Project 1 : Face Completion and Movie Recommendation Challenge

Vincent Matthys
vincent.matthys@ens-paris-saclay.fr

This is the first project of the course *Unsupervised learning* given by René Vidal. The following assignment was related to §3.1 of his textbook (*1*). Please note that the derivation presented here are extracted from his book.

1 Low Rank Matrix Completion

The implementation can be found in the *Implementations* part of the jupyter notebook.

2 Face completion

In this section, we try to complete an assumed low rank matrix, X, composed of images (168x192 pixels) of the face of individual 1, a.k.a. *YaleB01* of the Yale B dataset, under different illumination conditions. As we will verify, those images lie near a lower dimensional subspace. Our goal here is to complete the matrix of images X sampled uniformly at random, with the lower rank possible, and so find the matrix A minimizing :

$$\begin{aligned} \underset{A}{\text{minimize}} \quad & rank(A) \\ \text{subject to} \quad & \mathcal{P}_\Omega(A) = \mathcal{P}_\Omega(X) \end{aligned} \tag{1}$$

where $P_\Omega(X)$ denotes the observed entries.
The problem (2) can be relaxed as explained in the textbook (*1*) to this one :

$$\begin{aligned} \underset{A}{\text{minimize}} \quad & \tau\|A\|_* + \frac{1}{2}\|A\|_F^2 \\ \text{subject to} \quad & \mathcal{P}_\Omega(A) = \mathcal{P}_\Omega(X) \end{aligned} \tag{2}$$

And the solution can be computed iteratively by finding the saddle point of the Lagrangien $\mathcal{L}$:

$$\begin{cases} A_{k+1} = \underset{A}{\text{argmin}}\, \mathcal{L}(A, Z_k) \\ Z_{k+1} = Z_k + \beta \dfrac{\partial \mathcal{L}}{\partial Z}(A_{k+1}, Z_k)) \end{cases} \tag{3}$$

where Z is the lagrangian multiplier associated with the constraint. Finaly, the algorithm of low rank matrix completion by proximal gradient implemented before is simply computing the A for the saddle node.

The faces completion is shown in figure 1 for different percentage of missing entries and different τ. A quasi perfect completion, except in shadow areas next to the nose, is reached until 70 % of missing entries, with a minimal MSE for $\tau = 6 \times 10^5$ of 96.5. The results are comparable with the ones in the book, figure 3.2. We didn't reproduce the behaviour in large τ, as our algorithm, as implemented, didn't converge for values of τ to high. The same append for values of τ lower than 50000.

FIGURE 1 – Faces completion with low ranl matrix completion algorithm. Lines 1 to 5 : $\tau = [1, 2, 4, 6, 8] \times 10^5$. Columns 1 to 5 : %(missing entries) $= [30, 50, 70, 90] \times \%$

In figure 2 is presented the evolution of MSE in function of τ. It shows that the larger tau, the lower is the MSE, and better is the completion. It's a common result, as the relaxation of the problem is closer to the true one when τ is bigger.

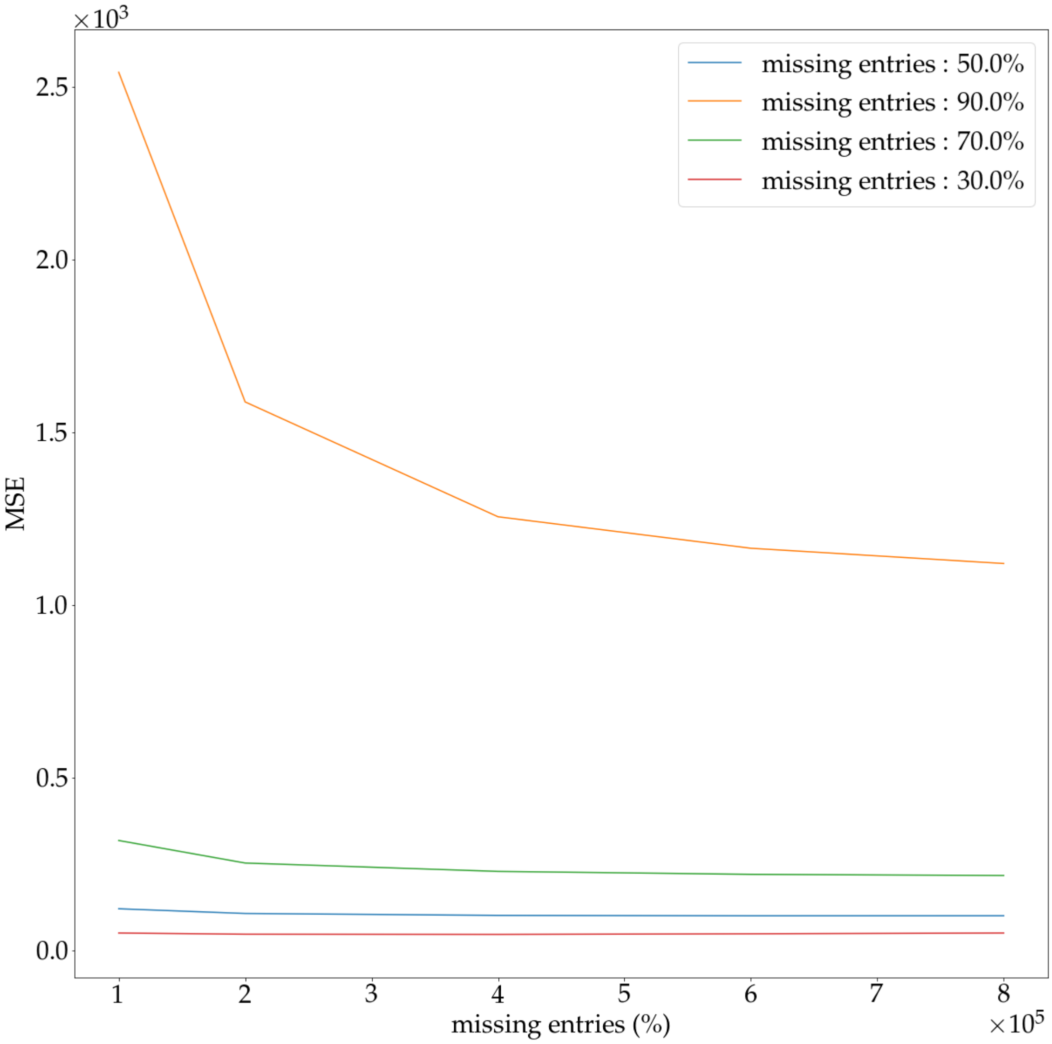

FIGURE 2 – MSE function of τ for the faces represented in 1

3 Movie recommandation Grand Challenge

For the movie recommandation Grand Challenge, the results are shown in table 1. All the results were computed for $\beta = 2$, and for 10 % of training data. It is not surprising to see that the completion works better (because we have a lower MSE) for the single-genre ratings, as the rank should be lower. Moreover, we noticed a high sensibility for the beta parameter, which didn't allow us to do a real grid search. The case $\beta = 2$ worked just fine.

Genres	τ	MSE
Horror	10^5	0.156
Romantic	10^4	0.177
Romantic + Horror	$1.2\ 10^3$	0.349

TABLE 1 – MSE on the test set held out for Horror ratings, Romantic ratings, and Romantic & Horror ratings

Références

1. R. VIDAL, Y. MA, S. S. SASTRY, in *Generalized Principal Component Analysis* (Springer, 2016), p. 25–62.