Invariance by intrinsic alignment

One can make use of the shape distance (8) in a segmentation process by adding it as a shape prior $E_{shape}(\phi) = d^2(\phi, \phi_0)$ is a weighted sum to the data term, which is in our case the Chan-vese functional (2) Minimizing the total energy

$$\begin{aligned} E_{total}(\phi) &= E_{cv}(\phi) + \alpha E_{shape}(\phi) \\ &= E_{cv}(\phi) + \alpha d^2(\phi, \phi_0) \qquad (9) \end{aligned}$$

With a weight $\alpha > 0$, induces an additional driving term which aims at maximizing the similarity of the evolving shape with a given template shape encoded by the function ϕ_0.

By construction this shape prior is not invariant with respect to certain transformations such as

translation, rotation and scaling of the shape represented by ϕ.

Iterative optimization of explicit parameters

image 간의invariance 를 측정하기 위한 방법 is to enhance the prior by a set of explicit parameters to account for

translation by μ, rotation by an angle θ and scaling by σ of the shape:

$$d^2(\phi, \phi_0, \mu, \theta, \sigma) = \int_\Omega (H(\phi(\sigma R_\theta(x - \mu))) - H\phi_0(x))^2 dx \qquad (10)$$

단점

1. optimization of the shape energy is done by local gradient descent.

   needs to determine an appropriate time step for each parameter, to guarantee stability of resulting evolution.

   balancing these parameters requires a careful tuning process.

2. the optimization of $\mu, \theta, \sigma \; and \; \phi$ is done simultaneously.

   In practice, however, it is unclear how to alternate between the updates of the respective parameters.

   In experiments, we found that the final solution depends on the selected scheme of optimization.

3. The optimal values for the transformation parameters will depend on the embedding function ϕ.

   An accurate shape gradient should therefore take into account this dependency.

   In other words, (10)을 ϕ 에 대해 미분 한 식이 should take into account

   how the optimal transformation parameters $\mu(\phi), \sigma(\phi) \; and \; \theta(\phi)$ vary with ϕ

In order to eliminate these difficulties associated with the local optimization of explicit transformation parameters,

we will in the following present an alternative approach to obtarin invariance.

We will show that invariance can be achieved analytically by an intrinsic registration process.

We will detail this for the cases of translation and scaling.

Extensions to rotation and other transformations are conceivable but will not be pursued here.

translation invariance by intrinsic alignment

Assume that the template shape represented by ϕ_0 is aligned with respect to its center of gravity.

Then we define a shape energy by

$$\begin{aligned} E_{shape}(\phi) &= d^2(\phi, \phi_0) \\ &= \int_\Omega (H\phi(x+\mu_\phi) - H\phi_0(x))^2 dx \end{aligned} \qquad (11)$$

where the function ϕ is evaluated in coordinates relative to its center of gravity μ_ϕ given by :

$$\mu_\phi = \int x\ h\phi\ \ dx\ \ with\ \ h\phi \equiv \frac{H\phi}{\int_\Omega H\phi\ dx} \qquad (12)$$

This intrinsic alignement guarantees that the distance (11) is invarianat to the location of the shape ϕ

Why?! μ_ϕ 가 중심점을 control? 해줘서?? 의미는 알겠는데 왜 그런지를 모르겠네

In contrast to the shape energy(10), no need to iteratively update an estimate of the location of the object of interest.

translation and scale invariance via alignment

given a template shape (represented by ϕ_0) which is normalized with respect to translation and scaling,

one can extend the above approach to a shape energy which is invariant to translation and scaling:

$$E_{shape}(\phi) = d^2(\phi, \phi_0) = \int_\Omega (H\phi(\sigma_\phi x + \mu_\phi) - H\phi_0(x))^2 dx \qquad (13)$$

where the level set function ϕ is evaluated in coordinates relative to its center of gravity μ_ϕ and

in units given by its intrinsic scale σ_ϕ defined as

$$\sigma_\phi = (\int (x-\mu)^2 h\phi \, dx)^{\frac{1}{2}}, where \ h\phi = \frac{H\phi}{\int_\Omega H\phi \, dx} \tag{14}$$

여기도 의미는 알겠는데 왜 그런지 잘 모르겠다

we will show that functional (13) is invariant with respect to translation and scaling of the shape represented by ϕ

Let ϕ be a level set function representing a shape which is centered and normalized such that $\mu_\phi = 0 \ and \ \sigma_\phi = 1$

Let $\tilde{\phi}$ by an (arbitrary) level set function encoding the same shape after

scaling by $\sigma \in \mathbb{R}$ and shifting by $\mu \in \mathbb{R}^2$: $H\tilde{\phi}(x) = H\phi(\frac{x-\mu}{\sigma})$

Indeed, center and intrinsic scale of the transformed shape are given by:

$$\mu_{\tilde{\phi}} = \frac{\int xH\tilde{\phi} \, dx}{\int H\tilde{\phi} \, dx} = \frac{\int xH\phi(\frac{x-\mu}{\sigma})dx}{\int H\phi(\frac{x-\mu}{\sigma})dx} = \frac{\int(\sigma x' + \mu)H\phi(x')\sigma dx'}{\int H\phi(x')\sigma dx'} = \sigma\mu_\phi + \mu = \mu, \qquad ₩$$

$$\sigma_{\tilde{\phi}} = (\frac{\int(x-\mu_{\tilde{\phi}})^2 H\tilde{\phi} \, dx}{\int H\tilde{\phi} \, dx})^{\frac{1}{2}} = (\frac{\int(x-\mu)^2 H\phi(\frac{x-\mu}{\sigma})dx}{\int H\phi(\frac{x-\mu}{\sigma})dx})^{\frac{1}{2}} = (\frac{\int(\sigma x' H\phi(x')dx')}{\int H\phi(x')dx'})^{\frac{1}{2}} = \sigma$$

The shape energy(13) evaluated for $\tilde{\phi}$ is given by:

$$\begin{aligned} E_{shape}(\tilde{\phi}) &= \int_\Omega (H\tilde{\phi}(\sigma_{\tilde{\phi}} x + \mu_{\tilde{\phi}}) - H\phi_0(x))^2 dx \\ &= \int_\Omega (H\tilde{\phi}(\sigma x + \mu) - H\phi_0(x))^2 dx \\ &= \int_\Omega (H\phi(x) - H\phi_0(x))^2 dx = E_{shape}(\phi) \end{aligned}$$

결국 $\tilde{\phi}$ 에 대한 $\mu, \sigma, Energy$ 가 ϕ 에 대한것과 동일 하다는 것이다.

Therefore, the proposed shape dissimilarity measure is invariant with respect to translation and scaling.

Note, however, that while this analytical solution guarantees and invariant shape distance, the transformation parameters μ_ϕ and σ_ϕ are not necessarily the ones which minimize the shape distance (10.

Extensions of this approach to a larger class of invariance are conceivable.

For example, one could generate invariance with respect to rotation by rootational alignment with respect to the (oriented) principal axis of the shape encoded by ϕ

We will not pursue this in the present work.

For explicit contour representations, an analogous intrinsic alignment with respect to similarity transformation was proposed in Cremers