Task 1

In this task, simulation of Geometric Brownian Motion (GBM) and mean reversal process are applied in Part 1 and Part 2, respectively. Due to the homogeneity of the simulation, the general simulation techniques are used in these two parts.

In this task, it is modeled to 1,000 paths when generating result of Part 1 and Part 2. This is to use the sufficiently large amount of data in computing certain parameters to gain higher accuracy. The usage of random number is required in generating the motion. In this case, `pylab.randn` is used in generating a set of normalized random numbers. `pylab` is a library defined in Python. All values generated by using the function above is then divided by $\sqrt{n}$, due to the usage of Central Limit Theorem in this task. The generations of random number takes place before the generation of GBM and mean reversal process takes place. Therefore, the generated random number is denoted by dB. In order to get a standard Brownian Motion, `dB.cumsum()` is used to cumulate the random numbers generated in dB to a series of numbers, B which denotes the current point after it goes up and goes down for given t. The generation of the paths will be described in each part, respectively.

Since it requires only 5 realization of GBM and mean reversal process, the first five paths are being selected and plotted into the graph generated by the two parts.

Part 1

Geometric Brownian Motion (GBM) follows the stochastic differential equation listed below,

$$\frac{dS_t}{S_t} = \mu\, dt + \sigma dB_t$$

where the solution of this stochastic differential equation is:

$$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B_t}$$

This solution can be expressed in an expected function and variance function, *E(X)* and *Var(X)*, where X is a random variable. The solution is very similar to moment generating function of a normal random variable, denote this by X, $X \sim N(\mu, \sigma^2)$,

$$M_X(t) = E(e^{tX}) = e^{\mu t + \frac{\sigma^2 t^2}{2}}, -\infty < t < \infty$$

apply some modification by letting *X* be a stochastic random variable, the moment generating function of *X(t)*, where $X(t) \sim N(\mu t, \sigma^2 t)$,

$$M_{X(t)}(s) = E\left(e^{sX(t)}\right) = e^{\mu s t + \frac{\sigma^2 t s^2}{2}}, -\infty < s < \infty$$

therefore:

$$E\big(S(t)\big) = S_0 e^{\left(\mu+\frac{\sigma^2}{2}\right)t}$$

$$Var\big(S(t)\big) = {S_0}^2 e^{2\mu t+\sigma^2 t}(e^{\sigma^2 t} - 1)$$

can be easily proven from the moment generating function mentioned above. (Sigman, 2006) By applying the formulas mentioned above, with these parameters: $S(0) = 39, \mu = \frac{0.1}{39}, \sigma = \frac{0.26}{39}, t = 3$

$$E\big(S(3)\big) = 39.30270718, Var\big(S(3)\big) = 0.205985312$$

Since the solution of the stochastic differential equation is known, instead of using iterations, the actual solution is presented.

This part is generated from Python.

```
Computation of mean, variance, P[S(3) > 39] and E[S(3)|S(3) > 39] from the
simulation:
Mean, E[S(3)] =  39.3014866876
Variance, Var[S(3)] =  0.0693010343703
P[S(3) > 39] = 0.876
E[S(3)|S(3) > 39] = 39301.4866876

Theoretical variance:
Variance, Var[S(3)] =  0.205985316648
```

Computation of mean, $E(S(3))$ is theoretically taken from the end of each path of the generated GBM and take the average of it. The mean from the simulation is approximately equal to the mean calculated using the above formula. This is due to the difference of the simulation with the exact formula as it is always depends on the dB generated from the initial stage. At the same time, the variance, $Var(S(3))$ also generated from the last column from the variable. However, due to the large number of paths generated (1,000 paths), the variance shall converge to the actual variance. The reason that the variance is less than the actual variance is due to the variance computed is based on the final values obtained only but the actual variance is computed based on t and other variables.

The computation of probability, $P[S(3) > 39]$ varies depending on the amount of the last column of S generated each time. In the code, the usage of Boolean data type is to check the condition whether the element of S is satisfying the condition of more than 39 or not. The `pylab.sum()` function helps in adding up the total amount of the condition when it is true. Then, the definition of probability is used in computing the value. The computation of conditional expectation, $E[S(3)|S(3) > 39]$ is not quite true based on the computation. The equations below show the reason on why the conditional expectation is the sum of all last column of S in the code.

$$E[S(3)|S(3) > 39] = \sum S(3)P[S(3)|S(3) > 39]$$

$$= \sum S(3) \frac{P[S(3) \cap S(3) > 39]}{P[S(3) > 39]}$$

$$= \sum S(3) \frac{P[S(3) > 39]}{P[S(3) > 39]}$$

$$= \sum S(3)$$

Since there are 1,000 paths of GBM generated, therefore, it is equal to the mean times 1,000 (the amount of paths). The expected value is using summation notation as the amount is countable.

For the convenience of reference, Appendix 1 shows a single run of the Python file "gbm.py".

Part 2

The mean reversal process is a stochastic differential equation that can be reverted to a solution that has this pattern of equation that follows Ornstein-Uehlenbeck process.

Differential equation:

$$dX_t = \alpha(\theta - X_t)dt + \sigma dB_t$$

Solution:

$$X_t = \theta(1 - e^{-\alpha t}) + X_0 e^{-\alpha t} + \sigma \int_0^t e^{\alpha(s-t)} dB_s$$

However the differential equation that is used to simulate in this task is hard to apply this. Therefore, numerical approach is possible in Python. From this stochastic differential equation, apply some modification to make iteration possible.

$$dR(t) = \big(0.064 - R(t)\big)dt + 0.27R(t)dB(t)$$

Let $dR(t) = R(t_k) - R(t_{k-1}), dt = t_k - t_{k-1}, dB(t) = B(t_k) - B(t_{k-1})$

$$R(t_k) - R(t_{k-1}) = \big(0.064 - R(t_{k-1})\big)(t_k - t_{k-1}) + 0.27R(t_{k-1})(B(t_k) - B(t_{k-1}))$$

$$R(t_k) = R(t_{k-1}) + \big(0.064 - R(t_{k-1})\big)(t_k - t_{k-1}) + 0.27R(t_{k-1})(B(t_k) - B(t_{k-1}))$$

This is then expressed in Python as:

```
for c in range(n):
      R[:,c+1] = R[:,c] + (0.064 - R[:,c]) * dt + 0.27 * R[:,c] * (B[:,c+1] -
      B[:,c])
```

`dt` is defined to be the total period divided by number of parts. Since $(t_k - t_{k-1})$ is always equal to `dt`, the code is using `dt` in the iterations. Again, `B` is generated from standard Brownian motion as mentioned in the beginning of the description of this task.

The mean, $E[R(1)]$ is calculated using the basic definition of expected value, by summing up the last column of R in the matrix and divided by the number of paths generated. The generation of the probability is similar to the way the probability is generated in the previous part.

Appendix 2 shows a single run of the Python file of "mr.py".

Task 2

Part 1

There are thirty (30) stocks which are the components of the FTSE Bursa Malaysia KLCI Index. Below is the table of stocks of components.

Stock Name	Stock Code	Stock Sector	Weightage in FTSE KLCI	PE Ratio	Net Market Capital (Billion)
AMMB Holdings	1015	Finance	0.016835	8.85	16.970
Astro Malaysia Holdings	6399	Service	0.015636	28.19	15.761
Axiata Group	6888	Service	0.054907	24.51	55.347
British American Tobacco (Malaysia)	4162	Consumer Products	0.018298	20.13	18.445
CIMB Group Holdings	1023	Finance	0.046323	17.82	46.694
Digi.com	6947	IPC	0.041498	21.02	41.830
MISC	3816	Service	0.034984	16.00	35.264
Genting	3182	Service	0.030710	19.13	30.956
Genting Malaysia	4715	Service	0.025036	21.17	25.237
Hong Leong Bank	5819	Finance	0.025476	11.91	25.680
Hong Leong Financial	1082	Finance	0.016126	9.97	16.255
IHH Healthcare	5225	Service	0.048370	63.63	48.758
IOI	1961	Plantation	0.027041	65.53	27.258
Westports Holdings	5246	Service	0.013802	26.58	13.913
Kuala Lumpur Kepong	2445	Plantation	0.023849	29.28	24.040
Malayan Banking	1155	Finance	0.087340	12.92	88.040
Maxis	6012	Service	0.049016	30.14	49.409
PETRONAS Chemicals Group	5183	Industrial Products	0.050237	21.82	50.640
Petronas Dagangan	5681	Service	0.020292	37.02	20.455
Petronas Gas	6033	Industrial Products	0.042401	22.79	42.741
PPB Group	4065	Consumer Products	0.018088	18.13	18.233
Public Bank	1295	Finance	0.073175	15.78	73.761
RHB Capital	1066	Finance	0.018977	9.27	19.129
SapuraKencana Petroleum	5218	Service	0.014207	12.10	14.321
Sime Darby	4197	Service	0.052252	20.93	52.671
Telekom Malaysia	4863	Service	0.024717	33.22	24.915
Tenaga Nasional	5347	Service	0.068753	9.15	69.304
UMW Holdings	4588	Consumer Products	0.011892	20.41	11.987
YTL Corp	4677	Service	0.017133	15.56	17.270
KLCC	5235SS	REITS	0.012627	13.65	12.728

Table 1: Components of FTSE Bursa Malaysia KLCI with the Stock Code, Sectors, Weightage in the Index, P/E Ratio and Net Market Capital in billions of Ringgit. Information adapted from http://www.malaysiastock.biz/Listed-Companies.aspx on July 26, 2015.

Part 2

In this part, Kuala Lumpur Kepong (KLK) (2445.KL) is chosen as the stock for the calculation of 5-day moving average as well as the correlation coefficient between itself and FTSE Bursa Malaysia KLCI Index.

In this part, instead of extracting the 5-day moving average out of the data type of DataFrame, another column for calculating 5-day moving average is added to that DataFrame named `data`. Besides that, implementation of `pandas` in Python allows the calculation of moving average, in Python is called `rolling_mean`, in which it accepts inputs as a column of data from data type DataFrame and an integer that gives the amount of "rolls" for each iteration, which is meant for the amount of input requires for the moving average. For example, if it requires a 10-day moving average, the input receives ten (10) elements in the array and divides it by 10. This can be calculated faster than using loops in the computation.

When computing correlation coefficient of the data, an array of strings is set up to denote the requirement of the data downloader to download "^KLSE" and "2445.KL" on the values of closing only for the specified dates. After receiving the data, using the `closing.corr()` to compute the correlation matrix. `closing` is an array with data type DataFrame and `corr` asks Python to compute the correlation matrix of the data that `closing` had received. The `closing.corr()`, which is denoted by `correlation` in "download_data.py", gives a symmetrical matrix. If an input of `correlation` is typed after the computation, observation will be as follows:

```
           2445.KL     ^KLSE
2445.KL   1.000000  0.318993
^KLSE     0.318993  1.000000
```

It is logical to obtain this result. As `2445.KL` is identical to `2445.KL`, as well as `^KLSE` is identical to `^KLSE`, both of their correlation with respect to themselves are always perfectly positively correlated, therefore the diagonal part is always 1. When goes to the non-diagonal part, the correlation between `2445.KL` and `^KLSE` is 0.318993, which is the same when they swap their positions. This means that 2445.KL and FTSE Bursa Malaysia KLCI Index are weak-positively correlated with respect to each other. For example, if there is an increase in the stock price of KLK, there will be generally a slight increase in FTSE Bursa Malaysia KLCI Index. In taking out the value from the matrix, the "slice" technique is used. The `printer` slices the second column out of the correlation matrix and use `printer` as the basic of printing the correlation coefficient to the display screen.

The way of creating the graph is different from the way used in the previous task. The call of the storage DataFrame is used instead of asking `pylab` in doing so. `data['5-day Moving Average'].plot()` plots the values on the `5-day Moving Average` column to the graph with the index (horizontal axis) as date labeled in the `data` with default features. The usage of `title` and `color` is to write a title on the graph and to change the colour from its default settings, respectively.

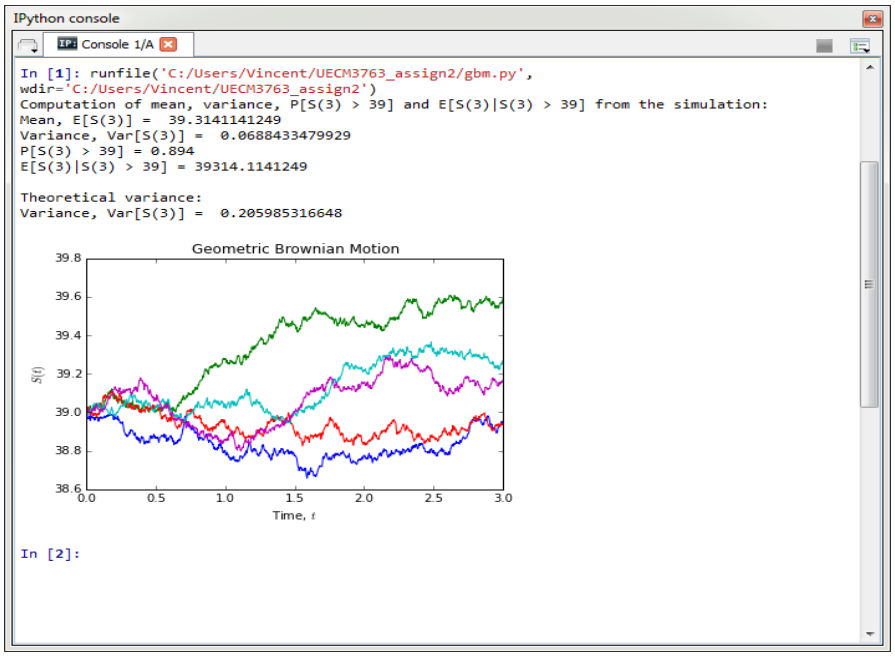
Please refer to Appendix 3 for the 5-day moving average plot by the Python file "download_data.py".

References

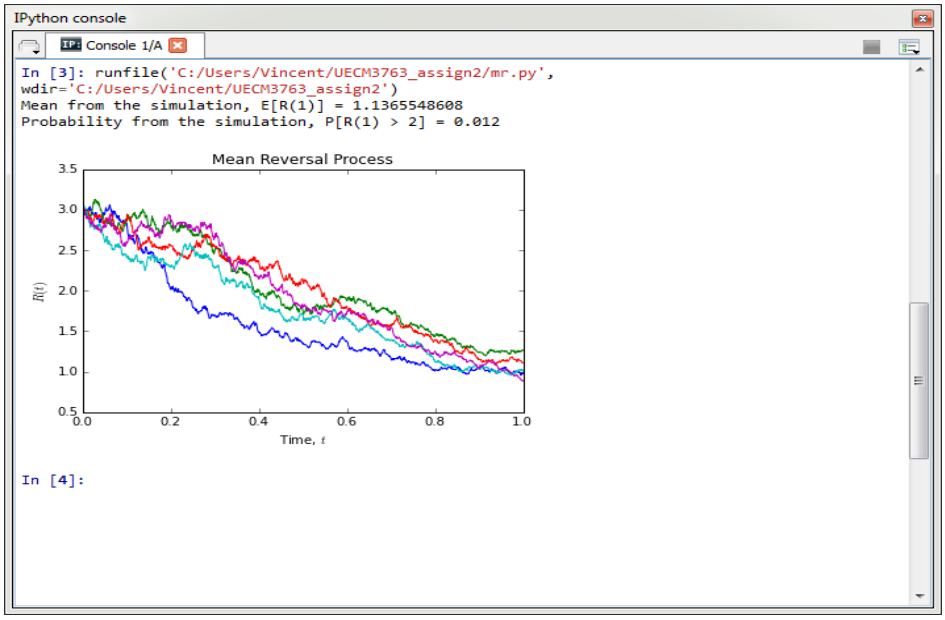
Malaysiastock.biz. (n.d.). *KLSE listing companies.* Retrieved July 26, 2015, from http://www.malaysiastock.biz/Listed-Companies.aspx

Sigman, K. (2006). Geometric Brownian motion. *Back to our study to geometric BM.* Columbia University: New York. Retrieved July 25, 2015, from http://www.columbia.edu/~ks20/FE-Notes/4700-07-Notes-GBM.pdf

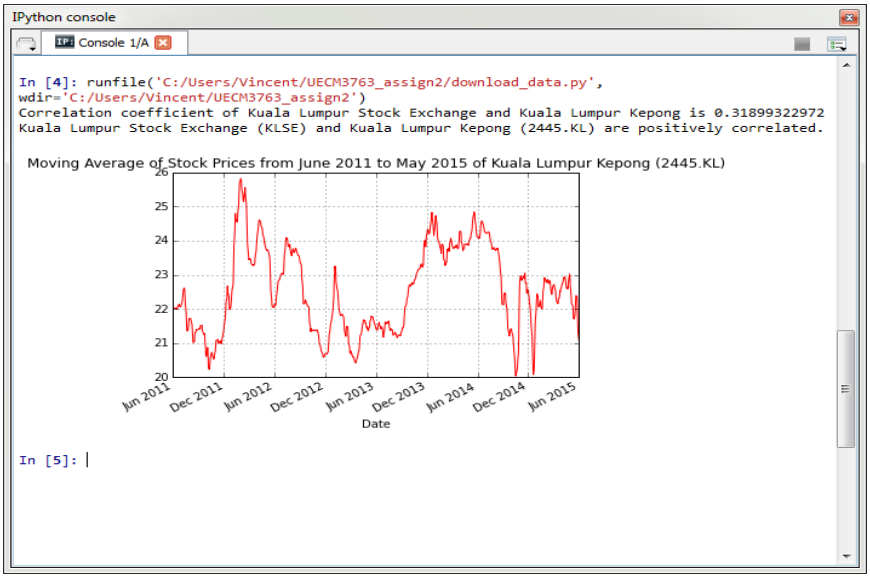
Appendix



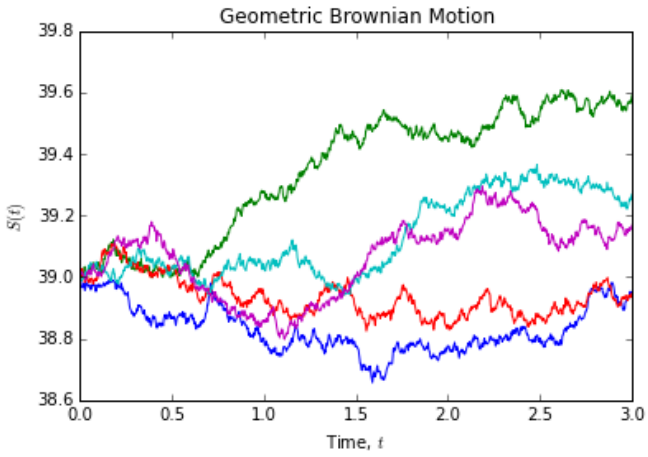
Appendix 1: Result generated from the Python file "gbm.py"



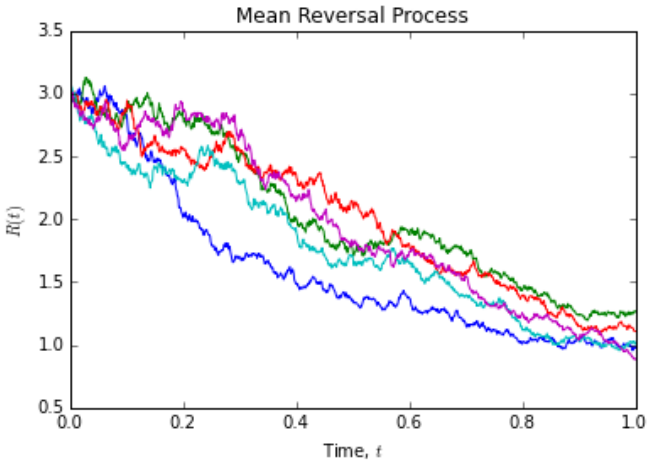
Appendix 2: Results generated from the Python file "mr.py"



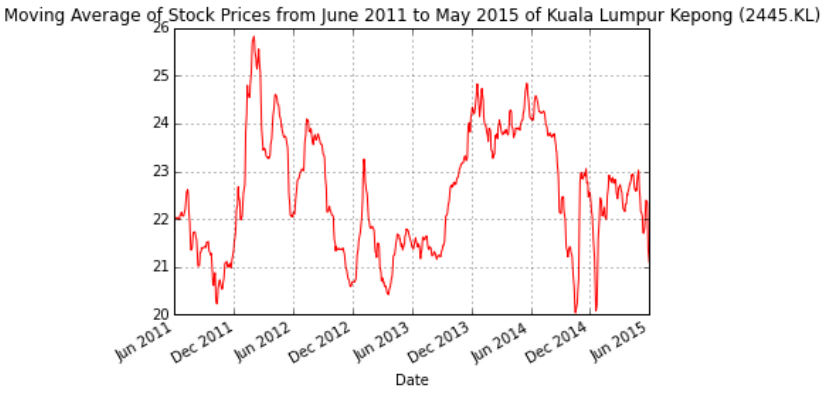
Appendix 3: Result generated from the Python file "download_data.py"



Appendix 4: Graph of 5 realizations of Geometric Brownian Motion (GBM)



Appendix 5: Graph of 5 realizations of Mean Reversal Process



Appendix 6: Graph of 5-day moving average of Kuala Lumpur Kepong (2445.KL) from 1 June 2011 to 1 June 2015