CSCI-UA 480.4: APS
Algorithmic Problem Solving

Greedy Algorithms

Instructor: Joanna Klukowska

created based on materials for this class by
Bowen Yu and materials shared by the authors of
the textbook Steven and Felix Halim

Greedy

- solving problems by making locally (at each step) optimal choices with the hope of obtaining optimal solution overall
- when does it work
    - problem has optimal sub-structure: optimal solution for the problem contains optimal solutions for sub-problems
    - problem has a greedy property: optimal choices made at the moment lead to otpimal overall solution (no need to go back and explore an alternative)
- examples

Challenge: Making Change

Task

Given an amount V (in cents) and a list of denominations of n coins find the way to make the change with the minimum number of coins. Assume that you have unlimited supply of each coin (for each given denomination).

Challenge: Making Change

Example

- n = 4
- denominations = {25, 10, 5, 1} cents
- desired amount V = 47 cents

solution:

- start with the largest denomination and use as many coins as possible
  47 - 25 = 22
- use the next highest denomination
  22 - 10 = 12
  12 - 10 = 2
- the remaining amount is too small for the denomination of 5, so skip it
- use the last denomination
  2 - 1 = 1
  2 - 1 = 0

so we need 5 coins: 25, 10, 10, 1, 1

Challenge: Making Change

This problem posseses both characteristics that make the greedy solution applicable here:

- the solutions to its subproblems are optimal, for example
  - for 22 cents, the otpimal solution would be 4 coins: {10, 10, 1, 1}
  - for 12 cents, the optimal solution would be 3 coins: {10, 1, 1}
- the solution has a greedy property: we use the largest possible coin first, if we used a smaller one instead, we would have to replace that coin with more than one coin in the alternative solution.

Challenge: Making Change

This problem posseses both characteristics that make the greedy solution applicable here:

- the solutions to its subproblems are optimal, for example
  - for 22 cents, the otpimal solution would be 4 coins: {10, 10, 1, 1}
  - for 12 cents, the optimal solution would be 3 coins: {10, 1, 1}
- the solution has a greedy property: we use the largest possible coin first, if we used a smaller one instead, we would have to replace that coin with more than one coin in the alternative solution.

Note

This approach does not work for all denominations. It does work for American coins of {25, 10, 5, 1} cents.

Problems to work on

- Station Balance
- Watering Grass
- Dragon of Loowater

Station Balance

Task

Given a number of chambers $1 \leq C \leq 5$ and a number of specimens $1 \leq S \leq 2C$ and a list of M masses for each specimen, determine how to distribute the specimens between the chambers so that their *imbalance* is as small as possible.

Restriction: each chamber can store 0, 1, or 2 specimens.

The *imbalance* is calculated as

$$\text{Imbalance} = \sum_{i=1}^{C} |X_i - A|$$

where A is the average weight that each chamber needs to carry $A = (\sum_{i=M}^{S} M_j)/C$ and X_i is the total weight of the specimens at chamber i.

Station Balance

Example

C = 3
S = 4
M = {5, 1, 2, 7}

The average weight A = (5+1+2+7)/3 = 5

A possible assignment (not optimal):

C1 = 7, 5
C2 = 2
C3 = 1

Imbalance = | 7+5 - 5| + |2 - 5| + | 1- 5| = 14

Station Balance

Example

C = 3
S = 4
M = {5, 1, 2, 7}

The average weight A = (5+1+2+7)/3 = 5

A possible assignment (not optimal):

C1 = 7, 5
C2 = 2
C3 = 1

Imbalance = | 7+5 - 5| + |2 - 5| + | 1- 5| = 14

Observation 1: If there is an empty chamber in the final solution, than it is always beneficial (in the sense that the new solution is not worse than the original) to move one specimen from a chamber with two specimens to the empty one. Otherwise, the empty chamber contributes more to the imbalance than the alternative assignment.

Observation 2: If S > C, then S - C specimens must be placed into chambers that already contain other specimens.

Station Balance

Solution

- If S <= C, then place each specimen in its own chamber. Some chambers might be empty.

- If S > C, then

    - create 2C - S dummy specimens with weight zero
    - sort 2C specimens (including the dummy ones)
    - pair the specimens from the beginning and end of the sorted list (i.e., smallest with largest, second smallest with second largest, ... until all are used) and place each pair in a chamber

Station Balance

Solution

- If S <= C, then place each specimen in its own chamber. Some chambers might be empty.
- If S > C, then
  - create 2C - S dummy specimens with weight zero
  - sort 2C specimens (including the dummy ones)
  - pair the specimens from the beginning and end of the sorted list (i.e., smallest with largest, second smallest with second largest, ... until all are used) and place each pair in a chamber

Example

C = 3, S = 4, M = {5, 1, 2, 7}

The average weight A = (5+1+2+7)/3 = 5

M with dummy specimens = { 5, 1, 2, 7, 0, 0}

sorted M with dummy specimens = {0, 0, 1, 2, 5, 7}

Optimal specimens to chamber assignment:

C1 = 7
C2 = 5
C3 = 1, 2

Imbalance = | 7 - 5| + |5 - 5| + | 3 - 5| = 4