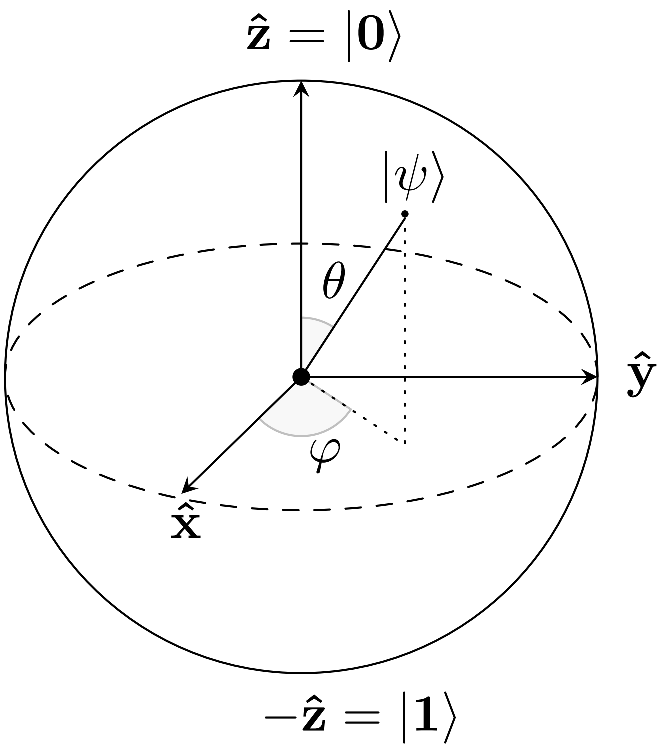
Report intership - 4A
Polytech Clermont-Ferrand - GP4A (2023 - 2024)
Engineering Physics Department

Entanglement in a spin chain



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Abstract

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Quantum information XY-Γ chain

Keywords :

Acknowledgements

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Glossaries

1. Introduction

Quantum entanglement, a fundamental feature of quantum mechanics, was first recognized in the early 20th century by Einstein, Podolsky, Rosen, and Schrödinger [3]. This phenomenon describes a situation where the quantum states of two or more particles become intertwined, such that the state of one particle cannot be described independently of the state of the other, no matter the distance separating them.

Entanglement become a practical resource in quantum information science, underpinning technologies such as quantum cryptography [6], quantum teleportation [1], and quantum computing [7]. These applications exploit entanglement to perform tasks that are impossible with classical systems.

Despite its utility, entanglement is a fragile and complex phenomenon, challenging to detect and manipulate. The study of entanglement involves understanding its properties, methods for its detection, and strategies for its quantification and manipulation. These efforts are crucial for advancing our ability to harness entanglement for practical applications, ensuring that it can be effectively used as a resource in quantum communication and computation.

The work presented here go will study the simplification of the Kitaev model call $XY - \Gamma$ model because this model have a good futur to build a quantum computer (cf paper XY-Gamma for the ref about Kitaev) I this model we will see how the entanglement propagate and quantify the entanglement.

1.1 Model XY-DMI

Introduction on the Heinsenberg model + Introduction on the DMI + spin chain +qubits

The XY-Γ model [4] or XY model is a devivative of the

with a Dzyaloshinskii–Moriya interaction (DMI) [5, 2] is one-dimentionel spin chain.

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_{XY} + \hat{\mathcal{H}}_{\Gamma}, \tag{1}$$

where

$$\hat{\mathcal{H}}_{XY} = J \sum_{n=1}^{L} \left[\left(\frac{1+\delta}{2} \right) \sigma_n^x \sigma_{n+1}^x + \left(\frac{1-\delta}{2} \right) \sigma_n^y \sigma_{n+1}^y \right] + h \sum_{n=1}^{L} \sigma_n^z, \tag{2}$$

and

$$\hat{\mathcal{H}}_{\Gamma} = \Gamma \sum_{n=1}^{L} \left(\sigma_n^x \sigma_{n+1}^y + \gamma \sigma_n^y \sigma_{n+1}^x \right). \tag{3}$$

Here, $\sigma^x, \sigma^y, \sigma^z$ are the Pauli matrices, J is the exchange constants, δ is the anisotropy parameter, h is the strength of the transverse field, and D characterizes the amplitude of the off-diagonal exchange interactions with γ being the relative coefficient of these couplings.

eff

- **XY term** : $\hat{\mathrm{H}}_{XY}$ describes the nearest-neighbor interactions along the x and y directions with anisotropy δ and a transverse magnetic field h.
- Γ **term or Dzyaloshinskii-Moriya interaction** : $\hat{\mathrm{H}}_{\Gamma}$ introduces an additional interaction that mixes the x and y components of neighboring spins, breaking mirror symmetry (Because the particle at position n does not have the same orientation of spin as the particle at position $n+1$, as represented by the term $\sigma_n^x \sigma_{n+1}^y$, where the particle at position n has a spin oriented in the x direction and the particle at position $n+1$ has a spin oriented in the y direction.).

1.2 Entanglement metric

cf the video

2. Methodology

Methodology

Numerical approch

Algo : Solve Schrodinger EQUATION

Algo : Found Concurrence

Analytical approch

Hilbert space reduction (cf intro report for Guillermo)

3. Results And Discussion

Results And Discussion

3.1 Entanglement in a spin chain with 2 qubits

3.1.1 Numerical approch

3.1.2 Analytical approch

3.2 Entanglement in a spin chain with N qubits

3.2.1 Entanglement in a spin chain with 3 qubits

3.2.2 Entanglement in a spin chain with 8 qubits

3.2.3 Analytical approch

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