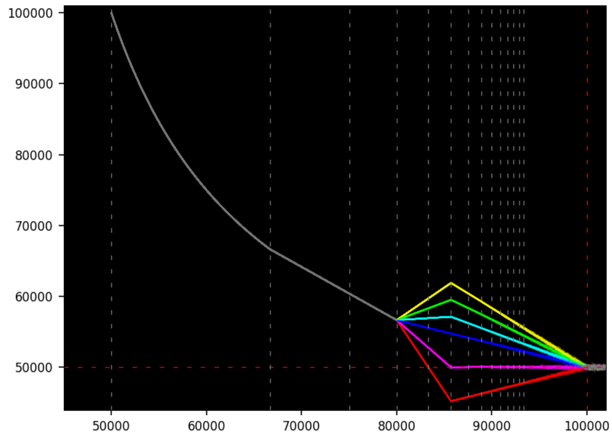
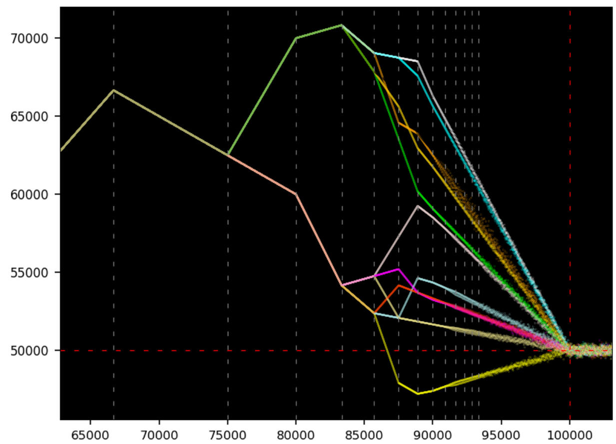
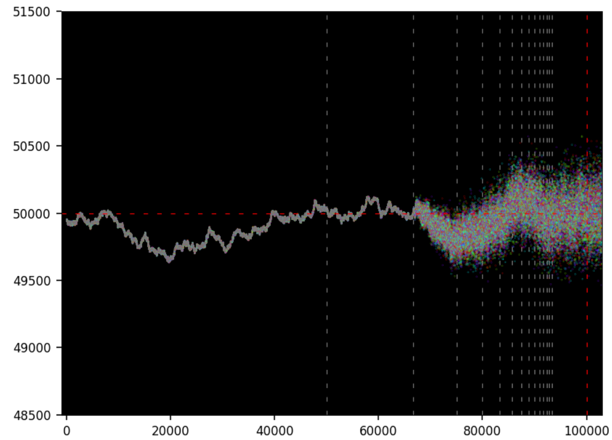
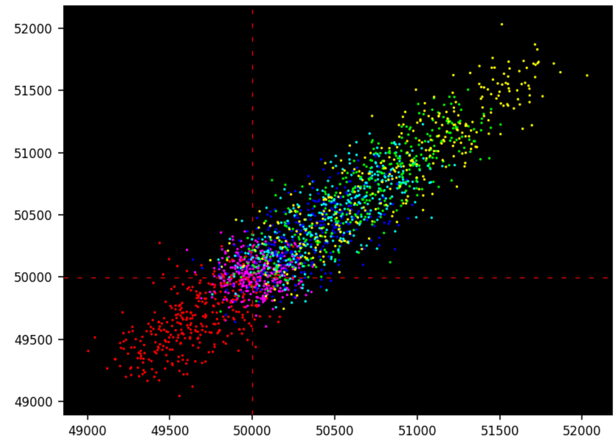
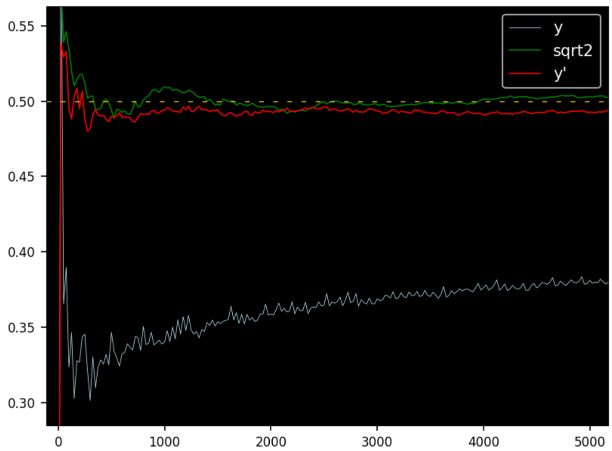
Breakthroughs on the Digit Distribution of Classic Constants

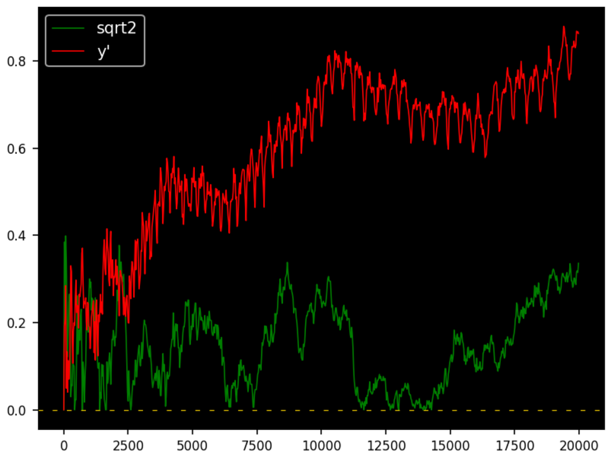












Vincent Granville, Ph.D. | BondingAI.io | Version 2.0, April 2026

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Introduction

Since the first edition entitled "0 and 1 – From Elemental Math to Quantum AI" and released in early 2025, a lot of progress has been made. Fascinating new results have been uncovered and proved by the author, many still leading to interesting quantum dynamics. In 100 pages, the new material presented here goes far beyond any articles and books published so far on the topic.

This second edition offers a trip deep into the most elusive and fascinating multi-century old conjecture in number theory: are the binary digits of the fundamental math constants evenly distributed? No one even knows if the proportions of '0' and '1' exist, for any of them: it could oscillate indefinitely between 0% and 100%. This new edition includes a new chapter on testing randomness with a much simplified version of Weyl's criterion. It also features a breakthrough result regarding the binary digit distribution, stating that the proportion of 1 must lie between $\frac{5}{16}$ and $\frac{11}{16}$ for a large class of numbers including all the standard mathematical constants such as π, e or $\sqrt{2}$. The details, with a hard, computer-assisted proof, are in the new chapter 5 and published here for the first time. In another example, I use quadratic dynamical systems on a matrix space with Chebyshev polynomials to unearth beautiful results.

This book is written in simple English even when covering advanced topics, avoiding jargon and advanced mathematics when not necessary. It is offered with enterprise-grade Python code for scientific and high performance computing with the Gmpy2 library, numerous high-quality illustrations, a comprehensive clickable index and bibliography, along with efficient algorithms not taught in any classroom or textbook. The target audience includes professionals in computer science, physics, AI, machine learning, engineering, quantitative finance, and related fields, as well as students and beginners with one year of exposure to college-level mathematics and Python.

The book opens up new fundamental research areas in theoretical and computational number theory, numerical approximation, dynamical systems, quantum dynamics, and the physics of numbers. It has a strong emphasis on applications: automated pattern detection and theorem proving with AI, agent-based modeling, building a universal unbiased pattern-rich synthetic dataset, cryptography (fast, strong random number generators based on irrational numbers), dynamical systems with chaos detection and isolation, computer-intensive simulations, and high performance computing to handle numbers such as $2^n + 1$ at power 2^n with $n = 10^6$.

Each chapter is self-contained and can be read separately from the others. Compared to the first version, this second edition contains significantly more material, including results published here for the first time. In particular, chapters 6 and 7 are new additions and contain a mix of theory, applications, and off-the-beaten path problems with solution. Quantum states and the Riemann zeta function are central themes in each of them. The section on signal processing and discrete convolution is a very strong, practical introduction to the topic, serving as a cheat sheet for practitioners or as a solid presentation for beginners, summarizing in a few pages material usually spread over several chapters.

About the author

Vincent Granville is a pioneering AI scientist and mathematician, co-founder at DataScienceCentral (acquired by TechTarget in 2020), co-founder and AI lead at Bonding AI, author and patent owner. He worked with Visa, Wells Fargo, eBay, NBC, Microsoft, CNET and several startups. He is also one of top AI influencers working with NVIDIA, and publish a GenAI newsletter with 200,000 subscribers.

Vincent is a former post-doc at University of Cambridge. He published in *Journal of Number Theory*, *Journal of the Royal Statistical Society* (Series B), and *IEEE Transactions on Pattern Analysis and Machine Intelligence*. He is the author of multiple books, available here, including "Synthetic Data and Generative AI" (Elsevier, 2024). Vincent lives in Washington state, and enjoys doing research on stochastic processes, dynamical systems, probabilistic and computational number theory.