Database Systems Extra Credit HW #2

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February 2026

Practice Problem for Homework 2 Question 4

Question 4 [10 points]. Convert the following set of functional dependencies to minimal basis.
Show all steps:

$F = \{ A \to BC,\ B \to D,\ A \to D,\ D \to E,\ A \to E,\ CE \to F,\ AF \to G,\ DG \to H,\ H \to I \}.$

Solution

We compute a minimal basis by performing the following steps:

1. Split functional dependencies so that each has a single attribute on the RHS.
2. Remove extraneous attributes from the LHS.
3. Remove redundant functional dependencies.

Step 1: Split RHS attributes

We split dependencies with multiple attributes on the RHS:

$$A \to BC \ \Rightarrow \ A \to B,\ A \to C$$

After splitting and removing trivial dependencies, we obtain:

$F_1 = \{ A \to B,\ A \to C,\ B \to D,\ A \to D,\ D \to E,\ A \to E,\ CE \to F,\ AF \to G,\ DG \to H,\ H \to I \}$

Step 2: Remove extraneous attributes from LHS

We examine only dependencies with multiple attributes on the LHS.

$CE \to F$: Check whether C is extraneous by computing E^+. Since $E^+ = \{E\}$ and does not include F, C is not extraneous. Similarly, $C^+ = \{C\}$ does not include F, so E is not extraneous. Thus, $CE \to F$ remains unchanged.

$AF \to G$**:** Check whether A is extraneous by computing F^+. Since $F^+ = \{F\}$ does not include G, A is not extraneous. Check whether F is extraneous by computing A^+:

$$A^+ = \{A, B, C, D, E\}$$

which does not include G, so F is not extraneous. Thus, $AF \to G$ remains unchanged.

$DG \to H$**:** Neither D nor G can be removed since neither D^+ nor G^+ contains H. Thus, $DG \to H$ remains unchanged.

No LHS attributes are extraneous.

Step 3: Remove redundant dependencies

Check redundancy of $A \to D$**:** Remove $A \to D$ and compute A^+ using the remaining dependencies:

$$A^+ = \{A\} \xrightarrow{A \to B} \{A, B\} \xrightarrow{B \to D} \{A, B, D\}$$

Since $D \in A^+$, the dependency $A \to D$ is redundant and can be removed.

Check redundancy of $A \to E$**:** Remove $A \to E$ and compute A^+:

$$A^+ = \{A, B, C, D\} \xrightarrow{D \to E} \{A, B, C, D, E\}$$

Since $E \in A^+$, the dependency $A \to E$ is redundant and can be removed.

All remaining dependencies: The rest of the dependency are independent and not redundant so there are no need to remove any more.

Final Minimal Basis

$$\boxed{F_{\min} = \{\, A \to B,\ A \to C,\ B \to D,\ D \to E,\ CE \to F,\ AF \to G,\ DG \to H,\ H \to I \,\}}$$