Report intership - 4A

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Engineering Physics Department

Quantum Superconducting circuit

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Contents

1. Introduction of superconducting

The superconducting state is a phase of matter or, more precisely, a second-order phase transition of matter at a temperature T_c that induces different properties. The historical property is the low resistance ($R < 10^{-5}\Omega$) discovered by Heike Kamerlingh Onnes.

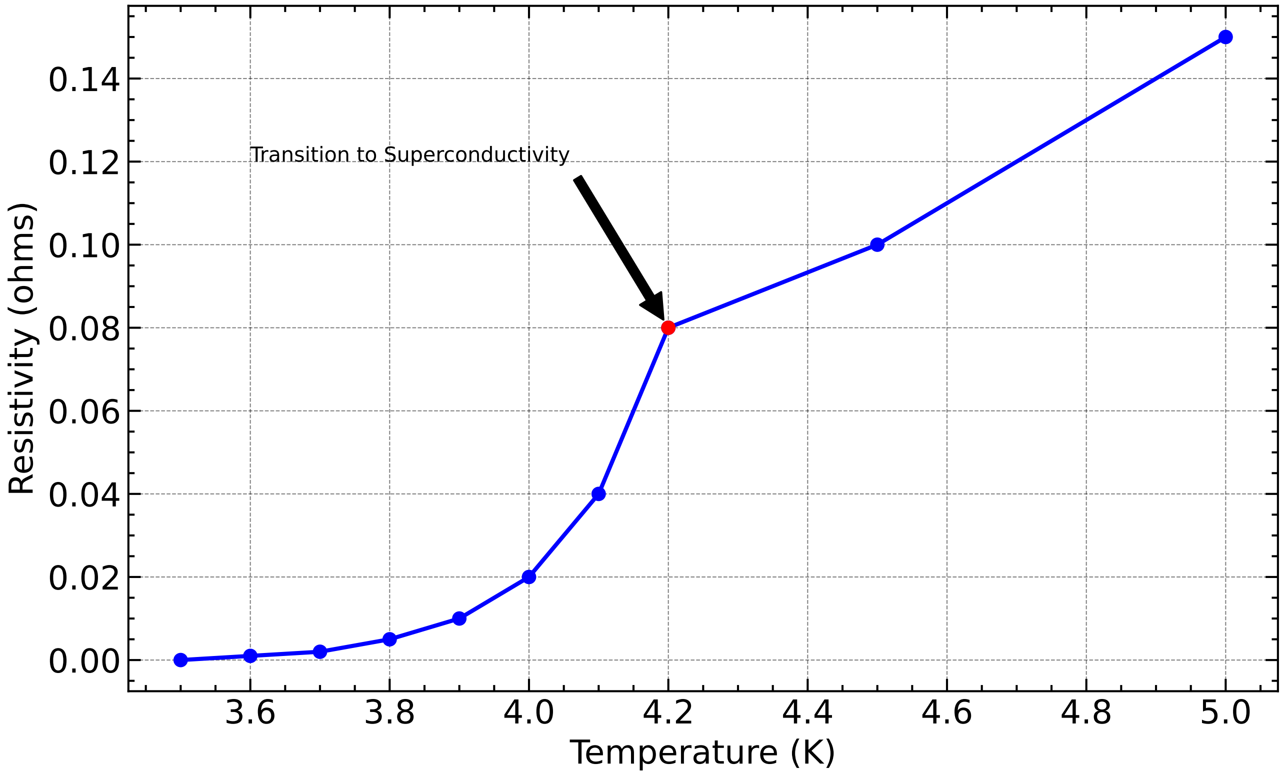


Figure 1.1: Mercury superconducting transition

In 1933, in Berlin, Walther Meissner and Robert Ochsenfeld showed that the magnetic field B is "expelled" from superconductors. This means that when subjected to an external magnetic field, superconductors divert the field lines so that the magnetic field vanishes inside. The superconducting material behaves as a perfect diamagnet [1] p.20.

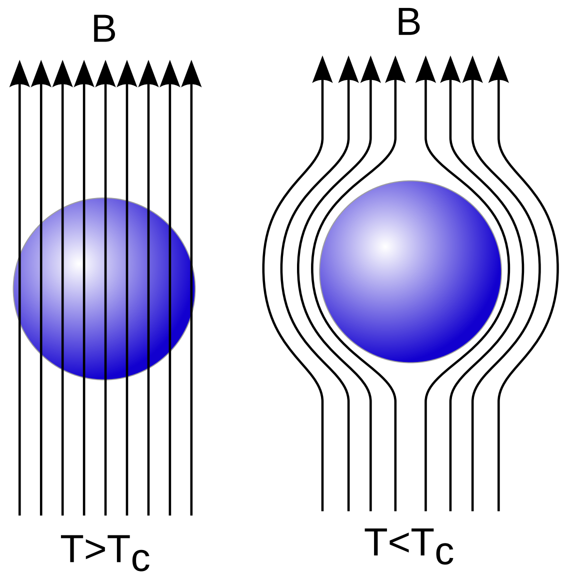
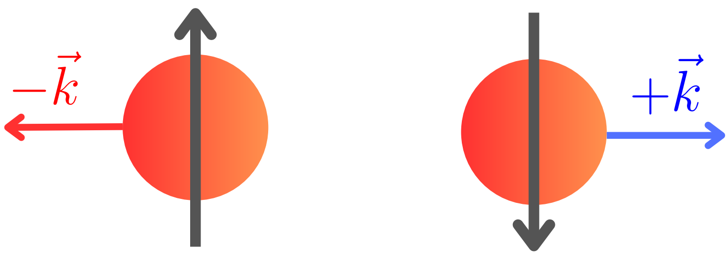


Figure 1.2: Diagram of the Meissner-Ochsenfeld effect. Magnetic field lines **B**, represented as arrows, are excluded from a superconductor when it is below its critical temperature T_c [1].

Definition: Cooper pairs

Is a pair of two fermions with a opposite spin and momentum



Theorem: Type of particule

The Cooper pairs are bosons

Proof. The spin of the Cooper in equal to zero, by consequese of the statistical theorem the Cooper pairs are bosons. □

The microscopic wave function of the system of Cooper pairs

$$\psi(\vec{r}_1, \vec{r}_2, \cdots, \vec{r}_N, t) = \varphi(\vec{r}_1, t)\varphi(\vec{r}_2, t)\cdots\varphi(\vec{r}_N, t) \quad (1)$$

and $\varphi(\vec{r}_i, t)$: describes the wavefunction of a single Cooper pair at the time t and the position $\vec{r}_i$

Is mean Field approach ! (or the wave function is a seperable state in quantum information)

Definition: Mean Field Theory

The main idea of MFT is to replace all interactions to any one body with an average or effective interaction, sometimes called a molecular field.[1] This reduces any many-body problem into an effective one-body problem.

1.1 Current density and the LONDON equation

Hypothesis :

COOPER pairs will be considered in this chapter as "particles" of mass m_p and charge q_p double those of an electron, and whose density n_p is half that of superconducting electrons:

$$m_p = 2m\,; \quad q_p = -2e\,; \quad n_p = \frac{n_s}{2}$$

And why now experiementaly at the temperature T_c the Cooper pairs condensate we can write the macroscopic wave function of the system of many Cooper pairs in polar representation

$$\Psi(\vec{r},t) = \sqrt{n_s(\vec{r},t)}e^{i\theta(\vec{r},t)}$$

where :

- n_s : density of charge carrier (Cooper pair)
- θ : superconducting phase

Proposal: Hamiltionan of charge particles

The Hamiltonian for a system of charged particles in an electromagnetic field is given by:

$$\hat{H} = \sum_i \left(\frac{1}{2m_i} \left(\hat{\mathbf{p}}_i - q_i \mathbf{A}(\hat{\mathbf{r}}_i) \right)^2 + q_i \phi(\hat{\mathbf{r}}_i) \right) + \frac{1}{2} \sum_{i \neq j} \frac{q_i q_j}{4\pi\epsilon_0 |\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j|} \tag{2}$$

- $\hat{H}$ is the Hamiltonian operator for the system.
- m_i is the mass of the i-th particle.
- $\hat{\mathbf{p}}_i$ is the momentum operator of the i-th particle.
- q_i is the charge of the i-th particle.
- $\mathbf{A}(\hat{\mathbf{r}}_i)$ is the vector potential of the electromagnetic field at the position $\hat{\mathbf{r}}_i$ of the i-th particle.
- $\phi(\hat{\mathbf{r}}_i)$ is the scalar potential of the electromagnetic field at the position $\hat{\mathbf{r}}_i$ of the i-th particle.
- ϵ_0 is the permittivity of free space and $|\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j|$ is the distance between the i-th and j-th particles.

Proof. cf book of Electrodynamics □

Theorem: London equations

The London equations describe the electromagnetic properties of superconductors. One of the London equations can be written as:

$$\nabla \times \mathbf{J} = -\frac{n_s e^2}{m}\mathbf{B} \tag{3}$$

where:

- $\mathbf{J}$ is the superconducting current density,
- n_s is the density of superconducting electrons,
- e is the charge of an electron,
- m is the mass of an electron,
- $\mathbf{B}$ is the magnetic field.

Proof. □

Rq: A major triumph o f the equations is their ability to explain the Meissner effect, wherein a material exponentially expels all internal magnetic fields as it crosses the superconducting threshold.

2. Quantum phase transition

Definition: Ground state

Definition: Bose–Einstein condensate (BEC)

In condensed matter physics, a Bose–Einstein condensate (BEC) is a state of matter that is typically formed when a gas of bosons at very low densities is cooled to temperatures very close to absolute zero ($-273.15°$C or $-459.67°$F or $0\,$K). Under such conditions, a large fraction of bosons occupy the same ground state.
Video BEC

Definition: Charge superfluid

3. Lagrangian and Hamiltonian mechanics

Definition: Lagrangian (L)

$$L = T - V$$

where T is the kinetic energy and V is the potential energy.

Definition: Hamiltonian (H)

$$H = \dot{q}_i p_i - L$$

where $\dot{q}_i$ are the generalized velocities and p_i are the canonical momenta.

Definition: Euler-Lagrange Equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

Definition: Canonical Momenta (p_i)

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

Definition: Hamilton's Equations of Motion

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

Definition: Phase Space

The space of canonical variables (q, p).

Definition: Cyclic Coordinates

Coordinates that do not appear in the Lagrangian, leading to conserved canonical momenta.

Definition: D'Alembert's Principle

$$\sum(\dot{p}_i - F_i) \cdot \delta r_i = 0$$

Definition: Virtual Displacement (δr_i)

An infinitesimal displacement consistent with the constraints, carried out at a fixed time.

Important Theorems

Theorem: Hamilton's Principle

The motion of a system extremizes the action S:

$$\delta S = \delta \int_{t_1}^{t_2} L \, dt = 0$$

This leads to the Euler-Lagrange equations.

Theorem: Noether's Theorem

Every differentiable symmetry of the action corresponds to a conservation law.

Theorem: Liouville's Theorem

The phase space distribution function is conserved along the trajectories of the system.

Key Concepts

Definition: Newtonian Mechanics

Describes the dynamics of particles using vector spatial coordinates or generalized coordinates.

Definition: Lagrangian Mechanics

Focuses on the Lagrangian function and provides a scalar approach to dynamics, making it easier to handle different coordinate systems and constraints.

Definition: Hamiltonian Mechanics

Introduces canonical momenta and phase space, providing a different formalism that is especially useful in statistical mechanics and quantum mechanics.

References

[1] Philippe Mangin and Rémi Kahn. *Superconductivity*. Springer International Publishing, Cham, 2017.