LEVEL SET SEGMENTATION

originally introduced in the community of computational physics as a means of propagating interfaces, the level set method has become a popular framework for image segmentation.

The central idea is to implicitly represent a contour C in the image plane $\Omega \in \mathbb{R}^2$ as the zero-level of an embedding function $\phi : \Omega \Rightarrow \mathbb{R}$

$$C = \{x \in \Omega | \phi(x) = 0\} \qquad (1)$$

Rather than directly evolving the contour C, one evolves the level set function ϕ.

Advantages

1. don't need to to deal with control or marker points (and respective regarding schemes to prevent overlapping)
2. the embedded contour is free to under go topological changes such as splitting and merging which makes it well-suited for the segmentation of multiple or multiply-connected objects.

In the present paper, we use a level set formulation of the piecewise constant Mumford-Shah functional.

In particular, a two-phase segmentation of an image $I : \Omega \Rightarrow \mathbb{R}$ can be generated by minimizing the functional

$$E_{cv}(\phi) = \int_\Omega (I - u_+)^2 H\phi(x) dx + \int_\Omega (I - u_-)^2 (1 - H\phi(x)) dx + \nu \int_\Omega |\nabla H\phi| dx \qquad (2)$$

With respect to the embedding function ϕ

$H\phi \equiv H(\phi)$ denotes the Heaviside step function

u_+ is the mean intensity in the positive region

u_- is the mean intensity in the negative region

first two terms : minimizing the gray value variance in the separated phases,

last term : a minimal length of the separating boundary.

Gradient descent with respect to ϕ amounts to the evolution equation:

$$\frac{\partial \phi}{\partial t} = -\frac{\partial E_{cv}}{\partial \phi} = \delta_\epsilon(\phi)[\nu \, div(\frac{\nabla \phi}{|\nabla \phi|}) - (I - u_+)^2 + (I - u_-)^2] \qquad (3)$$

a smooth approximation δ_ϵ (Dirac delta) of the delta function which allows the detection of interior boundaries.

In the corresponding Bayesian interpretation. (베이지안 해석으로 (2)의 3번째 length term이 사전 확률로 보아질수 있다)

the length constraint given by the last term in (2) corresponds to a prior probability

which induces the segmentation scheme to favor contours of minimal length.

But what if we have more informative prior knowledge about the shape of expected objects?

we will construct a shape prior which statistically approximates an arbitrary distribution of training shapes.