1. (CLRS 16.1-2) Suppose that, in the activity-selection problem, instead of selecting the first activity to finish, we select the last activity to start which is compatible with all previously selected activities. Show that this approach exhibits the greedy-choice and optimal-substructure properties.

2. (CLRS 16.1-3) Not just any greedy approach to the activity-selection problem produces a maximum-size set of mutually compatible activities. Give examples to show that the following approaches do not always yield optimal solutions:

   (a) Selecting an activity of least duration from among those that are compatible with previously selected activities.

   (b) Selecting the compatible activity overlapping the fewest other remaining activities.

   (c) Selecting the compatible activity with the earliest running time.

3. (CLRS 16.2-5) Given a list $(x_0, ..., x_{n-1})$ of points on the real line, the problem is to determine the smallest set of closed intervals, each of length one, whose union contains all the given points. (Assume the list has distinct elements sorted in increasing order.)

   (a) Describe a greedy approach to this problem and the subproblem it leaves.

   (b) Show that your approach satisfies the greedy-choice and optimal-substructure properties.

   (c) Implement your approach in pseudocode.

   (d) Analyze the runtime and memory consumption of your algorithm.

4. (CLRS 16.4-1, 16.4-3, 16.4-4) Show that, in each the following, $(S, \mathcal{I})$ is a matroid.

   (a) S is a finite set, and $\mathcal{I}$ is the collection of subsets of S with size at most k (for some integer $0 \leq k \leq |S|$).

   (b) S is a finite set, $(S, \mathcal{I}')$ is a matroid, and $\mathcal{I} = \{A : S \backslash A \text{ contains a maximal element of } \mathcal{I}'\}$.

   (c) S is a finite set, $\{S_1, ..., S_k\}$ is a partition of S into k nonempty disjoint subsets, and $\mathcal{I} = \{A : |A \cap S_i| \leq 1 \text{ for all } 1 \leq i \leq k\}$.

5. Consider the 0-1 knapsack problem. Define $S = \{a_0, ..., a_{n-1}\}$ to be the set of objects, W to be the weight limit on the knapsack, and $\mathcal{I}$ to be the collection of subsets $A \subseteq S$ such that $\sum_{a_i \in A} w_i \leq W$.

   Letting $w(a_i) = v_i$, is $(S, \mathcal{I})$ necessarily a weighted matroid? Prove or give a counterexample.