Shape distances for level sets

The first step in deriving a shape prior is to define a distance or dissimilarity measure for two shapes encoded by the level set functions $\phi_1 \ and \ \phi_2$

We shall briefly discuss three solutions to this problem.

In order to guarantee a unique correspondence between a give shape and its embedding function ϕ , we will in the following assume that ϕ is $signed \ distance \ function$ i.e $\phi > 0$ inside the shape, $\phi < 0$ outside and $|\nabla\phi| = 1$ a.e.

to project a given emedding function onto the space of signed distance functions

Given two shapes encoded by their signed distance functions $\phi_1 \ and \ \phi_2$, a simple measure of their dissimilarity is given by their L_2 distance in Ω :

$$\int_\Omega (\phi_1 - \phi_2)^2 dx \qquad (4)$$

이 메저의 단점 : 상대적으로 동일 한 사진도 image domain이 커지면 동일하다고 인식을 못함

Remedy

It was proposed to constrain the integral to the domain where ϕ_1 is positive :

$$d_1^2(\phi_1, \phi_2) = \int_\Omega (\phi_1 - \phi_2)^2 H\phi_1(x) dx \qquad (5)$$

This mearues can be further improved by normalizing with respect to the area

where ϕ_1 is positive and by symmetrizing with respect to the exchanges of $\phi_1 \ and \ \phi_2$

The resulting dissimilarity measure,

$$d_{1s}^2(\phi_1, \phi_2) = \int_\Omega (\phi_1 - \phi_2)^2 \frac{h\phi_1 + h\phi_2}{2} dx \qquad (6)$$

$$h\phi \equiv \frac{H\phi}{\int H\phi dx}$$

Constitutes a pseudo-distance on the space of signed distance functions

Although the requirement of symmetry may appear to be a theoretical formality,

such symmetry considerations can have very relevant practical implications.

in particular, asymmetric measures of the form (5) do not allow to impose prior shape information outside the evolving shape (i.e in areas where $\phi_1 < 0$). =>(메저 (5)는 바깥 모양을 고려 못해줌)

Figure 1 shows an example of two circles which only differ by the fact that the second shape has a spike.

The measure (5) gives the same distance between the two shape

because it only takes into account shape discrepancy inside the first shape.

In contrast, the symmetric variant (6) also takes into account shape discrepancies within the second shape.

It gives a more informative measure of the shape dissimilarity and therefore allows for more powerful shape prior

1. (4)에서 ϕ_1 를 the contour C_1 로 표현 (i.e to the area where $\phi = 0$:)

$$d_2^2(\phi_1, \phi_2) = \oint_{C_1} \phi_2^2 dC_1 = \int_\Omega \phi_2^2(x)\delta(\phi_1)|\nabla\phi_1|dx \tag{7}$$

Due to the definition of the signed distance function,

this measure corresponds to the distance of the closest point on the contour C_2 (given by $|\phi_2|$) integrated over the entire contour C_1

As with (5), this measure suffers from not being symmetric.

The measure in(7) for example will only take into account points of contour C_2 which are sufficiently close to contour C_1, distant (and possibly disconnected) components of C_2 will be ignored.

2. A symmetric variant of (7) is given by:

$$d_{2s}^2(\phi_1, \phi_2) = \oint_{C_1} \phi_2^2 \, dC_1 + \oint_{C_2} \phi_1^2 \, dC_2 = \int_\Omega (\phi_2^2(x)|\nabla H\phi| + \phi_1^2(x)|\nabla H\phi_2|)dx$$

Further normalization with respect to the contour length is conceivable.

3. to compute the dissimilarity of two shapes represented by their embedding functions ϕ_1 and ϕ_2 is to compute the area of the set symmetric differene.

$$d^2(\phi_1, \phi_2) = \int_\Omega (H\phi_1(x) - H\phi_2(x))^2 dx \tag{8}$$

3번 장점 많아서 distance between two shapes=measure 를 3번으로 택할거다

properties (1) nonnegative, (2) symmetric, (3) triangle inequality.

embedding function의 sign에만 depend 하기 때문에 level set method 의 철학에 더 부합한다.

don't need to constain the two level set functions to the space of signed distance functions.

It can be shown that L^∞ and $W^{1,2}$ norms on the signed distance functions induce equivalent topologies as the metric(8)

Since the distance(8) is not differentiable, we will consider an approximation of the Heaviside function H by a smooth(differentiable) version H_ϵ H_2 말하는 거겠지