

# CSCI-UA 480.4: APS

## Algorithmic Problem Solving

### Non-Linear Data Structures

Instructor: Joanna Klukowska

created based on materials for this class by  
Bowen Yu and materials shared by the authors of  
the textbook Steven and Felix Halim

# Binary Trees

# Balanced Binary Search Tree

- binary tree
- for each node
  - all the values in the left subtree are smaller than the value in that node
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- AVL tree, Red-Black tree

# Binary Heap / Priority Queue

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- building a heap from a collection of values is `O(n)` !!!

Why?

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- Direct Addressing Table (DAT) is the simplest form of a hash table in which *keys* are the indexes - it is simply an array

# Challenge

For each of the following problems 1) come up with tests that could be used for the implementation, 2) come up with an algorithm(s) that can be used to solve the problem, 3) what is the performance of your algorithms?

- Given  $n$  integers in arbitrary order find the  $k$  largest ones.
- Given a root to a binary tree containing  $n$  nodes, determine if it is a binary search tree?
- Given a binary search tree containing  $n$  nodes, output the elements with values in the range  $[a..b]$  in ascending order.
- Given a binary search tree containing  $n$  nodes, output the values in the leaves in ascending order.
- A basic max-heap does only supports additions and removal of the max element. How would you implement the following two operations:
  - $change\_value(i, new\_value)$  - modify the value at a specified index to the new\_value
  - $delete(i)$  - deletes the value at a specified index

