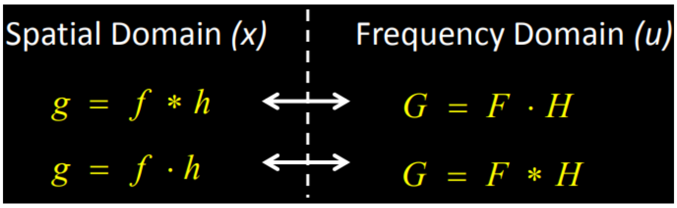
- And the 2D ***Discrete FT***:

$$F(k_x,k_y) = \frac{1}{N} \sum_{x=0}^{x=N-1} \sum_{y=0}^{y=N-1} f(x,y) e^{-i\frac{2\pi(k_x x+k_y y)}{N}}$$

            4. Spectra
                a. even vs. odd
                b. we care about the power rather than the phase
                c. the power is especial important when we talk about the presence of Fre in any image.
    a. Now

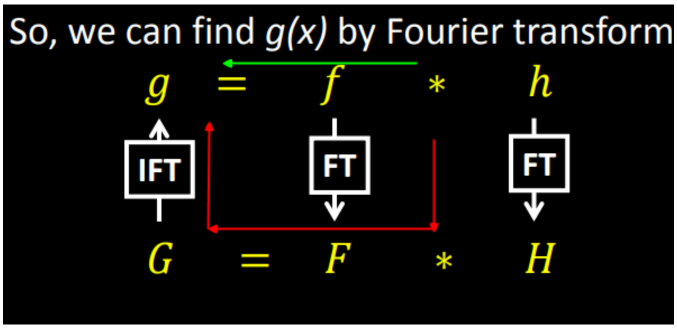
i. to relate the Fourier transform to convolution
ii. how convolution in space is multiplication in frequency

1. Fourier Transform and Convolution
    a. convolution in one domain is equal to the multiplication in the other domain



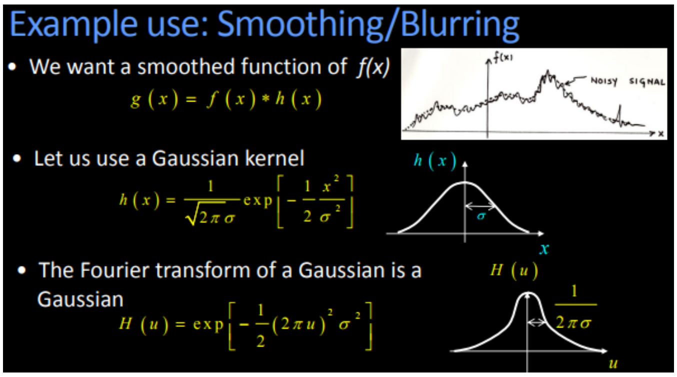
i. it's useful for aliasing

1. FFT

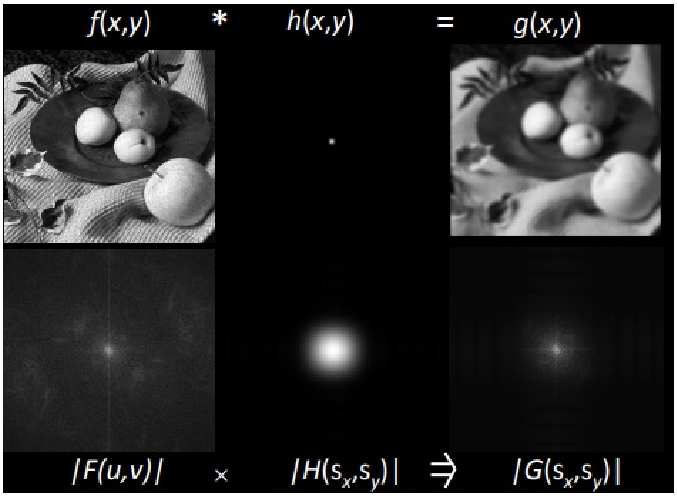


a. when you have big masks/filters or large images to do the convolution, the red way is much faster than the green way via FFT & IFFT

1. Smoothing and Blurring
    a. a Gaussian kernel
        i. the Fourier transform of a Gaussian is a Gaussian whose variance is 1 over sigma squared.
        ii. the Fourier transform of a fat Gaussian is a skinny one, and the Fourier transform of a skinny Gaussian is a fat one.
    b. the key idea is the original image multiply a Gaussian which attenuate the high Fre.
        i. if we convolve a image with an impulse, we get the original image. Gaussian is flatter than the impulse, so it keeps the low Fre more than the high Fre.

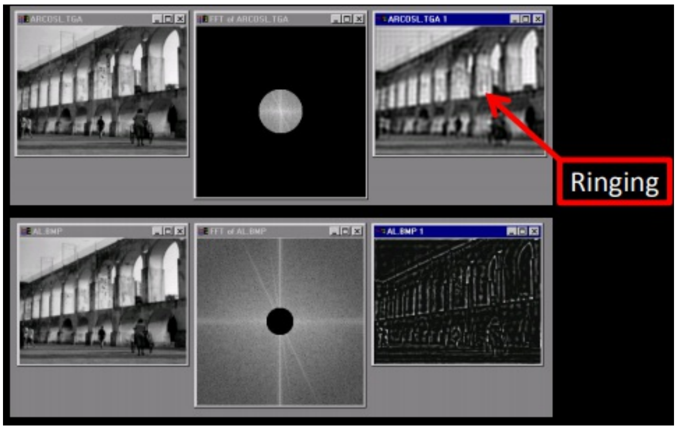


5. 2D Example

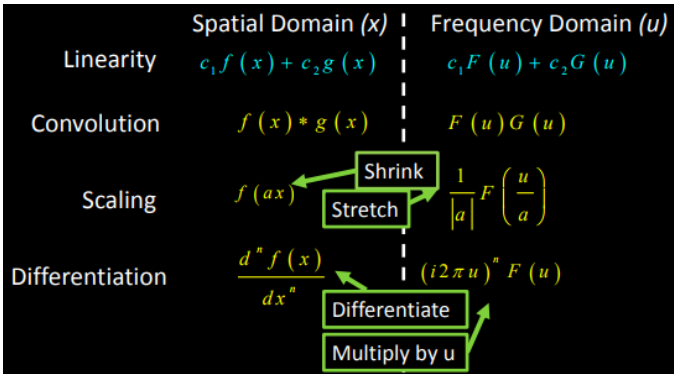


a. it keeps the low Fre more than the high Fre.

1. Low and High Pass Filtering

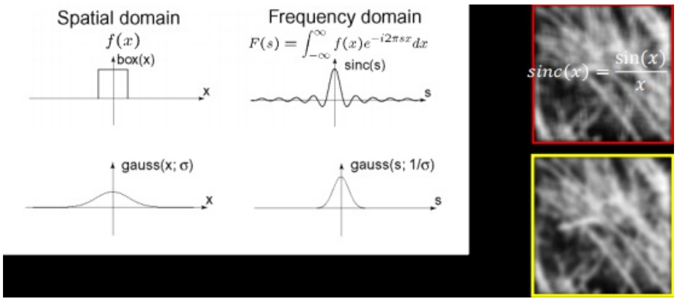


7. Properties of Fourier Transform



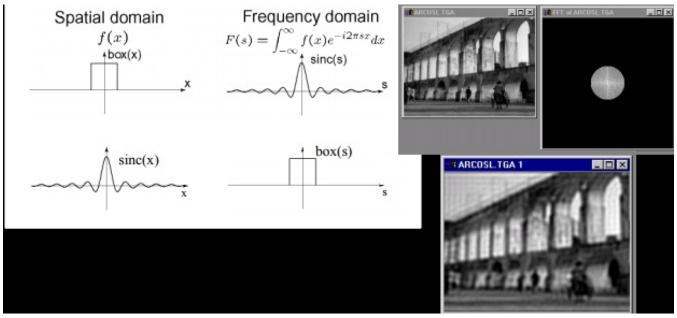
a. for differentiation, when you take this derivative, you multiply the high frequency components. Which means the higher frequencies get accentuated.

1. Fourier Pairs from Szeliski
   a. why Gaussian filter is better than box filter
      i. because the FT of box filter is infinite in Fre domain, so there're high Fre left in image, the wired edges. But Gaussian attenuates them well.



b. The ringing effect(in space or Fre)

i. a box in Fre domain corresponds to a sinc in spatial domain, or vise versa, just the same problem as before.



c. So it’s nice to use Gaussian filters instead of box