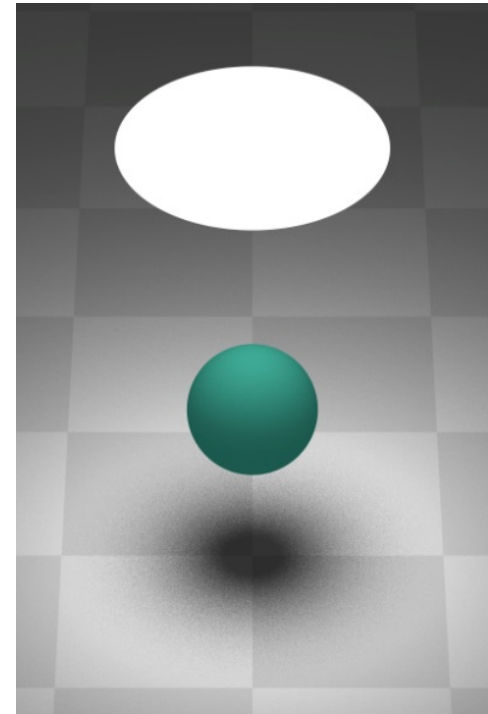


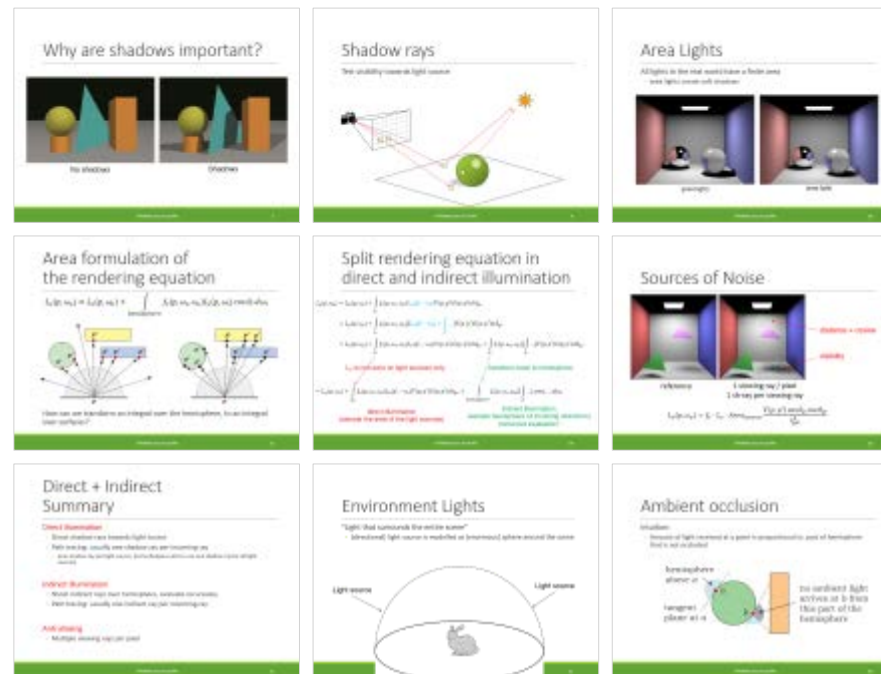
# Direct Illumination



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FUNDAMENTALS OF COMPUTER GRAPHICS  
PHILIP DUTRÉ  
DEPARTMENT OF COMPUTER SCIENCE

# Overview Lecture

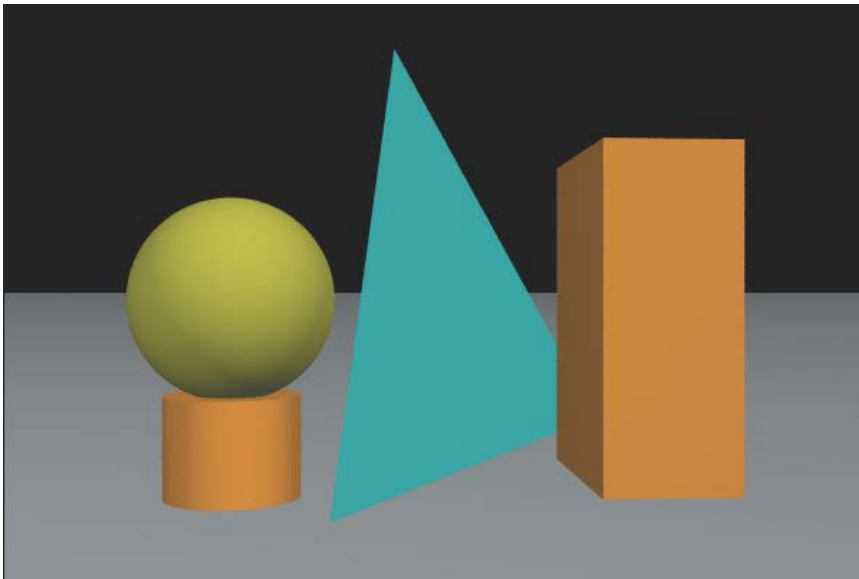


Relevant sections in book: Chapter 16, 17, 18. Useful: Chapters 5 and 7.

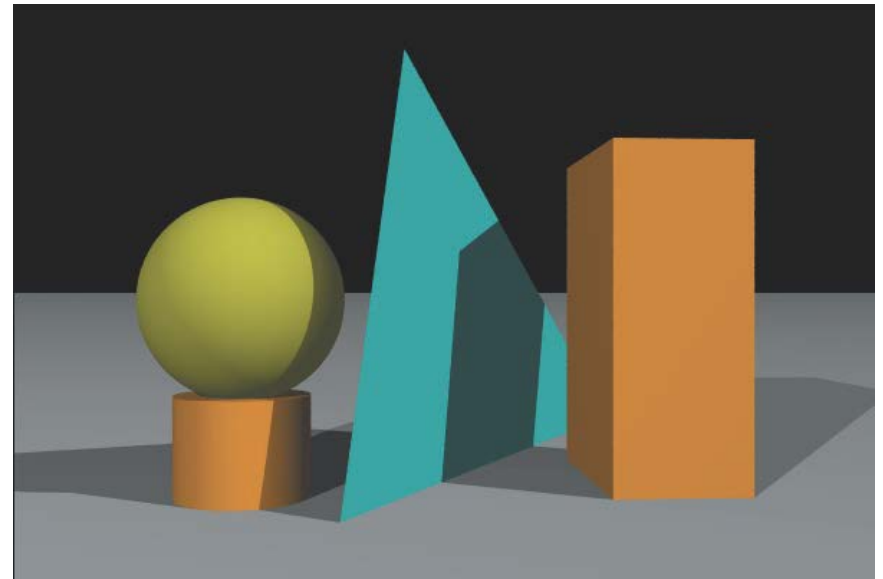
(Illustrations from Ray Tracing From The Ground Up, Physically-Based Rendering, Fundamentals of Computer Graphics)  
(Page numbering might skip some slides due to 'hidden' slides in my presentation.)

# Why are shadows important?

---



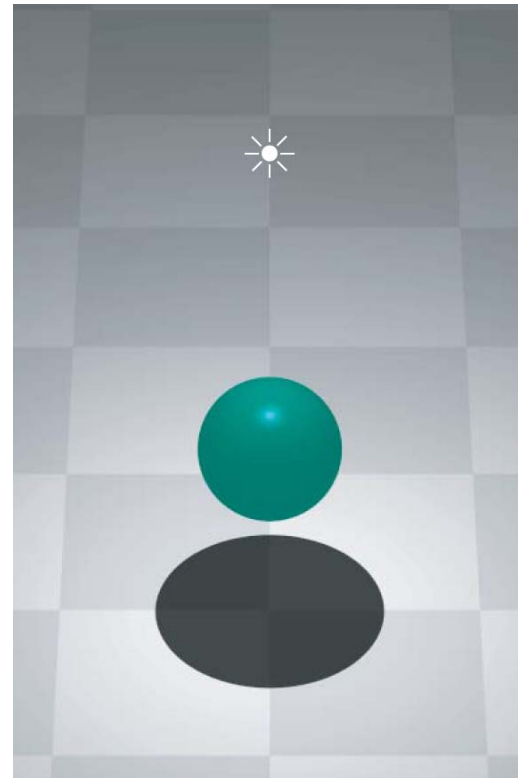
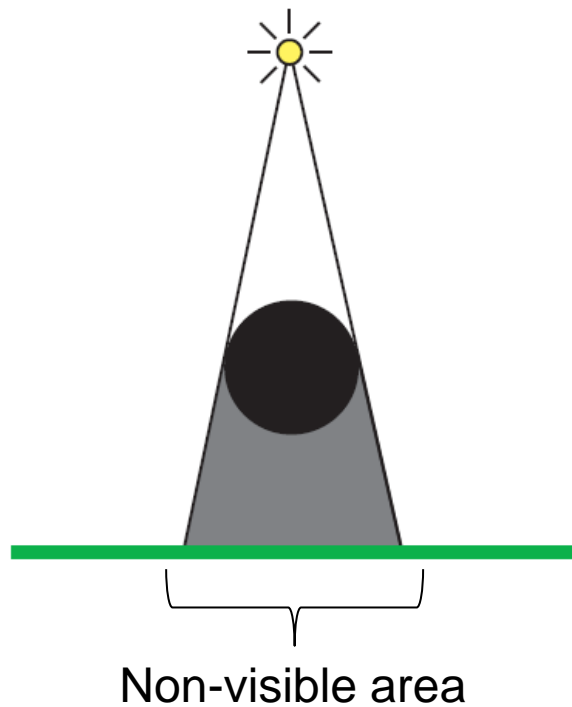
No shadows



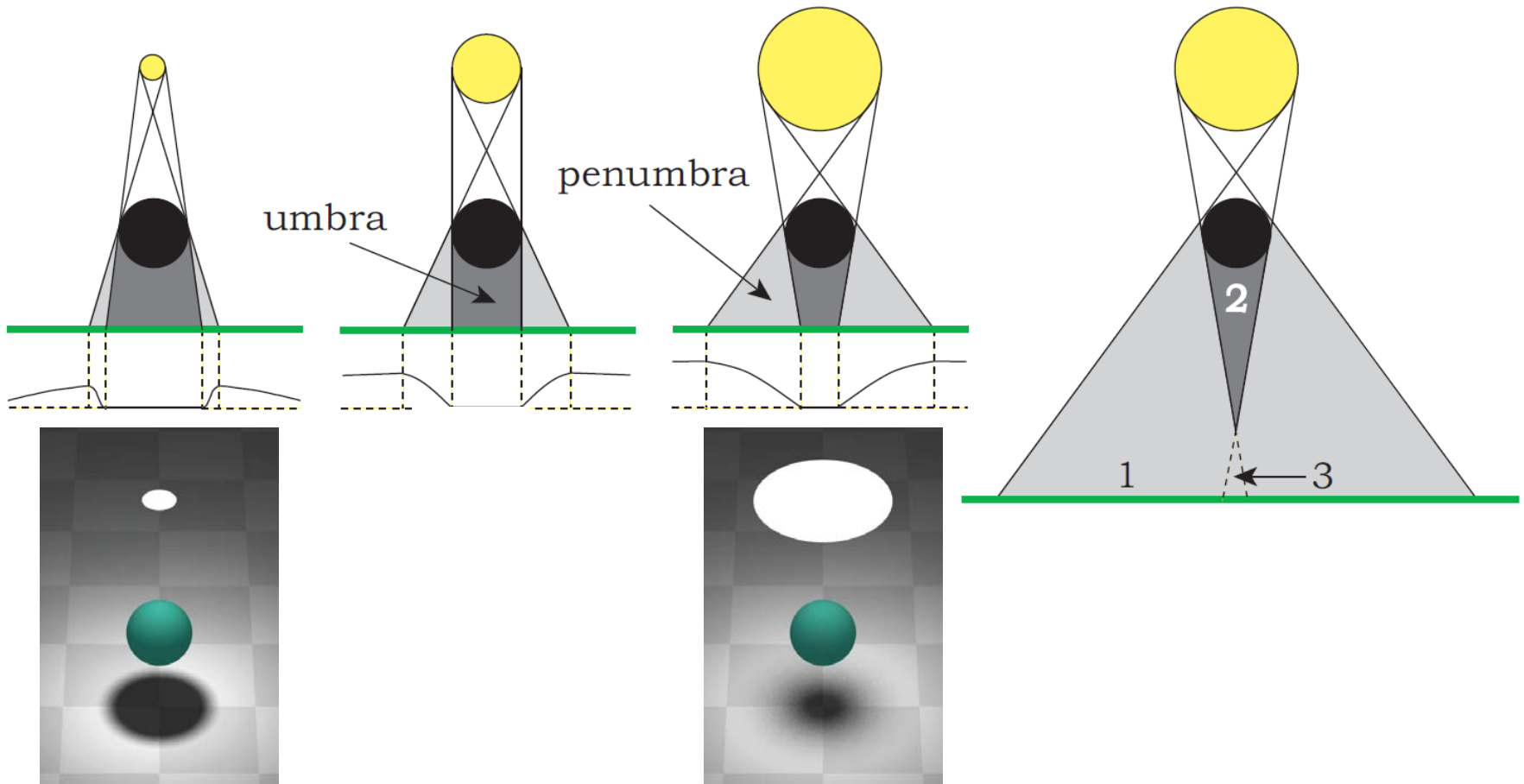
Shadows

# Definitions w.r.t shadows

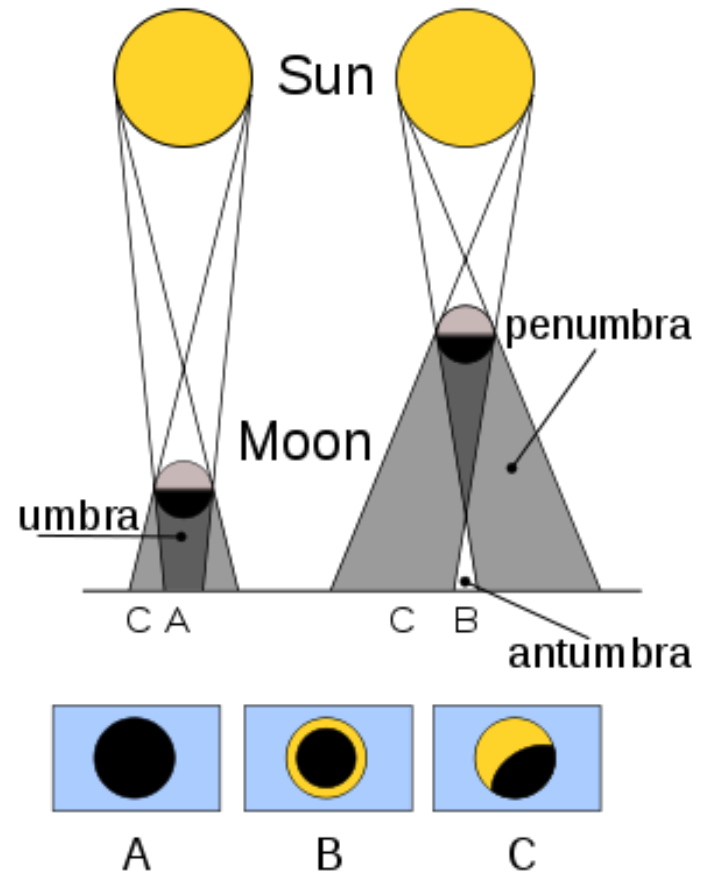
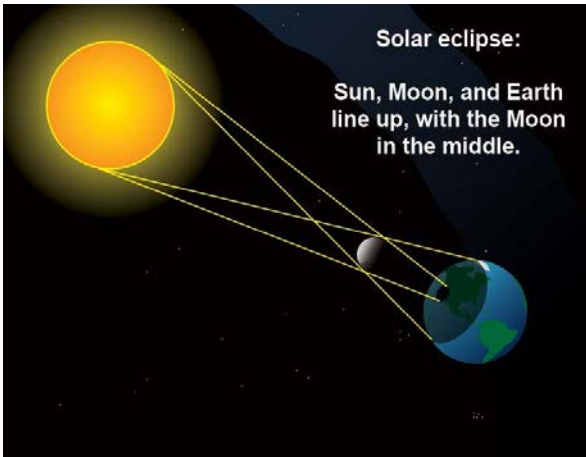
---



# Definitions w.r.t shadows



# Cfr. Solar Eclipse



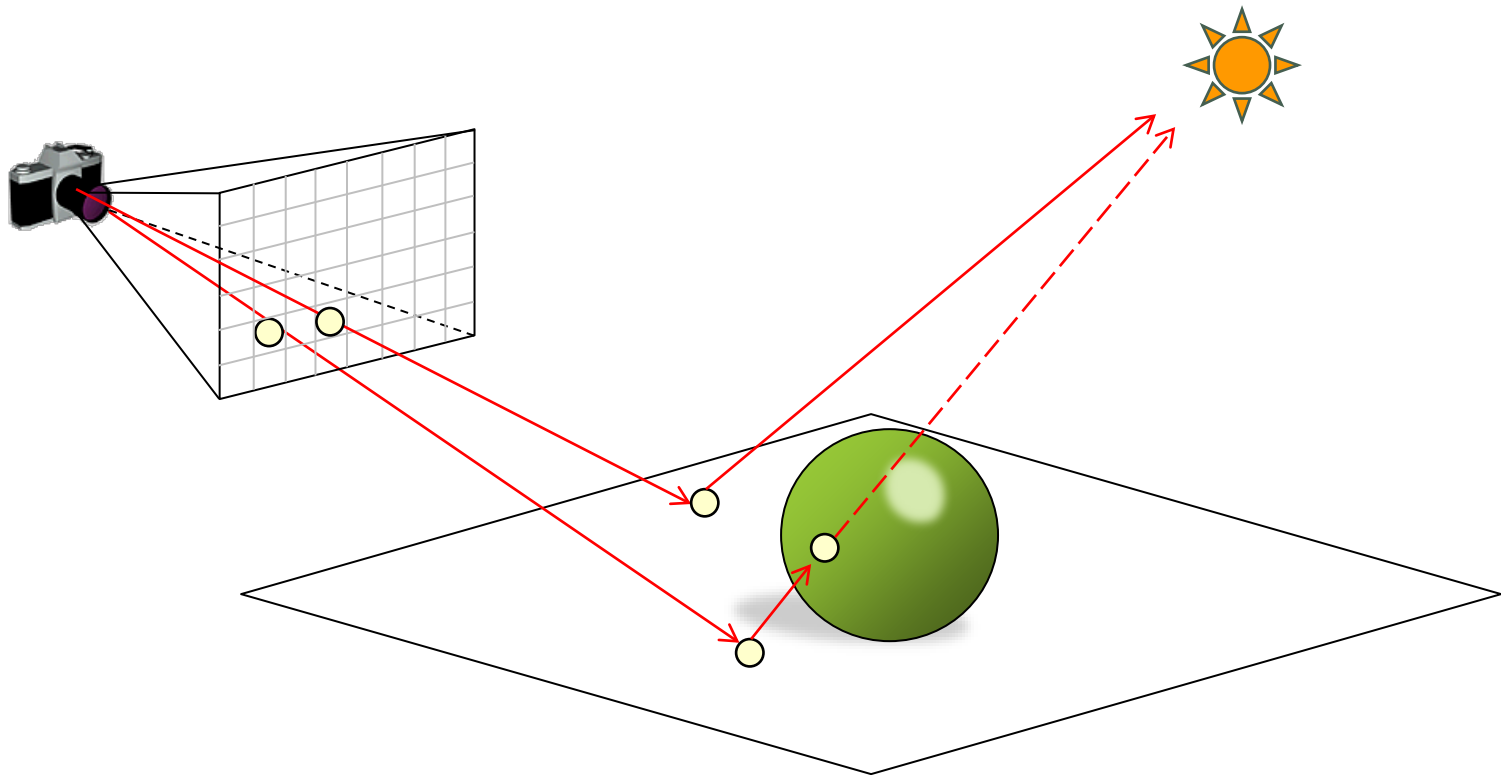
# What do we need to compute shadows / direct illumination?

---

1. Determine what parts of the light source(s) are visible
  - Geometric operation ... compute visibility
  - Tracing “shadow rays”
2. Compute the intensity of the illuminated areas
  - Direct illumination ... compute intensity
  - Compute illumination integral using Monte Carlo

# Shadow rays

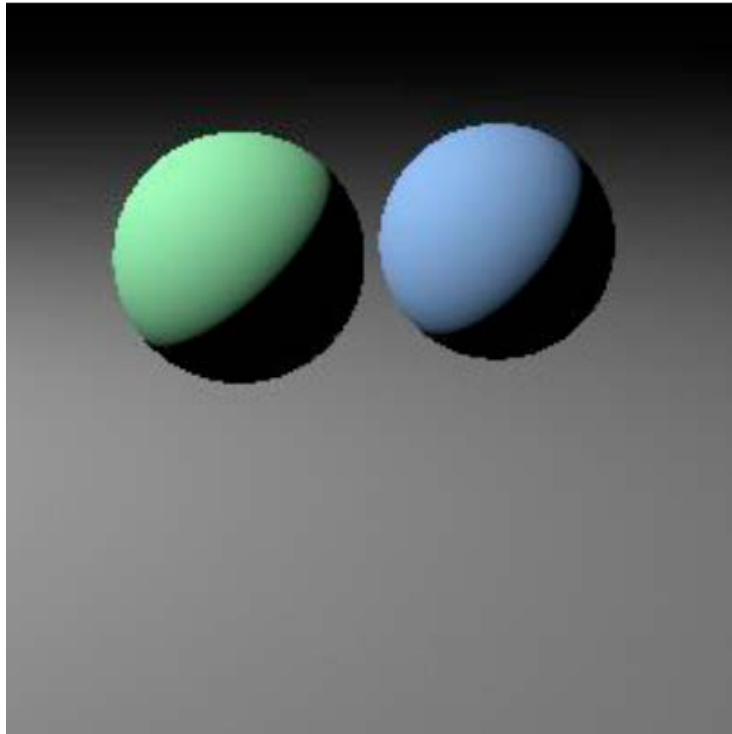
Test visibility towards light source



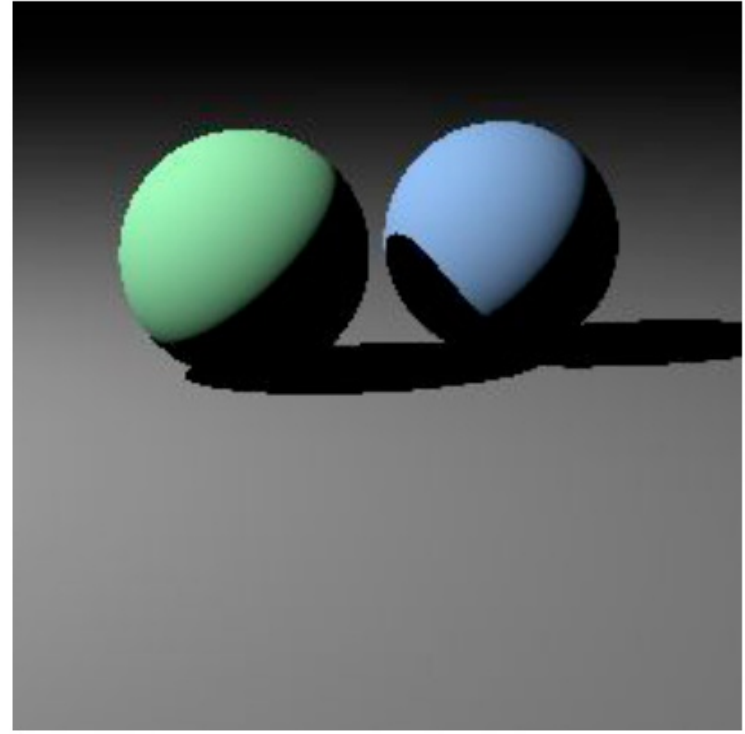


# Shadow rays

---



Diffuse shading model (no shadows)

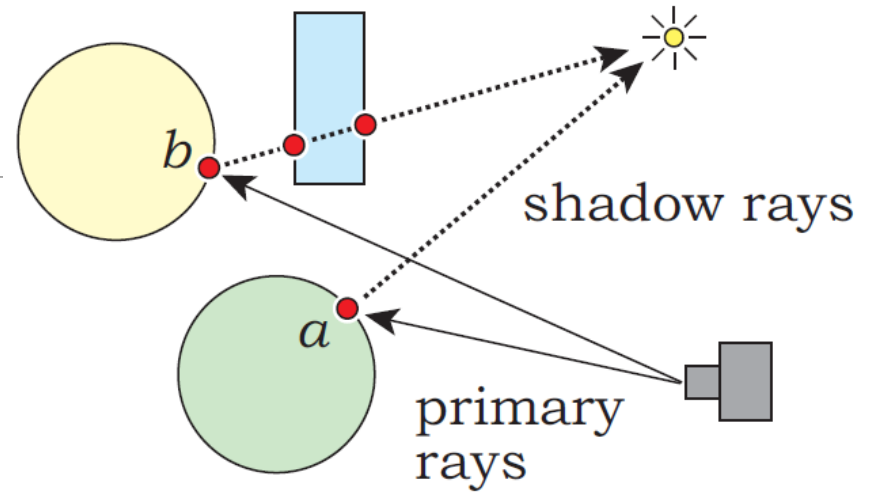


Diffuse shading model (shadows)

# Shadow rays

Shadow Ray:

- **origin**: point to be shaded
- **direction**: towards the light source

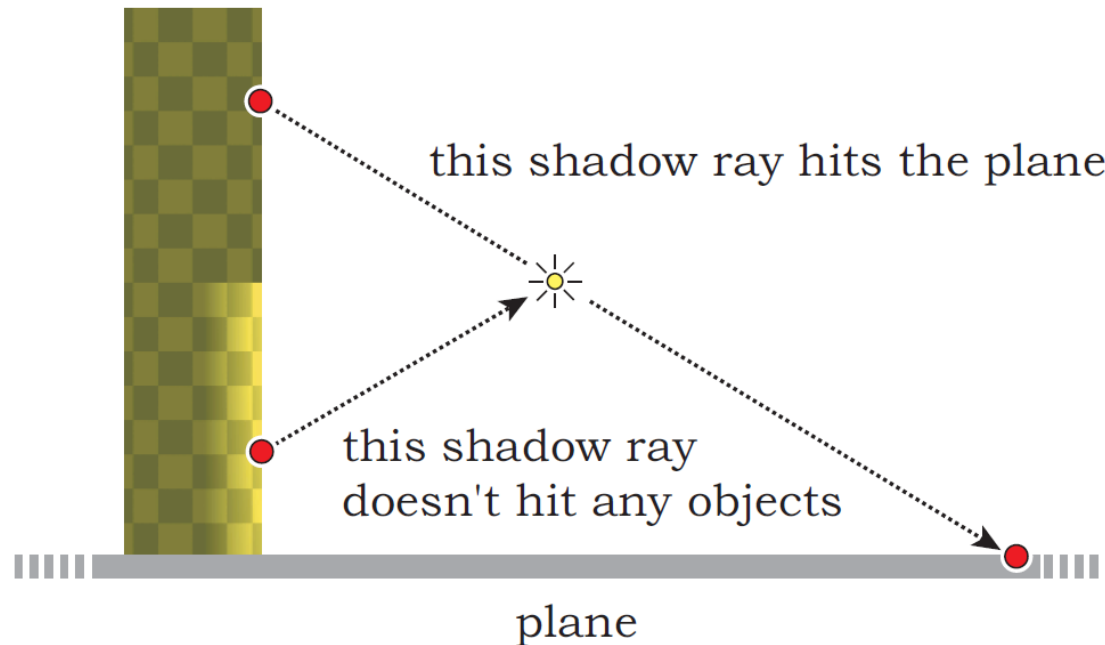


If we find **any** point between surface point to be shaded and the light source → surface point is in shadow

- finding the closest intersection is not needed
- (more efficient than viewing rays; while-loop to intersect all objects can be early-aborted when we find any hitpoint)

# Shadow rays: problems

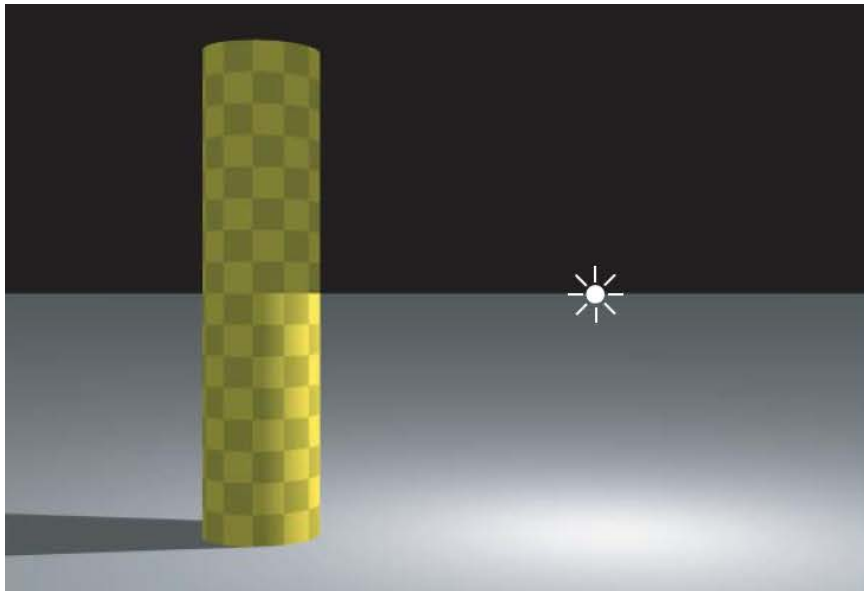
Geometric element casting the shadow must be located between the light source and point to be shaded



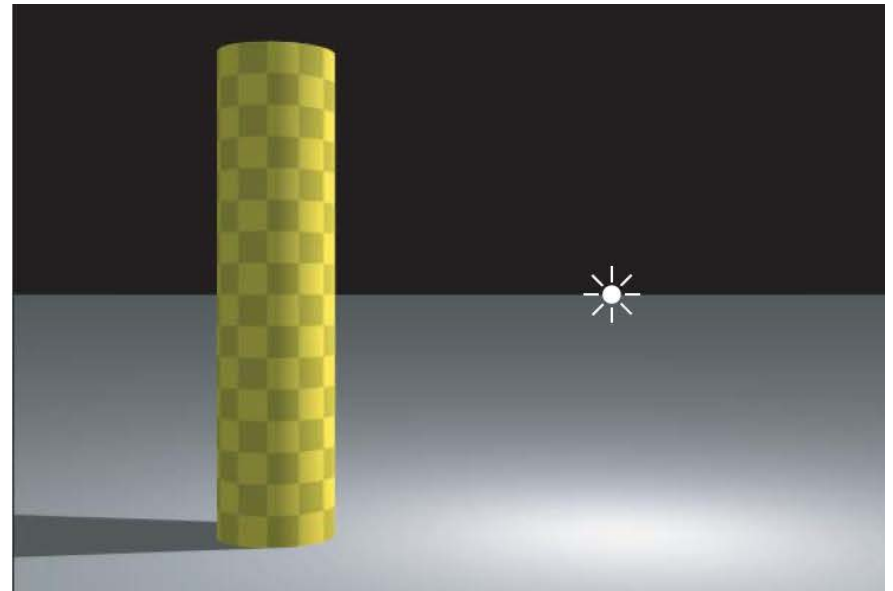
# Shadow rays: problems

---

Geometric element casting the shadow must be located between the light source and point



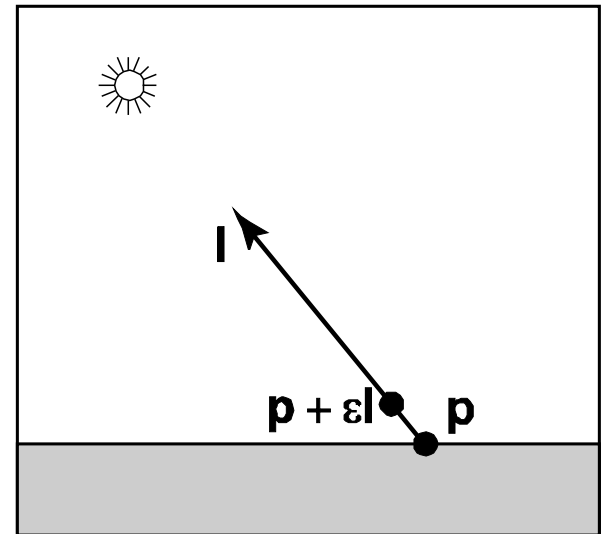
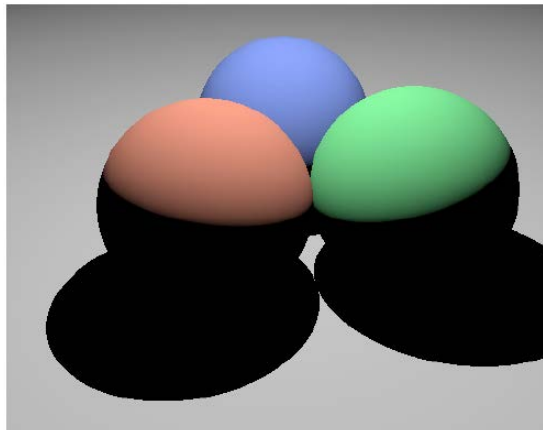
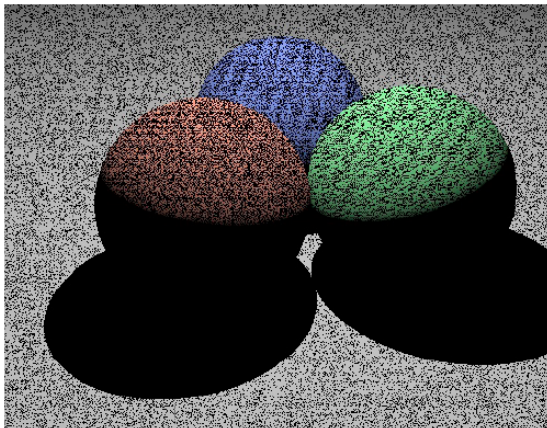
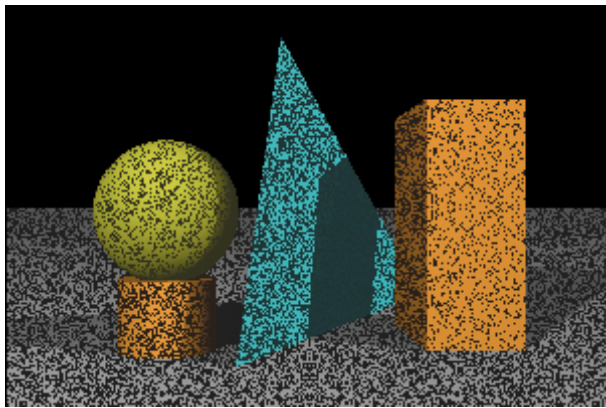
(plane casts shadow on top-half of cylinder)



(correct shadows)

# Shadow rays: problems

## Self-intersection of shadow rays



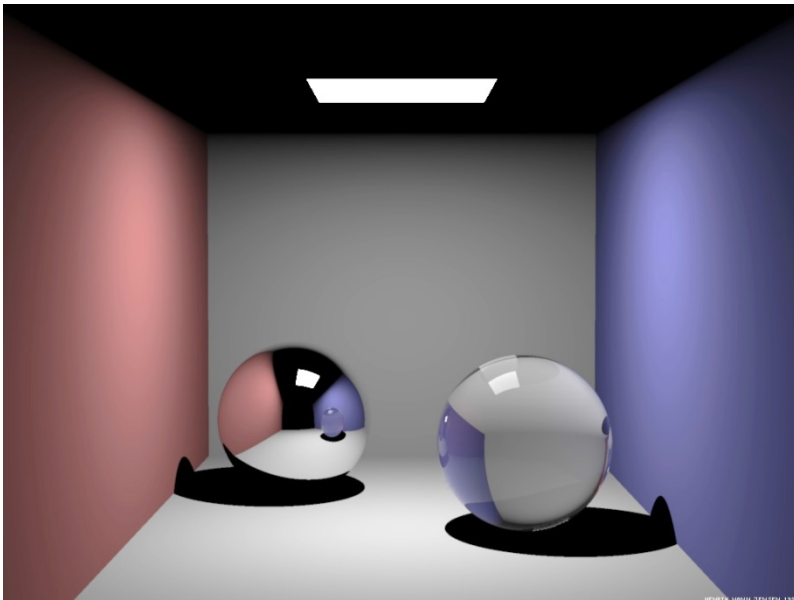
(Why is this not a problem for camera rays?)

# Area Lights

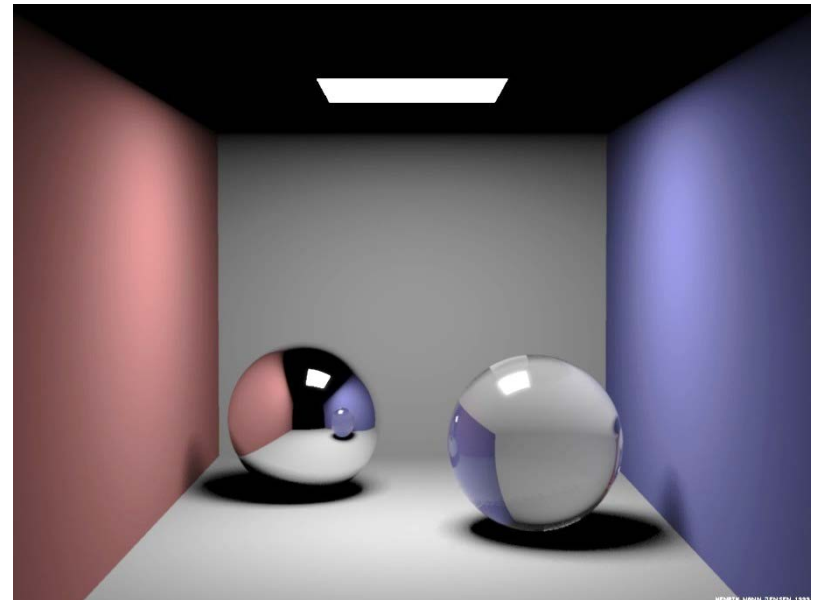
---

All lights in the real world have a finite area

- area lights create soft shadows



(point light)



(area light)

# Area Lights

---

Rendering Equation:

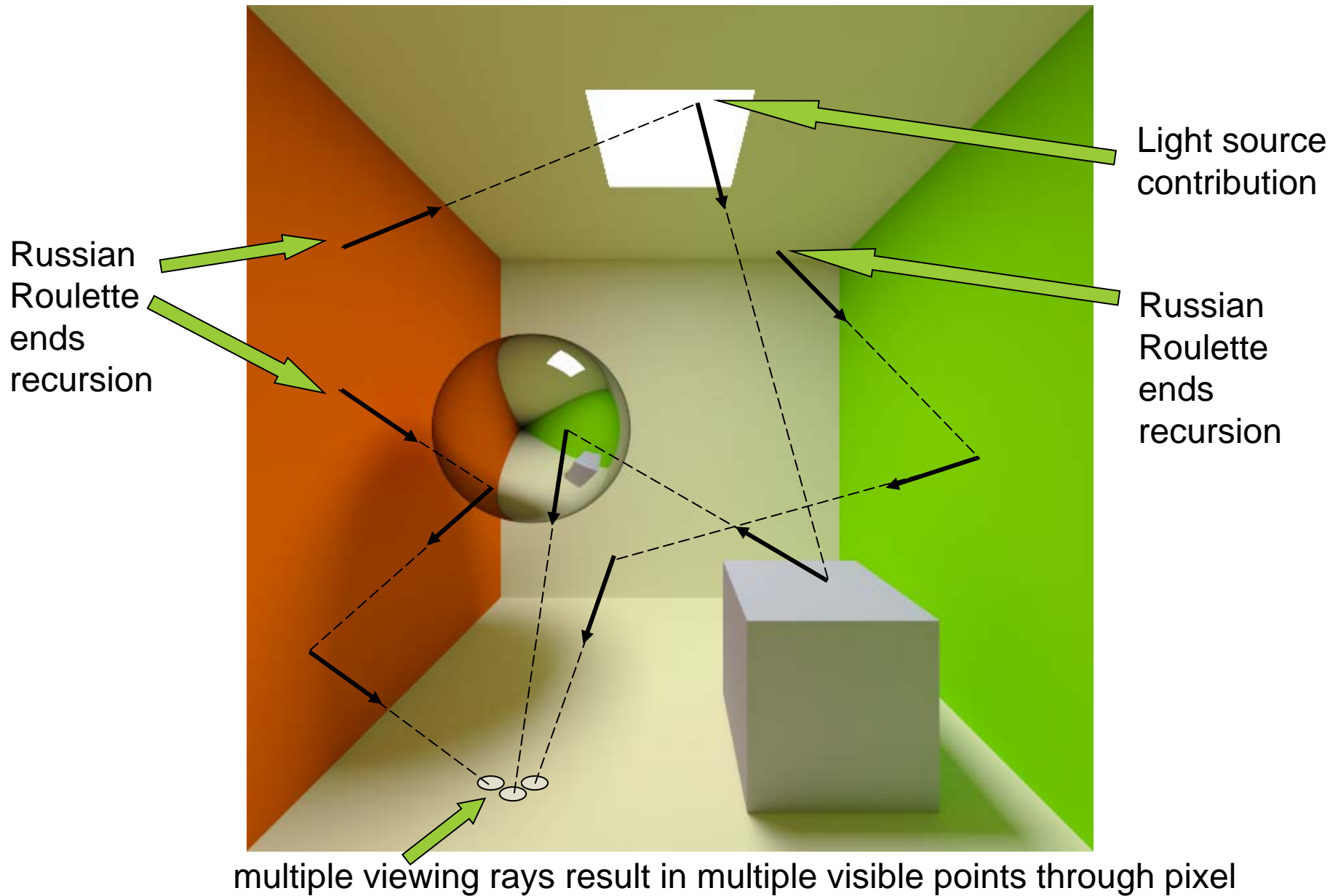
$$L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{\text{hemisphere}} f_r(p, \omega_i, \omega_o) L_i(p, \omega_i) \cos \theta_i d\omega_i$$

Path tracing algorithm (see previous lecture):

- Generate random directions over hemisphere
- Terminate paths using Russian Roulette

But we can do better for direct illumination:

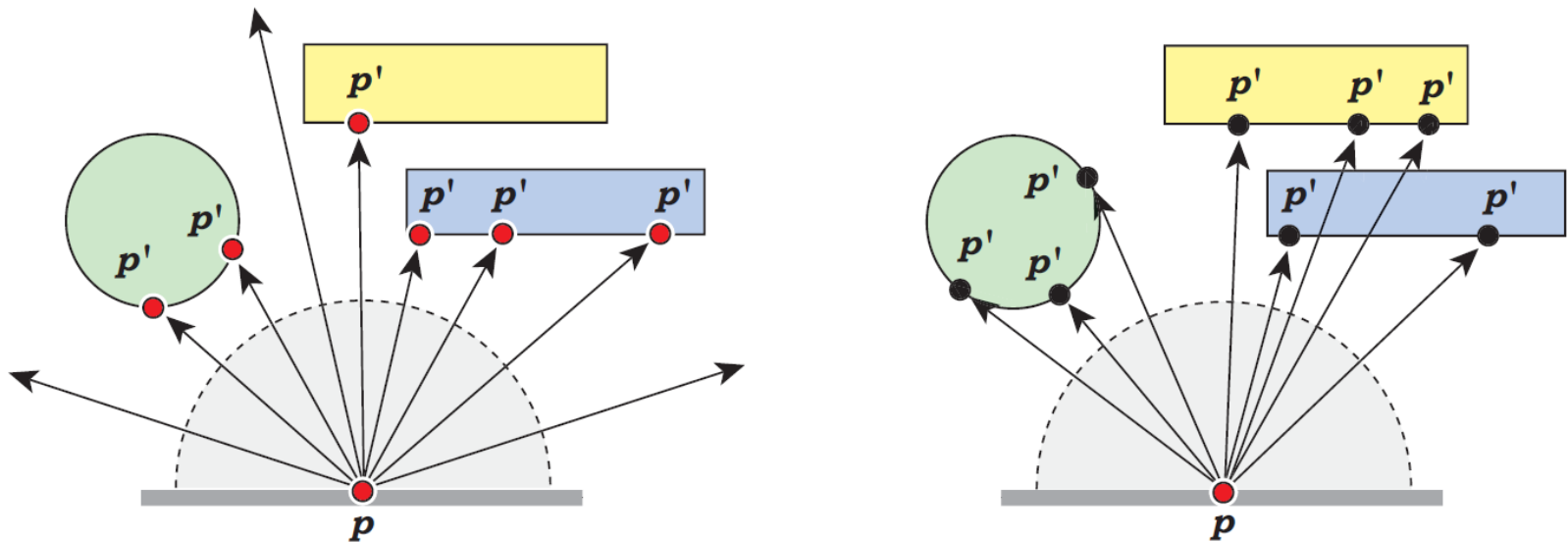
- We know where the light is coming from ...  
... light is coming from the light sources!





# Area formulation of the rendering equation

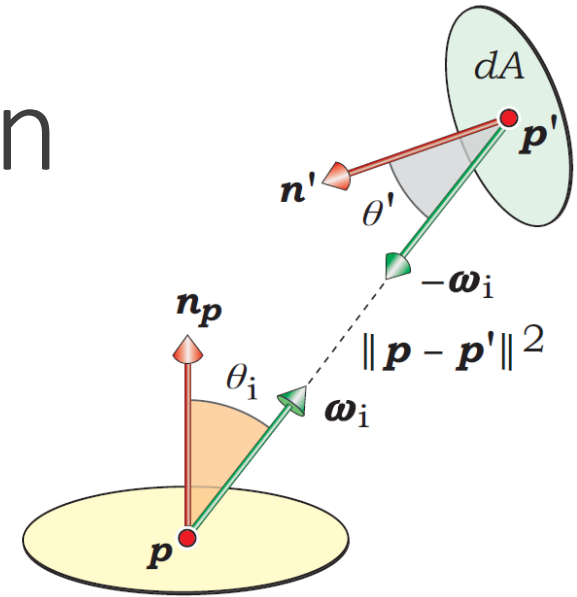
$$L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{\text{hemisphere}} f_r(p, \omega_i, \omega_o) L_i(p, \omega_i) \cos \theta_i d\omega_i$$



How can we transform an integral over the hemisphere, to an integral over surfaces?

# Area formulation of the rendering equation

transform solid angle to area:  $d\omega_i = \frac{\cos \theta' dA_{p'}}{\|p - p'\|^2}$



$$\begin{aligned}
 L_o(p, \omega_o) &= L_e(p, \omega_o) + \int_{\text{hemisphere}} f_r(p, \omega_i, \omega_o) L_i(p, \omega_i) \cos \theta_i d\omega_i \\
 &= L_e(p, \omega_o) + \int_{A_{\text{visible}}} f_r(p, \omega_i, \omega_o) L_i(p, \omega_i) \cos \theta_i \frac{\cos \theta' dA_{p'}}{\|p - p'\|^2} \\
 &= L_e(p, \omega_o) + \int_A f_r(p, \omega_i, \omega_o) L_o(p', -\omega_i) V(p, p') G(p, p') dA_{p'}
 \end{aligned}$$

visible surface points  $p'$  to  $p$

all surface points  $p'$  in scene

radiance emitted by  $p'$

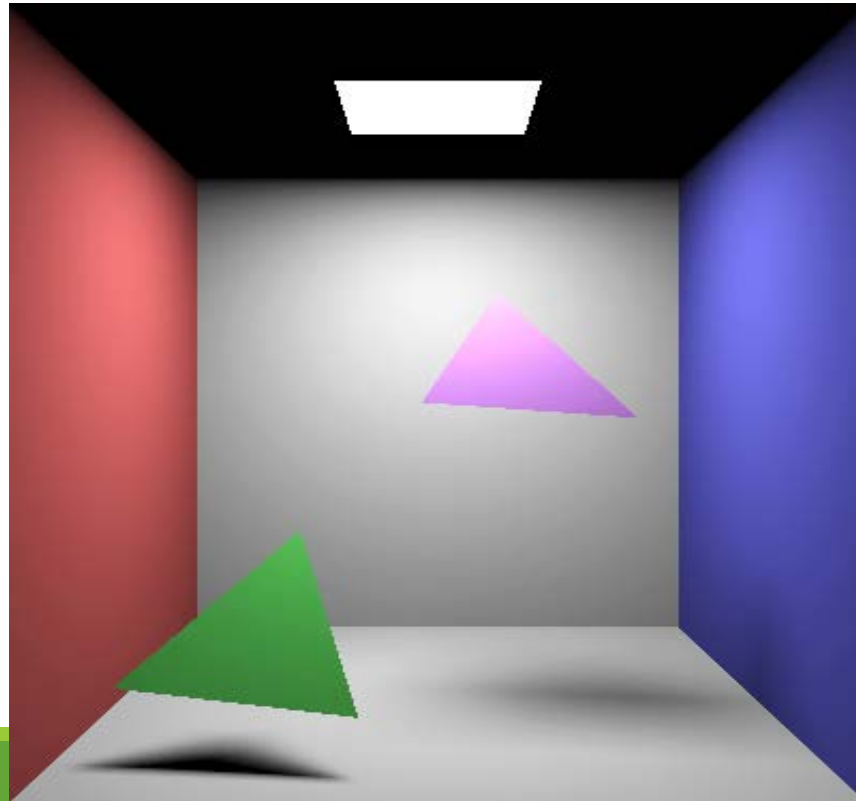
visibility between  $p$  and  $p'$

# Area formulation of the rendering equation

---

What if we apply MC to this formulation?

$$L_o(p, \omega_o) = L_e(p, \omega_o) + \int_A f_r(p, \omega_i, \omega_o) L_o(p', -\omega_i) V(p, p') G(p, p') dA_{p'}$$



# Split rendering equation in direct and indirect illumination

$$\begin{aligned}
 L_o(p, \omega_o) &= L_e(p, \omega_o) + \int_A f_r(p, \omega_i, \omega_o) L_o(p', -\omega_i) V(p, p') G(p, p') dA_{p'} \\
 &= L_e(p, \omega_o) + \int_A f_r(p, \omega_i, \omega_o) [L_e(p', -\omega_i) + \int_A \dots] V(p, p') G(p, p') dA_{p'} \\
 &= L_e(p, \omega_o) + \int_A f_r(p, \omega_i, \omega_o) L_e(p', -\omega_i) V(p, p') G(p, p') dA_{p'} + \int_A f_r(p, \omega_i, \omega_o) \left[ \int_A \dots \right] V(p, p') G(p, p') dA_{p'}
 \end{aligned}$$

$L_e$  is non-zero on light sources only

transform back to hemisphere

$$= L_e(p, \omega_o) + \int_{A_L} f_r(p, \omega_i, \omega_o) L_e(p', -\omega_i) V(p, p') G(p, p') dA_{p'} + \int_{\text{hemisphere}} f_r(p, \omega_i, \omega_o) \left[ \int_A \dots \right] \cos(\dots) d\omega_i$$

direct illumination  
(sample the area of the light sources)

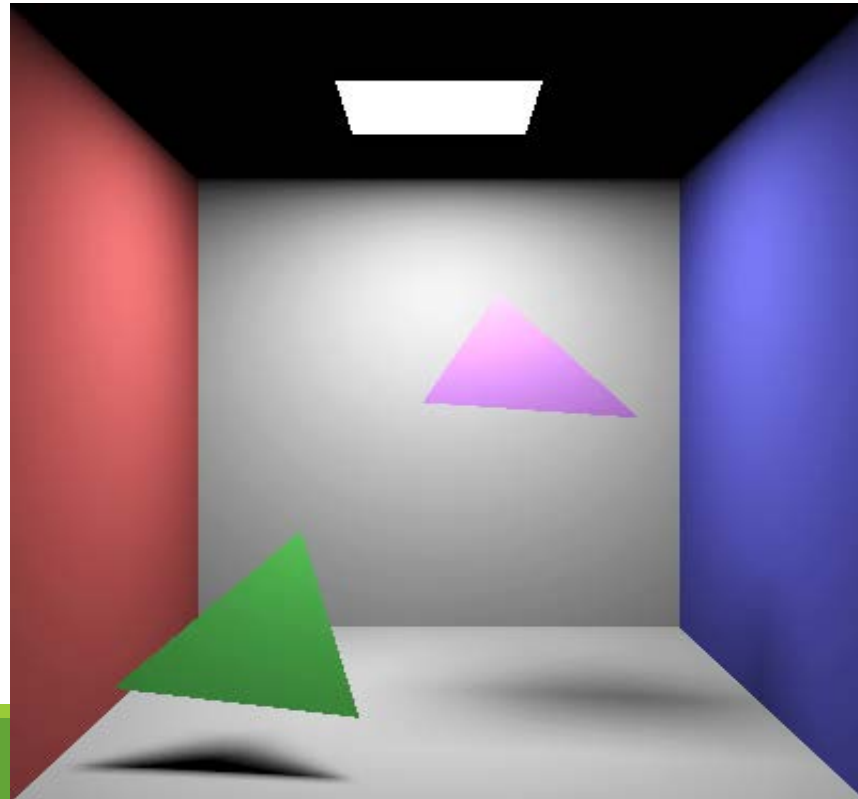
indirect illumination  
(sample hemisphere of incoming directions)  
(recursive evaluation)

# Direct illumination only ...

---

What if we apply MC to this formulation?

$$L_o(p, \omega_o) = \int_{A_L} f_r(p, \omega_i, \omega_o) L_e(p', -\omega_i) V(p, p') G(p, p') dA_{p'},$$

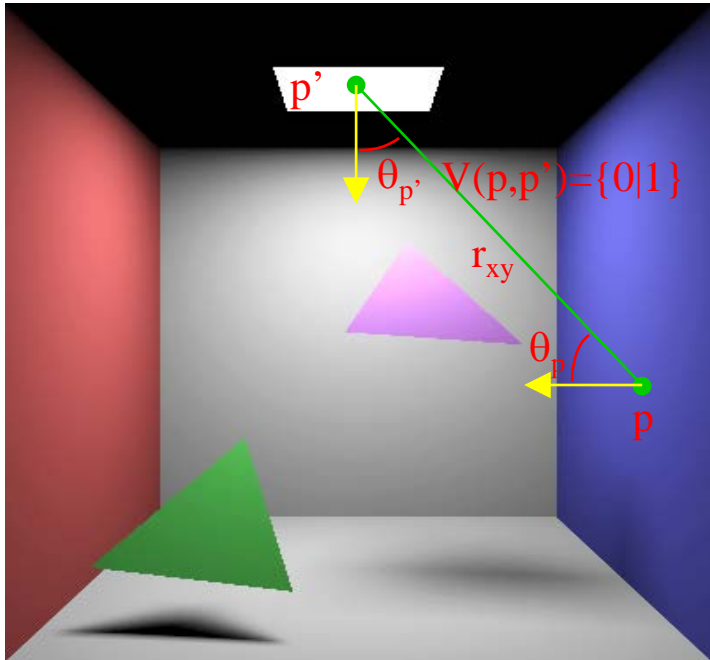


# Direct diffuse illumination

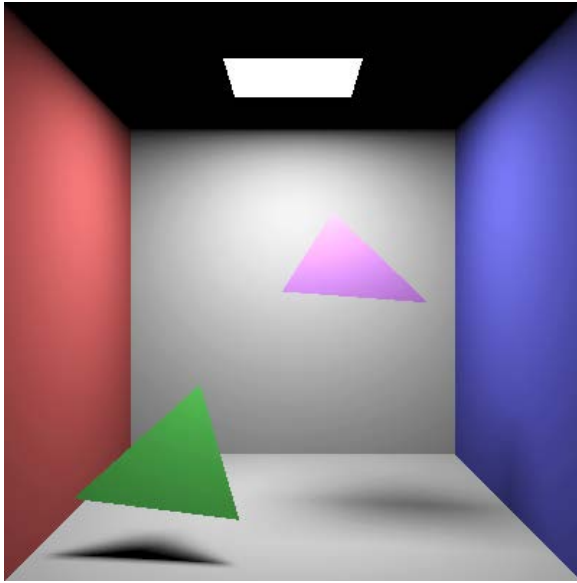
Generate random points on surface of the light source, using a probability density function  $pdf(p')$

$$L_o(p, \omega_o) = \int_{A_L} f_r(p, \omega_i, \omega_o) L_e(p', -\omega_i) V(p, p') G(p, p') dA_{p'},$$

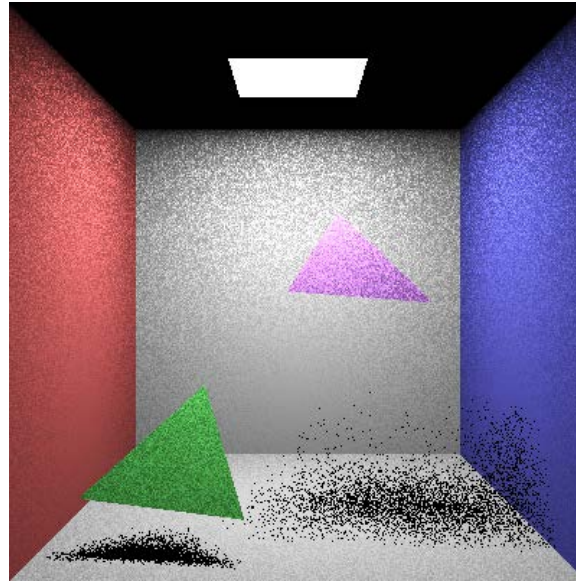
$$\begin{aligned}
&= f_r \cdot L_e \int_{A_L} V(p, p') \frac{\cos \theta_p \cos \theta_{p'}}{r_{pp'}^2} dA_{p'} \\
&\approx \frac{f_r \cdot L_e}{N} \sum_{j=1}^N \frac{V(p, p') \cos \theta_p \cos \theta_{p'}}{pdf(p') r_{pp'}^2}
\end{aligned}$$



# Direct diffuse illumination



reference

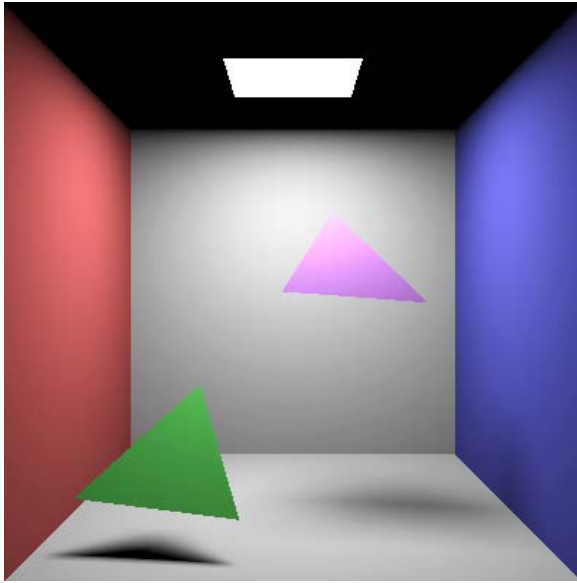


1 point on light source (1 shadow ray)

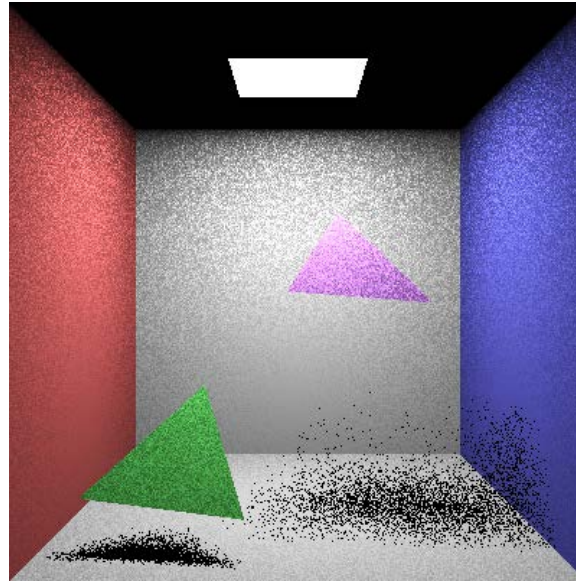
$$pdf(p') = \frac{1}{Area_{source}}$$

$$L_o(p, \omega_o) \approx f_r \cdot L_e \cdot Area_{source} \frac{V(p, p') \cos \theta_p \cos \theta_{p'}}{r_{pp'}^2}$$

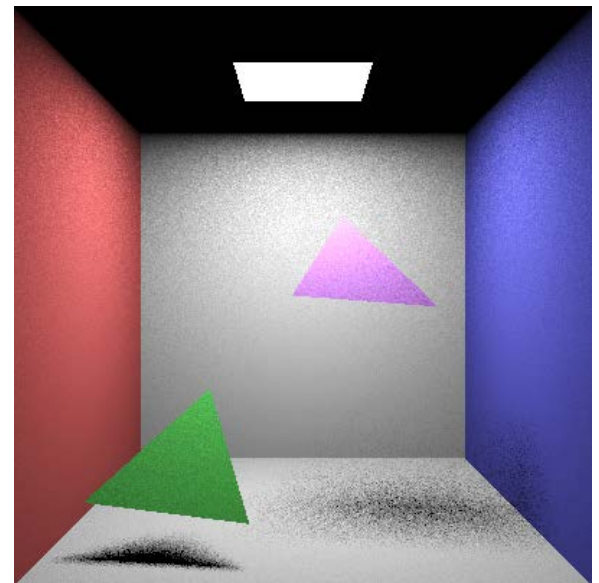
# Direct diffuse illumination



reference



1 shadow ray



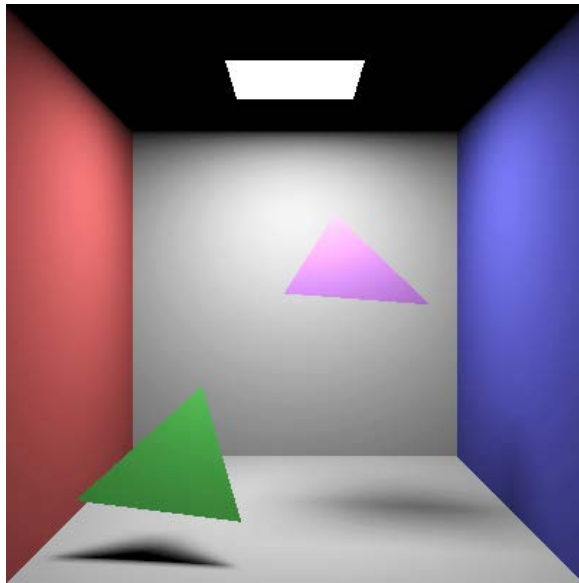
9 shadow rays

$$pdf(p') = \frac{1}{Area_{source}}$$

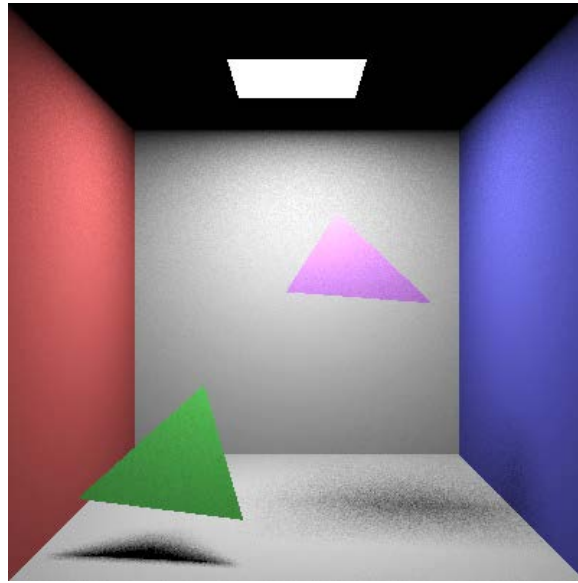
$$L_o(p, \omega_o) \approx f_r \cdot L_e \cdot Area_{source} \frac{V(p, p') \cos \theta_p \cos \theta_{p'}}{r_{pp'}^2}$$



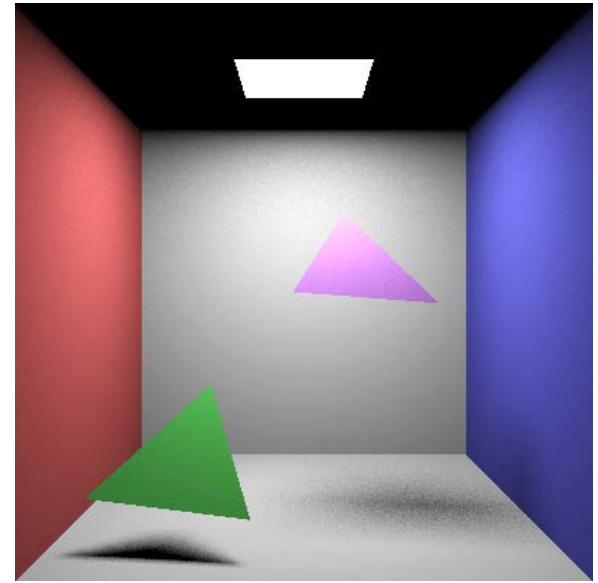
# Direct diffuse illumination



reference



36 shadow rays



100 shadow rays

$$pdf(p') = \frac{1}{Area_{source}}$$

$$L_o(p, \omega_o) \approx f_r \cdot L_e \cdot Area_{source} \frac{V(p, p') \cos \theta_p \cos \theta_{p'}}{r_{pp'}^2}$$

# Choices for Direct illumination

---

## Sampling scheme for viewing rays

- “How to select viewing rays within pixel area?”

## Sampling scheme for shadow rays

- “How to select shadow rays over the light source?”

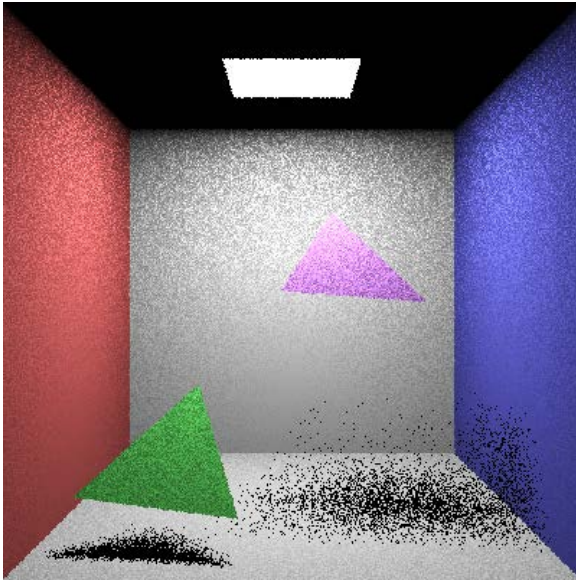
## ‘path tracing’:

- Exactly 1 shadow ray per viewing ray
  - Power of Monte Carlo: 1 sample in multi-dimensional space
    - 2 random numbers for position in pixel
    - 2 random numbers for position of shadow ray on light source
- ➔ 1 single quadruplet of random numbers

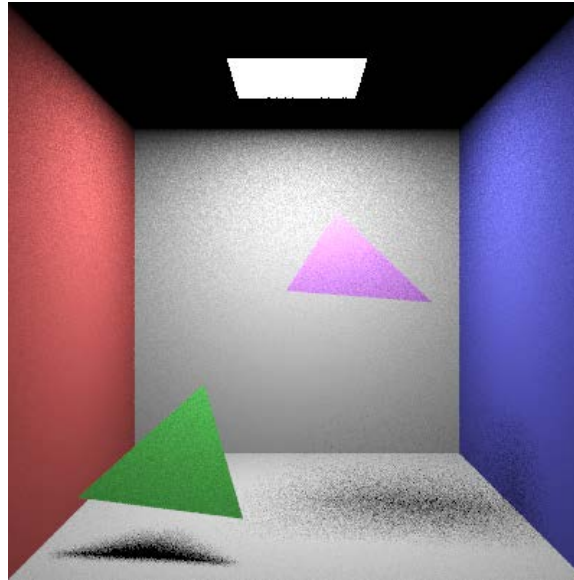
# Path tracing

## Direct illumination

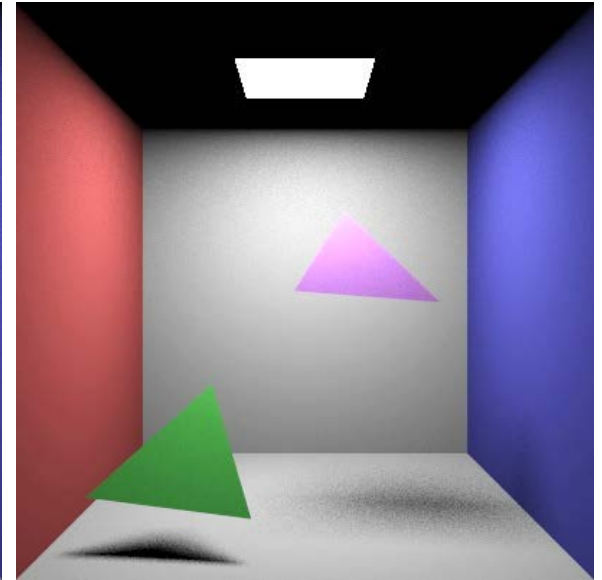
---



1 viewing ray / pixel  
1 sh-ray per viewing ray



10 viewing rays / pixel  
1 sh-ray per viewing ray

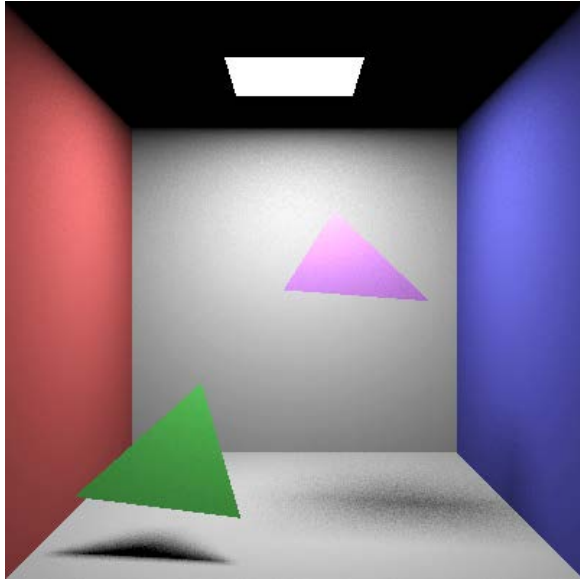


100 viewing rays / pixel  
1 sh-ray per viewing ray

# Path tracing

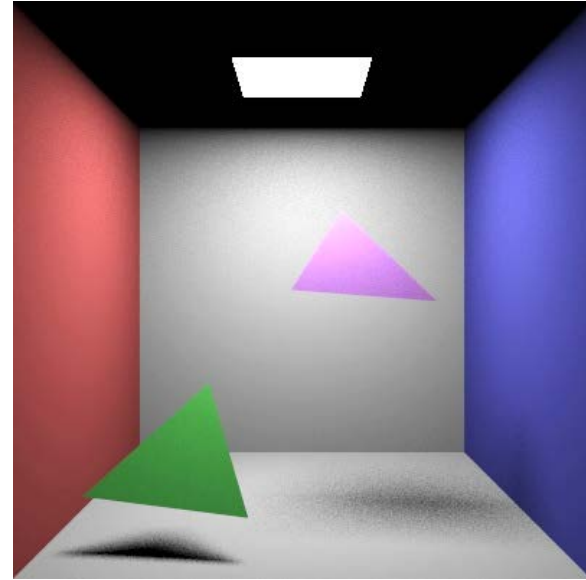
## Direct illumination

---



1 centered viewing ray  
100 random shadow rays per  
viewing ray

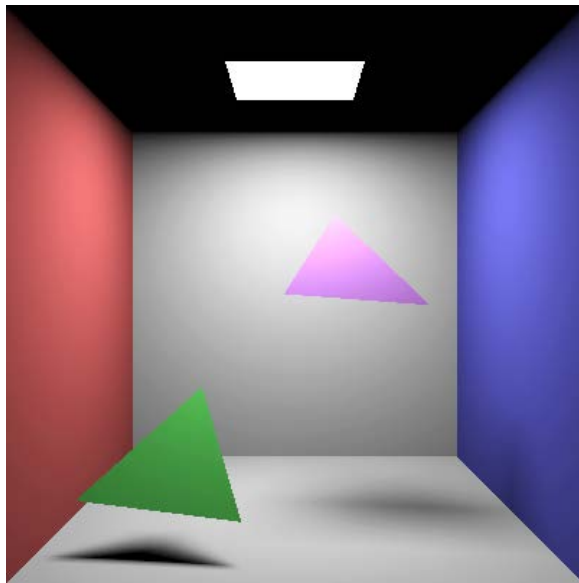
101 rays total per pixel



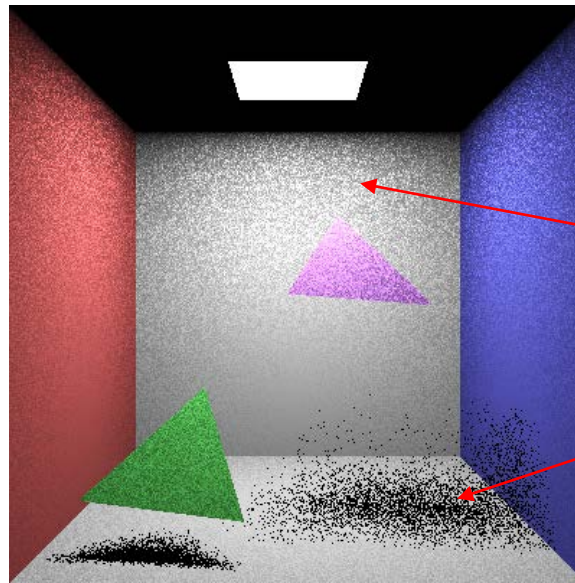
100 random viewing rays  
1 random shadow ray per  
viewing ray

200 rays total per pixel

# Sources of Noise



reference



distance + cosine

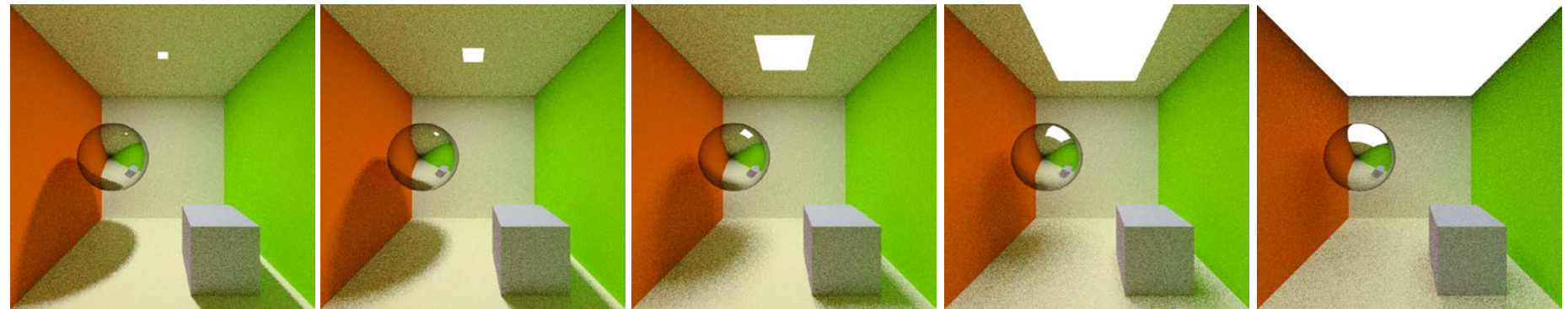
visibility

1 viewing ray / pixel  
1 sh-ray per viewing ray

$$L_o(p, \omega_o) \approx f_r \cdot L_e \cdot Area_{source} \frac{V(p, p') \cos \theta_p \cos \theta_{p'}}{r_{pp'}^2}$$

# Sources of Noise

Size of light source



$$L_o(p, \omega_o) \approx f_r \cdot L_e \cdot Area_{source} \frac{V(p, p') \cos \theta_p \cos \theta_{p'}}{r_{pp'}^2}$$

# Direct + Indirect Summary

---

## Direct illumination

- Shoot shadow rays towards light source
- Path tracing: usually one shadow ray per incoming ray
  - (one shadow ray per light source, but techniques exist to use one shadow ray for all light sources)

## Indirect illumination

- Shoot indirect rays over hemisphere, evaluate recursively
- Path tracing: usually one indirect ray per incoming ray

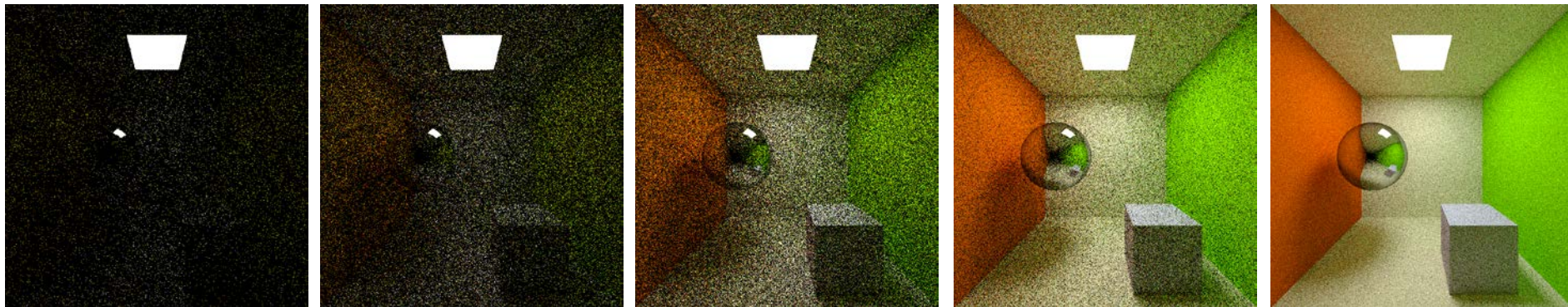
## Anti-aliasing

- Multiple viewing rays per pixel



# Direct + Indirect Summary

Path tracing, no explicit sampling of light sources



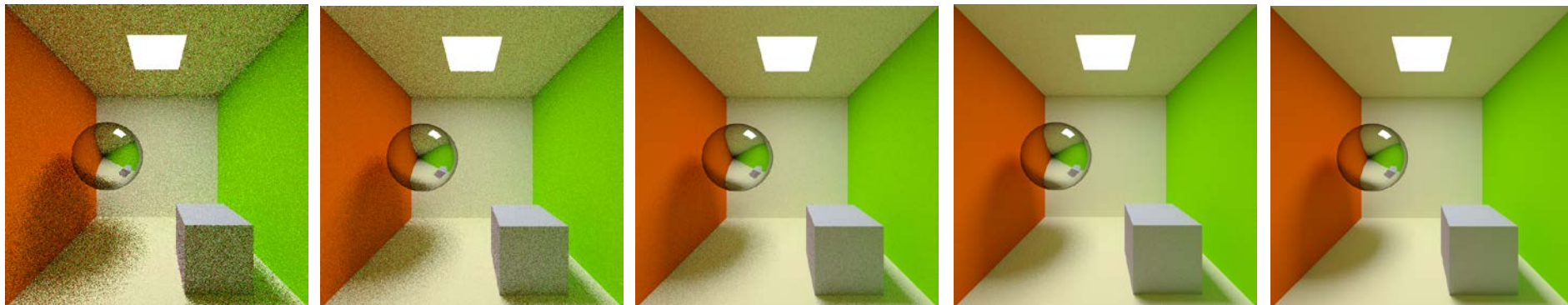
1 path per pixel

4

16

64

256



Path tracing, direct + indirect computed separately

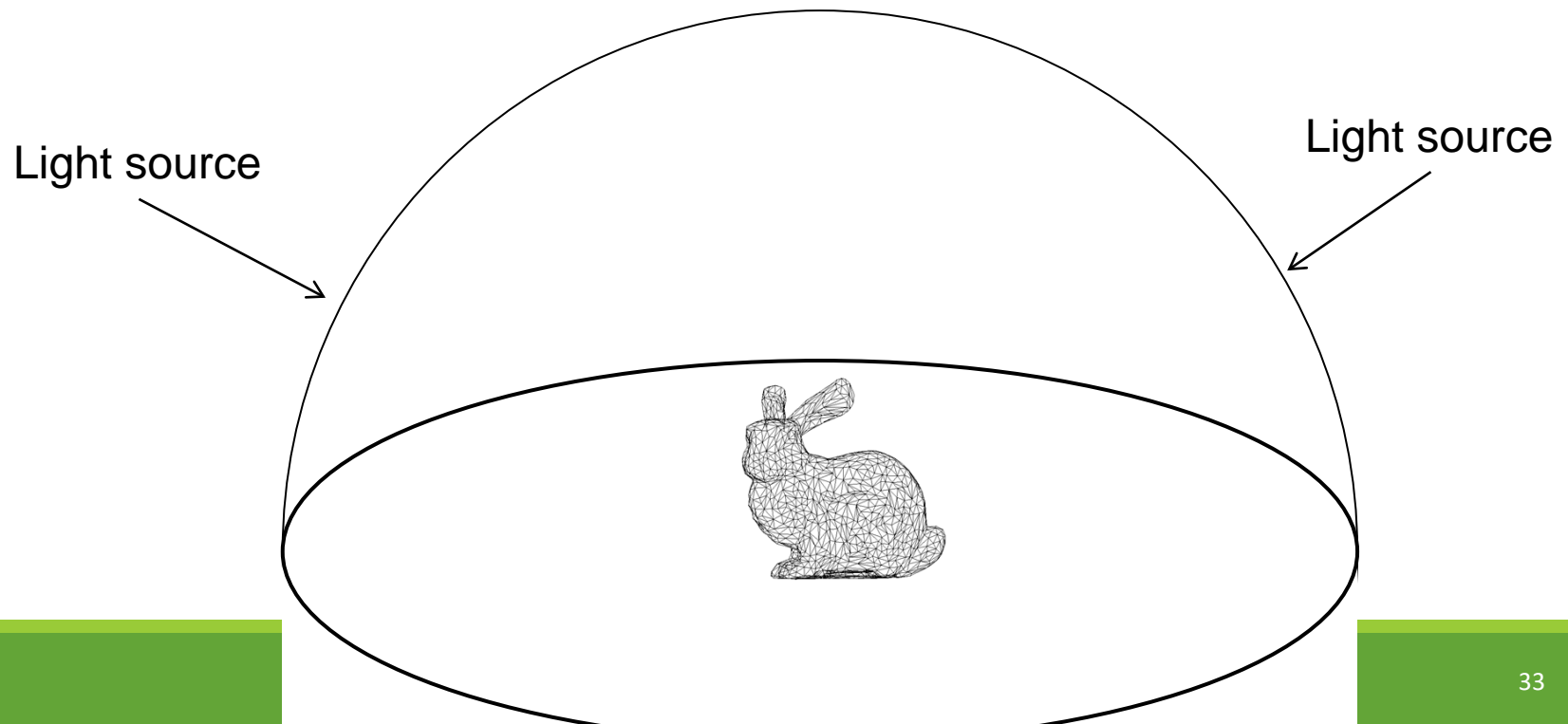


# Environment Lights

---

“Light that surrounds the entire scene”

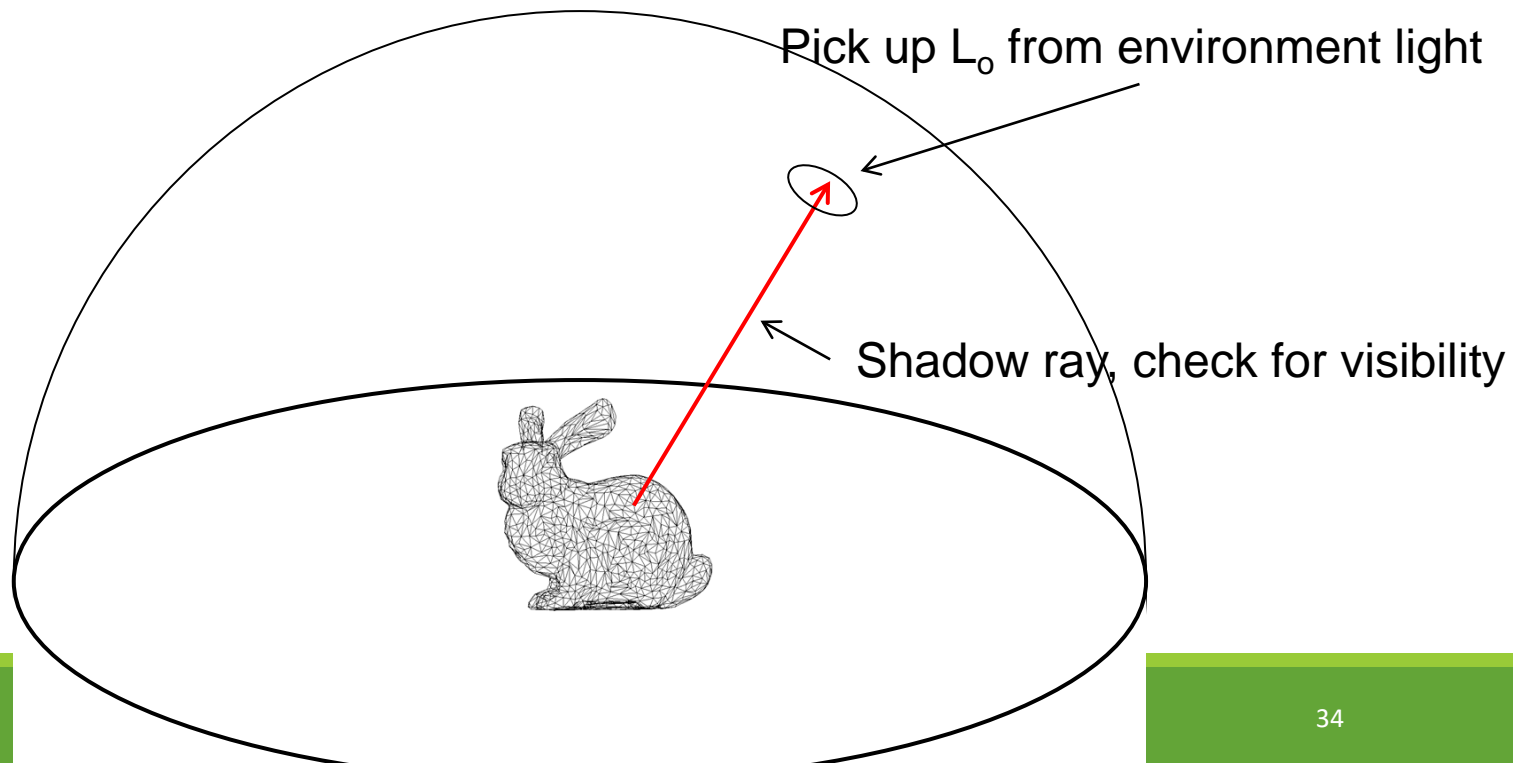
- (directional) light source is modelled as (enormous) sphere around the scene



# Environment Lights

Write direct illumination as integral over hemisphere directions:

$$L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{\text{hemisphere}} f_r(p, \omega_i, \omega_o) L_o(-\omega_i) V(\omega_i) \cos \theta \, d\omega_i$$



# Environment Lights



(a)



(b)



(b)



(c)

**Figure 12.15:** Changing just the environment map used for illumination gives quite different results in the final image; (a) using a midday skylight distribution and (b) using a sunset environment map.

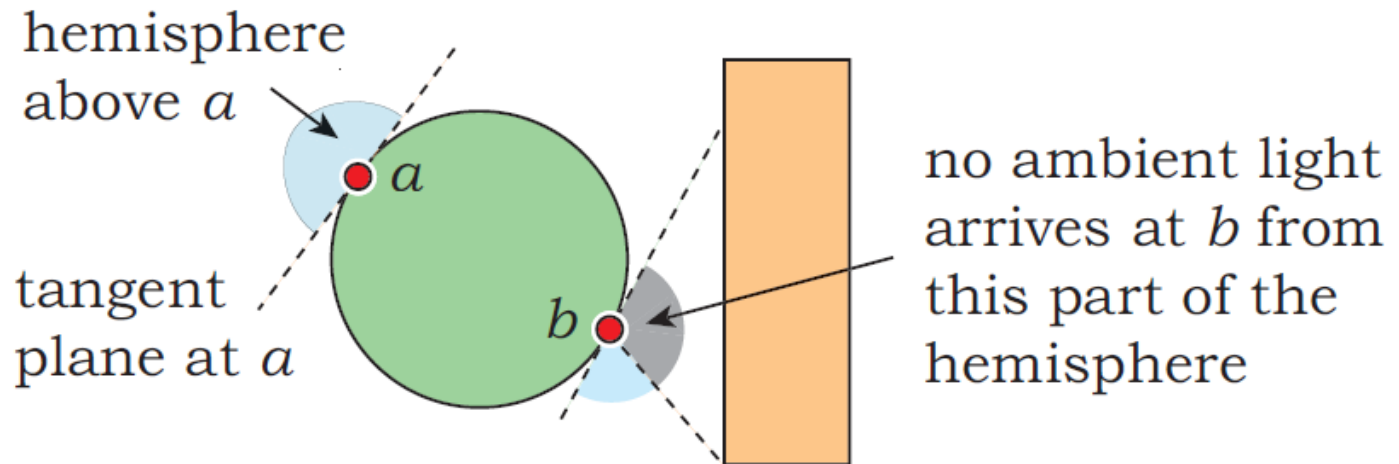
**Figure 12.16: Environment Maps Used for Illumination in Figures 12.14 and 12.15.** (a) Morning, (b) midday, and (c) sunset sky. (The bottom halves of these maps aren't shown here, since they are just black pixels.)

# Ambient occlusion

---

Intuition:

- Amount of light received at a point is proportional to part of hemisphere that is not occluded



# Ambient occlusion: formulation

---

$$L_o(p, \omega_o) = [L_e(p, \omega_o)] + \int_{\text{hemisphere}} f_r(p, \omega_i, \omega_o) L_i(p, \omega_i) \cos \theta_i d\omega_i$$



Incoming light equal intensity from all directions  
but some light might be blocked

$$L_o(p, \omega_o) = L_i \int_{\text{hemisphere}} f_r(p, \omega_i, \omega_o) V(p, \omega_i) \cos \theta_i d\omega_i$$



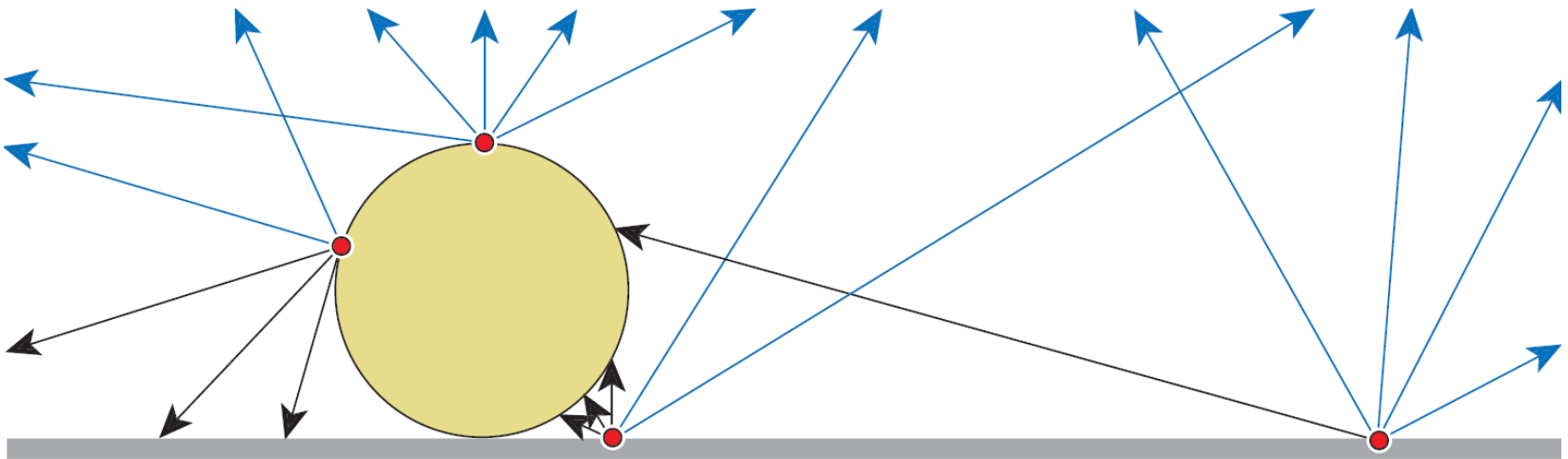
Diffuse brdf (brdf = constant)

$$L_o(p, \omega_o) = f_r \cdot L_i \int_{\text{hemisphere}} V(p, \omega_i) \cos \theta_i d\omega_i$$

# Ambient occlusion: formulation

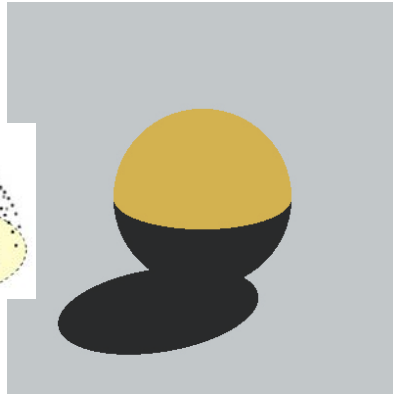
$$L_o(p, \omega_o) = f_r \cdot L_i \underbrace{\int_{\text{hemisphere}} V(p, \omega_i) \cos \theta_i d\omega_i}_{\text{}}$$

- Can be pre-computed (e.g. at vertices)
- Only dependent on geometry

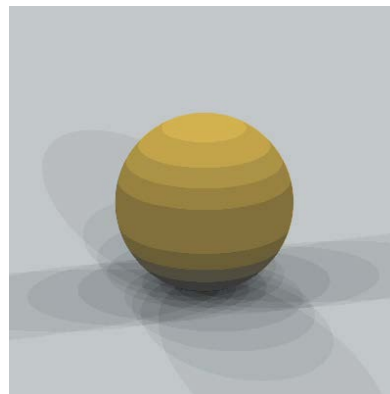


# Ambient occlusion: sampling

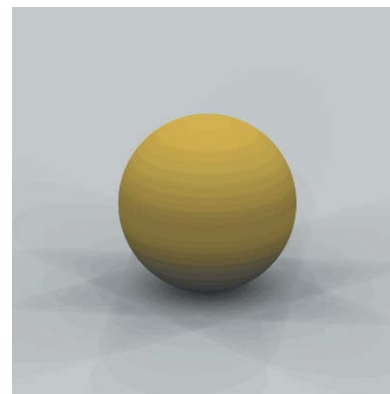
Regular sampling of the hemisphere



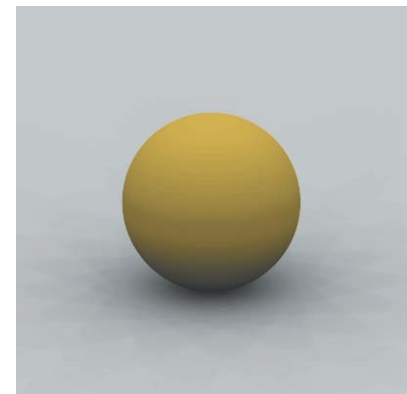
1 sample



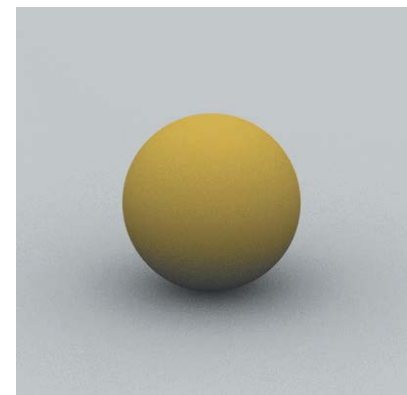
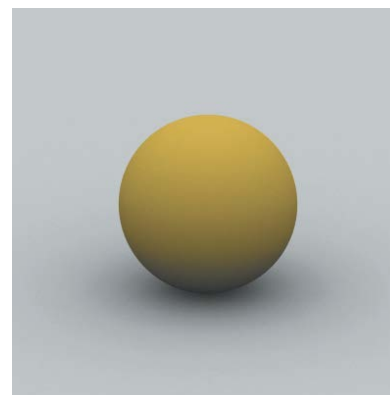
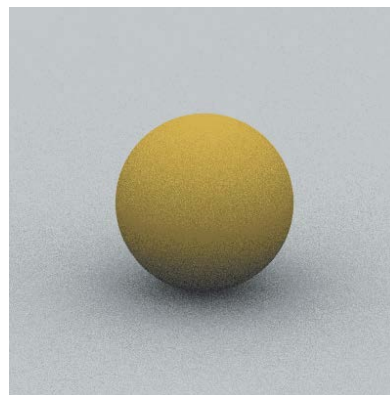
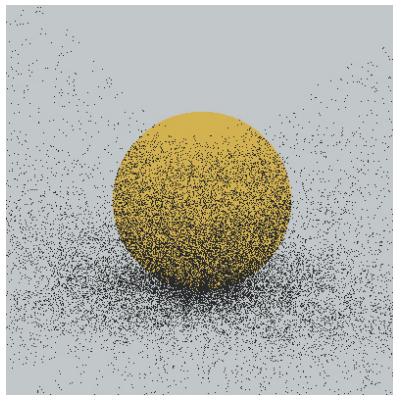
16 samples



64 samples



256 samples



Monte Carlo (random) sampling of the hemisphere

# Ambient occlusion: result

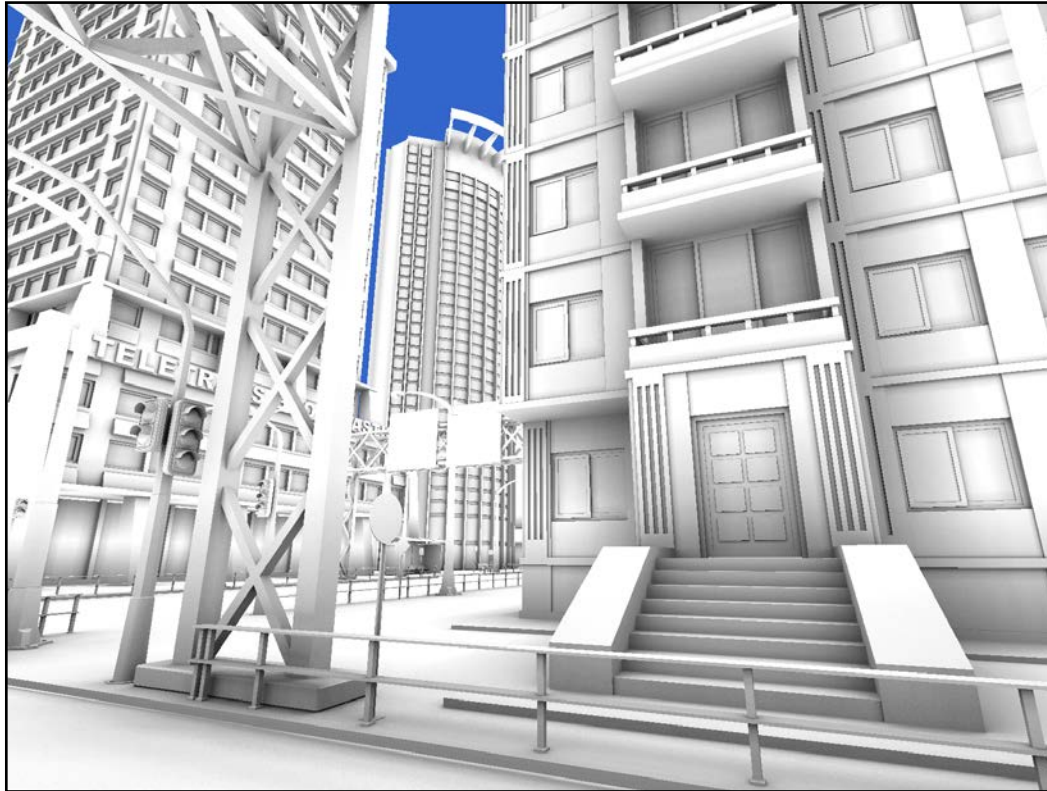


<http://forums.3dtotal.com/showthread.php?t=75339>



# Ambient occlusion: result

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*Two Methods for Fast Ray-Cast Ambient Occlusion*  
Samuli Laine and Tero Karras (EGSR 2010)