

3A-L2 Perspective imaging

2017/11/24 16:58

1. Intro

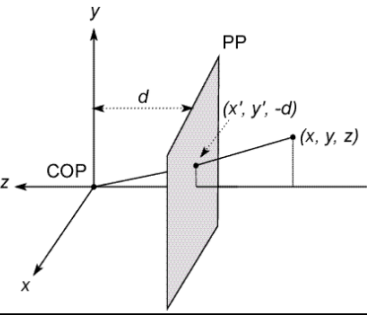
- a. math behind geometry and configuration of cameras.
- b. pinhole model
 - i. all the rays are in focus
 - ii. the reason we're doing that is to try to make our images be more like they were generated from just some really uber pin-hole camera. Since camera is not a perfect pin-hole model.

c. modeling projection

2. Coordinate System

- a. fundamental to the notion of imaging is projection operation.

- We will use the pin-hole model as an approximation
- Put the optical center (Center Of Projection) at the origin
- STANDARD (x,y) COORDINATE SYSTEM
- Put the image plane (Projection Plane) in front of the COP (why?)
- The camera looks down the **negative** z axis



- b. put the image plane in front of the coordinate system.
 - i. it's mathematically convenient because this way our images don't get inverted
 - ii. so it's (x', y', -d)
 1. the distance d from the origin to the image plane

1. Modeling Projection

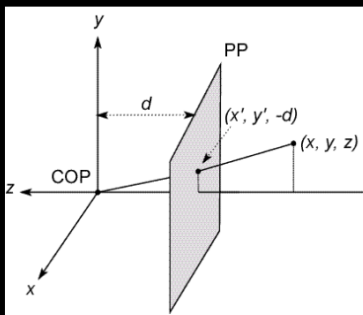
Projection equations

$$(X, Y, Z) \rightarrow (-d \frac{X}{Z}, -d \frac{Y}{Z}, -d)$$

We get the projection by throwing out the last coordinate:

$$(x', y') = (-d \frac{X}{Z}, -d \frac{Y}{Z})$$

Distant objects are smaller



- a. use similar triangles to compute the coordinates
- b. the origin of the image is in the center
- c. Z's effect: the farther the distance, the smaller the image.
- d. When objects are very far away, the real X and real X can be huge. If I move the camera (the origin) those numbers hardly change. Since the thing really matters is the angle

1. Homogeneous Coordinates

- a. The projection operator is not a linear transformation, which

brings inconvenience

b. In order to make it linear, we introduce another coordinate,
Homogeneous Coordinates

i. 加一维，作为被除因子

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image
(2D) coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene
(3D) coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

(this makes homogenous coordinates invariant under scale)

5. Perspective Projection

a. multiplication under the HC is linear now. During computation, we keep the additional dim. When we need the image, we convert it back.

b. f is the focal length, the distance from the origin to the image plane. It's the d talked above.

Projection is a matrix multiply using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f \end{bmatrix} \Rightarrow \left(f \frac{x}{z}, f \frac{y}{z} \right) \Rightarrow (u, v)$$

c. How does scaling the projection matrix change the transformation?

i. invariant

1. Geometric Properties of Projection

a. points to points, so lines to lines

2. Parallel Lines

a. All the lines except those parallel with the image plane converges at certain point

Parallel lines converge in math too...

Line in 3-space

$$x(t) = x_0 + at$$

$$y(t) = y_0 + bt$$

$$z(t) = z_0 + ct$$

Perspective projection of the line

$$x'(t) = \frac{fx}{z} = \frac{f(x_0 + at)}{z_0 + ct}$$

$$y'(t) = \frac{fy}{z} = \frac{f(y_0 + bt)}{z_0 + ct}$$

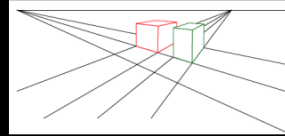
In the limit as $t \rightarrow \pm\infty$ we have (for $c \neq 0$):

$$x'(t) \rightarrow \frac{fa}{c}, \quad y'(t) \rightarrow \frac{fb}{c}$$

8. Vanishing Points

Vanishing points

- Each set of parallel lines (=direction) meets at a different point

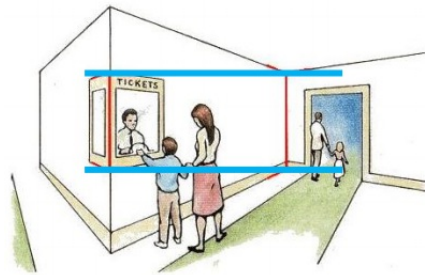


- Sets of parallel lines on the same plane lead to collinear vanishing points.
 - The line is called the horizon for that plane

- Good ways to spot faked images
 - scale and perspective don't work
 - vanishing points behave badly

9. Human Vision

- We're very sensitive to this structure of parallel lines and what they convey to us. your brain automatically wants to undo that projection transformation



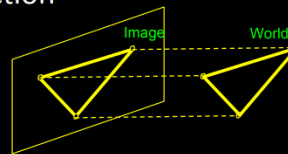
10. Other Models

- Orthographic or parallel projection

- Special case of perspective projection

- Distance from the COP to the image plane is infinite
 - => Both f and Z are very large

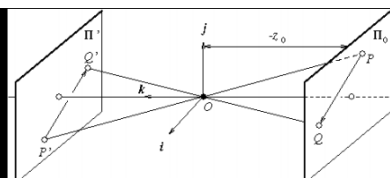
- Good approximation for telephoto optics
- Also called "parallel projection":
 $(x, y, z) \rightarrow (x, y)$



- a special case of perspective projection. where the distance from the center of projection to the image plane is infinite and my object is infinite, [z & x is infinite]

- Weak perspective

- *Perspective effects, but not over the scale of individual objects*
- Collect points into a group at about the same depth, then divide each point by the depth of its group
- Advantage: easy
- Disadvantage : only approximate



$$(x, y, z) \rightarrow \left(\frac{fx}{z_0}, \frac{fy}{z_0} \right)$$

- a special case of perspective projection, where each group of objects has its own scale factor

	3-d point	2-d image position
(1) Perspective:	(x, y, z)	$\rightarrow \left(\frac{fx}{z}, \frac{fy}{z} \right)$
(2) Weak perspective:	(x, y, z)	$\rightarrow \left(\frac{fx}{z_0}, \frac{fy}{z_0} \right)$
(3) Orthographic:	(x, y, z)	$\rightarrow (x, y)$