Kernel Density estimation in the level set domain

In the previous sections, we have introduced a translation and scale invariant shape enerhy and demonstrated its effect on the reconstruction of a corrupted version of a single familiar silhouette the pose of which was unknown

In many practical problems, however, we do not have the exact silhouette of the object of interest.

Reasons

1. The object of interest may be three-dimensional.

   Rather than trying to reconstruct the three dimensional object(which generally requires multiple images and the estimation of correspondence), one may learn the two dimensional appearance from a set of sample views, A meaningful shape dissimilarity measure should then measure the dissimilarity with respect to this set of projections.

   We refer to Cremers as an example.

2. The object of interest may be one object out of a class of similar objects (the class of cars or the class of tree leaves)

   Givne a limited number of training shapes sampled from the class, a useful shape energy should provide the dissimilarity of a particular silhouette with respect to this class.

3. Even a single object, observed from a single view-point, may exhibiy strong shape deformation-the deformation of a gesticulating hand or the deformation which a human silhouette undergoes while walking.

   In many cases, possibly because the camera frame rate is low compared to the speed of the moving hand or person, one is not able to exrract a model of the temporal succession of silhouettes.

   In this paper, we will assume that one can merely generate a set of stills corresponding to various(randomly sampled) views of the object of interest for different deformations: Figure 4, show such sample views for the case of a walking person.

   In the following, we will demonstrate that without being able to construct a dynamical model of the walking process one can exploit this set of sample views in order to improve the segmentation of a walking person.

In the above cases, the construction of appropriate shape dissimilarity measures amounts to a problem of density estimation. In the case of explicitly represented boundaries, this has been addressed by modeling the space of familiar shapes by linear subspaces(PCA) and the related Gaussian distribution, by mixture models or nonlinear representation via simple models in appropriate features spaces

For level set based shape representations, it was suggested to fit a linear subspace to the sampled signed distance functions. Alternatively, it was suggested to represent familiar shapes by the level set function encoding the mean shape and a Gaussian fluctuation at each image location.

These approaches were shown to capture some shape variability but have limitations

1. They rely on the assumption of a Gaussian distribution which is not well suited to approximate shape distributions encoding more complex shape variation.

2. they work under the assumption that shapes are represented by signed distance functions.

   Yet, the space of signed distance functions is not a linear space

   Therefore, in general, neither the mean nor the linear combination of a set of signed distance functions will correspond to a signed distance function.

In the following, we will propose an alternative approach to generate a statistical shape dissimilarity measure for level set based shape representations.

It is based on classical methods of kernel density estimation(non-parametric) and overcomes the above limitations.

Given a set of training shapes $\{\phi_i\}_{i=1..N}$- such as those shown in Figure 4= we define a probability density on the space of signed distance functions by intergrating the shape distances (11) or (13) in a P-R kernel density estimator

$$\mathcal{P} \propto \frac{1}{N} \sum_{i=1}^{N} \exp\big(-\frac{1}{2\sigma^2} d^2(H\phi, H\phi_i)\big) \qquad (21)$$

The kernel density estimator is. among the theoretically most studied density esimation methods.

It was shown to. onverge to. he tru distribution in the limit of. nfinite samples (and $\sigma \to 0$), the asymptotic convergence rate was studied for different choieces of kernel functions.

It should be pointed out that the theory of. lassical nonparametric density estimation was developed for the. ase of. inite0dimesional data.

For a general formalism to model probability dentities on infinite dimensional spaces, we refer the. eader to. he. heory of. aussian processes

In our case, an extension to infinite-dimensional objects sucha as level set surfaces $\phi : \Omega \to \R$ could be tackled by considering discrete approximations $\{\phi_{ij} \in \R\}_{i=1,..,N,j=1,..,M}$ of these surfaces at increasing levels of spatial resolution and studying the limit of. nfinitesimal grid size (i.e $N, M \to \infin$)

Alternatively, given a. inite number of samples, one can apply classical density estimation techniques efficiently in. he. inite-dimensional subspace spanned by the training data.

There exist extensive studies on how to. ptimally choose the. ernel width σ, based on. Symptotic expansions such as the parametric method, heuristic estimates or maximum likelihood optimization by cross validation.

We refer to. and G S for a. etailed discussion.

For this work, we. imply fix σ^2 to be the. ean squatred nearest-neighbor distance:

$$\sigma^2 = \frac{1}{N} \sum_i =1^N \min_{j \neq i} d^2(H\phi_i, H\phi_j). \qquad (22)$$

The intuition behind this choice is that the width of the Gaussians is chosen such that on the average the next training shape is within one standard deviation.

Reverting to kernel density estimation resolves the drawbacks of existing approaches to shape models for level set segmentation discussed above.

1. The silhouettes of a rigid 3D object of a deformable object with few degrees of freedom can be expected to form fairly low-dimensional manifolds.

   The kernel density estimator can capture these without imposing the restrictive assumption of a Gaussian distribution.

Figure 5, shows a 3D approximation of our method: We simply porjected the embedding functions of 100 silhouettes of a walking person onto the first three eigenmodes of the distribution.

The projected silhouette data and the kernel density estimate computed in the 3D subspace indicate that the underlying distribution is not Gaussian.

The estimated distribution (indicated by an isosurface) shows a closed loop which stems from the fact that the silhouettes were drawn from an essentially periodic process.

2. Kernel density estimator were show to converge to the true distribution in the limit of infinite(independent and identically distributed) training samples.

   In the context of shape representations, this implies that our approach is capable of accurately representing arbitrarily complex shape deformations.

3. By not imposing a linear subspace, we circumvent the problem that the space of shapes(and signed distance functions) is not a linear space. In other words: Kernel density estimation allow to estimate distributions on non-linear manifolds. In the limit of infinite samples and kernel width σ going to zero, the estimated distribution is more and more contrained to the manifold defined by the shapes.