# Active Contours Without Edges Review

## (기존의 모델들은 gradient를 사용하여 contours를 찾았고 더 좋은 모델을 찾았다)

Abstract -

In this paper, we propose a new model for active contours to detect objects in a give image, based on techniques of curve evolution,

Mumford-Shah functional for segmentation and level sets.

Our model can detect objects whose boundaries are not necessarily defined by gradient.

We minimize an energy which can be seen as a particular case of the minimal parition problem,

In the Level set formulation, the problem becomes a "mean-curvature flow" - like evolving the active contour, which will stop on the desired boundary.

However, the stopping term does not depend on the gradient of the image, as in the classical active contour models, but is instead realted to a particular segmentation of the image.

we will give a numerical algorithm using finite differences.

Finally, we will present various experimental results and in particular some examples for which the classical snakes methods based on the gradient are not applicable.

Also, the initial curve can be anywhere in the image. and interior contours are automatically detected.

## Introduction

The basic idea in active contour models or snakes is to evove a curve, subject to constraints from a given image $u_0$, in order to detect objects in that image.

For instance, starting with a curve around the object to be detected, the curve moves toward its interior normal and has to stop on the boundary of the object.

Let $\Omega$ be a bounded open subset of $\mathbb{R}^2$ , with $\partial\Omega$ its boundary. Let $u_0 : \bar{\Omega} \to \mathbb{R}$ be a given image, and $C(s) : [0, 1] \to \mathbb{R}^2$ be a parameterized curve.

In the classical snakes and active contour models([][][3],[4],[9],[13]), an edge-detector is used, depending on the gradient of the image $u_0$ to stop the evolving curve on the boundary of the desired object.

We briefly recall these models next. Snake model[9] is $\inf_C \ J_1(C)$ ,where

$$J_1(C) = \alpha \int_0^1 |C'(s)^2| ds + \beta \int_0^1 |C''(s)| ds - \lambda \int_0^1 |\nabla u_0(C(s))|^2 ds \qquad (1)$$

Here, $\alpha, \beta, \lambda$ are positive parameters, $\alpha, \beta$ control the smoothness(internal energy) of contour, $\lambda$ controls the contour toward the object(external energy) in the image. by minimizing the energy(1) $J_1(C)$, we are trying to locate the curve at the points of maxima $|\nabla u_0|$ , acting as an edge-detector, while keeping a smoothness in the curve.

A general edge detector can be defined by a positive and decreasing function $g$ , depending on the gradient of the image $u_0$, such that $\lim_{z \to \infty} g(z) = 0$ . For instance

$$g(|\nabla u_0(x,y)|) = \frac{1}{1 + |\nabla G_\sigma(x,y) * u_0(x,y)|^p} \quad , \quad p \geq 1$$

,where $G_\sigma * u_0$, a smoother version of $u_0$, is the convolution of the image $u_0$ with Gaussian $G_\sigma(x,y)$. The function $g(|\nabla u_0|)$ is positive in homogeneous regions, and zero at the edges.

In problems of curve evolution, the level set method and in particular the motion by mean curvature of Osher와 Sethian[19] have been used extensively, because it allows for cusps, corners, and automatic topological changes.

Moreover, the discretization of the problem is made on a fixed rectangular grid.

The curve $C$ is represented implicitly via a Lipschitz function $\phi$, by $C = \{(x,y) | \phi(x,y) = 0\}$ , and the evolution of the curve is given by the zero-level curve at the time $t$ of the function $\phi(t,x,y)$.

Evolving the curve $C$ in normal direction with speed $F$ amounts to solve the differenctial equation[19]

$$\frac{\partial \phi}{\partial t} = |\nabla \phi| F, \;\; where \;\; \phi(0,x,y) = \phi_0(x,y)$$

Where the set $\{(x,y) | \phi_0(x,y)\}$ defines the initial contour.

When $F = div(\nabla \phi(x,y) / |\nabla \phi(x,y))$ is the curvature of the level curve of $\phi$ passing through $(x,y)$ , a particular case is the motion by mean curvature,

The equation becomes

$$\frac{\partial \phi}{\partial t} = |\nabla \phi| div(\frac{\nabla \phi}{|\nabla \phi|}), \;\; t \in (0, \infty), x \in \mathbb{R}^2$$

$$\phi(0,x,y) = \phi_0(x,y), \;\; x \in \mathbb{R}^2$$

$$div(\frac{\nabla \phi}{|\nabla \phi|}) = \frac{\partial \frac{\nabla \phi_t}{|\nabla \phi_t|}}{\partial t} + \frac{\partial \frac{\nabla \phi_x}{|\nabla \phi_x|}}{\partial x} + \frac{\partial \frac{\nabla \phi_y}{|\nabla \phi_y|}}{\partial y}$$

$$\therefore \frac{\partial \phi}{\partial t} = |\nabla \phi| * (\frac{\partial \frac{\nabla \phi_t}{|\nabla \phi_t|}}{\partial t} + \frac{\partial \frac{\nabla \phi_x}{|\nabla \phi_x|}}{\partial x} + \frac{\partial \frac{\nabla \phi_y}{|\nabla \phi_y|}}{\partial y}),\ \ t \in (0, \infty),\ c \in \mathbb{R}^2$$

A geometric active contour model based on the mean curvature motion is given by the following evolution equation[3]:

$$\frac{\partial \phi}{\partial} = g(|\nabla u_0|)|\nabla \phi|(div(\frac{\nabla \phi}{|\nabla \phi|}) + v),\ \ in\ \ (0, \infty) \times \mathbb{R}^2 \quad (2)$$

$$\phi(0, x, y) = \phi_0(x, y)\ \ in\ \ \mathbb{R}^2$$

Where, $g(|\nabla u_0|)$ edged-fnt with p=2, $0 \leq v$ is constant, $\phi_0$ is initial level set fnt.

Its zero level curve moves in the normal direction with speed $g(|\nabla u_0|)(curv(\phi)(x, y) + \mu)$ and thereofre stops on the desired boundary, where $g$ vanishes($i.e.\ g = 0$).

The constant $v$ is a correction term chosen so that the quantity $(div(\nabla \phi(x, y)/|\nabla \phi(x, y)|) + v|)$ remains always positive.

This constant $v$ may be interpreted as a forve pushing the curve toward the object, when the curvature becomes null or negative. Also, $v > 0$ is a constraint on the area inside the curve, increasing the propagation speed.

Two other active contour model based on level sets were proposed in[13], again using the image gradient to stop the curve.

### The first one is

$$\frac{\partial \phi}{\partial t} = |\nabla \phi|(-v + \frac{v}{M_1 - M_2)}(|\nabla G_\sigma * u_0| - M_2))$$

$$\phi(0, x, y) = \phi_0(x, y)\ \ in\ \ [0, \infty) \times R^2$$

Where, $v$ is constant, and $M_1$ and $M_2$ are the maximum and minimum values of the magnitude of the image gradient $|\nabla G_\sigma * u_0|$.

Again, the speed of the evolving curve becomes zero on the points with hightest gradients, and therefore the curve stops on the desired boundary, defined by strong gradients.

### The second model [13] is similar to the geometric modef[3]. with $p = 1$.

Other related works are [14] and [15].

The geodesic model[4] is

$$\inf_C J_2(C) = 2\int_0^1 |C'(s)| * g(|\nabla u_0(C(s)|)ds \qquad (3)$$

This is a problem of geodesic computation in a Riemannian space, according to a metric induced bt the image $u_0$.

Solving the minimization probelm(3) consists in finding the path of minimal new length in that metric.

A minimizer C will be obtained when $g(|\nabla u_0(C(s)|)$ = 0, i.e., when the curve C is on the boundary of the object.

The geodesic active contour model(3) from[4] also has a level set formulation.

$$\frac{\partial \phi}{\partial t} = |\nabla \phi|(div(g(|\nabla u_0|)\frac{\nabla \phi}{|\nabla \phi|}) + vg(|\nabla u_0)), \ \ in \ \ [0,\infty) \times \mathbb{R}^2 \qquad (4)$$

$$\phi(0,x,y) = \phi_0(x,y) \ \ in \ \ \mathbb{R}^2$$

Because all these classical nakes and active contour models rely on the edge-function $g$, depending on the image gradient $|\nabla u_0|$, to stop the curve evolution, these models can detect only objects with edges defined by gradient.

In practice, this discrete gradients are bounded and then the stopping function $g$ is never zero on the edges, and the curve may pass through the boundary, especially for the models in[3],[13]-[15].

If the image $u_0$ is very noisy, the the isotropic smoothing Gaussian has to be strong, which will smooth the edges too.

In this paper, we propose a different active contour model, without a stopping edge-function, i.e. a model which is not based on the gradient of the image $u_0$ for the stooping process.

The stopping term is based on Mumford-Shah segmentation techniques[18].

In this way, we obtain a model which can detect contours both with or without gradient, for instance objects with very smooth boundaries or even with discontinuous boundaries.

In addition, our model has a level set formulation, interior contours are automatically detected, and the initial curve can be any where in the image.

### The outline of the paper is as follows.

In the next section we introduce our model as an energy minimization and discuss the relationship with the Mumford-Shah functional for segmentation.

Also, we formulate the model in terms of level set functions and compute the associated Euler-Lagrange equations.

In section3 we present an iterative algorithm for solving the problem and its discretization.

In section 5 we validate our model by various numerical results on synthetic and real images, showing the advantages of our model described before, and we end the paper by a brief concluding section.

Other related works are[29],[10],[26] and [24] on active contours and segmentation,[28] and [11] on shape reconstruction from unorganized points, and finally the recent works[20] and [21] where a probability based geodesic active region model combined with classical gradient based active contour techniques is proposed.