

2C-L3 Aliasing

2017/11/23 12:13

1. Intro

a. Previously

i. Fourier basis – basis set

1. decomposing functions in terms of sinusoidal basis.

ii. Fourier Transform

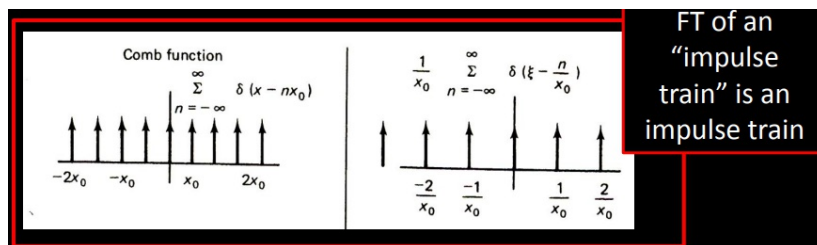
1. multiplication in one domain \Leftrightarrow convolution in the other domain

b. Today

i. How frequency explains the notion of aliasing.

ii. Multiplication in the spatial domain

2. Fourier Transform Sampling Pairs



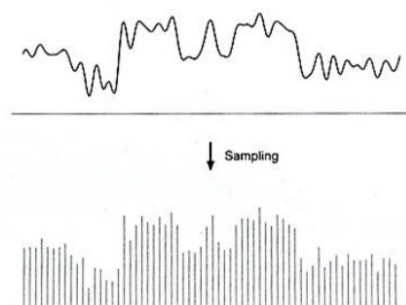
a. FT of a impulse train is another impulse train

b. for two impulses, the further in space, the closer in Fre.

1. Sampling and Reconstruction

a. Sampling

i. to solve How to store and compute with continuous functions?



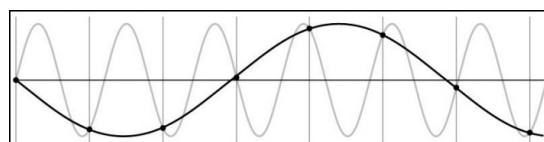
b. Reconstruction

i. Making samples back into a continuous function

ii. kind of "guessing"

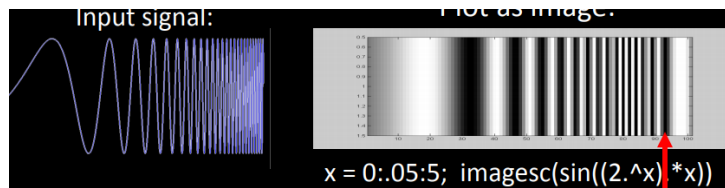
1. Aliasing

a. Aliasing: signals "traveling in disguise" as other frequencies



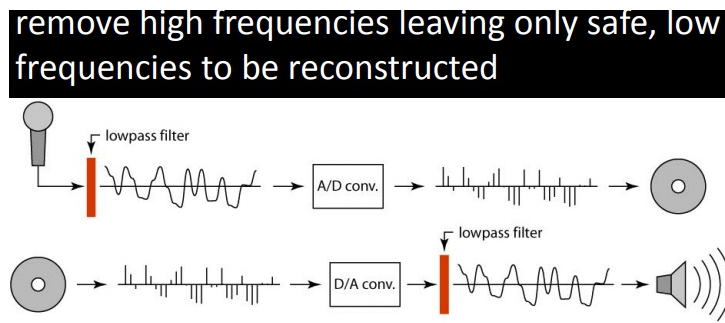
b. caused by Undersampling

- i. if a sine wave is undersampled, Low frequency is always indistinguishable from higher frequencies
- ii. the sampling Fre is too slow to capture the high Fre, e.g. wheel, propeller



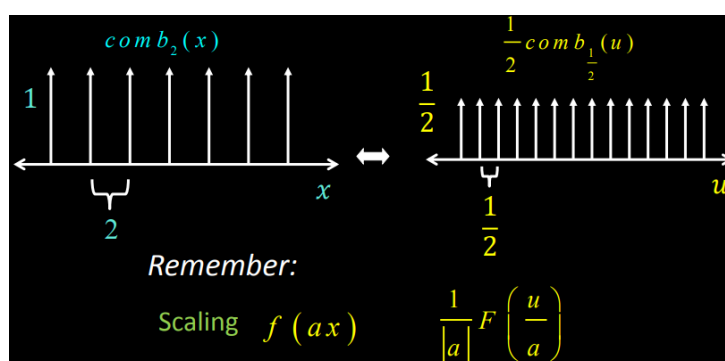
5. Antialiasing

- a. Sample more often
 - i. • Join the Mega-Pixel craze of the photo industry
 - ii. • But this can't go on forever
- b. Lowpass Filter: Make the signal less "wiggly"
 - i. • Get rid of some high frequencies
 - ii. • Will lose information
 - iii. • But it's better than aliasing



6. Impulse Train and Bed of Nails

- a. space and Fre is inverse proportion
- b. Impulse Train



c. Bed of Nails

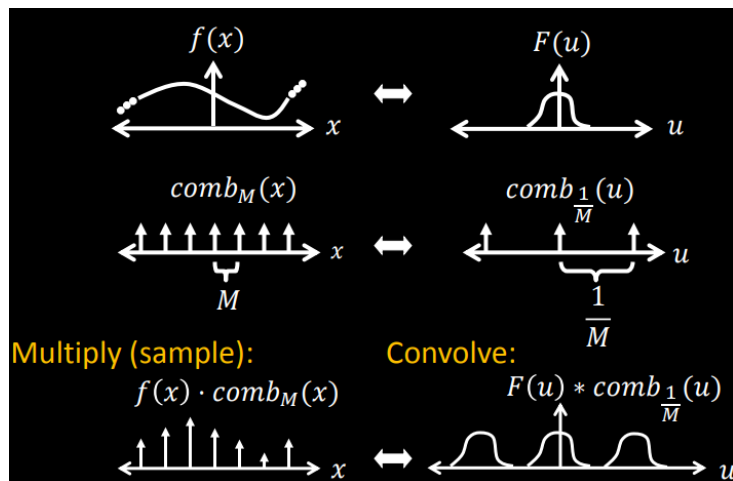
• Fourier Transform of an impulse train is also an impulse train:

$$\sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \delta(x - kM, y - lN) \Leftrightarrow \frac{1}{MN} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \delta\left(u - \frac{k}{M}, v - \frac{l}{N}\right)$$

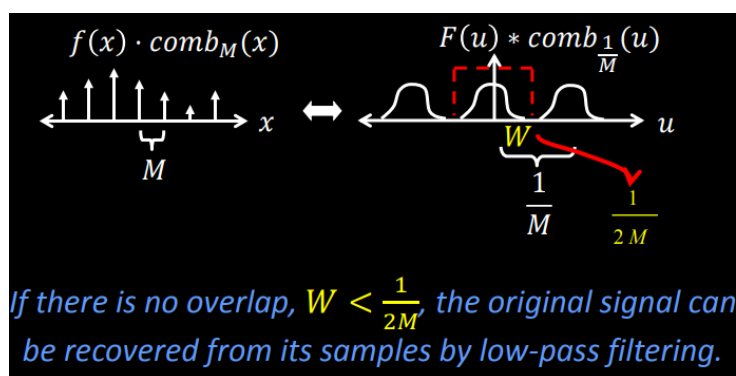
$comb_{M,N}(x,y)$ $comb_{\frac{1}{M}, \frac{1}{N}}(u,v)$

As the comb samples get further apart, the spectrum samples get closer together!

7. Sampling Theorem for low Fre.



a. sampling is just multiplying the signal with a comb in the space, while a convolution in Fre.



If there is no overlap, $W < \frac{1}{2M}$, the original signal can be recovered from its samples by low-pass filtering.

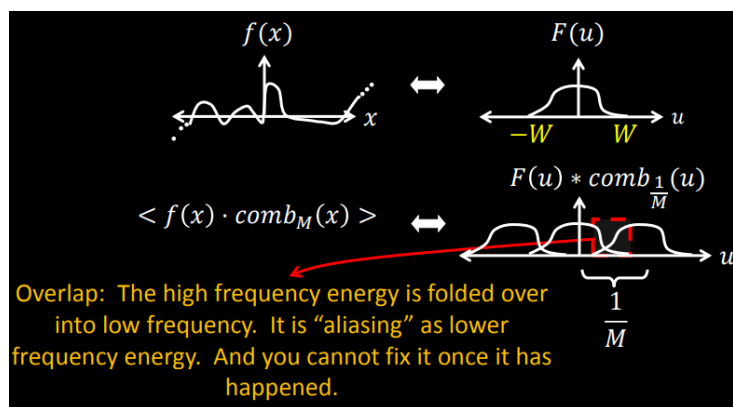
b. for the sampled signal, as long as the red box contains all its Fre and no overlap, then the original signal can be recovered back.

i. Nyquist criterion: $f_s > 2W$

1. Sampling High Frequency Signal

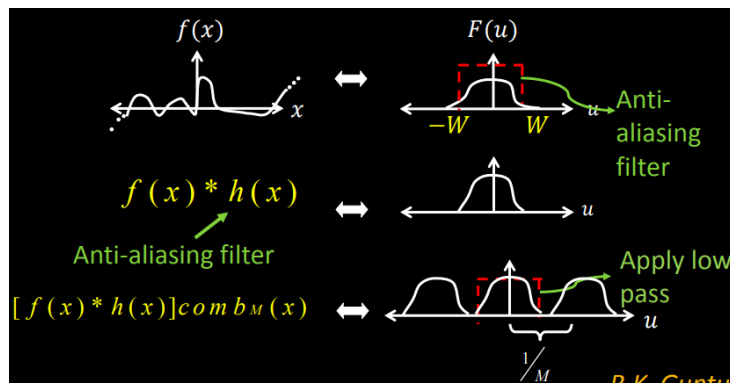
a. aliasing

i. The high frequency energy is folded over into low frequency. It is “aliasing” as lower frequency energy. And you cannot fix it once it has happened.



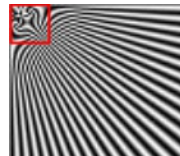
Overlap: The high frequency energy is folded over into low frequency. It is “aliasing” as lower frequency energy. And you cannot fix it once it has happened.

ii. we can fix it via reducing M , but the M can't be infinitely small while the Fre can be infinitely big.
iii. So we use an Anti-aliasing filter

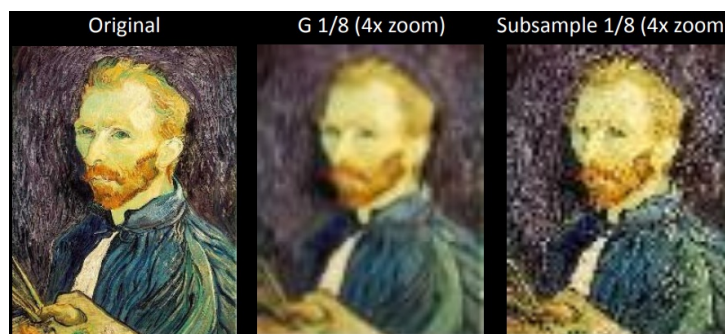


1. to remove the high Fre. e.g. Gaussian
2. for sampling, filter the HF before applying the comb, and for reconstruction, filter HF afterwards. Since we know there shouldn't be any HF in the original signal.

1. Aliasing in Images - resize the image
 - a. aliasing e.g.



- b. resize the image
 - i. Intuitive approach: take one pixel out of 8 both in row and column
 1. then we get the subsample one, which is ugly, since there's aliasing
 - ii. Better approach: apply the original image with Gaussian and then do the sampling, which is just a blurred image, a nicer one.
 1. $G 1/8$ can be achieved by $G 1/2 \Rightarrow G 1/4 \Rightarrow G 1/8$ or with a big G directly to $G 1/8$



10. Campbell-Robson Contrast Sensitivity
 - a. The higher the frequency the less sensitive human visual system is
11. Image Compression
 - a. Lossy Image Compression (JPEG)
 - i. DCT helps save more bits for Low Fre, but less bits

for HF

- DCT enables image compression by concentrating most image information in the low frequencies

Quantization Table

3	5	7	9	11	13	15	17
5	7	9	11	13	15	17	19
7	9	11	13	15	17	19	21
9	11	13	15	17	19	21	23
11	13	15	17	19	21	23	25
13	15	17	19	21	23	25	27
15	17	19	21	23	25	27	29
17	19	21	23	25	27	29	31