# ISDA 609 - Mathematical Modeling Techniques for Data Analytics Course Project

## Project Team

Team members consists of

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3. Vincent Ying
4. A project report will be completed with all the points below addressed. Due by 12/14/14.

* Analysis of the problem
* Methodology used for each problem
* Algorithm and Details for solving each problem
* Results from each project
* Conclusions to be drawn from the results

## Project 1

### Project 5.3.3 (page 201):

Craps - Construct and perform a Monte Carlo simulation of the popular casino game of craps. The rules are as follows:

There are two basic bets in craps, pass and don't pass. In the pass bet, you wager that the shooter (the person throwing the dice) will win; in the don't pass bet, you wager that the shooter will lose. We will play by the rule that on an initial roll of 12 (``boxcars''), both pass and don't pass bets are losers. Both are even-money bets.

Conduct of the game:

* Roll a 7 or 11 on the first roll: Shooter wins (pass bets win and don't pass bets lose).
* Roll a 12 on the first roll: Shooter loses (boxcars; pass and don't pass bets lose).
* Roll a 2 or 3 on the first roll: Shooter loses (pass bets lose, don't pass bets win).
* Roll 4, 5, 6, 8, 9, 10 on the first roll: This becomes the point. The object then becomes to roll the point again before rolling a 7.
* The shooter continues to roll the dice until the point or a 7 appears. Pass bettors win if the shooter rolls the point again before rolling a 7. Don't pass bettors win if the shooter rolls a 7 before rolling the point again.

Write an algorithm and code it in the computer language of your choice. Run the simulation to estimate the probability of winning a pass bet and the probability of winning a don't pass bet. Which is the better bet? As the number of trials increases, to what do the probabilities converge?

**Summary**:

The purpose of this project is to create a Monte Carlo simulation for craps and utilize the results of the simulation to determine which bet to make to maximize the chance of winning.

**Analysis of the Problem**:

The craps dice game is base on a pair of dice and wagering if the shooter either wins or loses. A simulation of dice rolls will be needed to determine the probability of the shooter winning/losing. This simulation will provide the best choice in placing a bet.

**Dataset Description**:

All data used in this project been provided for. The rules of the game give rise to probabilities to be used. Simulation of the dice throws will be done to determine how to wager. No data collection from external sources would be necessary.

**Methodologies**:

1. Simulation of Probabilistic Behavior

The R language will be used in the Monte Carlo simulation. A probability will be generated for whether a shooter wins or loses.

1. Sequential Decisions and Conditional Probabilities

A decision tree with expected value of winning on each roll will be generated to determine the best course of action.

**Results To Be Obtained**:

The result from the Monte Carlo simulation and the path with the highest expected value in the decision tree will be compared for selection of the best wager. Show if either modeling method is better suited for this type of problem.

**Results**:

The results here with any associated tables/graphs.

**Conclusion**:

The conclusion lives here.

## Project 2

### Project 9.4.2 (page 376):

Retirement and Social Security. Should US citizens build their own retirement through 401Ks or use the current Social Security program? Build models to be able to compare these systems and provide decisions that can help someone to plan a better retirement.

**Summary**:

This purpose of this project is to determine the best way to maximize savings for retirement. 401K investment returns and Social Security payout from contributions will be analyzed and compared.

**Analysis of the Problem**:

401K investment accounts provide a mix of securities that can be withdrawn later in life without penalty. The Social Security program is funded by taxes that are contributed by every worker in America. We will take the amount that is contributed to Social Security program as working capital and try to maximize the returns until the start of retirement.

Retirement Statistics For Americans (2014):

• Average retirement age is 62 years old.

• Average retirement length is 18 years.

• Medical costs over a 20 year period for a couple over 65 is $215,000.

• 35% of Americans over 65 rely completely on Social Security.

• 36% of Americans don't save anything for retirement.

From the statistics gathered, a large portion of Americans are not saving enough for retirement. For those that do plan for retirement, there are a variety of ways to save and consider for retirement.

Social Security For Retirement:

OASI (Old-Age and Survivors Insurance) Trust Fund:

• The trust fund is supplied by Social Security Taxes that are involuntarily levied.

SSA Contributions:

• SSA tax rate has been around 5% for the employed and 10% for the self-employed.

• There is also a maximum taxable earnings, currently at $113,700 for 2013.

SSA Benefits:

• Benefits payouts can start at age 62, but delayed retirement (up to age 70) can increase payout.

• Benefit calculation is inflation adjusted with the past highest 35 years in earnings.

• Benefit payouts are inflation adjusted and can also be claimed by spouse upon death.

Social Security is only meant to be supplemental income during retirement, but many do make use of it as their sole source of income.

401k For Retirement:

401(k) Defined-Contribution Pension:

• First established in 1978, it allows taxpayers to defer taxes on income for retirement.

401(k) Contribution:

• The maximum amount that can be contributed is currently $17,500.

• 401K Contribution is voluntarily made with a possible company match.

401(k) Withdrawals:

• Withdrawal without penalty starts at 59.5 years of age.

• There is a required minimum distribution (RMD) starting at 70.5 years or start retirement.

Since contributions to 401K compound over time, it is best to for a participant to start saving early and maximize contribution into the plan whenever possible.

Comparison of SSA and 401k:

SSA:

• Since SSA is funded by payroll taxes, contribution is automatically taken care by the government.

• There is a projected shortfall of the trust fund in 2037. However, increasing payroll taxes and benefit reductions would alleviate this concern.

401(k) Defined-Contribution Plan:

• Since a larger portion of one's income can be contributed to a 401K, it should be the main source of income for those with access to it.

• The returns of a 401K plan is dependent on the investment choices made.

• Management fees with the choice of investments made also needs to be considered.

**Dataset Description**:

Data collection from external sources will be necessary. The main source will be from data.gov for the Social Security system. We will need the amount that is contributed by workers and the payout amount upon retirement.

Data will also need to be collected for 401K return information. Return rates for various investment profiles will be needed. The amount that can be withdrawn upon retirement and length of payouts will be required to compare both systems.

**Methodologies**:

1. Modeling with Decision Theory

Decision Theory can be used to determine the investment profile for a 401K account. The optimization criterion of interest would be Maximin Criterion for a modest return and lowest risk in a retirement account. However, the alternative criterion Expected Value, Coefficient of optimism, and Minimax regret will also be considered.

1. Optimization of Discrete Models with the Simplex method

The investment mix for a 401K account can be an optimization problem. Most optimization problems can be solved through the Simplex method. We will use Simplex method to determine the best mix of security types in 401K for accounts.

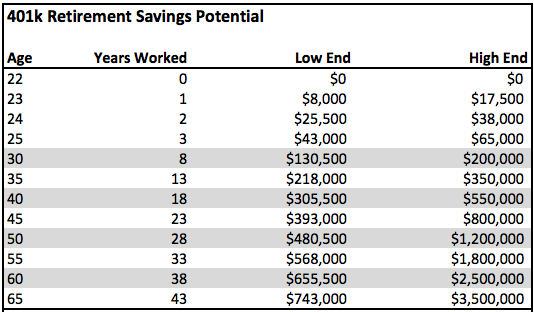
Assumptions for Modeling:

* Start working at age 22 and live until the average age of 85.45.
* Start contribution to 401k at 22 with initial contribution of $8,000 and $17,500 every subsequent year.
* 401k Rate of Growth is 7% compounded until start of withdrawal.

Effect of Early, Normal, and Delayed Retirement on SSA Benefit Amounts (2012)

|  |  |  |
| --- | --- | --- |
| **Age** | **Percent Benefit** | **Monthly Payout** |
| **62** | 70 | 1104 |
| **63** | 75 | 1183 |
| **64** | 80 | 1262 |
| **65** | 86.66 | 1367 |
| **66** | 93.33 | 1472 |
| **67** | 100 | 1577 |
| **68** | 108 | 1703 |
| **69** | 116 | 1829 |
| **70** | 124 | 1955 |

401k Balance Throughout Contribution Period



**Results To Be Obtained**:

The best investment mix for a 401K will be determined through the use of both modeling methods. The results of these two models will be compared against the sole use of the Social Security program. This will provide several options for towards saving for retirement. The model with the best results will be selected to maximize a person's savings over their career towards retirement.

**Results**:

Combined Income From SSA Benefits and 401k Withdrawals

(When starting 401k withdrawal at the age of 62.)

|  |  |  |
| --- | --- | --- |
| **Age** | **Monthly**  **Payout** | **Total (k)** |
| **62** | 3826 | 3924 |
| **63** | 3835 | 3932 |
| **64** | 3841 | 3938 |
| **65** | 3851 | 3948 |
| **66** | 3859 | 3957 |
| **67** | 3864 | 3963 |
| **68** | 3872 | 3970 |
| **69** | 3876 | 3975 |
| **70** | 3877 | 3976 |

Combined Income From SSA Benefits and 401k Withdrawals

(When starting 401k withdrawal at the same time as SSA withdrawal.)

|  |  |  |
| --- | --- | --- |
| **Age** | **Monthly**  **Payout** | **Total (k)** |
| **62** | 3826 | 3924 |
| **63** | 3835 | 3932 |
| **64** | 3841 | 3938 |
| **65** | 3851 | 3948 |
| **66** | 3859 | 3957 |
| **67** | 3864 | 3963 |
| **68** | 3872 | 3970 |
| **69** | 3876 | 3975 |
| **70** | 3877 | 3976 |

**Conclusion**:

Planning and saving for retirement is a basic optimization problem.

With regards to SSA planning, delay the start of SSA benefit payouts as long as possible (up to age 70) to maximize the payout amount throughout retirement.

With regards to 401k planning, to maximize savings for retirement is to maximize contributions into a 401K account as early as possible on the planning horizon and to contribute as long as possible. Start withdrawal of the 401K account earlier, at the start or before retirement.

The contribution and returns on Social Security, 401K, and any other retirement account needs to be considered in combination to better plan for retirement.

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