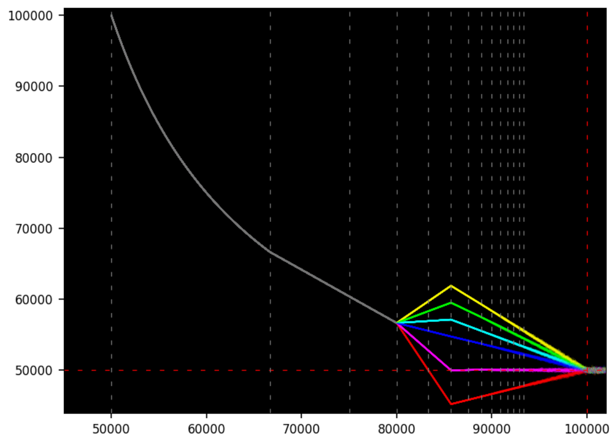
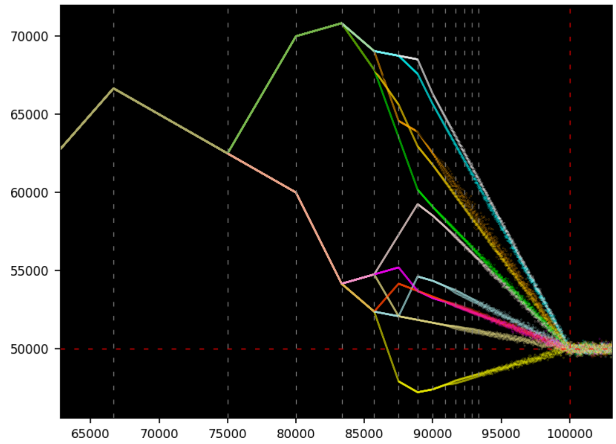
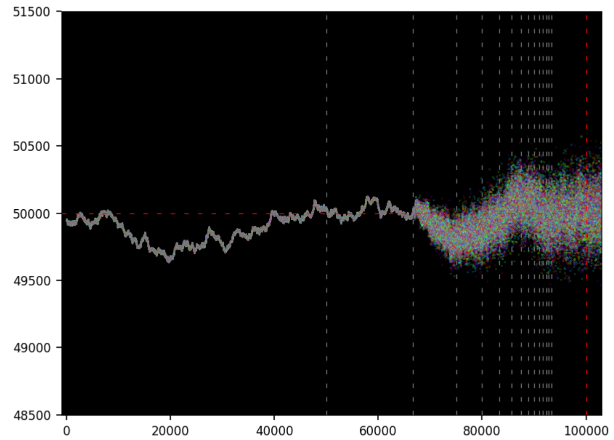
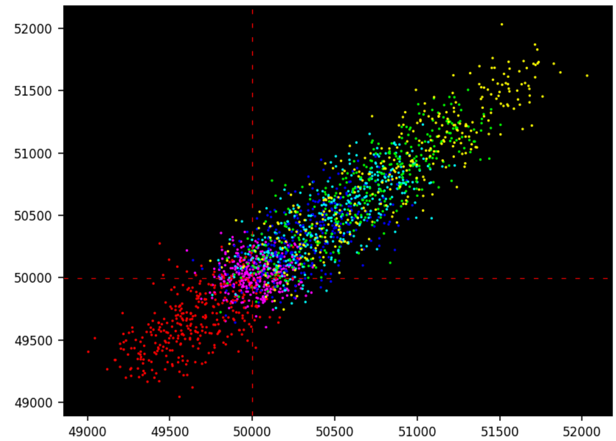
0 and 1

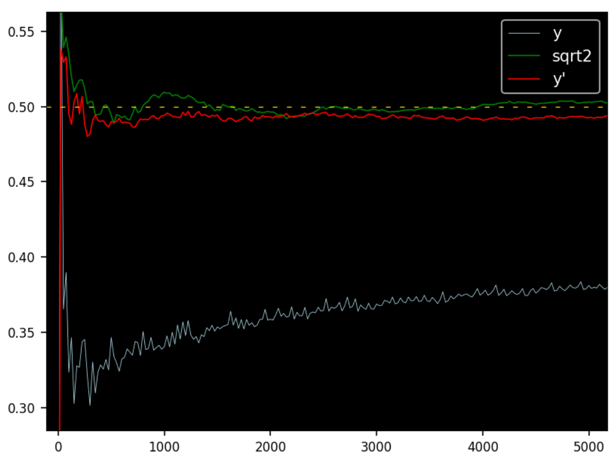
From Elemental Math to Quantum AI

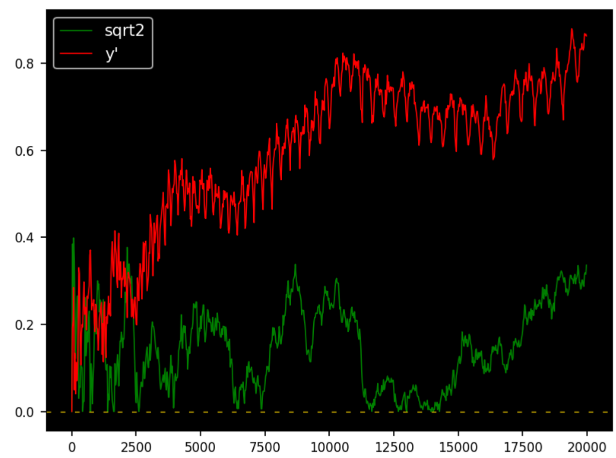












Vincent Granville, Ph.D. | BondingAI.io | Version 1.0, May 2025

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Introduction

It all started with the number 1. This book offers a trip deep into the most elusive and fascinating multi-century old conjecture in number theory: are the binary digits of the fundamental math constants evenly distributed? No one even knows if the proportions of '0' and '1' exist, for any of them: it could oscillate indefinitely between 0% and 100%.

After a quick read, you might be convinced that I solved it. Yet, very smooth functions visibly converging to the expected 50% ratio, time after time, are all but smoke and mirrors for the experienced practitioner. They are known to have unexpected, large singularities in extremely rare cases. And if you look at the curves with a microscope, such as Figure 3.6, there is still a bit of lingering randomness even after using advanced chaos removal techniques. Throughout this book, I guide the reader, warning about these caveats and dead ends while pointing to promising directions to work on the conjecture.

My novel approach sets a clear path towards solving the problem, for the first time ever. Building on solid foundations spanning across multiple disciplines, I share a number of spectacular victories along the way. For instance, this new and unexpected result: all numbers that can be written as

$$\frac{a_1}{2^1} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \cdots$$

where (a_n) is an increasing sequence of positive integers with sub-linear growth rate, have a proportion of '1' equal to zero in their infinite binary digit expansion.

I hope this document becomes a reference on the topic, and serves as a launchpad towards a final resolution. It is written in simple English, jargon-free, even when covering advanced topics. With enterprise-grade Python code along with efficient algorithms not taught in any classroom or textbook. To benefit from the material, you need beginner experience with Python, and the equivalent of a first year college course on calculus. The book is targeted to professionals in AI, machine learning, engineering, physics, scientific computing, operations research, computer science, and Fintech. Students with an analytical mindset will discover original, useful material to enhance their creativity, learning experience, while gaining strong exposure to professional code and applications.

This book also opens up new research areas in theoretical and computational number theory, numerical approximation, dynamical systems, quantum dynamics, and the physics of numbers. With a strong emphasis on applications: automated pattern detection and theorem proving with AI, agent-based modeling, building a universal unbiased pattern-rich synthetic dataset, cryptography (fast, strong random number generators based on irrational numbers), dynamical systems with chaos detection and isolation, computer-intensive simulations, and high performance computing to handle numbers such as $2^n + 1$ at power 2^n with $n = 10^6$.

Each chapter is self-contained and can be read separately from the others. Chapters are listed in chronological order. The last one features the most recent research and discoveries. The first one is more technical, focusing on the foundations; it can be skipped initially if you are mainly interested in the applications.

About the author

Vincent Granville is a pioneering AI scientist and mathematician, co-founder at DataScienceCentral (acquired by TechTarget in 2020), co-founder and AI lead at Bonding AI, author and patent owner. He worked with Visa, Wells Fargo, eBay, NBC, Microsoft, CNET and several startups. He is also one of top AI influencers working with NVIDIA, and publish a GenAI newsletter with 200,000 subscribers.

Vincent is a former post-doc at University of Cambridge. He published in *Journal of Number Theory*, *Journal of the Royal Statistical Society* (Series B), and *IEEE Transactions on Pattern Analysis and Machine Intelligence*. He is the author of multiple books, available here, including "Synthetic Data and Generative AI" (Elsevier, 2024). Vincent lives in Washington state, and enjoys doing research on stochastic processes, dynamical systems, probabilistic and computational number theory.